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Many thanks for your assistance
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**Abstract**

In *The Calculus of Consent* (<CitationRef CitationID="CR6">1962</CitationRef>: 235) Buchanan and Tullock assert: (1) *ceteris paribus*, while a coalition controlling less than a majority of voters may control in *either* chamber, the greater the difference in the bases of representation in the two houses, the less likely is any given coalition of voters to control a majority of the seats in *both* chambers; (2) the potential of cross-chamber logrolls (on issues of unequal intensity) increases the likelihood that a minority may effectively control policy making. We link these ideas to social theory approaches to bicameralism and for the empirical study of legislatures.

**Keywords**

Bicameralism – Representation – Majority rule – Supermajorities – Coalitions

**Footnotes**

A much earlier version of this paper was presented at the Liberty Fund Conference on “The ‘Calculus of Consent’,“ Santa Cruz, California, June 23–25, 1988. Partial support of this research came through SSHRCC research grant #410-2007-2153 (Stanley Winer and Stephen Ferris, co-PIs). Grofman’s work was also supported by the Jack W. Peltason (Bren Foundation) Endowed Chair, University of California, Irvine, and by the UCI Center for the Study of Democracy.
Towards a theory of bicameralism: the neglected contributions of the calculus of consent

Bernard Grofman · Thomas Brunell · Scott L. Feld

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Abstract In The Calculus of Consent (1962: 235) Buchanan and Tullock assert: (1) ceteris paribus, while a coalition controlling less than a majority of voters may control in either chamber, the greater the difference in the bases of representation in the two houses, the less likely is any given coalition of voters to control a majority of the seats in both chambers; (2) the potential of cross-chamber logrolls (on issues of unequal intensity) increases the likelihood that a minority may effectively control policy making. We link these ideas to social theory approaches to bicameralism and for the empirical study of legislatures.

Keywords Bicameralism · Representation · Majority rule · Supermajorities · Coalitions

1 Introduction: the effects of bicameralism

While The Calculus of Consent is truly a co-authored work, Chap. 16, “The Bicameral Legislature,” is one of the chapters which bear most clearly the hand of Gordon Tullock. Chapter 16 has another much less happy distinction. It is perhaps the most neglected of A much earlier version of this paper was presented at the Liberty Fund Conference on “The ‘Calculus of Consent’,” Santa Cruz, California, June 23–25, 1988. Partial support of this research came through SSHRC research grant #410-2007-2153 (Stanley Winer and Stephen Ferris, co-PIs). Grofman’s work was also supported by the Jack W. Pel tason (Bren Foundation) Endowed Chair, University of California, Irvine, and by the UCI Center for the Study of Democracy.

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all the original contributions of that remarkable volume. The *Calculus of Consent* is well remembered for such seminal ideas as (1) the two-stage model of social choice, and (2) the efficiency of logrolling in allowing for reconciling majority rule with differential intensity of preferences across different issues. But, as Wuffle (1986) once observed, no article is likely ever to be remembered for more than one idea, and no book (with the possible exception of Anthony Downs’s, *An Economic Theory of Democracy*) is likely to ever be remembered for more than two ideas. Thus, the ideas about bicameralism in Chapter 16 are too little known.

Chapter 16 is, in our view, of great importance, containing one of the more important insights in *The Calculus of Consent*, namely that the effects of bicameralism on representation are primarily a function of the degree of overlapping of the “interest-group coalitions in each house” (Buchanan and Tullock 1962: 235). More specifically, *ceteris paribus*, while a minority of voters who comprise a majority of the voters in a majority of the constituencies can control a chamber, the greater the difference in the bases of representation in the two houses, the less likely is any given coalition of voters going to be able to control a majority of the seats in both chambers. Moreover, Buchanan and Tullock also note that, when cross-chamber logrolling is possible (on issues of unequal intensity), it is easier for a coalition controlling less than a majority of voters to still be able to control a majority of legislative seats in both chambers, thus linking bicameralism to the critical discussion in B&T of the normative aspects of logrolling.

The aim of this essay is threefold:

First and foremost we review the main theoretical results of the discussion of bicameralism in Chap. 16 and reformulate some of them in a more precise algebraic notation. Also, we derive a further result that, under plausible assumptions about the overlap among the winning coalitions in each chamber, the greater the size of the super majority required for passage in each chamber the greater the efficiency in Buchanan and Tullock cost-benefit terms of a bicameral legislative as compared to a unicameral legislative.

Second, we look at the practical implications of those results for the operation of bicameralism in the United States. In particular, we look at five constituency characteristics that are politically important, and compare mean and median on these characteristics across districts in the U.S. House of Representatives and across states in the U.S. Senate. And, drawing on

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1While there are a number of public choice scholars who have cited to the chapter’s discussion of bicameralism (e.g., Crain and Tollison 1977; Grofman et al. 1991; Tsebelis and Money 1997; Diermeier and Myerson 1999; Borchering 2002; Brauminger 2003; Mueller 2002), B&T’s discussion of bicameralism is not cited in some of the more important theoretically or empirically oriented books on legislative behavior or representation written by political scientists (e.g., Sartori 1968: Chap. 8; Shugart and Carey 1992).

