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Transforming reflectance spectra into Munsell color space by using prime colors

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Independent researchers have proved mathematically that, given a set of color-matching functions, there exists a unique set of three monochromatic spectral lights that optimizes luminous efficiency and color gamut. These lights are called prime colors. We present a method for transforming reflectance spectra into Munsell color space by using hypothetical absorbance curves based on Gaussian approximations of the prime colors and a simplified version of opponent process theory. The derived color appearance system is represented as a 3D color system that is qualitatively similar to a conceptual representation of the Munsell color system. We illustrate the application of the model and compare it with existing models by using reflectance spectra obtained from 1,269 Munsell color samples.

A quantitative model recently was presented by D'Andrade and Romney (1) for using the standard Stockman and Sharpe (2) human cone sensitivity functions to transform physical reflectance spectra of Munsell color chips into perceptual space as represented by the Munsell color system. Their transformations include quantitative estimates of the opponent process weights needed to transform cone activations (that involve a nonlinear cube root function) into Munsell color space coordinates. We will show that the use of hypothetical absorbance curves based on prime colors in place of human cone sensitivity functions leads to a model simpler than the one they propose.

Our quantitative model for the transformation of reflectance spectra into the perceptually based Munsell color space employs a simplified opponent process and requires three sets of data: (*i*) estimates of three prime color absorbance curves, (*ii*) Cartesian coordinates for the perceptual Munsell color space, and (*iii*) the reflectance spectra of 1,269 Munsell color chips. In what follows, we first describe the data sets and then present a theoretical model of color appearance and apply it to the reflectance spectra of the color chips. Comparisons with other color appearance systems, including the international standard that best fits the perceptual Munsell system, the Commission International de l'Eclairage (CIE) L*a*b* (3) system, then are presented.

Discussion

Prime Colors. Prime colors are the three wavelengths at which monochromatic lights maximize the tristimulus gamut (the range of visible colors) with minimal power consumption and optimal color balance. Thornton (4) summarizes his research (5, 6) that led to the empirical discovery of prime color wavelengths. Since then, it has been mathematically proved that (given a set of color -matching functions) three wavelength numbers uniquely identify the prime colors (7–10). Because a variety of color-matching functions exists, there is some minor difference in the exact wavelength numbers reported. For example, Brill *et al.* (ref. 8, p. 34) report "the modal wavelengths for the CIE 2° standard observer functions were found to be 447 nm, 541 nm, and 604 nm; and for the 10° standard observer, 446 nm, 538 nm, and 600 nm; modern data for six human observers show 450 nm, 533 nm, and 611 nm." Brill and Worthey (9) recently have made more precise calculations and report "Precise prime color sets have been calculated: (603, 538, 446) for the 2-degree observer, and (600, 536, 445) for the 10-degree observer." The modal wavelengths found by Hornæs *et al.* (10), based on the Stiles and Burch (11) 1959 color- matching functions, are reported to be 447.9, 536.5, and 600.0 nm.

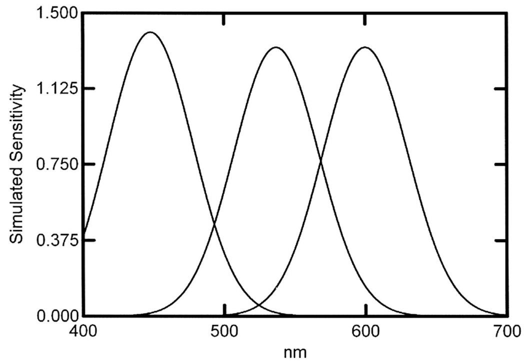

Fig. 1. The three simulated prime color sensitivity curves with peaks at 448, 537, and 600 nm and SDs of 30 nm normalized to have equal areas of 100.

