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INCUMBENCY ADVANTAGE, VOTER LOYALTY AND THE BENEFIT OF THE DOUBT*

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ABSTRACT

The standard Downsian model of two-party competition cannot account for the well-known fact that incumbents rarely lose elections. This paper modifies the model to take account of the tendency for voters to give the incumbent the 'benefit of the doubt'; i.e. voters may vote for an incumbent even when they prefer the policies of the challenger. We show that if all of the voters give the incumbent a very small benefit of the doubt, or if only a few voters give a larger benefit of the doubt, the incumbent may become invulnerable to defeat by any challenger, especially if his position is a relatively central one relative to voter ideal points. We also show that taking account of voter willingness to give some form of benefit of the doubt to incumbents also helps us to understand why politics deviates from the standard Downsian model in two other ways: (1) tweedledum-tweedledee politics is uncommon, and (2) incumbents often win by much larger than bare majorities. Data on attitudes toward government jobs policy and on US involvement in Central America, as reported in the 1984 NES survey, are used to illustrate that the more centrally located the incumbent, the smaller the benefits of the doubt that are required to ensure re-election. In an Appendix we present a number of new and useful theorems about the stability properties of situations in which voters provide incumbents with some form of benefit of the doubt. Our results show how voter willingness to give incumbents benefit of the doubt can make incumbents, especially centrally located ones, virtually impossible to defeat. Thus, 'benefits of the doubt' serve to reduce competition and deter entry.

KEY WORDS • benefit of the doubt • incumbency • rational choice • re-election • voter loyalty

It is a well-known fact of contemporary American politics that incumbents (especially in the US House of Representatives, but also in state and local legislative bodies) are rarely defeated for re-election, and that many incumbents win by sizeable margins. This fact is in direct contrast to the predictions of the basic Downsian model (Downs, 1957) that, when voting is along a left-right dimension, candidates zeroing in on the policy preferences of

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the median voter will generate competitive politics. A further challenge to the Downsian model is posed by the findings of Sullivan and Uslaner (1978) that, in marginal districts, the incumbent won 95 percent of the time even though the incumbent was closer to the median (mean) issue position of the voters in less than half of the districts. The fact that incumbents have such resilience in spite of their theoretical vulnerability has led researchers to try to account for the popularity of incumbents and their ability to withstand electoral challenge.

The four basic (though by no means mutually exclusive) accounts given for incumbency resilience are:

1. Incumbents can make a plausible claim that, because of seniority and experience, they will be more effective in securing benefits for the district than will their opponent (Fiorina and Noll, 1979; cf. Cox, 1986).

2. Incumbents buy loyalty with personal service, and build name recognition with the trappings and perquisites of office and the media attention and free postage that comes therewith; these advantages are of increased value in the present era of media-based politics (Fiorina, 1977).

3. 'The party is over', i.e. an ongoing dealignment of the American political party system has weakened voter loyalties to party labels (Broder, 1972; Burnham, 1975; cf. Wattenberg, 1986) and made service records and simple name recognition even more important cues for voter choice, thus creating bulwarks of individual protection which allow incumbents of one party to withstand statewide or national tides in favor of the other party's candidates.

4. In a world of uncertainty, voters may refuse to 'change horses' unless there is extraordinary reason for expecting a substantial improvement (Grofman and Uhlaner, 1985; Hardin, 1987). As an ABC newsman stated during a Los Angeles election night telecast on 9 November 1986: 'It's the comfort zone that all incumbents get to live in. If it's not broke, don't fix it.'

Each of these accounts provides reasons for the voters to support the incumbent apart from the incumbent's policy positions. In addition, it has been claimed that gerrymandering has reduced the heterogeneity of congressional districts (Mayhew, 1974; Tufte, 1973), and thus the candidate associated with the predominant bloc of voters in the district is virtually certain to be elected.

In this paper we develop implications of a 'rational choice' model operating in a policy space when some or all of the voters have non-policy reasons for supporting the incumbent. As in the usual Downsian story we posit that voters choose the candidate whose proposed policies are closest to the voter's own most preferred positions, but we modify the standard model by positing that the ith voter treats the distance between herself and the incumbent as less than it actually is. A voter might vote for an incumbent even while recognizing that the incumbent's policies are slightly less desirable than the

challenger's; or the voter may perceive the incumbent as having a more favorable policy position than he actually has (Markus and Converse, 1979; Page and Jones, 1979; cf. Chapman, 1967, 1968; Wittman, 1983). In either case, the voter acts as if she is giving the incumbent a certain benefit of the doubt. (Note that to simplify the presentation we adopt the arbitrary convention of referring to the candidates as 'he' and the voters as 'she'.)

It is well known from formal voting theory (e.g. McKelvey, 1976, 1979; Plott, 1967) that in two or more dimensions no position is secure against challenge.[1] However, it is also well known (Sloss, 1973) that, when some of the voters give some benefit of the doubt to the status quo (incumbent) when it is paired against any other alternative, the cumulative effect of these benefits of the doubt may be great enough to guarantee that the status quo will not be beaten by any other alternative. Results to this effect are also provided in Wilson and Herzberg (1988). We go beyond these earlier results in three important ways: two theoretical and one empirical.

First, we show the importance of a central location. If an incumbent is located 'centrally', even a small 'benefit of the doubt' from all of the voters may be sufficient to make the incumbent invulnerable to challengers. Alternatively, it is often sufficient for the incumbent to receive a somewhat larger benefit of the doubt from a relatively small subset of the voters. While a large enough benefit of the doubt from a large enough subset of the voters could make any spatial location invulnerable, an incumbent in a 'central' position requires less of a benefit of the doubt from fewer voters than does an incumbent in a more extreme position.

Second, we show that even when the benefits of the doubt are insufficient to guarantee the invulnerability of an incumbent, they are nonetheless important in limiting the challenging positions that can beat the incumbent to a few positions relatively far from the position of the incumbent - thus avoiding tweedledum-tweedledee politics.

Third, on the empirical side, we use data on voter attitudes toward two distinct issues, government jobs policy and US involvement in Central America (as reported in the 1984 NES election survey) to illustrate the impact of benefits of the doubt on the re-electability of incumbents. For a sample of 1569 voters, we show that even quite small benefits of the doubt are enough not only to ensure that some incumbents would be invulnerable, but also to ensure that certain centrally located points will defeat any challenger by a substantial margin.

1. Indeed, from this literature, one might expect that the challenger would be in an advantageous strategic position; while the incumbent is a known commodity with a relatively fixed policy position (fixed by previous campaigns and a legislative voting record), the challenger has greater flexibility to locate him/herself closer to the median voter. Even in one dimension, it is unlikely that the incumbent's position will fall exactly at the median voter (especially if that position changes over time); and in two or more dimensions, there will always be other spatial locations preferred by a majority of voters.

