CUMULATIVE VOTING IN CORPORATE ELECTIONS: INTRODUCING STRATEGY INTO THE EQUATION*

AMIHAI GLAZER**
DEBRA G. GLAZER***
BERNARD GROFMAN****

I. INTRODUCTION

Cumulative voting, a concept introduced more than one hundred years ago, is widely used in corporate elections in the United States.[1] Currently, approximately twenty states require and thirteen states permit its use in corporate elections.[2] The purpose of cumulative voting is to permit minority interests to gain proportional representation on the board of directors roughly commensurate with their share of ownership. In contrast, under straight majority voting, a simple majority of the shareholders is able to elect the entire board of directors while the minority finds itself unrepresented.[3]

---

*This research was partially supported by Grant #SES 80-7915, National Science Foundation, Political Science Program.

**Assistant Professor of Economics, University of California, Irvine; A.B. 1974, Cornell University; Ph.D. 1978, Yale University.

***Attorney, Los Angeles, California; A.B. 1974, Cornell University; J.D. 1980, Yale University.

****Professor of Political Science, University of California, Irvine; A.B. 1966, Ph.D. 1972, University of Chicago.

1. Cumulative voting was first introduced into the context of corporate elections by a provision in the Illinois State Constitution adopted in 1870. ILL. CONST. art. XI, § 3. The proponents of cumulative voting included the Minority Representation Society, a group led by the publisher of the *Chicago Tribune*, Joseph Medill. During the 1860s, the press and certain segments of the public became indignant over the excesses and frauds of certain railroad managements. These groups denounced the "rings" which controlled many of the railroad companies and defrauded minority stockholders. *See* Wolfson v. Avery, 6 Ill.2d 78, 126 N.E.2d 701 (1955); Campbell, *The Origin and Growth of Cumulative Voting for Directors*, BUS. LAW., Apr. 1955, at 3-6.

2. 5 W. FLETCHER, CYCLOPEDIA OF THE LAW OF PRIVATE CORPORATIONS § 2048 nn.6-8 (rev. perm. ed. 1976).

3. Arguments for and against cumulative voting are set forth in Williams, *Cumulative Voting*, 33 HARV. BUS. REV., May-June 1955, at 108, 111. Among the arguments advanced in favor of cumulative voting are: (1) cumulative voting is equitable because

The essence of cumulative voting is easily described: each shareholder has a number of votes equal to the number of shares he will vote multiplied by the number of directors to be elected.[4] The shareholder may cast all his votes for a single director or distribute them among several candidates. By distributing votes properly, minority shareholders may attain representation on the board of directors roughly proportional to the number of shares they own. Under straight voting, the other commonly used method of electing directors, each shareholder simply votes the number of shares he owns for each director nominated. Obviously, under this system, a bare majority shareholder (e.g., a fifty-one percent owner) can elect the entire slate of directors, whereas a holder of a minority interest (e.g., a forty-nine percent owner) can elect none.

Indeed, it has become common in recent years for corporate managements, in an effort to discourage takeover bids, to propose amendments to their corporate charters (in states where cumulative voting is only permissive, not manadatory) which would eliminate the right of cumulative voting.[5] Some manage-

---

stockholders should have the opportunity for representation on the board of directors in proportion to their holdings; (2) minority representation does not interfere with the principle of majority rule since the number of directors elected by each group will vary with its proportion of ownership; (3) it is important that minority interests have a voice on the board since stockholders and management often have different goals; and (4) since corporate and securities laws generally create a balance of power in favor of insiders and controlling interests, some countervailing power in the hands of outside minority interests is desirable.

Arguments voiced against cumulative voting include the following: (1) cumulative voting tends to partisan representation on the board, which is inconsistent with the notion that a director properly represents all interest groups in a corporation; (2) disharmony on the board dissipates the energies of management and leads to an atmosphere of uncertainty at the top level; (3) cumulative voting is often used by persons who are motivated by narrow, selfish interests rather than by the broader interests of all stockholders; and (4) each meeting of a board of directors chosen by cumulative voting becomes a skirmish in a long-run fight for control of the corporation, leading the board to neglect its real functions of leadership for the entire corporation and thereby damaging corporate interests.