For our hypothetical absorbance curves, we use Gaussian distributions with means at the prime peaks (rounded to the nearest nanometer) reported by Hornæs *et al.* (10): 448, 537, and 600 nm. The question arises as to the appropriate width of distribution, as measured by the SD, to use in the model. Whereas color-matching functions are based on additive color processes produced by adding monochromatic lights, the receptors of light reflected from surfaces are based on subtractive color processes. The receptors our hypothetical absorbance curves are modeling have negligible reflectivity but very high absorption over a limited range and are largely transparent outside of this range. They are characterized by rather wide distributions as, for example, in the case of the cone sensitivity curves in human vision. For our theoretical model, we have arbitrarily chosen to use distributions with a SD between that observed for the short- and medium-wavelength cone sensitivity curves as defined by Stockman and Sharpe (2). Our final simulated prime color curves consist of Gaussian distributions with means of 600, 537, and 448 nm and SDs of 30 nm on the interval of 400–700 nm. The final result of this procedure is shown in Fig. 1, where the curves are normalized to have equal areas, arbitrarily set at 100 to give all of the receptors equal effects.

The Munsell Perceptual Color System. The perception of the color of a single-color chip observed on a neutral background derives from the spectral characteristics of the light that stimulates the retina. In the case of such a physical chip, the appearance is given by the spectral intensity of the product of the object reflectance

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Abbreviation: CIE, Commission International de l'Eclairage.

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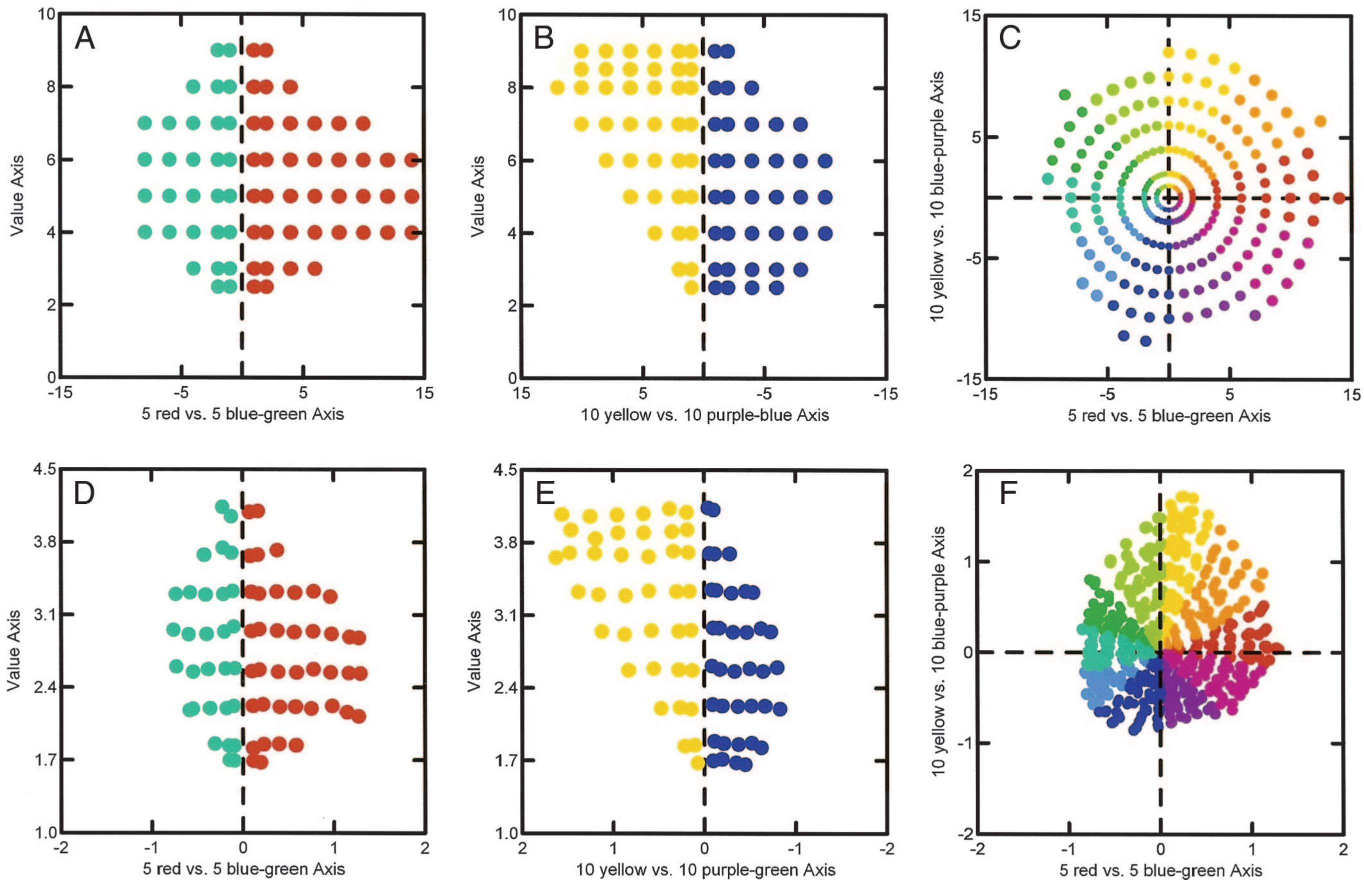