Our approach is also related to earlier work (especially that by Kamlet et al., 1985; and by Chapman, 1967, 1968; and Wittman, 1983), on party loyalty as a voter 'bias' (cf. Budge and Fairlie, 1977: Ch. 6); but we advance previous research which posits 'zones of indifference' by providing illustrations for two-dimensional spatial majority-rule voting games of the remarkably small magnitude of the benefit of the doubt that will be sufficient to ensure invulnerability to challenge. We also link our work to results provided by Wilson and Herzberg (1988) on agenda games with exogenous costs.

We begin by fully analyzing situations including three voters. In particular, we offer a construction to find the point which achieves stability with the minimum benefit of the doubt and show that this point is not, in general, located at one of the standard 'centers' of the space, e.g. the Copeland winner (Grofman et al., 1987); the center of the yolk (Feld et al., 1987; McKelvey, 1986), or the center of gravity. Next, we examine some five-voter situations that have been used in experimental voting games, to further illustrate the large impact of small benefits of the doubt. In the Appendix, we offer a general construction to find the benefit of the doubt sufficient to make any given point a core.

Three-voter Situations

It is important to recognize that unless three voters are located on a single line, every incumbent can be beaten by challengers in the absence of some incumbent benefits of the doubt. A challenger can always move in the direction of one pair of voters or another to obtain a majority of the votes against the incumbent. Thus, if voting is policy motivated, an incumbent requires some benefit of the doubt to have any chance of being re-elected, unless his challenger makes an error in choosing his policy position.

It is readily apparent that, if all voters give an incumbent sufficient benefit of the doubt, that incumbent can beat all challengers; the most obvious condition is where each voter gives sufficient benefit of the doubt to the incumbent as if to treat him as located at the voter's own ideal point. The stability that arises from such an extreme situation is obvious – and empirically vacuous (cf. Sloss, 1973). The strength of the present theory comes from showing that stability arises from substantially less benefit of the doubt being given; moreover, in general, an incumbent location can be stable even when each voter has some other candidates preferred to the incumbent.

Figure 1 provides an illustration of the large impact of a small benefit of the doubt. It shows three voters (A, B and C) distributed in a two-dimensional issue space. The figure also shows the *BOD point* (benefit of the doubt point), •, the point requiring the least benefit of the doubt from all of the voters to ward off all challengers. The radius of the circle

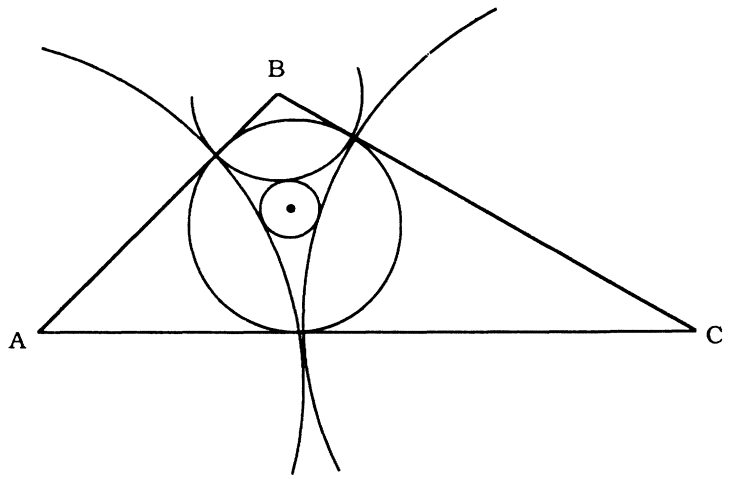

Figure 1. Large Impact of a Small Benefit of the Doubt

surrounding •, *the BOD circle*, is the magnitude of the benefit of the doubt, b, that is required for that point to be invulnerable to challenge. The additional arcs in the figure help our understanding of how it is that small b is sufficient to make • invulnerable to challengers. For • to lose to some other point, X, a majority (two) of the three voters must prefer X to •. Voter A prefers all points at least b closer to A than is •; these points are enclosed by a circle around A that is tangent to the BOD circle around •. Similarly, the circle around B tangent to the BOD circle includes all points that B prefers to • subject to the benefit of the doubt of b. It can be seen that the A circle and the B circle do not overlap (b is taken to be incrementally large enough so that the two circles do not touch). Therefore, even though A will vote for some candidates against •, and B will vote for some candidates against •, there is no candidate that both A and B will vote for against •. The same is true for A and C, and B and C. Therefore, no majority votes for any candidate over •; • can successfully withstand all challenges.

A proof that the construction in Figure 1 provides the point of minimum BOD radius is given in the Appendix. The arcs in Figure 1 can be used to find the BOD point, and the radius of the BOD circle of that point. It should be apparent that the BOD circle of the BOD point is always small and centrally located within the triangle created by three-voter ideal points. For example, the BOD radius of the BOD point of an equilateral triangle is .077 of the length of a side; i.e. each voter must only give a benefit of the doubt of less than 8 percent of the distance between herself and any other voter. That does not seem much to ask! For three voters the BOD radius of the BOD point can be no more than .125 of the smallest distance between any

pair of voters. In the example in Figure 1, the BOD radius is approximately .05 of the shortest distance between voter ideal points.

However, as shown in Figure 1, the BOD point, although relatively central to the Pareto set, does not correspond to any of the usual notions of the 'center' of the space, e.g. the center of the yolk (Feld et al., 1987; McKelvey, 1986), the Copeland Winner (Grofman et al., 1987) or the center of gravity. As we show in the Appendix, for the three-voter case, the BOD point corresponds to what Wuffle et al. (1989) refer to as the 'finagle point', or what Owen calls the point of 'least perturbation' (Owen, 1988). However, this is not true in general. Rather the radius of the finagle circle provides an upper bound for the BOD radius.

In three-voter situations, we have shown that if an incumbent is careful to locate at the ideal spatial location, the BOD point, he requires very little benefit of the doubt from the voters to defeat challengers. While these specific results are applicable only in situations including three voters, the substantive point that the BOD required for stability is small generalizes to situations with any number of voters. We provide empirical examples in the next sections.

However, an incumbent may not be able to find the ideal location; and even if he or she knows it, there may be other constraints (e.g. the preferences of fellow party members and the need to win their support in primaries and their financial support in the general election, the candidates' previous records, or simply their own personal beliefs) that prevent them from locating at the BOD point (cf. Wittman, 1983). What happens if they locate somewhere else? The simple geometry of the three-voter situation provides a clear example of how the required benefit of doubt increases as incumbents locate further from the BOD point itself.

If an incumbent is given more than the minimum benefit of the doubt necessary for stability at the BOD point, then there is a 'region of invulnerability' surrounding the BOD point. An incumbent located at a point within the region of invulnerability cannot be beaten. Even a small benefit of the doubt can lead to a large region of invulnerability. Figure 2 shows a region of invulnerability, for the example from Figure 1, when the benefit of the doubt is twice the minimum (still only .10 of the shortest distance between the voter ideal points). An incumbent moving away from the BOD point, in any direction, requires a greater BOD to be invulnerable. However, even if the incumbent locates away from the optimal central location, the required benefit of the doubt may still be surprisingly small.