4. CAL. CORP. CODE § 708(a) (West 1977) defines cumulative voting as follows:

> Every shareholder . . . entitled to vote at any election of directors may cumulate such shareholder's votes and give one candidate a number of votes equal to the number of directors to be elected multiplied by the number of votes to which the shareholder's shares are normally entitled, or distribute the shareholder's votes on the same principle among as many candidates as the shareholder thinks fit.

5. The abolition of cumulative voting is only one strategy aimed at stopping take-

ments have even suggested reincorporating in Delaware, a state in which the corporate law generally favors management and in which cumulative voting is not required.[6]

The use of cumulative voting does not, by itself, assure that optimal representation on the board will be achieved. Minority shareholders will attain representation on the board of directors only if the minority interests correctly determine their voting strategies. Similarly, the majority's miscalculation in cumulating its votes can prove costly. For example, in the 1883 election of the Sharpsville Railroad Company Board of Directors, the minority not only gained representation on the board of directors, but control as well:

> The majority group, with 53 percent of the votes, chose a strategy of dividing its votes equally among a full slate of six candidates, and the minority group, with 47 percent of the shares, chose a strategy of dividing its votes equally among a slate of four. The minority group easily captured the four directorships it contested, with each of its candidates getting 11.8 percent of the vote to the 8.8 percent for each of the majority group's candidates. In fact, if the minority group had pursued the bolder strategy of contesting five directorships, it could have won all by giving 9.4 percent of all votes to each of its candidates.[7]

---

overs, or at least making them slower and more difficult. Other proposals, colloquially known as "shark repellants," include "fair price" rules that require a tender offeror to pay an equivalent price for all shares (thereby prohibiting a raider from offering a generous amount for a controlling stake in a target's stock and then offering a lower amount for the remainder); "supermajority" rules that require a raider to obtain a vote significantly greater than a mere majority in favor of an unfriendly takeover; and a "staggered" or "classified" system to elect directors, whereby only a minority of directors, and not the entire board, comes up for election each year. Indeed, use of the staggered board has proven a useful device for mitigating the effect of minority representation provided by cumulative voting. *See* Sell and Fuge, *Impact of Classified Corporate Directorates on the Constitutional Right of Cumulative Voting*, 17 U. PITT. L. REV. 151 (1956); Adkins, *Corporate Democracy and Classified Directors*, 11 BUS. LAW., Nov. 1955, at 31.

6. For example, at its annual meeting of shareholders held April 25, 1983, the management of the Union Oil Company of California, a company incorporated in California (where cumulative voting is required), presented for shareholder approval a proposal to reincorporate the company in Delaware (where cumulative voting is only permissive). But query whether this corporate action will render cumulative voting nonmandatory in light of the applicability of certain provisions of California's general corporation law to pseudo-foreign corporations. CAL. CORP. CODE § 2115 (West 1977). This statute, as applied to mandatory cumulative voting for pseudo-foreign corporations, was recently upheld as constitutional in Wilson v. Louisiana Pacific Resources, Inc., 138 Cal. App. 3d 216, 187 Cal. Rptr. 852 (1982).

7. S. BRAMS, GAME THEORY AND POLITICS 116 (1975). The Pennsylvania Supreme

II. THE CLASSIC FORMULA

The Sharpsville Railroad Company election illustrates the most important decision in a cumulative voting contest: how to cumulate votes to elect the maximum number of directors. Several authors have devised mathematical formulae purporting to solve this problem, and these formulae are repeatedly cited in legal literature.[8]

The most commonly cited formula for finding the minimum number of shares that a shareholder or group of shareholders must own to elect a given number of directors is given by Cole as follows:[9]

$$S_I = \frac{S \times D_I}{D + 1} + 1 \qquad \text{(Equation 1).}$$

In this formula:

S_I = number of shares owned by some shareholder or group of shareholders (Bloc I);
D_I = number of directors Bloc I desires to elect;
S = total number of shares voting at the meeting; and
D = total number of directors to be elected at the meeting.

