Research Note: The Accuracy of Group Majorities for Disjunctive and Conjunctive Decision Tasks

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For complex group decision tasks which can be described in terms of establishing the truth value of conjunctive (and) or disjunctive (or) composite propositions, the advantage for group judgmental accuracy of disaggregating the decision task into separate individual evaluations of the truth value of each component atomic proposition in the composite is investigated. For true propositions it is shown that disaggregation is preferable for conjunctive tasks, but not for disjunctive tasks; while the reverse is true for false propositions.

In this paper we shall discuss how a group decision task can best be structured for different types of fact-finding situations. In particular we shall consider differences between the situation where an affirmative finding on any one of a number of different factors is sufficient to generate an overall affirmative judgment versus that where an affirmative finding on each of a number of different factors is necessary to generate an overall affirmative judgment. This question is directly relevant to group/jury decision making on complex factual issues. For example, in conspiracy trials there are usually several elements of proof required to establish the presence of a conspiracy, all of which must be shown. In other cases, e.g., civil rights trials brought under Section II of the Voting Rights Act of 1965 (as amended in 1982), either a proof of intent to dilute minority voting strength or a proof that an electoral plan has the effect of diluting minority voting strength is sufficient for a violation of the Act to be shown.

Consider a group (such as a jury) confronting a decision task which is to establish the truth value of a composite statement either at the conjunctive form \( x_1 \land x_2 \land \ldots \land x_n \), where \( \land \) represents and, or of the

This research was supported by NSF Grant SES 80-07915. We acknowledge the assistance of the staff of the Word Processing Center of the School of Social Sciences, University of California, Irvine. Guillermo Owen provided very helpful comments on an earlier version of this paper, which had been prepared for delivery to the panel on "Jury Decision-Making," Law and Society Association Annual Meeting, June 3–6, 1982, Toronto. Requests for reprints should be sent to Dr. Bernard Grofman, School of Social Sciences, University of California, Irvine, CA 92717.
disjunctive form \( x_1 \lor x_2 \ldots \lor x_n \) where \( \lor \) represents or. (The components, \( x_i \), of these statements may themselves be composites and may include such other logical symbols as negation, but we shall without great loss of generality disregard such complications in our exposition.) We may imagine two alternative procedures which the group (jury) could use. In the first procedure, each member of the group would be asked to assess the truth value of the composite expression (voting either T or F, i.e., yes, if the proposition is true, or no, if it is not), and these individual judgments would then be pooled in some fashion to form the group consensus as to the truth value of the composite expression (i.e., of the truth value of \( x_1 \lor x_2 \lor \ldots \lor x_n \) or \( x_1 \land x_2 \land \ldots \land x_n \), whichever might be under consideration). In the second procedure, each group member would be asked to evaluate the truth value of each of the atomic components \( x_1, x_2, \ldots, x_n \), and the set of individual componential judgments would then be pooled in some fashion to form the group consensus as to the truth value of the composite expression. To make the calculations more readily tractable, we shall posit a particularly simple form of establishing group consensus, that of ordinary majority rule. While real-life juries often operate under supramajoritarian procedures (especially unanimity), the social psychological literature on jury decision making provides evidence that even under a de jure unanimity rule most (>90%) of all jury decision making can be predicted by considering only the pre-deliberation verdict preferences of the jury majority (Kalven & Zeisel, 1966; cf. Davis, 1973; Grofman, 1976, 1977; Lempert, 1975). Moreover, the majority rule case is of considerable interest in its own right as the most common decision mechanism for deliberate bodies in the United States and elsewhere.

Deriving our inspiration from the probabilistic ideas of Condorcet (1785; see Black, 1958; Grofman, 1975, 1979, 1981; Grofman and Feld, 1984; Grofman & Owen, 1985), we shall posit that the \( i \)th group member (juror) has a probability \( p_{ij} \), \( 0 \leq p_{ij} \leq 1 \), of correctly assessing the truth value of the proposition \( x_j \) and that juror competences across propositions are independent. Thus, the probability that the \( i \)th juror correctly assesses the truth value of conjunctive true proposition \( "x_1 \land x_2" \) is simply \( p_{i1} \cdot p_{i2} \), and the probability the \( i \)th juror correctly assesses the truth value of the true disjunctive proposition \( "x_1 \lor x_2" \) is simply \( 1 - (1 - p_{i1}) (1 - p_{i2}) \). Without real loss of generality we can confine ourselves to a comparison of the \( "x_1 \land x_2" \) and \( "x_1 \lor x_2" \) cases. Also without great loss of generality we can initially consider a group of size \( N = 3 \). Let us simplify further by positing homogeneous abilities, i.e., \( p_{ij} = p_{kj} \) for all \( i,k \); and atomic propositions of identical difficulty levels, i.e., \( p_{ij} = p_{jl} \) for all \( j,l \). Neither of these assumptions is really critical, but both together serve to dramatically simplify the formulas to be considered.
Let us first look at group judgments about "$x_1 \land x_2$." If the group considers the true composite "$x_1 \land x_2$," the probability that any individual juror correctly assesses the truth value of "$x_1 \land x_2$" is $p_{i1} \cdot p_{i2}$. Let us denote $p_{i1} \cdot p_{i2}$ as $p$. Hence, by our earlier simplifying assumptions, $p_{i1} = p_{i2} = p^{\frac{1}{2}}$. The probability that for a group of size $N = 3$, a group majority ($m = (N + 1)/2$) correctly evaluates the truth value of "$x_1 \land x_2$" can be shown to be given by

$$
\sum_{h=2}^{3} \binom{3}{h} p^h (1 - p)^{3-h} = 3p^2 - 2p^3.
$$

(1)