We can use the three-voter situation once again to show how candidates can become invulnerable to challengers by obtaining some benefit of the doubt from only a *subset* of the voters. Certainly, if a majority of the voters give any incumbent sufficient benefit of the doubt, the incumbent can beat all challengers. Moreover, a sufficient benefit of the doubt from a minority of the voters is always sufficient to ensure the invulnerability of a candidate

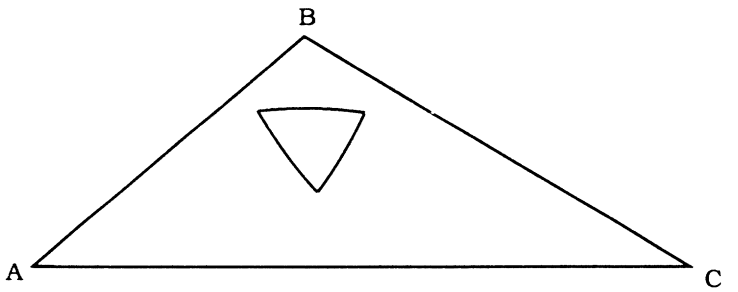

Figure 2. A Region of Invulnerability When the Benefit of the Doubt Is Twice the Minimum

who is properly located. In the three-voter example, suppose that a candidate locates on the line between B and C; and that A alone gives him some benefit of the doubt. Figure 3 shows a benefit of the doubt from A sufficient to ensure that there are no challengers that could obtain a majority against that incumbent. Thus, even a benefit of the doubt from A alone (larger, of course, than would be needed if some of the other voters were also giving some benefit of the doubt) is sufficient to ensure a carefully located incumbent's re-election.

In general, it should be apparent that for any particular incumbent location, as the number of voters giving a benefit of the doubt increases, the size of the benefit of the doubt required to ensure stability can only decrease.

In the Appendix we provide a general construction to find the value of the benefit of the doubt needed to make any given point invulnerable. The intuition underlying this construction is that we seek a BOD radius large

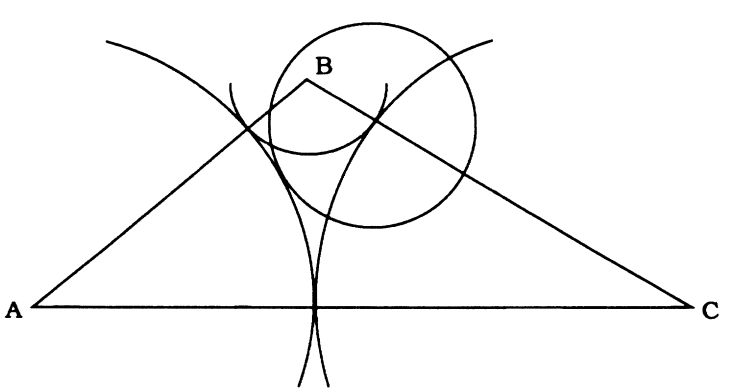

Figure 3. A Benefit of the Doubt Sufficient to Ensure No Challengers with a Majority against the Incumbent

enough to 'suborn' each minimal winning coalition. We construct the set of the points that is preferred by *all* members of that minimal winning coalition to the given point x. This set will be a convex region of the space. We find the maximum circle that can be inscribed in this region. The radius of that circle gives us the benefit of the doubt needed to be given x so as to assure that that minimal winning coalition has no point that they prefer to x given that benefit of the doubt.

We have written a computer program (in BASIC) to find the BOD radius of any point, and we have used this program to calculate BOD values for the 1569 voter example we describe later (copies of this program are available upon request from the authors). To further elucidate the properties of benefit of the doubt in an intuitive fashion, we next consider some specific five-voter situations.

Experimental Voting Games With Five Voters

Work in game theory has inspired economists and political scientists to conduct experiments in simulated voting situations. The most common experimental voting games involve five-voter ideal points. With five-voter ideal points, there are two commonly studied generic types of distributions of voter ideal points: (a) one with all voters distributed on the periphery, and (b) one with four voters on the periphery and one 'inside' the quadrilateral bounded by the other voters. From the set of distributions that have been used in experiments, we selected one specific distribution of each type to illustrate the potential significance of benefits of the doubt. Other formal properties of these particular distributions of voter ideal points have been analyzed, and so may be used for comparison (cf. Fiorina and Plott, 1978; Wilson and Herzberg, 1984).

Figure 4 shows the distributions of voter ideal points in one five-voter situation, and the size of the generalized benefit of the doubt that is required to ensure the invulnerability of an incumbent (status quo point) located at one central internal point. In the Fiorina–Plott game shown in Figure 4, the voter ideal points are located at $A(51, 59)$, $B(30, 52)$, $C(25, 72)$, $D(62, 109)$ and $E(165, 32)$. Recall that each voter regards the incumbent as if he were located on the outside of the circle nearest the voter ideal point. In the Fiorina–Plott game, we find that a benefit of the doubt of as little as 1.8 is sufficient to ensure the invulnerability of a central point. Wilson and Herzberg (1984) use a game with voter ideal points distributed at the vertices of a symmetric pentagon. For a pentagon inscribed in a circle with a radius of 60 units (e.g. $A(120, 60)$, $B(79, 117)$, $C(11, 95)$, $E(79, 3)$ – not shown), a benefit of the doubt of only 3 units is sufficient to ensure the invulnerability of an incumbent located at the center. This is a very small benefit of the doubt, especially in comparison with the distances between

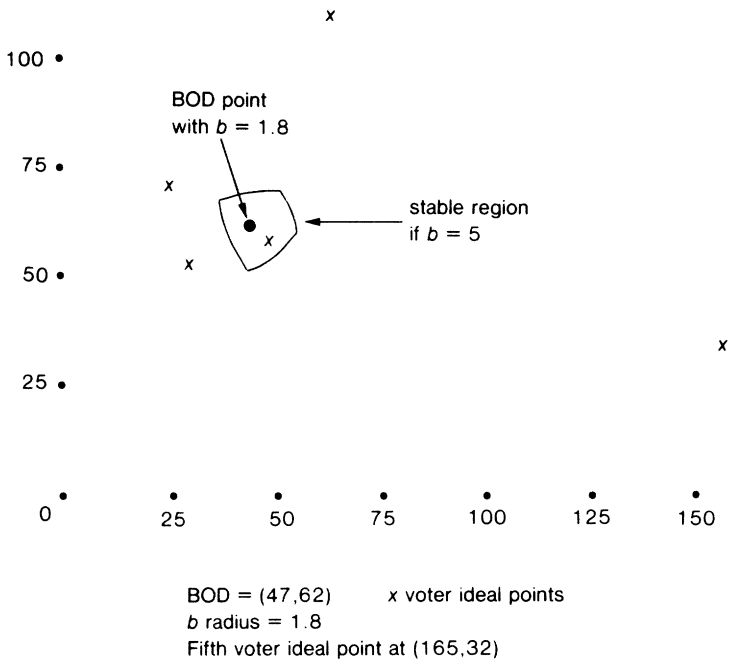

Figure 4. Fiorina–Plott Five-voter Game, No Core (arcs shown are approximate)

voter ideal points: each voter ideal point is at least 70 units from each other ideal point.