2The first tier involves the adoption of a constitution. The second level specifies the voting rule for each different domain of political decision-making choice. While constitutional choice requires unanimous consent, the constitution is also allowed to provide for future decision-making by $k$ of $N$ voting rules (for various $k < N$), with the choice of $k$ as reflecting a trade off between the relative costs/benefits of being able to allow actors to achieve ends they desire, on the one hand, and being able to prevent the imposition on them of costs to which they did not consent, on the other.

3Wuffle went on to suggest that most articles are remembered for fewer than one idea, and that even the count of ideas remembered from *An Economic Theory of Democracy* almost never exceeds three (convergence to the median voter position in two-party competition, rational ignorance, and information shortcuts).

4Chapter 16 is important not merely for its discussion of bicameralism, *per se*. It treats bicameralism as part of an integrated discussion of veto, veto override, and committee games in which we are asked to look not only at the rules for electing members of a legislature but also at the rules for making decisions within a legislature. For space reasons we will not discuss these other more general topics (see brief discussion in Grofman 2000).
Our own earlier work (Grofman et al. 1991; Brunell 1999), we link these findings to empirical results about changing patterns of the relative policy liberalism of the House and the Senate, and to differences in the party composition of the two chambers.

Third, we link the theory in Chap. 16 to recent work in spatial social choice on bicameral voting. If constituencies in the two chambers do not come from identical distributions (over, say, a two-dimensional issue space), then majority rule instability that is manifest within each chamber taken separately, may not be found when we consider the bicameral voting game. In effect, bicameralism functions in a way that is analogous to super-majoritarian decision-making, and creates a set of stable (undominated) outcomes.

2 The central arguments in Chap. 16 of the calculus of consent

2.1 The calculus of consent and the discussions in the Federalist Papers about bicameral legislatures

We begin our discussion of the fundamental insights in Chapter 16 by comparing the Buchanan and Tullock approach to that in Federalist # 62.

Buchanan and Tullock note (1962: 236) that: “if the bases of representation can be made significantly different in the two houses, the institution of the bicameral legislature may prove to be an effective means of securing a substantial reduction in the expected external costs of collective action without incurring as much added decision costs as a more inclusive rule would involve in a single house.” In Federalist # 62 Madison downplays the possibility that the House and the Senate would agree on legislation that would be injurious to the public welfare. The basic logic is that “... the improbability of sinister combinations will be in proportion to the dissimilarity in the genius of the two bodies ....” Certainly, this sounds remarkably like the Buchanan and Tullock claim that the external costs of sub-majority coalitions (factions) will be reduced in a bicameral legislature to the extent that the bases of representation in the two houses are different since this increases the size of the necessary winning coalition. However, we must be cautious in directly equating Madison’s views with those of Buchanan and Tullock, even though they are certainly closely related.

Buchanan and Tullock (1962: 236) assert that because “the two-house system will involve considerably higher decision-making costs than the single-house system, given the same rules for choice under each alternative,” then “unless the two-house system is expected to produce some offsetting reduction in external costs, there is little reason for its rational support.” Or, to put it another way, in terms of the cost-benefit framework used by Buchanan and Tullock, “unless the bases of representation are significantly different in the two houses, there would seem to be little excuse for the two-house system. In contrast, the Federalist Papers provide a much broader set of justifications for bicameralism than does The Calculus of Consent.

In the Federalist Papers, a number of differences in the “genius” of the two houses are identified—not just differences in representational base—including the fact that one house was popularly elected and the other indirectly elected, the longer term length of the upper house (intended to insulate it for immediate electoral pressures and permit its members time for deliberation and the opportunity for considering a long-run time horizon), the greater eligibility requirements for the upper chamber, and the greater size of the lower chamber (intended from the inception, because of its status as the “popular” chamber). Moreover, for the Federalist authors, the justification for bicameralism is rooted in a notion of divided and balanced government, in which a golden mean is sought between monarchic (energy,
secrecy, unity of command and control), aristocratic (enlightenment, reflection, historical perspective), and democratic (responsiveness to the popular will) elements.

Madison states, the rationale for bicameralism in the *Federalist* as follows:

“In Republican government the legislative authority necessarily predominates. The remedy for this inconvenience is to divide the legislature into different branches: and to render them by different modes of election and different principles of action, as little connected with each other as the nature of their common functions and their common dependence on the society will admit.”

Other arguments for bicameralism in the *Federalist Papers* show how certain specific features of the Senate, e.g., its longer term, staggered elections, and its mode of indirect election, contribute to the likelihood that Senate members will be wiser and more mature than House members and that the Senate will, in the famous phrase, act as “the saucer to the House’s cup”, cooling the “passions” of the more “popular” body by interposing a period of calm and delay. In the next section we restate and clarify the central argument in Buchanan and Tullock (1962) on the link between the differences in the bases of representation and the likelihood of inter-chamber agreement.