In cases where S_I includes a fractional term (e.g., 5¼), Cole states that S_I should be taken to be the largest whole number less than that given by the formula (e.g., 5).[10]

Court upheld the Sharpsville Railroad election as simply the exercise of a constitutional right. Pierce v. Commonwealth, 104 Pa. 150 (1883). Several other reported decisions have dealt with situations in which, due to miscalculation on the part of the majority, cumulative voting permitted minority stockholders to gain control of the board of directors. *See* Dulin v. Pacific Wood & Coal Co., 103 Cal. 357, 35 P. 1045 (1884), *aff'd on rehearing*, 103 Cal. 359, 37 P. 207 (1894)(per curiam) ; Chicago Macaroni Mfg. Co. v. Boggiano, 202 Ill. 312, 67 N.E. 17 (1903); *In re* P.B. Mathiason Mfg. Co., 122 Mo. App. 437, 99 S.W. 502 (1907); State *ex rel* Price v. DeBrul, 100 Ohio St. 272, 126 N.E. 87 (1919); Schwartz v. State *ex rel.* Schwartz, 61 Ohio St. 497, 56 N.E. 201 (1908).

8. Essentially the same formulae are used in the following works: Cole, *Legal and Mathematical Aspects of Cumulative Voting*, 2 S.C.L.Q. 225 (1950) [hereinafter cited as Cole]; W. CAREY, CASES AND MATERIALS ON CORPORATIONS at 285 (4th ed. 1969); H. HEHN, LAW OF CORPORATIONS at 364 n.11 (2d ed. 1970); C. WILLIAMS, CUMULATIVE VOTING FOR DIRECTORS 40-42 (1951); Mills, *The Mathematics of Cumulative Voting*, 1968 DUKE L.J. 28. For convenience, we will refer only to the formulae given by Cole, which appears to be the source used by the other authors.

9. Cole, *supra* note 8, at 229.

10. *Id.* at 230 n.19, 233, 239 n.31. Mills, *supra* note 8, offers an improved formula which allows for voting by fractional shares, and which in other cases yields results simi-

For example, suppose the total shares outstanding and voting is five, the total number of directors to be elected is eleven, and that Bloc I attempts to elect seven directors. Using Cole's formula, Equation 1, the number of shares Bloc I would need to own to elect seven directors is calculated as follows:

$$S_I = \frac{5 \times 7}{11 + 1} + 1 = 3^{11}/_{12}, \text{ or } 3.$$

In other words, using Cole's formula, Bloc I should be able to elect seven directors if it owns three shares.

Unfortunately, Cole's formula suffers from three flaws. First, it occasionally yields erroneous results in situations involving only two competing blocs of shareholders. Second, it ignores the possibility that a bloc's optimal strategy may be to vote for more candidates than it is certain of electing, and instead addresses only the question of how many directors a bloc can be *assured* of electing. Third, Cole's formula may yield erroneous results in situations involving more than two competing blocs of shareholders. These issues are discussed and resolved in the remaining portion of this Article.[11]

III. UPDATING COLE

This section addresses the basic issue of the number of directors each of *two* competing blocs can elect.[12] This issue is addressed through a hypothetical shareholder election designed to demonstrate that Cole's solution may at times be incorrect. Although Cole's solution may be correct in *other* situations, shareholders and their counsel would be well advised to use the

lar to Cole's Equation 1. Since the legal literature refers almost exclusively to Cole's formula, this Article shall do the same.

11. These solutions have appeared in the game theory and political science literature. *See* Glasser, *Game Theory and Cumulative Voting*, 5 MGMT. SCI. 151 (1959); S. BRAMS, *supra* note 7, at 111; B. GROFMAN, *A Review of Macro-Election Systems*, GERMAN POLITICAL YEARBOOK (R. Wildenmann ed. 1975); Balinski and Young, *Stability, Coalitions, and Schisms in Proportional Representation Systems*, 72 AM. POL. SCI. REV. 848 (1978). Only the first work cited discusses cumulative voting, and even it fails to address the issues raised in the final two sections of this paper..

12. Throughout this paper it is presumed that each bloc decides how to cumulate its votes based on the assumption that other blocs are cumulating their own votes in the most advantageous manner. The same approach is used in the game theory literature under the rubric "maximin" strategies ("*maxi*mize the *min*imum number of directors that will be elected"). *See* D. LUCE & H. RAIFFA, GAMES AND DECISIONS 67 (1957).

method described below because it never fails.

Recall the previous hypothetical the total number of directors to be elected was eleven and the total number of shares voting was five. Cole's formula yields the result that a bloc owning three shares can be *certain* of electing seven directors. This result is incorrect.