On the other hand, if the decision task is disaggregated, the probability that an individual juror will correctly evaluate the truth value of any given component, $x_i$, is simply $p^{\frac{1}{2}}$, and the probability that the group majority will correctly assess that component is $3p - 2p^2$ (see Eq. (1)). For the conjunctive task, where components are independent, the probability that the group majority will correctly assess the truth value of "$x_1 \land x_2$" is the product of its judgmental accuracies on each component, and thus (under our simplifying assumptions) is given by

$$
(3p - 2p^{\frac{1}{2}})^2.
$$

(2)

We can show that, for all $p$ ($0 < p < 1$),

$$
(3p - 2p^{\frac{1}{2}})^2 > 3p^2 - 2p^3,
$$

(3)

since, after some straightforward algebraic simplification (e.g., dividing both sides through by $p^2$), Eq. (3) reduces to

$$
1 + p > 2 \sqrt{p},
$$

(4)

which is always true for $p$: $0 < p < 1$. (The strict inequality is required because we have divided by $p$ which, were it 0, would have been impermissible.) The implication of Eq. (4) is quite clear. If the conjunctive proposition is true, in dealing with a conjunctive decision task under majority rule procedures, it is better to disaggregate the components and require a judgment on the truth value of each component before assessing the truth of the composite conjunctive expression. Of course, the difference between the judgmental accuracy implied by Eq. (1) vs that of Eq. (3) will be relatively minimal for $p > .5$ especially as $p \to 1$ (see Eq. (4)).

Now let us look at the "$x_1 \lor x_2$" case. We shall repeat our earlier simplifying assumptions so as to have $p_{i1} = p_{i2} = p^{\frac{1}{2}}$ for all $i,j$ and again look only at a group of size 3. The probability that any individual juror correctly assesses the truth value of the truth proposition "$x_1 \lor x_2$" is $1 - (1 - p^{\frac{1}{2}})^2$. For the composite assessment task, the probability that
the group majority correctly evaluates the truth value of the true proposition \( x_1 \lor x_2 \) is simply
\[
3\left(1 - (1 - p^{\frac{1}{2}})^2\right)^2 - 2\left(1 - (1 - p^{\frac{1}{2}})^2\right)^3.
\] (5)

On the other hand, if we disaggregate, the probability that an individual juror will correctly evaluate any given component \( x_i \) is again simply \( p^{\frac{1}{2}} \) and the probability that the group majority will correctly assess that component is \( 3p - 2p^{\frac{3}{2}} \); but for the disjunctive task, where components are independent, the probability that the group majority will correctly assess the truth value of \( x_1 \lor x_2 \) will then be given by
\[
1 - \left(1 - (3p - 2p^{\frac{3}{2}})\right)^2.
\] (6)

We may show that for \( 0 < p < 1 \), the expression in Eq. (6) always exceeds that in Eq. (5). Hence, for majority rule decisions, accuracy on disjunctive tasks is opposite to that for conjunctive tasks—for true propositions, it is always preferable to ask jurors to evaluate the composite decision problem and then to pool the composite evaluations, rather than to disaggregate. Again, however, for \( p > \frac{1}{2} \), especially as \( p \to 1 \), the differences between Eqs. (5) and (6) are not that large. The policy implications of these findings exactly parallel those for the conjunctive case, only with the nature of the recommendations reversed.

If instead of looking at true propositions we look at false propositions, we obtain by symmetry (since the formulas are otherwise identical) the opposite result: that under majority decision making for a disjunctive task, it is better to disaggregate the components and require a judgment on each component before judging the whole; while for conjunctive tasks judgment of the composite is to be preferred. Thus, if either our a priori probability is greater than one half that a proposition is true, or we are more concerned with falsely rejecting a true proposition than with wrongly accepting a false one, then we should act as if the proposition were true and disaggregate decision making for conjunctive tasks and deal with evaluations of the composite issue for disjunctive tasks. On the other hand, if our a priori probability is greater than one half that a proposition is false, or we are more concerned with wrongly accepting a false proposition than with wrongly rejecting a true one, then we should act as if the proposition were true and disaggregate decision making for disjunctive but not conjunctive tasks.

Although we have formally proved the result only for the two-com-
ponent case, extension to the $n$-component case is straightforward. We can show the result for two of the $n$ components, and then proceed by adding one new component each time. Some preliminary exploration suggests that for the majority rule case, for false (true) propositions superiority in accuracy of the aggregated (disaggregated) over the disaggregated (aggregated) decision making increases as we increase the number of component factors which must be found present for an affirmative judgment to be reached. Extension of the results to the case where group members/jurors are not identical in their competence can be done by taking advantage of the result (Grofman, Owen, & Feld, 1983) that, where the $p_{ij}$ values are symmetrically distributed or where $N$ is large, as long as all $p_{ij} \geq \frac{1}{2}$, using $\bar{p}_{ij}$, the group’s mean competence, to calculate the probability that a group majority will obtain the correct answer provides a very good approximation to the true group majority competence, i.e., we may treat the mean competence value as if it were the competence of each member of the group.

REFERENCES


RECEIVED: May 16, 1983