In the succeeding section we will look at the connections between Buchanan and Tullock’s work on bicameralism and the recent literature on supramajoritarian, weighted, and compound voting games in the spatial context, especially Hammond and Miller (1987), Miller et al. (1996), and the Tsebelis and Money (1997) on the stability properties of bicameralism. In the final section we will look at recent empirical work on the U.S. House and Senate. There we will focus on ways in which the Buchanan and Tullock perspective on differences in the bases of representation between the two houses of the U.S. Congress might be operationalized, and what its implications are for partisan and policy differences between the chambers.

### 2.2 The basic model

Buchanan and Tullock consider two polar bases of interest group representation in the two chambers: in the first, there is “complete diversity.” In Buchanan and Tullock’s (1962: 237) words “The only requirement for complete diversity is that the members of a constituency of a representative in one house be distributed evenly over all of the constituencies for the other house.” We translate this as requiring that the distribution of constituency characteristics be the same in the districts in each house and be independent of that in the other house. The other polar type is “complete identity” in the constituency basis of the two houses; i.e., every district in each chamber is mirrored by a proportional number of districts of the same constituency characteristics in the other chamber. Buchanan and Tullock also consider cases intermediate between these two polar types.

A crosscutting type of classification scheme involves the nature of the set of issues that are being considered. Here the two polar types are “equal intensity” issues, where the majority and the minority are equally intense in their concerns about the bill’s passage or defeat,

5 Of course, once members of the U.S. Senate were no longer elected by state legislatures but directly elected by the people of their state, the differences in motivation and accountability between House members and Senators lessened, but these other differences do remain.

6 See also Hammond and Miller (HamMil1989).

7 A third body of potentially relevant literature, that dealing with the relative power of the two houses and the power of the legislature vis-à-vis other branches of government (see e.g., Brams 1989) takes us beyond the scope of this essay.
Table 1  Size of potential winning majorities in chambers using simple majority

<table>
<thead>
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<th>Equal intensity</th>
<th>Unequal intensity (logrolling possible)</th>
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<td>Unicameral legislature</td>
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<tr>
<td>Identical constituencies in the two chambers</td>
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<tr>
<td>Maximally divergent constituencies in the two chambers</td>
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and “unequal intensity” issues, the most important case of which is when the minority is more intense than the majority. (If the majority is the more intense, of course, the bill can be expected to pass.)

2.3 Simple majority

We show in Table 1 Buchanan and Tullock’s principal conclusions about the implications for representation of each of the six cells generated by this $3 \times 2$ typology, under the assumptions that each chamber uses simple majority rule and that there are a very large number of voters.

We see from Table 1 that, with simple majority voting in each chamber, the proportion of voters needed to control both chambers will range from $1/4$ to $1/2$ within a single chamber. The size of winning voter coalitions also ranges from $1/4$ to $1/2$. We also see from Table 1 that, ceteris paribus, unicameral and bicameral chambers will differ only for issues of unequal intensity.

When we look at the unequal intensity case, in a single chamber legislature, a bare majority of voters in a bare majority of constituencies would be sufficient to logroll a winning coalition. Thus, in the first row of the last column, we have shown $1/4$. For the case shown in the second row of the last column, where the bases of representation in the two chambers are identical, again $1/4$ of the voters could, in principle, enact legislation of their choice.

In contrast, in the third row of the last column, where the basis of representation in the two chambers is maximally diversified, roughly $7/16$ of the voters would, on average, be in a winning coalition in this cell. The key argument is that “the agreement finally reached will represent the minimum number of voters required to form that effective coalition which involves a minimum of bargaining costs”), and thus the expected coalition would not be as small as $1/4$ “even on the assumption of fully rational behavior on the part of all members” (1962: 240).

The argument has to do with the threat power of members of a barely winning coalition, who would seek to extort excess gains to preserve their membership in the coalition, given that to replace them one would either need to find some other voter who was pivotal in the same House and Senate district, or to find two voters, one pivotal in the House district and one pivotal in the Senate district. If we assumed that those pivotal in only one chamber would be half as expensive as those pivotal in two, then we may assume that the winning coalition will consist of the $1/4$ of the voters needed to control one chamber plus the $1/4$ of the voters needed to control the other chamber less the overlap among these two sets. If the two coalitions are unrelated, this will be given by

$$1/4 + 1/4 - (1/4 \times 1/4) = 7/16$$

(1)
The formula in (1) represents a situation in which, in a bare majority of districts in one house there is a bare majority of voters in the coalition, and the same is true for a bare majority of the voters in the other house, but there is “only a random overlap between the voters in the coalitions which control the majority in each house” (1962: 240; also see their Fig. 22 on p. 241). 8

2.4 Super-majoritarian decision-making

Although the discussion is rather elliptical, Buchanan and Tullock discuss the consequences for coalition formation of bicameral legislature supramajoritarian decision-making. The question becomes important in their view only for the case of unequal intensity issues. For unequal intensity issues, even a two-chamber legislature has an effective majority of voters needed for agreement which is less than 1/2, since only a majority of the voters in k of the N constituencies are needed to agree. For the unequal intensity case, Buchanan and Tullock (1962) argue that the effective majority in a single chambered legislature operating under a de jure decision rule of $j/n$ is only $j/2n$. Thus, in terms of voter coalitions, in logrolling around issues of unequal intensity, a one-chamber legislature using a 7/8 rule can be thought of as acting if it were using a 7/16 rule. On the other side of the coin, a bicameral legislature operating under a simple majority rule in each chamber requires the same proportion of voters to reach agreement (7/16) on unequal intensity issues as does a one-chamber legislature under a de jure 7/8 rule (cf. Table 1).