The bloc owning three shares is designated "Bloc I." The reamining two shares are owned by a competing group, "Bloc II." Cole states that Bloc I can be *certain* of electing seven directors, regardless of Bloc II's voting strategy. But suppose Bloc II (owning two shares and having the right to cast twenty-two (2 x 11) votes) casts five votes for each of two candidates and four votes for each of three candidates.

Contrary to Cole, Bloc I cannot be certain of electing seven directors under these circumstances. Bloc I can be certain of electing seven directors only if it casts five votes for each. However, Bloc I has only thirty-three votes (three shares x eleven directors to be elected); thus it cannot cast the five votes for each of the seven candidates necessary to guarantee the election. Instead, Bloc I can be certain of winning six seats by casting five votes for each of five candidates and eight votes for the sixth candidate. Alternatively, Bloc I can cast five votes for each of five candidates and four votes for the two other candidates. In the latter case, Bloc I would be assured of winning five seats and, depending on the rule used to break ties, would also elect none, one, or both of the candidates receiving four votes each. (Remember that Bloc II also cast four votes for each of three candidates.) In any event, Bloc I cannot be guaranteed of winning seven seats.[13]

The correct solution (in this and in all other two-bloc elections) is that given by Glasser,[14] who was, apparently, the first author to explicitly apply game theoretic notions to cumulative voting. Glasser discovered that this sort of problem cannot be solved by using only one equation; rather, a correct solution requires iterative methods. He argues that if Bloc I wishes to guarantee seating to at least D_I directors out of a total of D directors to be elected, it must give its D_Ith candidate more votes than

13. Mills, *supra* note 8, at 38-42, recognizes that Cole's formula may be invalid in the case of ties, but he offers no explicit solution to overcome this problem.

14. Glasser, *supra* note 11.

Bloc II can possibly give its $(D + 1 - D_I\text{th})$ candidate. Hence, to elect D_I candidates Bloc I must have a number of votes sufficient to satisfy the following inequality, (Inequality 1):

$$\text{Integer } \frac{D \times S_I}{D_I} \text{ is greater than Integer } \frac{D \times S_{II}}{D + 1 - D_I}.$$

The mathematical symbol "Integer $\frac{D \times S_I}{D_I}$" means that one first calculates the value of $\frac{D \times S_I}{D_I}$ and drops any fractional part thereof; for example, if $\frac{D \times S_I}{D_I} = 5\frac{1}{2}$, integer $\frac{D \times S_I}{D_I} = 5$. A similar operation should, of course, be performed on the right hand side of the inequality, that is, on "Integer $\frac{D \times S_{II}}{D + 1 - D_I}$." Thus, the maximum number of directors Bloc I can be certain of electing is the largest value of D_I for which the above inequality is true.

In the hypothetical shareholder election, Bloc I wishes to determine the maximum number of directors it can be certain of electing. Recall that:

D (total number of directors to be elected) = 11
S_I (number of shares owned by Bloc I) = 3
S_{II} (number of shares owned by Bloc II) = 2.

To reach the needed determination by using an iterative procedure, successively larger values of D_I are inserted into the inequality, and this insertion stops when the inequality is not satisfied:

(1) $D_I = 1$: Integer $\frac{11 \times 3}{1} = 33$ is greater than

$$\text{Integer } \frac{11 \times 2}{11 + 1 - 1} = 2$$

(2) $D_I = 2$: Integer $\frac{11 \times 3}{2} = 16$ is greater than

$$\text{Integer } \frac{11 \times 2}{11 + 1 - 2} = 2$$

(3) $D_I = 3$: Integer $\frac{11 \times 3}{3} = 11$ is greater than

$$\text{Integer } \frac{11 \times 2}{11 + 1 - 3} = 2$$

(4) $D_I = 4$: Integer $\dfrac{11 \times 3}{4} = 8$ is greater than

Integer $\dfrac{11 \times 2}{11 + 1 - 4} = 2$

(5) $D_I = 5$: Integer $\dfrac{11 \times 3}{5} = 6$ is greater than

Integer $\dfrac{11 \times 2}{11 + 1 - 5} = 3$

(6) $D_I = 6$: Integer $\dfrac{11 \times 3}{6} = 5$ is greater than

Integer $\dfrac{11 \times 2}{11 + 1 - 6} = 3$

(7) *But* $D_I = 7$: Integer $\dfrac{11 \times 3}{7} = 4$ is *equal* to

Integer $\dfrac{11 \times 2}{11 + 1 - 7} = 4.$

Thus, Bloc I can be guaranteed of winning *six* seats, not seven as suggested by Equation 1.