The actual experiments typically allow voters to continue to replace the status quo with new alternatives by majority vote indefinitely until a decision for cloture is reached. Subjects are given more money for outcomes that are closer to their own ideal points; however, small differences in distance generally make small differences in money outcomes. Thus, small benefits of the doubt in distance are equivalent to small benefits of the doubt in money. We suggest that subjects may vote for the status quo over alternatives that pay only slightly (e.g. 25 cents) more.

Thus, small benefits of the doubt can result in stable central outcomes in such five-voter games. Furthermore, if the benefits of the doubt are even slightly greater than the minimum required to make one point invulnerable to challenge, then many more points become invulnerable to challenge. For example, Figure 4 shows the set of points that are invulnerable when given a benefit of the doubt by each voter of 5 units. It should be apparent that this is a fairly large region. We suggest that one reason that results of experiments using these games vary is the wide-ranging stability that results from even small benefits of the doubt.

Wilson and Herzberg (1988), following Hoffman and Packel (1980) and

others, introduce costs into their experimental voting games by requiring voters to spend money to modify the status quo. They show that sufficient 'access costs' to the agenda can make the status quo invulnerable to defeat. However, while they do provide a number of quite informative examples, they do not provide a general algorithm to find the minimum (benefit of the doubt) 'agenda access cost' sufficient to make any given point a core, or to find the point which requires the minimum 'agenda access cost' (benefit of the doubt) to become a core. Rather they characterize their construction in terms of finding the 'thickest petal' of the win set of a point. We believe, however, that, mathematically, their approach is isomorphic to that taken in the Appendix.

Heterogeneous Benefits of the Doubt

Benefits of the doubt can result in even more widely varied outcomes when the benefits of the doubt are not the same for all voters. Unevenly distributed benefits of the doubt may lead to non-central stable outcomes, as shown in Figure 3. When one or a few voters give larger benefits of the doubt, stable outcomes can be far from central.

There are situations where a benefit of the doubt from even one voter may be sufficient to make some point stable. For example, in all five- and six-voter situations, sufficient benefit of the doubt from any one voter is enough to make a carefully located incumbent invulnerable. In general, when 25 percent of the voters give sufficient benefit of the doubt, there are always some points that are invulnerable to challenge.

It is important to note that a benefit of the doubt could be negative. For example, if there is a 'throw the rascals out' sentiment, this effectively could create a negative benefit of the doubt. It is also possible that some voters will give the incumbent a positive benefit of the doubt, while others will give him a negative benefit of the doubt. Any negative benefits of the doubt obviously make it more difficult for an incumbent to be re-elected. However, even with some antipathies, an incumbent with sufficient positive benefits of the doubt might be invulnerable to challengers.

Benefit of the Doubt: An Empirical Example

In 1984, the Survey Research Center at the University of Michigan included questions on the National Election Study NES Survey about government involvement in guaranteeing jobs and US involvement in Central America. Respondents were asked to indicate their opinions on a 7-point Likert Scale ranging from extremely favorable toward government help to jobseekers (1), to the view that people should get jobs on their own (7); and their opinion on government involvement in Central America from much more

(1), to much less (7). The cross-tabulation of these two variables can be taken to indicate the location of voters in a two-dimensional issue space. We have chosen these issues as ones potentially relevant to candidate choice, subject to the constraint that attitudes on these issues would be relatively independent of one another. The substantive results we obtain for these particular two issues appear to be quite typical of what we have found for other pairs of issue positions.

Table 1 shows the cross-tabulation; the median voter position on job policy was the middle of the scale (4); the median position on Central America was slightly partial to less involvement (5). We define the 'generalized median' to be the spatial location determined by the medians on these two dimensions (4, 5).

Although the generalized median is majority preferred to all positions directly above it or below it, or directly to the left or right of it, it is not majority preferred to certain other positions. If the generalized median is voted against an alternative slightly down and to the left, say, (4.1, 4.9), the results are 59 percent against and 41 percent in favor of the generalized median.[2]

Nevertheless, an incumbent located at the generalized median can be re-elected if he can obtain even a small benefit of the doubt from all of the voters (which we refer to as a *generalized benefit of the doubt*), or

Table 1. Jobs Policy versus Central American Policy

Government Guaranteed Jobs		US Involvement in Central America							
		More 1	2	3	4	5[a]	6	Less 7	Total
Government help	1	26	6	7	19	14	16	77	165
	2	7	10	14	18	24	35	20	128
	3	2	15	25	35	45	44	35	201
	4[a]	10	29	43	97	70	66	63	378
	5	12	25	43	62	73	63	28	306
	6	15	23	37	55	45	35	22	232
On own	7	24	13	18	36	12	18	38	159
Total		96	121	187	322	283	277	283	1569

Source: 1984 NES Study.

[a] Indicates the median for each of the attitudes separately.

2. Note that all results in this section are based upon approximations that were derived by limiting feasible alternatives to a finite grid with unit of .02. Calculations compared each alternative against the given status quo (incumbent). These calculations were repeated with various benefits of the doubt, beginning with zero and incremented by .01. For purposes of presentation, results were rounded.

a larger benefit of the doubt from a few.[3] In this specific example, if all of the voters give an incumbent located at the generalized median a benefit of the doubt of only .14, no challenger is majority preferred to the incumbent.[4]

Majority Margins

For each generalized benefit of the doubt from 0 to .90, Table 2 shows the largest margin that any challenger can obtain against an incumbent located at the generalized median. The table indicates that, as the generalized BOD increases, the maximum margin of victory that any challenger can achieve against the incumbent declines. Once the BOD is at least .14, no challenger can obtain a majority against the incumbent. When the BOD is .90, then the incumbent beats every possible challenger by at least 72 to 28 percent.

Table 2. Incumbent Margin Against the Most Effective Challenger with Various Levels of the Generalized Benefit of the Doubt

Generalized Benefit of the Doubt	Percent Received by Incumbent at the Generalized Median Against the *Most* Effective Challenger
.0	41
.1	50
.2	54
.3	58
.4	59
.5	60
.6	66
.7	70
.8	70
.9	72

An alternative way for an incumbent to become invulnerable to challengers is to obtain the loyalty of a small random subset of the voters, say, by intervening to obtain social security benefits for their relatives or by convincing them of his competence in generating benefits for the district as a whole. For example, for the 1569 voters in Table 1, suppose that an

3. We should note that the generalized median is only one convenient centrally located point. In general, and in this situation in particular, the point that is made invulnerable by the smallest BOD will not be the generalized median itself, but will be a point nearby.