In the case of unequal intensity issues, a bicameral legislature with some degree of diversity in the bases of representation in its two chambers has a larger “effective majority” than does a single-chamber legislature. Thus, we can use bicameralism with majority rule in each chamber as an institutional tool to impose a requirement for increased voter agreement before unequal intensity issues can be passed, without at the same time paying the price of greater transaction costs within the legislature imposed by use of supramajoritarian decision rules. This is especially important since these transaction costs would have to be paid not merely to reduce the likelihood of passage of unequal intensity issues (likely to be high in external costs), but on all bills, including those which we might otherwise prefer to see passed by a simple majority.

Buchanan and Tullock make the important point that, in the mixed case, the costs of bargaining are lower than in the complete diversity case and, perhaps even more importantly, “this system greatly favors the voters who are arranged as to have the advantage of a sort of prefabricated bargain” i.e., voters who are the majority or near majority group in constituencies in both chambers. Writing before the passage of Baker v. Carr, they suggest for example, that “American farmers possess what amounts to a built-in coalition in the two

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8For equal intensity issues, logrolling will not occur, and thus: “Given that the electorate in each constituency is large and that there are quite a number of constituencies (which is the situation in real life), it is highly likely that a majority of the constituencies will have a majority reflecting a majority of the whole electorate” (Buchanan and Tullock 1962: 243). Hence, for a one-chamber legislature, for equal intensity issues, the expected proportion of voters in a winning coalition is 1/2, as is shown in parentheses in cells (1)–(3) in Table 1. If we posit diversity in constituency base little changes. “Again, if the number of voters is very large and the number of constituencies quite large, the laws of combinations and permutations would result in a majority of constituencies in both houses being in agreement with the majority of the whole population. . . . Cases in which the voters were distributed in such a way that they failed of a majority in one house or the other would be . . . relatively uncommon” (at p. 243). Thus, we have treated row one (cells (1)–(3)) as one in which the expected proportion of voters in the winning coalition is only trivially greater than 1/2.
Public Choice

houses of our legislature. This gives them a great advantage over less fortunately situated groups (1962: 246).\footnote{Buchanan and Tullock (1962: 248) also briefly discuss the advantage to groups whose constituency size is smaller than average, e.g., voters from small states.}

Buchanan and Tullock (1962: 244) point out that “(t)he advantage gained by the use of the two-house legislature... is rather dissipated by the simple majority method of voting” On the same page they go on to suggest that “departures from the simple majority voting rule, however, can improve the situation.” We shall make this suggestion more precise by generalizing the 7/16 result for simple majority rule bicameralism in the unequal intensity case given above to the case of any \(j/n\) rule with \(j \geq (n + 1)/2\).

For large \(n\), for a \(j/n\) rule, and for legislatures using simple majority, (1) generalizes to

\[
(j/n) - ((j/n)/2)^2
\]

Note that the limiting value of (2) is 3/4.

If \(j/n\) is 3/4 in each house, for example, then in a bicameral legislature, under the specified assumptions, if a bare majority of voters control each district in each chamber, but there is only random overlap among the voters in the winning coalition in each house, the effective majority needed for logrolling is \(3/4 - 9/64 = 39/64 (= 0.61)\).\footnote{This result is compatible with the numerical calculations on p. 242 of Calculus, but Buchanan and Tullock do not provide any derivation of the results they give.}

For the Buchanan and Tullock model, something like a decision rule of slightly over 0.58 in each chamber will give us a situation in which, under not unreasonable assumptions about voter distribution, exactly a bare majority of voters is needed for successful logrolling on unequal intensity issues, since\footnote{Recall that, were voters to be perfectly distributed for the purpose of minimizing the needed coalition size, we know that, for unequal intensity issues, \((j/n)/2\) is the minimum number of voters needed to control both chambers if \(j/n\) is the decision rule in each chamber.}

\[
(0.58) - (0.29)^2 \approx 0.50
\]

There are three critical points in the above analyses.

First, the two-chamber legislature differs from the single-chamber legislature in terms of the effective majority involved in the passage of legislation only in the case of unequal intensity issues (regardless of whether the voting is simple majority or supramajoritarian).

Second, “(e)ven in the two-house legislature the intense minority can pass its measures with less popular support than can an equal-intensity majority” (Buchanan and Tullock 1962: 244).