In general, the result given by Equation 1 will be incorrect whenever it differs from the result given by Inequality 1. The former will give correct results where the total number of shares voted is very large in relation to the number of vacant board seats.[15] However, Inequality 1 is *always* correct in the two-bloc case and is therefore the preferred method.

Inequality 1 illustrates an important feature of cumulative voting not previously mentioned in the literature: the number of directors a bloc can be certain of electing depends not only upon the proportion of total shares which it owns, but also upon on the *absolute* number of shares it owns. Since in the present example Bloc I owned three out of a total of five shares outstanding, Bloc I can be certain of electing six directors. However, if

15. If we assume that a bloc owns n shares, the discrepancy occurs whenever D x Integer $\frac{n}{D+1}$ is greater than n. In the large numbers case there is no such discrepancy. For if n is very large relative to D, then D x Integer $\frac{n}{D+1}$ is approximately $\frac{D \times n}{D+1}$, which is less than n, and therefore Equation 1 and Inequality 1 yield identical results.

Bloc I owns 3000 out of a total of 5000 shares outstanding, the result is different. For, where $D_I = 7$:

$$\text{Integer } \frac{11 \times 3000}{7} = 4714 \text{ is greater than}$$

$$\text{Integer } \frac{11 \times 2000}{11 + 1 - 7} = 4400,$$

and Bloc I can be certain of electing seven directors, not just six. This discrepancy should remind corporate management and its counsel that the absolute number of shares to be issued to each shareholder group as well as the relative proportional interests of each such group are important in planning a corporation's shareholder structure.

IV. Improving Cole

The previous section considered the problem of the number of directors either of two blocs can be certain of electing. But the question of how many candidates a bloc should vote for remains. Clearly, a bloc which can be guaranteed of winning D_I seats should vote for at least D_I candidates; but should it vote for exactly D_I candidates or more than that number?

This section presents an easily calculable solution to the problem of whether a bloc should vote for more candidates than it is certain of electing. In contrast to the previous sections, it is assumed in this section that a shareholder may cast fractional votes. The results, however, are also valid in situations where the number of shares a bloc owns is very large compared to the number of directors to be elected. For example, this section considers the example of a bloc casting 42½ shares out of a total of 100 shares; the same result could be obtained if such a bloc cast 425 shares out of a total of 1000 shares. But the results discussed in this section may not hold if the number of shares outstanding is small and fractional voting is not permitted.

The following example illustrates the proposition that a bloc may find it worthwhile to vote for *more* candidates than the number it can be certain of electing.[16] Suppose that of 100

16. This divergence between the minimax solution and a bloc's dominant strategy was first noted by S. BRAMS, *supra* note 7, at 111.

shares outstanding, Bloc I owns sixty-six shares and Bloc II owns thirty-four, and suppose that five directors are to be elected (that is, $S = 100$, $S_I = 66$, $S_{II} = 34$, $D = 5$). According to the Inequality 1 calculation, Bloc I can be certain of electing at least three directors by giving each of them $5 \times 66 = 110$ votes.[17] Similarly, Bloc II can be certain of electing at least two directors by giving each one $5 \times 34 = 85$ votes.[18]

Rather than nominating three candidates, however, Bloc I can divide its votes among *five* candidates, giving each one 66 votes. The number of directors Bloc I will thereby elect depends upon how Bloc II votes its shares. Consider, then, the voting outcome for each possible strategy which Bloc II might use:

(1) If Bloc II votes for only one candidate, giving him (5 x 34) = 170 votes, Bloc I will win 4 seats.
(2) If Bloc II votes for two candidates, giving each (5 x 34)/2 = 85 votes, Bloc I will win 3 seats.
(3) If Bloc II votes for three candidates, giving each (5 x 34)/3 = 56⅔ votes, then, since 66 is greater than 56⅔, Bloc I will win 5 seats.
(4) If Bloc II votes for four candidates, giving each (5 x 34)/4 = 42½ votes, then, since 66 is greater than 42½, Bloc I will