4. There are numerous reasons to believe that voters do provide some form of benefit of the doubt to candidates. For example, Norrander (1988) shows that, for presidential primaries, voters do not always vote for the candidate ideologically closest to them, even though, in aggregate, voters with a given ideological position often distribute their vote among candidates in a fashion generally consistent with ideological proximity. One explanation for this finding is that some voters give some candidates a non-policy-related benefit of the doubt.

incumbent located at the generalized median obtains the complete loyalty of a random 16 percent of the voters. Among the rest of the voters, Table 2 shows that the most effective challenger obtains 59.5 percent (to 40.5 percent) of the vote against the incumbent; i.e. a margin of 19 percent. This is 19 percent of the 84 percent remaining (non-loyal) voters, or 16 percent. This margin could be overcome with the 16 percent loyal votes. Thus, with 16 percent loyal voters, the incumbent would not lose even to the most effective challenger. Against a less effectively located challenger, even fewer 'loyalists' would be sufficient to ensure victory.

As in our discussions of three- and five-voter situations, various combinations of benefits of the doubt from different voters can ensure the invulnerability of an incumbent. The number of loyal voters required for invulnerability depends upon the margin obtainable by the most effectively located challenger. Specifically, if the most effective challenger can obtain X fraction of the votes, then a random $Y = (2X - 1)/2X$ loyalists will be sufficient to guarantee invulnerability.

As the candidate moves further from the 'center', there is the general tendency for him to require more benefit of the doubt in order to defend his position. The generalized BODs required for invulnerability of an incumbent located at each of several points near the generalized median ((3, 5), (5, 9), (4, 4) and (4, 6)) are .69, .32, .32 and .69, respectively. It is apparent that an incumbent located away from the 'center' requires more benefits of the doubt than one located at the generalized median. Nonetheless, the benefit of doubt required for points near the generalized median seems still to be of a magnitude that is readily possible to imagine voters giving to an incumbent, considering the extent of various incumbency non-policy advantages.

We would expect that incumbents will receive different benefits of the doubt from different voters. We believe that incumbents who can effectively marshall their incumbency advantages may be able to protect non-optimal but still relatively central positions against challengers, benefiting from large BODs from some voters and small BODs from others. However, for a given incumbency advantage, incumbents with more extreme positions will be more vulnerable to challengers.

Further Implications of Benefits of the Doubt

The central point of our analysis has been to show that incumbents can make themselves invulnerable by obtaining relatively small (positive) benefits of the doubt from voters. Moreover, benefits of the doubt can not only account for the longevity of incumbents, *but also help to account for their large electoral margins*. For example, Table 2 shows the assured margins that an incumbent located at the generalized median obtains as a function of the

generalized benefit of the doubt level. If the incumbent receives more than the minimum generalized BOD necessary for invulnerability, the incumbent is ensured of winning by increasingly large margins.

When incumbents have insufficient benefits of the doubt to ensure invulnerability, benefits of the doubt nonetheless reduce the range of alternatives that can beat the incumbent. Specifically, benefits of the doubt preclude the possibility of challengers beating the incumbent by locating immediately adjacent to him in one direction or another. When the voters give the incumbent any benefit of the doubt, they always prefer the incumbent to immediately adjacent candidates. In fact, a small benefit of the doubt *can force challengers to locate relatively far from the incumbent to beat him*. For example, for the distribution of voters in Table 1, with a benefit of the doubt of .04, the nearest challenger who can beat a candidate located at the generalized median is .28 from the generalized median.

This logic can be illustrated with a three-voter example. The lightly shaded area in Figure 5 shows the set of alternatives that are majority preferred to the incumbent, O. The dark shaded areas show the alternatives that are successful, subject to a particular generalized benefit of the doubt (as indicated by the circle around O). It is clear that the only alternatives that beat O subject to the given BOD are relatively far from O. In this way, benefits of the doubt discourage tweedledum–tweedledee politics, because a challenger with nearly identical issue positions to the incumbent cannot win. An incumbent of one party tends to be slightly to one 'side' of the center, leaving the positions that can successfully challenge to be primarily on the other side. Thus, benefits of the doubt from voters encourage Democrats and Republicans to take distinct positions on opposite 'sides' of the center of the issue space.

Figure 5 also shows that *a BOD that is insufficient to make an incumbent invulnerable nevertheless shrinks the set of positions from which a challenger can succeed*. Under these conditions it may be very difficult for a challenger to locate at a winning position, because he may not be able to find the position, or because other constraints (e.g. from his personal convictions, his record, his party supporters, etc.) may preclude him from occupying those few particular positions that would enable him to win.

Discussion

Loyalty and the generalized benefit of the doubt can be seen as two extreme forms of phenomena whereby an incumbent ensures re-electability by getting voter support outside of the policy domain. 'Loyalty' refers to commitment by a small number of voters, and 'generalized benefit of the doubt' refers to a small benefit of the doubt from all of the voters. In the real world,

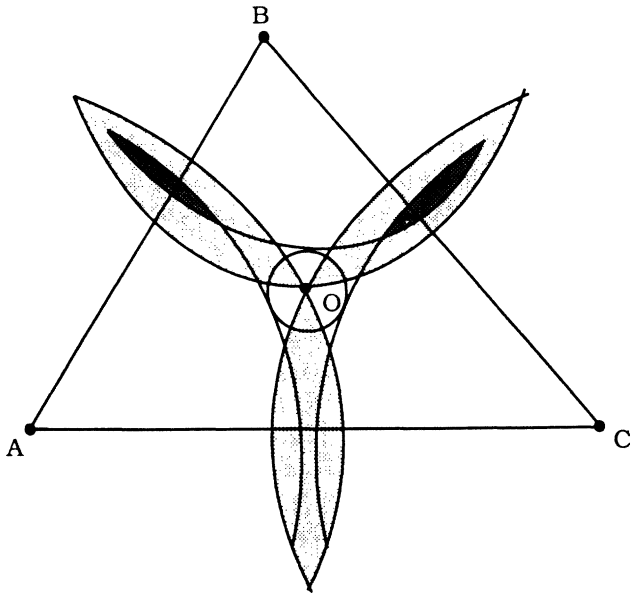

Figure 5. Alternatives in a Three-voter Example

voters will vary in the benefits of the doubt that they give to the incumbent; but it is realistic to expect that most voters will give the incumbent some benefit of the doubt.

Thinking about political competition in terms of voters giving incumbents benefits of the doubt enables us to account for features of American politics that the standard Downsian model does not adequately represent. In particular, we have shown how very small benefits of the doubt can insulate incumbents from electoral vulnerability, make possible their re-election by large margins, and make it likely that when they are defeated in a general election, they are defeated by candidates with distinctly different issue positions (as is true in elections for the United States Senate; cf. Grofman et al. forthcoming).

Incumbency benefits of the doubt cannot fully insulate incumbents who are far from the center of the electorate. When the incumbent is located near the 'center' of the issue space, it may be relatively easy to ensure re-electability by loyalty of a few and/or small benefits of the doubt from many; but if the incumbent is located far from the 'center', it may require much more non-issue support than may be practical to obtain. Thus, the special benefits of incumbency (e.g. seniority, reputation for bringing local benefits, avoiding uncertainty in the minds of voters, etc.) facilitate re-election, but do not completely insulate an incumbent from the pressures of Downsian politics.