Third, for unequal intensity issues, for \(j/n\) decision rules, for bicameral legislatures and maximum diversity in the constituency bases of the two houses, we can have effective majorities ranging from slightly under a majority (7/16, for \(j/n = 1/2\)) to well over a majority (3/4, for \(j/n = 1\)). Furthermore, we can express the relative efficiency for a given decision rule \((j/n)\), of a bicameral legislature as opposed to a unicameral one, in Buchanan and Tullock cost-benefit terms, as the difference between the value in (2) and \((j/n)/2\). The latter is the size of the voter coalition needed to control a single chamber legislature; the former is the size of the likely voter coalition needed to control a bicameral legislature. This difference is given by (3).

\[
(j/n) - ((j/n)/2)^2 - (j/n)^2 = (4jn - 5j)/4n^2
\]
This function is maximized as $j/n \to 1$, and approaches a value of $1/4$. For $j/n = 1/2$, it equals $3/16$; for $j/n = 3/4$, it equals $15/64$.

3 Linking Buchanan and Tullock to recent social choice work on bicameralism: spatial majority rule voting games

One of the advantages of bicameralism touted by the *Federalist Papers* was that it insured deliberation. But deliberation would seem to imply the potential for change. A rather different feature of bicameralism that has recently been investigated by social choice theorists (esp. Hammond and Miller 1987, ?1989) is its potential, for a fixed set of preferences, to establish a structure-induced equilibrium consisting of alternatives (policy positions) that once in place cannot be dislodged, i.e., such that, if a member of this set is chosen, it will be difficult or impossible to replace it with some other alternative.

These positions are in the core in a bicameral majority rule voting game because, although there are alternatives that a majority of senators prefer to them and there are alternatives that a majority of House members prefer to them, there are no alternatives that a majority of senators and a majority of representatives prefer to them. This stability of a bicameral majority rule voting game stands in contrast to the disequilibrium (majority rule cycling) characteristics of majority rule spatial voting games within a single committee or other legislative body (McKelvey 1976, 1979; Riker 1982). This insight can be linked to the insights of Buchanan and Tullock on the importance of differences in the bases of representation in the chambers by showing how divergent policy preferences are critical to the creation of a bicameral core, as well as to ideas in the *Federalist Papers*.

In particular, Hammond and Miller (1986: 21) have proposed an alternative translation into contemporary public choice terminology of our earlier quote from Madison, that “...the improbability of sinister combinations will be in proportion to the dissimilarity in the genius of the two chambers.” They suggest it is equivalent to stating that, “when the ideal points of the two chambers are sufficiently separated from each other, there will be a core to the bicameral game.” We shall show how that translation is in fact simply another way of conceptualizing the central point about bicameralism made by Buchanan and Tullock, namely that, with diversity in the bases of representation in the two chambers, bicameralism increases the “effective majority” needed to act.

We can imagine a one dimensional alignment of the voters in each chamber. As we move along the line we can identify the median voter in chamber 1 and the median voter in chamber 2. Positions in between these two medians are invulnerable to defeat, since any proposal to change from an alternative on this line segment in either direction can be defeated in at least one chamber, and this is true even for the median of the other chamber. If we look at choices based on voters, and posit unidimensionality, we can identify the median voter in each constituency in each chamber and then the chamber median, i.e., the median

---

12In a single chamber legislature governed by simple majority any policy supported by the votes of one majority can usually be upset by a policy supported by a different majority.

13Hammond and Miller also look at the role of veto rules and legislative veto overrides in creating stable outcomes, but that takes us beyond the scope of the present essay. The Hammond and Miller work on the effects of institutional rules is an important contribution to the literature on what has come to be called *structure-induced equilibria* (Shepsle 1979; Shepsle and Weingast 1981; Denzau and Mackay 1983; Krehbiel 1988; Feld and Grofman 1988). However, Hammond and Miller provide no empirical analysis of actual bicameral legislatures, while the empirical analyses in Tsebelis and Money (1997) are limited to France.
voter in the median district. What is generally true in each chamber is that the median voter in the median district need not be the overall median; thus the median voter in the two chambers need not coincide. Only when both median voters line up on the same side of an issue (i.e., on a vote between some alternative and the status quo) will legislation pass. In effect, this requires a supermajority of voters to be in agreement.

We can extend these intuitions to the case where alternatives are embedded in some multidimensional space rather than being unidimensional in character. That games with veto players have a core is well known (Schofield et al. 1988). It is also well known that committee systems which structure one-issue-at-a-time decision making will also create a core (Black and Newing 1951; Shepsle 1979; Shepsle and Weingast 1981; Feld and Grofman 1988). Results in Hammond and Miller (1987; see also ?HamMil1989) have to do with the stability of bicameralism games among sets of legislators in two chambers.14

First we state a central theorem about bicameralism in Hammond and Miller and then show how it can be reconceptualized. We will adapt the notation and terminology in the review essay on spatial social choice by Feld and Grofman (1987). Following Hammond and Miller, we illustrate the analysis in a two-dimensional issue-space and assume that each actor has an ideal point in this two-dimensional space and decides among alternatives according to which is closer to his/her ideal point.15 Proofs have been omitted.