17.

$$D_I = 3\text{: Integer } \frac{5 \times 66}{3} = 110 \text{ is \textit{greater} than}$$

$$\text{Integer } \frac{5 \times 34}{5 + 1 - 3} = 56$$

$$D_I = 4\text{: Integer } \frac{5 \times 66}{4} = 82 \text{ is \textit{less} than}$$

$$\text{Integer } \frac{5 \times 66}{5 + 1 - 4} = 85$$

18.

$$D_{II} = 2\text{: Integer } \frac{5 \times 34}{2} = 85 \text{ is \textit{greater} than}$$

$$\text{Integer } \frac{5 \times 66}{5 + 1 - 2} = 82$$

$$D_{II} = 3\text{: Integer } \frac{5 \times 34}{3} = 56 \text{ is \textit{less} than}$$

$$\text{Integer } \frac{5 \times 66}{5 + 1 - 3} = 110$$

win 5 seats.
(5) If Bloc II votes for five candidates, giving each (5 x 34)/5 = 34 votes, then, since 66 is greater than 34, Bloc I will win 5 seats.

Thus, Bloc I's optimal strategy is not to vote for three candidates, the maximum number it is certain of electing, but to vote for five candidates. By voting in this manner, Bloc I will be certain of electing at least three directors, but it may elect as many as five directors as illustrated in hypotheticals (3) through (5) above. In this example, Bloc I faces no downside risk if it votes for more directors than it is certain of electing. However, such a risk may exist in other cases as the method discussed below illustrates.

As demonstrated by the previous example, a bloc should determine not only the number of directors it is certain of electing, but also the number of directors for whom it should vote. Neither Cole's equation nor Inequality 1 addresses this issue.

The "D'Hondt Remainders Table"[19] offers a simple method for determining the number of candidates for whom a bloc should vote. This method is best illustrated by reference to the previous example. As before, let $S_I = 66$, $S_{II} = 34$, and $D = 5$. The appropriate D'Hondt Remainders Table is constructed by dividing the number of votes each bloc can cast by the integers 1 through D. This will indicate the number of votes each candidate will receive based on the number of shares controlled and the number of candidates for whom votes are cast.

TABLE 1

D'Hondt Remainders Table for 2-Bloc Cumulative Voting

	1	2	3	4	5
Bloc I	330	165	110	82½	66
Bloc II	170	85	56⅔	42½	34

The D'Hondt Table is first used to determine the number of

19. This technique is applied to proportional representation elections in B. GROFMAN, *supra* note 11, at 303, and to congressional apportionment by Balinski and Young, *supra* note 11.

directors each bloc is certain of electing. The largest entries in the table are circled, indicating D, the number of directors to be elected (in this illustration, five). The smallest circled entry in the top row appears in column 3, which means that Bloc I can guarantee itself three seats. The smallest circled entry in the bottom row appears in column 2, indicating that Bloc II can guarantee itself two seats. Each bloc should always nominate at least the number of candidates it is certain of electing.

In this case, however, Bloc I can safely nominate more than three candidates. That is, the fourth entry in the top row, 82½, is larger than the *first uncircled* entry in the bottom row, 56⅔. Hence, Bloc I can safely nominate four candidates. But should Bloc I nominate more than four candidates? Yes. The fifth entry in the top row, 66, is greater than the *first uncircled* number in the bottom row. Therefore, Bloc I should nominate five candidates, the same result obtained earlier through a somewhat different line of reasoning. If Bloc I divides its votes among five candidates each receiving 66 votes, and Bloc II votes for two candidates, Bloc I will still elect three candidates since Bloc II will exhaust its votes on its two candidates. If Bloc II divides its votes among three candidates instead of two, Bloc I will elect all five of its candidates since Bloc I can give 66 votes to each of its five candidates while Bloc II can only give 56⅔ votes to its three candidates. Bloc I cannot lose and may achieve a significant gain.

Generally, a bloc can safely vote for n directors if the nth entry in that bloc's row in the table is greater than the first uncircled entry in the competing bloc's row.

V. OPTIMAL STRATEGIES WHEN THERE ARE MORE THAN TWO BLOCS

The legal literature does not discuss cumulative voting in contests involving more than two competing blocs of shareholders. This section discusses strategies of cumulative voting in election contests involving at least three blocs.