The current approach suggests lines of both theoretical and empirical research. In terms of theory, the Appendix specifies an algorithm to find the BOD point (and its radius) for situations with three voters. A closed form solution is known only for this case, but we have also provided in the Appendix a general way to find the BOD radius of any point sufficient to make that point invulnerable to defeat, and we have a computer algorithm to find the BOD point in any situation of practical interest.

Further Research

Our empirical results were intended to be only an illustration of how even tiny benefits of the doubt have substantial consequences. Empirical research is needed to determine the benefits of the doubt actually extended by various segments of particular populations. For example, if some citizens receive services attributable to specific representatives, how does that affect the benefit of the doubt they are willing to give to these representatives? Do substantial public works projects voted to a district allow the representatives of that district greater ideological discretion? Also, the mathematics of 'benefits of the doubt' should be compared and contrasted with related geometric concepts such as the yolk (Feld et al., 1987; McKelvey, 1986).

In general, the recognition that choice among a set of options that can be thought of as alternatives in an n-dimensional issue space can be significantly altered by non-policy considerations might inform the study of a broad range of political contexts. For example, when a committee is designated to develop a proposal for the main body (especially one subject to a yes-no vote), then often the group will give that proposal a certain benefit of the doubt, in part because of the transaction costs of developing alternative proposals. This may help account for the often sizeable margins of passage of bills in Congress - contrary to the expectation of minimal winning coalitions (Linda Cohen, personal communication, 2 May 1988). Similarly, factors such as party loyalty or 'brand name' recognition (e.g. of a DuPont, a Danforth, a Lodge or a Kennedy) may be thought of as providing other sources of candidate benefit of the doubt (both positive and negative).

In this paper, however, our focus has been on benefit of the doubt as it relates to incumbents. We have shown how even *quite small* benefits of the doubt can *dramatically* change the nature of electoral competition and generate barriers to entry.

Appendix A

THEOREM 1. *For three voters, the point • in Figure A1 is the BOD point, the point with minimum BOD radius sufficient to make it invulnerable.*

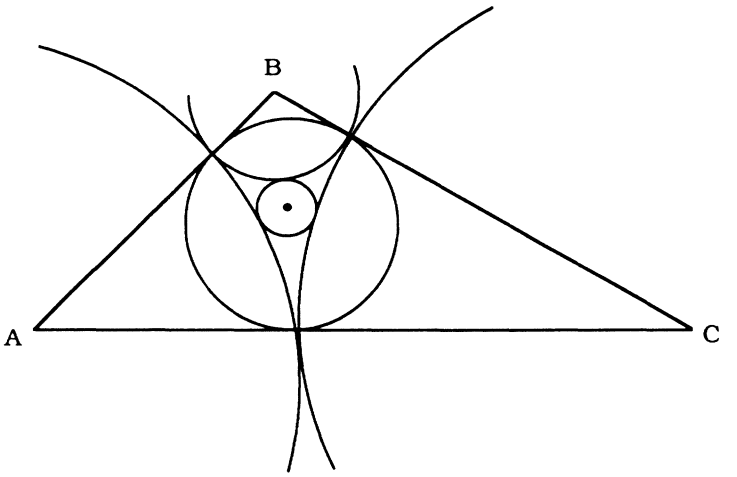

Figure A1

To prove the theorem, two lemmas are required.

Lemma 1: For three voters, the radius of the circle shown around • in Figure A1 is the BOD radius needed to make it invulnerable.

Proof of Lemma 1: It is apparent that if the point • is given a benefit of the doubt equal to b, no pair of voters jointly prefer any other points to •. Furthermore, with BOD radius less than b, it is apparent that the circles around the voter ideal points are incrementally larger and will overlap, leaving regions in which points are jointly preferred to •. Therefore, b is the smallest BOD that makes • defensible against all comers.

Lemma 2: Let A and B be two voters. For any point x, outside of an ellipse defined as the set of points, X, such that $A + B = \overline{AB} + 2b$, there exists a point z such that voter A and voter B both prefer z to x, even if x is given a benefit of the doubt of b.

Proof of Lemma 2: If $\overline{Ax} + \overline{Bx} > \overline{AB} + 2b$ (i.e. x is outside of the ellipse), then the circle around A with radius $\overline{Ax}$-b (describing the points preferred by A to x if x is given a benefit of the doubt equal to b), and the circle around B with radius $\overline{Bx}$-b (describing the points preferred by B to x if x is given a BOD radius of b) will overlap over the line AB as shown in Figure A2.

Now we can prove Theorem 1.

Proof of Theorem 1: Figure A3 shows the construction from Figure A1 with the ellipse drawn in corresponding to the BOD given by the radius of the circle around • tangent to the three arcs. It should be apparent by the construction in Figure A1 that point • is on all three ellipses; this can only occur when the three ellipses intersect at the point •. If we move in any direction from this intersection, we move outside of one or more of the ellipses (because there are no points within all three ellipses). Since any point is outside one ellipse, it is defeated by some point preferred by the two voters defining that ellipse (from Lemma 2). Thus, any other point requires a larger BOD. Q.E.D.

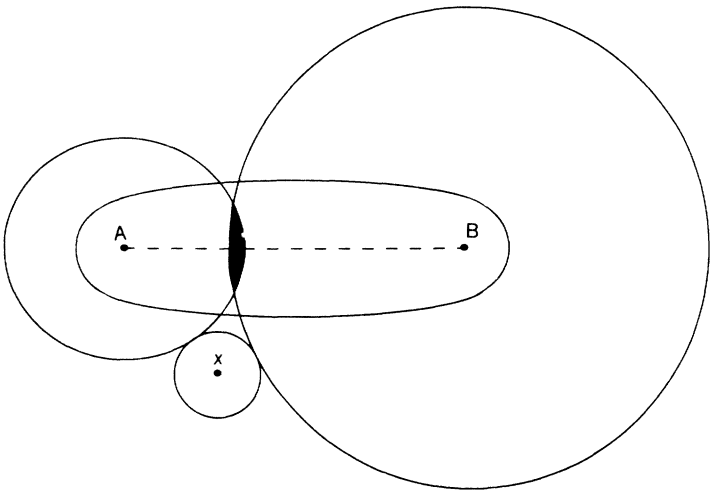

Figure A2. Ellipse Construction

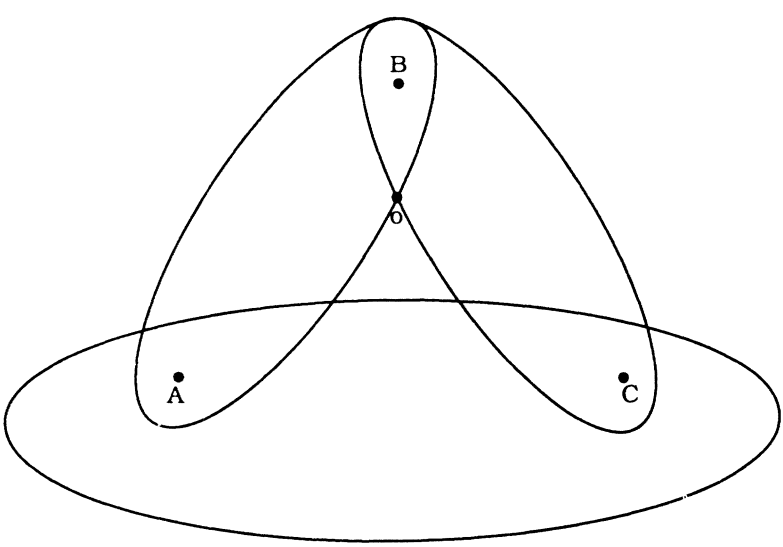

Figure A3. Construction Used to Prove Theorem 1

We now specify a general construction to find the benefit of the doubt needed to make any given point, x, unbeatable; this benefit of the doubt we label b_x.