We begin with some key definitions.

**Definition** A **median line** for a legislative chamber is one in which the number of legislator ideal points lying on or to a given side of the line constitute a majority of the chamber’s members, and the same is true for the other side of the line.

**Definition** A **bicameral median line** for a bicameral legislature is a line which is a median line for both chambers, i.e., one in which the number of legislator ideal points lying on or to a given side of the line constitutes a majority of the members of each chamber, and the same is true for the other side of the line.

**Lemma** Two groups must have at least one median line in common and (except at knife-edge) must have an odd number of median lines in common.

**Lemma** If there is a bicameral median line, then the ideal point of at least one voter in each chamber must lie on the line.

**Theorem 1** (Hammond and Miller 1987, Theorem 2) If a bicameral legislature has a unique bicameral median line, and there exists a point on that line such that none of the median lines for one of the chambers intersect the line above that point, and none of the median lines for the other chamber intersect the line below that point, then there exists a core.16

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14 They also look at bicameral games with veto overrides but we will not consider this extension to tricameral games.

15 Two dimensions capture most of the important politics in many parliaments.

16 An immediate, but not particularly helpful corollary to this theorem is that, in a bicameral legislature, “if the Pareto sets of the two chambers do not intersect, then there is a core.”
In general, if any such point exists, there may be more than one such point, and the set of such points will form a line segment. Such a point or line segment is what Miller and Hammond refer to as a *bicameral contract core*.

**Definition** The *star-angles* defined by a set of voter ideal points are the angles defined by the intersection of pairs of extremal median lines (a.k.a., *limiting median lines*), i.e., median lines which mark the transition from one winning coalition to another. The *star-figure* defined by a set of voter ideal points is defined by the limiting median lines which intersect at each star angle.

The next result holds only for an odd number of voters.

**Theorem 2** In a bicameral game without a core among the combined set of actors (i.e., the set of legislators taken as a whole), if there is a bicameral (contract) core then such a core includes that portion of the bicameral median line that is between the star-figures for each group (i.e., not in either star-figure). See Fig. 1, showing a three-member Senate and a five-member House.

We should also note that if one chamber has a core, the bicameral game must also have a core. It is also easy to see that

**Lemma** The core of a composite voting game is the union of the cores in the component games.

If we now turn from the legislator level to the voter level, the connection between this result and the Buchanan and Tullock result is the recognition that the minimal rule for the bicameral game can be expected to be greater than simple majority; i.e., unless the voter
coalition is large enough to control more than a bare majority of seats in each house, it simply will not be large enough to overturn the status quo.

Moreover, as recognized by Hammond and Miller (1987), their results connect to earlier results (e.g., Greenberg 1979) on the size of supramajoritarian role needed to guarantee a core and the work of Kramer (1977) on the min-max rule, the minimum supramajoritarian decision rule that, if required for passage of a new bill, would make at least one alternative invulnerable to overthrow if it were the status quo. If the “effective majority” imposed by bicameralism and a specific arrangement of voter ideal points in two dimensions is greater than 2/3, a core is guaranteed.

McKelvey et al. (1980) have also shown conditions sufficient for a core. These conditions are weak, e.g., a min-max rule just barely greater than 1/2 in two dimensions, one just barely greater than \((w - 1)/w\) more generally, where \(w\) is the number of dimensions. Thus we are led to expect that bicameral games virtually always have a core—at least if there is any real heterogeneity in the distribution of ideal points of members, which in our terms translates simply as heterogeneity in the distribution of constituency characteristics as they affect overlap in constituencies. Tsebelis and Money (1997: 85-90) have shown that, even if there is not a unique bicameral median line then, in two dimensions, the area bounded by the two lines that are tangent to both chamber yolks\(^ {17}\) (one from above and one from below) contains the bicameral uncovered set.\(^ {18}\) Thus, if the yolks of the two chambers are small, the area between these two tangent lines serves as a kind of “fat” median line, and can play much the same role as a core.

4 Observed constituency differences between the U.S. senate and the U.S. house and their implications for representation and policy stability

Buchanan and Tullock’s theoretical treatment of bicameralism has had virtually no empirical follow-up in the nearly five decades since its publication. To get a handle on the actual effects of bicameralism in the U.S., we will compare the actual distributions of demographic characteristics of the constituencies in the U.S. House and Senate, and then compare the distribution of the scores of Congressmen and Senators from various states in terms of their mean/median ideological propensities as evidenced by measures of roll-call voting patterns such as ADA scores. Our aim is to better understand how the different bases of representation in the two chambers can be expected to impact on the size/likelihood of blocking coalitions in the bicameral voting game. Here we draw on early empirical work by Froman (?Fro1965), Kernell (?Ker1973), and various other authors.