Consider the following election. There are three blocs, I, II, and III. The blocs' shareholdings are $S_I = 40$, $S_{II} = 25$, $S_{III} = 25$, so that $S = 90$, and D (the number of directors to be elected) = 5. According to Cole's formula, the maximum number of seats Bloc I is certain of winning is two.[20]

As long as Blocs II and III do not join forces to vote for the same candidates, Bloc I can elect more candidates by dividing its votes among *three* candidates, giving each one (5)(40)/3 = 66⅔ votes. If either of the other blocs divides its votes among two or more candidates, each candidate receives at most (5)(25)/2 = 62½ votes. Since this number is less than 66⅔ (the number of votes cast for each of Bloc I's three candidates), Bloc I is assured of filling three seats, rather than two, contrary to the outcome given by Cole's formula.

TABLE 2

D'Hondt Remainders Table for 3-Bloc Cumulative Voting

	1	2	3	4	5
Bloc I	200	100	66⅔	50	40
Bloc II	125	62½	41⅔	31¼	25
Bloc III	125	62½	41⅔	31¼	25

Once again the D'Hondt Remainders Table gives the correct answer. Table 2 is generated by dividing the number of votes which can be cast by Blocs I, II, and III (i.e., 200, 125, and 125) by the integers 1 through D (i.e., 1 through 5). Again, the five largest entries in the table are circled. This process indicates that Bloc I is guaranteed of electing three directors if each of the other blocs votes for a different set of candidates. Similarly, Blocs II and III are assured of electing one director each.

As stated previously, the D'Hondt Remainders Table indicates the number of directors a bloc is certain of electing as well as the number of directors for whom it should vote. The number in the fourth column of the top row, 50, is smaller than the largest of the uncircled entries in the other rows, 62½. Thus, Bloc I should *not* vote for four directors. Similarly, the number in the fifth column of the top row, 40, is less than the largest of the circled entries in the other rows, 125, so that Bloc I should not

20. According to Equation 1, a bloc would have to own at least 31 shares in order to elect two directors; to elect three directors, it would have to own at least 46 shares. Cole's equation therefore implies that Bloc I, with 40 shares, can be sure of electing only two directors.

vote for five directors. Instead, it should spread its votes evenly among *three* candidates, the number it is certain of electing. Following the same procedure for Bloc II, the first uncircled entry in the middle row, 62½, is not greater than the largest uncircled entry in the other rows, 62½. Bloc II and III should vote therefore for one candidate.

Suppose, however, that Blocs II and III vote for the same candidates pursuant to a voting agreement. How many candidates can Bloc I be certain of electing?

TABLE 3

D'Hondt Remainders Table for 3-Bloc Cumulative Voting

	1	2	3	4	5
Bloc I	200	100	66⅔	50	40
Blocs II & III	250	125	83⅓	62½	50

This time the D'Hondt Remainders Table is generated by placing Blocs II and III together in one row, treating them as one bloc owning fifty shares. The number of votes which can be cast by the two competing groups, is divided by the integers 1 through D (i.e., 1 through 5). The five largest entries in the table are circled yielding the result that Bloc I can be certain of electing two directors. Similarly, Blocs II and III can *jointly* elect three directors.[21]

Once again, each bloc should determine the number of directors for whom it should vote. The first uncircled entry in the top row, 66⅔, is greater than the first uncircled entry in the bottom row, 62½; therefore, Bloc I should vote for three directors. Blocs II and III together should vote for only three candidates since the first uncircled entry in the bottom row is less than the first uncircled entry in the top row.

VI. CONCLUSION

This Article has illustrated the difficulties that plague currently accepted cumulative voting strategies. Use of Inequality 1

21. Inequality 1 yields the same results if one assumes that $S_I = 40$ and $S_{II} = 25 + 25 = 50$.

yields correct results for a two-bloc contest. The D'Hondt Remainders Table yields correct results for contests involving any number of blocs when the number of shares is large. In addition, the latter method will indicate the number of candidates a bloc should nominate in both the two-bloc and multi-bloc cases. These techniques represent significant improvements over methods currently found in the legal literature and may prove helpful to the corporate practitioner.