Definition: A *petal* of a point, x, is the set of points that are preferred to x by a particular minimum winning coalition.

Thus, for Euclidean preferences, a petal corresponding to a particular minimal

winning coalition is just the intersection of some set of circles, where each circle is drawn through x and is centered at one voter ideal point in the minimal winning coalition.

Lemma 3: A petal is a convex region of the space.

Proof: Since a petal is an intersection of circles, it must be convex. Q.E.D.

Definition: The *win-set* of a point x is the union of the petals of x over the set of all minimal winning coalitions. Note that if the convex hull of a minimal winning coalition includes a point x, then the corresponding petal is empty; i.e. there is no alternative that *all* members of the given minimal winning coalition prefer to x.

THEOREM 2. *For any given point, x, the benefit of the doubt required to make x unbeaten by any other points is given by the radius of the largest circle that can be inscribed within a single 'petal' of the win-set of x.*

To prove this theorem requires several additional lemmas.

Lemma 4: The largest circle that can be inscribed within a petal must be tangent to the inside of one, two or three of the circles which define the petal (as shown in Figure A4).

For a given benefit of the doubt, b, a petal of the win-set of x is just the intersection of a set of circles, where each circle is drawn around the ideal point of a member of the winning coalition, with radius equal to its distance to x minus the quantity b.

Lemma 5: If a circle of radius b centered at q is tangent to the inside of a circle of larger radius that composes part of the boundary of the petal, then, when x is given a benefit of the doubt of b, this arc of the petal of the win-set of x passes through q.

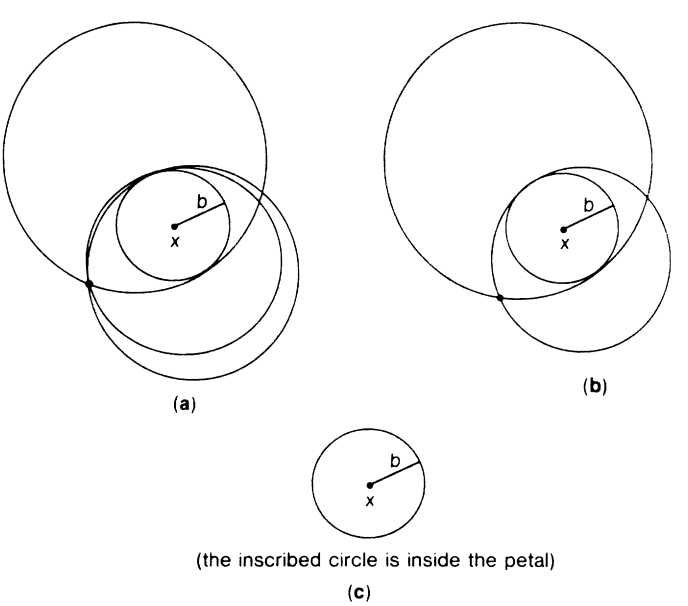

Figure A4. Construction Used to Prove Lemma

Proof of Lemma 6: If the smaller circle is tangent to the inside of the larger circle, then a diameter of the larger circle passes through q and the tangency point. If the radius of the larger circle is reduced by b, then the new circle passes through q.

Now we can prove Theorem 2.

Proof of Theorem 2: If the largest inscribed circle is tangent to three circles defining a petal, and x is given a benefit of the doubt of b, then the reduced petal is empty, because all three of the reduced circles pass through q (the center of the inscribed circle), and the intersection of three circles passing through a point is empty. Therefore, this benefit of the doubt is sufficient to ensure that the win-set is empty.

There are two other possibilities for the largest inscribed circle: it may be tangent to two circles (as shown in Figure A4(b)) or equal to one circle (as in Figure A4(c)). In either case, it is apparent that the shrinking of the circle(s) by the radius of the inscribed circle results in an empty petal.

Conversely, if a particular benefit of the doubt, b, results in an empty petal, then the members of that minimal winning coalition must have circles with empty intersection. Specifically, either one circle with reduced radius must be empty, two circles with reduced radius must have zero intersection, or three circles with reduced radius must have zero intersection. In any case, removing the benefit of the doubt can only increase the circle inscribed within that particular (previously empty) intersection to that specified by a circle of radius no more than b. Thus, the largest circle that can be inscribed within a petal can have a radius no larger than b. Q.E.D.

The constructions we use for this proof are very similar to that in Figure 2 in Wilson and Herzberg (1988). However, that paper deals only with the case where the petal of a win-set is defined by two intersecting circles. This is only one of three possible cases. Of course, the logic of the three cases shown in Figure A(4) is essentially identical.

Finally, we compare the BOD radius with an apparently similar construct, the finagle radius.

Definition: The *finagle radius*, f_x, of a point x is the radius of the minimum circle around that point such that some point in the circle defeats any point not in the circle.

Definition: The *finagle circle* around a point x is the circle around a point with radius equal to the finagle radius of a point.

More specifically, the finagle radius of a point is the radius of the largest circle that can be inscribed within the win-set of a point.

We now state an additional theorem.

THEOREM 3. *For any point*, x, $f_x \geqslant b_x$.

Proof of Theorem 3: Given f_x, for every y, there is a point z_y in the finagle circle around x that beats y. Therefore there are a majority of voters who prefer z_y to y, i.e. are closer to z_y than to y. With a benefit of the doubt f_x, every voter prefers x to every z_y, within the f circle. In particular, the majority that prefers z_y to y prefers x to z_y to y when x is given a benefit of the doubt of b_x. Therefore a majority prefers x to y with this BOD. Q.E.D.

When the petals of a win-set do not overlap, then the construction for the BOD radius and the finagle radius are identical. However, when two or more petals overlap, the BOD radius is the radius of the largest circle that can be inscribed within a single petal, while the finagle radius is the radius at the largest circle that can be

inscribed in any section at the win-set, which may include several overlapping petals.