One simple way to think about the basic point made by Buchanan and Tullock is that, if two chambers have a different representational base, a set of voters who together comprise a majority of the voters in a majority of the constituencies in one chamber will not, in general, comprise a majority of the voters in a majority of the constituencies in the other chamber. Thus, we would expect that some bills that pass one chamber will die in the other. This expectation is strongly confirmed.

\(^ {17}\) The yolk is the smallest circle tangent to all median lines (McKelvey 1986). For two alternatives \(s\) and \(y\), if \(y\) is further than 2 yolk radii from the center of the yolk than \(x\) is, then \(x\) is majority preferred to \(y\) (McKelvey 1986; Miller et al. 1989).

\(^ {18}\) The uncovered set can be thought of as a near core concept. An alternative is uncovered if it is majority preferred to every alternative either directly or at one remove (Feld et al., ?Feletal1988).
In each chamber well over 90% of all legislation introduced into Congress never makes it out of committee on to the floor, but bills which do get past committee tend to pass that chamber. In 1984, for example, about 80% of the bills on the House floor passed. Nonetheless, only roughly 50% of the bills that passed the House that year were enacted into law. Most of those bills were lost when they were referred to Senate committees and never heard of subsequently, although a few did die in conference or by presidential veto.19 In like manner, roughly half of all Senate bills die in the House.

Yet another obvious implication of the Buchanan and Tullock approach is that sometimes different parties (i.e., different coalitions of voters) will control each chamber. In fact, at the national level, from the 1930s until 1994, the House was thought to be inevitably Democrat, while the Senate was contestable and did change partisan control. Similarly, a remarkably high proportion of all states have divided legislatures, as high as 40% in some recent periods.

While Buchanan and Tullock are concerned to demonstrate that less than a majority of the voters may control both branches of a legislature when logrolling is possible, it is useful to turn to the other side of the ledger, the proportion of votes for passage we would expect on bills that pass both branches. If a bill passes one branch of the legislature with only a bare majority of votes, it is virtually certain that the voters who control the majority in the constituencies that supported the bill will not control a majority of the seats in the other chamber unless the two chambers have identical bases of representation. Thus, we would expect that bicameralism will create supraminimal coalitions in each house, since foresighted members will seek to put together a coalition that not only will win their own chamber but will constitute at least a (bare) majority in the other chamber as well.

To get a handle on how constituency differences between the U.S. House and the U.S. Senate can convert simple majority rule in each house into what is effectively super-majoritarian decision making, we examine the distribution across constituencies in the House and the Senate of demographic attributes such as percent black, percent non-white, percent of adults who have graduated high school, percent urban, and mean income—which are all important voter characteristics that differentiate constituencies and shape legislator preferences. There may be important differences between the two chambers in the mean value of these variables because the Senate is not apportioned on the basis of equal population. Moreover, there may be differences between the chambers even in the median values of important constituency variables because of “natural” geographic effects or because of gerrymandering in the House.20

We show in Table 2 the frequency distribution of these five attributes. There are several facts which strike one from Table 2. First, certain demographic groupings are underrepresented or overrepresented in the Senate as compared to the House. For example, the median state (unweighted) has consistently (over four decades) had a higher black proportion than the median House district, while the typical House district has been consistently more urban than the typical state. On the other hand, in characteristics like high school graduates, House-Senate differences seem minimal.

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19 Of course, we have to be careful how to interpret such data, since differences between chambers will, in an anticipatory fashion, affect the nature of the legislation that is introduced in each chamber.

20 A review of the gerrymandering literature would, however, take us too far afield from our present concerns. Here we simply note that one effect of gerrymandering is to skew distributions of voters in such a way that the median voter in the median constituency is not the same as the overall median voter, and the Senate is accordingly insulated from the deliberate manipulation of constituency difference effects since state boundary remain fixed. However, differences between the characteristics/preferences of the median voter in the median constituency may also occur because of “natural” geographic concentration effects.
Table 2 Means and medians for demographic and socioeconomic variables in the U.S. Senate and House (first value shown in the mean, the second is a median)

<table>
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<tbody>
<tr>
<td>Percent High School Graduate</td>
<td>41.8 53.3</td>
<td>67.9 76.7</td>
<td>40.6 52.6</td>
<td>67.1 76.4</td>
<td>40.6 51.8</td>
<td>66.4 74.9</td>
<td>40.6 51.8</td>
<td>66.4 74.9</td>
</tr>
<tr>
<td>Income</td>
<td>5568 9031</td>
<td>16677 28280</td>
<td>5630 9533</td>
<td>16572 28951</td>
<td>5377 9165</td>
<td>16640 29177</td>
<td>5108 9606</td>
<td>17114 30700</td>
</tr>
<tr>
<td>Percent Black</td>
<td>4.8 6.34</td>
<td>6.6 7.25</td>
<td>4.4 5</td>
<td>5.5 5.4</td>
<td>9 9.04</td>
<td>9.14 10.1</td>
<td>10.5 11.4</td>
<td>11.8</td>
</tr>
<tr>
<td>Percent Urban</td>
<td>62.4 66.3</td>
<td>67.1 68.8</td>
<td>68 74.4</td>
<td>77.1 79.3</td>
<td>61.5 65.8</td>
<td>67 67.6</td>
<td>69.5 73.4</td>
<td>73.7 75.2</td>
</tr>
<tr>
<td>Percent NonHispanic White</td>
<td>92.3 90.6</td>
<td>88.6 86.7</td>
<td>94.6 93.7</td>
<td>87.2 86.8</td>
<td>89.6 88.1</td>
<td>85.5 83.5</td>
<td>88.8 87.6</td>
<td>81.3 80.5</td>
</tr>
</tbody>
</table>