In the three-voter case, petals at the win-set of points with pareto set do not overlap. Therefore, the finagle radius and BOD radius are the same. In particular for three voters,

THEOREM 4. *For the case of three voters, the finagle radius (f_x) of any point x, within the pareto set is equal to the BOD radius (b_x) of that point.*

Proof: Compare the construction proof here with that in Wuffle et al. (1989).

References

Broder, David S. (1972) *The Party's Over: The Failure of Politics in America*. New York: Harper and Row.

Budge, Ian and Dennis Fairlie (1977) *Voting and Party Competition*. New York: John Wiley.

Burnham, Walter Dean (1975) 'Insulation and Responsiveness in Congressional Elections', *Political Science Quarterly* 90(3): 411–36.

Chapman, David (1967) 'Models of the Working of Two-Party Electoral System, I', *Papers on Non-Market Decision-Making*: 19–38.

Chapman, David (1968) 'Models of the Working of a Two-Party Electoral System, II', *Public Choice* 1: 19–38.

Cox, Gary W. (1986) 'On the Rationality of the Incumbency Cue'. Unpublished manuscript, University of Texas, Austin.

Cox, Gary W. (1987) 'Electoral Equilibrium under Alternative Voting Institutions', *American Journal of Political Science* 31(1): 82–108.

Downs, Anthony (1957) *An Economic Theory of Democracy*. New York: Harper.

Feld, Scott L. and Bernard Grofman (1988) 'Majority Rule Outcomes and the Structure of Debate in One-Issue-at-a-Time Decision Making', *Public Choice* 59: 167–76.

Feld, Scott L., Bernard Grofman, Richard Hartley, Mark O. Kilgour and Nicholas Miller (1987) 'The Uncovered Set in Spatial Voting Games', *Theory and Decision* 23: 129–56.

Feld, Scott L., Bernard Grofman and Nicholas Miller (1988) 'Centripetal Forces in Spatial Voting: On the Size of the Yolk', *Public Choice* 59: 37–50.

Ferejohn, J.A., R.D. McKelvey and E.W. Packel (1984) 'Limiting Distributions for Continuous State Markov Models', *Social Choice and Welfare* 1: 45–67.

Fiorina, Morris (1977) 'The Case of the Vanishing Marginals: The Bureaucracy Did It', *American Political Science Review* 71: 177–81.

Fiorina, Morris and Roger Noll (1979) 'Majority Rule Models and Legislative Elections', *Journal of Politics* 41: 1081–104.

Fiorina, Morris and Charles Plott (1978) 'Committee Decisions Under Majority Rule: An Experimental Study', *American Political Science Review* 72: 575–98.

Grofman, Bernard, Robert Griffin and Amihai Glazer (forthcoming) 'Identical Geography, Different Parties: See What a Difference Party Makes', in R.J. Johnston, F. Shelley and P. Taylor (eds) *Developments in Electoral Geography* (title tent.). London: Routledge.

Grofman, Bernard, Guillermo Owen, Nicholas Noviello and Amihai Glazer (1987) 'Stability and Centrality of Legislative Choice in the Spatial Context', *American Political Science Review* 81(2): 539–53.

Grofman, Bernard and Carole Uhlaner (1985) 'Metapreferences and Reasons for Stability in Social Choice: Thoughts on Broadening and Clarifying the Debate', *Theory and Decision* 19: 31–50.

Hardin, Russell (1987) 'Rational Choice Theories', in Terence Ball (ed.) *Idioms of Inquiry: Critique and Renewal in Political Science*. New York: SUNY Press.

Hoffman, Elizabeth and Edward Packel (1980) 'A Stochastic Model of Voting with Exogenous Costs: Theory and Experiments', *Behavioral Science* 27: 43-56.

Kamlet, Mark, Jack Citrin and Frank Levy (1985) 'Satisficing Behavior in Spatial Voting Models', *Political Methodology* 11: 19-41.

Markus, Gregory B. and Philip E. Converse (1979) 'A Dynamic Simultaneous Equation Model of Electoral Choice', *American Political Science Review* 73(4): 1055-70.

Mayhew, David R. (1974) 'Congressional Elections: The Case of Vanishing Marginals', *Polity* 6: 295-315.

McKelvey, Richard D. (1976) 'Intransitivities in Multidimensional Voting Models and Some Implications for Agenda Control', *Journal of Economic Theory* 12: 472-82.

McKelvey, Richard D. (1979) 'General Conditions for Global Intransitivities in Formal Voting Models', *Econometrica* 47: 1085-112.

McKelvey, Richard D. (1986) 'Covering, Dominance, and Institution Free Properties of Social Choice', *American Journal of Political Science* 30(2): 283-315.

McKelvey, Richard D. and Peter C. Ordeshook (1987) 'A Decade of Experimental Research on Spatial Models of Elections and Committees', Division of the Humanities and Social Sciences, California Institute of Technology, Social Science Working Paper No. 657, Pasadena, October.

Norrander, Barbara (1988) 'The Role of Ideology in Presidential Nominations', Paper presented at Midwest Political Science Association Convention, Chicago, 14-16 April.

Owen, Guillermo (1988) 'Spatial Voting Games', Paper presented at the Conference on Game Theory, Marseilles, France, April 1988.

Page, Benjamin and Calvin C. Jones (1979) 'Reciprocal Effects of Policy Preferences, Party Loyalties and the Vote', *American Political Science Review* 73(4): 1071-89.

Plott, Charles R. (1967) 'A Notion of Equilibrium and Its Possibility Under Majority Rule', *American Economic Review* 57: 787-806.

Simon, Herbert (1955) 'A Behavioral Model of Rational Choice', *Quarterly Journal of Economics* 69: 99-118.

Sloss, Judith (1973) 'Stable Outcomes in Majority Rule Voting Games', *Public Choice* 15: 19-48.

Sullivan, John L. and Eric M. Uslaner (1978) 'Congressional Behavior and Electoral Marginality', *American Journal of Political Science* 22: 536-53.

Tufte, Edward (1973) 'Determinants of Midterm Congressional Elections', *American Political Science Review* 69: 812-26.

Wattenberg, Martin (1986) *Decline of American Political Parties 1952-1984*. Cambridge: Harvard University Press.

Wilson, Rick K. and Roberta Q. Herzberg (1984) 'Voting is Only a Block Away: Theory and Experiments on Blocking Coalitions', Prepared for presentation at the Annual Meeting of the Public Choice Society, Phoenix, Arizona, 28-31 March.

Wilson, Rick K. and Roberta Q. Herzberg (1988) 'Effects of Agenda Access Costs in a Spatial Committee Setting', Prepared for presentation at the Annual Meeting of the Public Choice Society, Tucson, Arizona, 27-29 March 1988.

Wittman, Donald (1983) 'Candidate Motivation: A Synthesis of Alternative Theories', *American Political Science Review* 77: 142-57.

Wuffle, A., Scott L. Feld, Guillermo Owen and Bernard Grofman (1989) 'Finagle's Law and the Finagle Point, a New Solution Concept for Two-Candidate Competition in Spatial Voting Games', *American Journal of Political Science* 33(2): 348-75.

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