From a Downsian perspective (Downs ?Dow1957), information on the median rather than the mean constituency is what we want. We see from Table 2 that the median State is about 62%–68% urban (with the exact percentage depending upon decade); while the median House district is about 68%–79% urban, with the differences greater at present than in earlier periods. Thus, to create a winning coalition around urban issues in both chambers would be easier in the House than in the Senate. And Senators from the less urban states would need side-payments or some type of logroll to support a pro-urban bill.

This would suggest that the Senate should be less liberal than the House. However, with respect to ADA scores and related measures of liberalism, there is a considerable early literature in political science (with a seminal piece by Froman (?Fro1963); and an important follow-up by Jacobson 1973) that makes an argument that constituency distribution differences in characteristics such as urbanism and percent minority will make the Senate more liberal.

The intuition is that characteristics like percent non-white tend to have a more sharply skewed distribution in the House than in the Senate, i.e., more states have a non-trivial African-American proportion than is the case for House districts (Froman ?Fro1963; 80; Jacobson ?1971). However, while it is true that the difference between medians and means for blacks and Hispanics is greater in the House than in the Senate, mean-median differences between chambers in other categories are minimal to nonexistent. (See Table 2.)

From roughly 1960 until 1980 the Senate median member was more liberal than the House median member (Grofman et al. 1991), but when the GOP takes over the Senate in 1980 that chamber leapfrogs the House. After the 1986 election, though, the Senate has usually been just slightly more liberal than the House, but the differences are usually trivial. To try to explain the finding that, until recently, the Senate was noticeably more liberal than the House we need to take into account party composition.21

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21It is clear (McCubbins and Sullivan 1984; Grofman et al. 1991; cf. Fenno 1978; Bullock and Brady 1983) that it is not the median voter in the geographic constituency but rather the median voter in the representative’s electoral constituency (those who actually vote for him) that is critical in constraining a representative’s legislative policy stance. Given tendencies toward party-line voting in the electorate, this means that the representative’s party affiliation plays a critical intervening role.
We would expect that, the more Democratic the chamber, the more liberal it would be. In the Senate, in states that elect senators of opposite parties, the Democratic senator from the state is on average roughly 40 points more liberal in ADA terms than is the Republican (Grofman et al. 1991).  

Another interesting question about the effects of bicameralism is how the range of ideological positions is constrained or expanded by the nature of the number and location of constituency units, e.g., in the House as opposed to the Senate. We would expect that a statewide constituency narrows the range of ideological variation of representatives, as compared to the more numerous and less heterogeneous congressional districts within the state, especially once we control for party. But this difference is impacted by the way in which constituency boundaries are drawn. If constituencies are “arranged” to be homogeneous, there will be a substantial range of position reflected by the various legislators and conflicts among divergent views must be resolved within the legislature. In contrast, if constituencies are “arranged” to be heterogeneous, i.e., to mirror statewide characteristics, then we might expect that more representatives will all have relatively “centrist” views. Increasingly, constituencies in the House are drawn to concentrate supporters of each party, and, as the party support bases become more distinct as they have over the past several decades, we get a much greater range of variation in House constituency characteristics than we do for states.

We can get a good sense of the how the diversity issue plays within states, by comparing the voting patterns of House members within a state to the behavior of that state’s senators, controlling for party. Using data from 1960–2006, the mean absolute difference in DW-NOMINATE scores of senators of the same party from a given state is only 0.112 points (0.105 for Democratic senators and 0.122 for Republican senators).  

5 Conclusion

The simple point with which we wish to end this paper is that the theoretical analysis offered in Chap. 16 of The Calculus of Consent offers an as yet almost entirely unexplored goldmine of inspiration for empirical work. It is not just the U.S. Congress that is available to study; there are 49 states with bicameral legislatures!

References


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22 As noted in an earlier footnote, the “real” constituency of a representative is his election constituency and that can be expected to be non-centrist, especially if there is a bimodal distribution of ideology within the constituency.

23 During the same time period, the mean range in the DW-NOMINATE scores of Democratic congressional delegations within the same state is 0.316, while for Republican congressional delegations within a state the mean range is 0.259. Similar results obtain for other roll-call voting measures such as that generated by the Americans for Democratic Action (ADA). For example, in California, even two decades ago, before party differences had sharpened quite as much as they have at present, Democratic congressmen in 1987 had ADA scores ranging from 55 to 100, while Republican congressmen from that state had ADA scores ranging from 0 to 25.
Public Choice