Fig. 2. Location of color chips in the conceptual Munsell color system (*A*–*C*) and the prime color model (*D*–*F*). Color coding is not accurate and is meant to indicate only general hue quality of the color of the chips with no indication of value or chroma.

and the source radiance. Note that even though the prime colors were derived from color-matching functions obtained from experiments by using monochromatic lights, in this paper, we generalize the application of the prime colors to reflectance spectra. We use reflectance spectra from a sample of 1,269 Munsell color chips as examples of surfaces that provide widely distributed stimuli with the structure described below. The Munsell system is a perception-based 3D system designed to present an arrangement of the full sample of colors in equal perceptual intervals. Perfected in studies spanning >100 years, it represents a reasonable and well known description of the structure of human color perception of a single chip on a neutral background (3, 12).

In the Munsell system, each color is described in terms of three attributes: value, hue, and chroma. The full set of color chips in the Munsell system may be represented as a somewhat irregular sphere as shown in Fig. 2 *A*–*C*. The colors in the plot, although arbitrary, are meant to reflect the major hue sectors named below. The true colors range from dark to light on the vertical value axis and from achromatic to higher chroma or saturation as color chips depart from the locus of the chroma circle. Viewed from above, as in Fig. 2*C*, the various hues are seen as 40 "spokes" radiating out from the achromatic central axis. The color samples increase in chroma or saturation with distance from the central axis (the scale values of chroma are 1, 2, 4, 6, 8, 10, 12, and 14) and, therefore, the circles of increasing size represent increasing values of chroma (samples with equal chroma values but differing in value or lightness are stacked on top of each other in Fig. 2*C*). The hues are divided into 10 equally spaced sectors labeled red, red-purple, purple, purple-blue, blue, blue-green, green, green-yellow, yellow, and yellow-red. The four spokes in each hue sector represent finer hue gradations, called areas in the Munsell system, labeled 2.5, 5, 7.5, and 10. Area 5 is usually the most representative hue. Fig. 2*A* shows a full plot of the two pages of the Munsell color system that constitute the spokes of the 5 red vs. 5 green-blue axis, whereas Fig. 2*B* shows those of the 10 yellow vs. 10 blue-purple axis. The central axis of value in Fig. 2 *A* and *B* is achromatic and runs in equal perceptual steps of lightness from black (*Lower*) to white (*Upper*) on a scale of 0 to 10. To plot the location of the conceptual Munsell color chips in Cartesian space, the 40 hues are positioned around the circumference of a circle, beginning with 5 red at 0° and moving clockwise in 9° steps for each of the other 39 hues. The position of each color chip then was calculated by standard trigonometric functions. This transformation from polar to Cartesian coordinates is outlined in more detail in D'Andrade and Romney (1).

The Reflectance Spectra of Munsell Color Chips. Examples of the reflectance spectra of colored surfaces from the atlas of Munsell (13) color chips (matte edition) are shown in Fig. 3. These spectra are a subset of the 1,269 spectra used to test the models presented later in the paper. The data were obtained from the web site www.it.lut.fi/ip/research/color/database/database.html. The eight panels of reflectance spectra represent eight equally spaced hues from the Munsell color circle. Each panel represents all of the chips from one of the atlas' pages. The four pages illustrated in Fig. 2 *A* and *B* appear in Fig. 3. All chips on a given page have identical hues but differ in value (lightness) and chroma (saturation); value varies directly with the mean

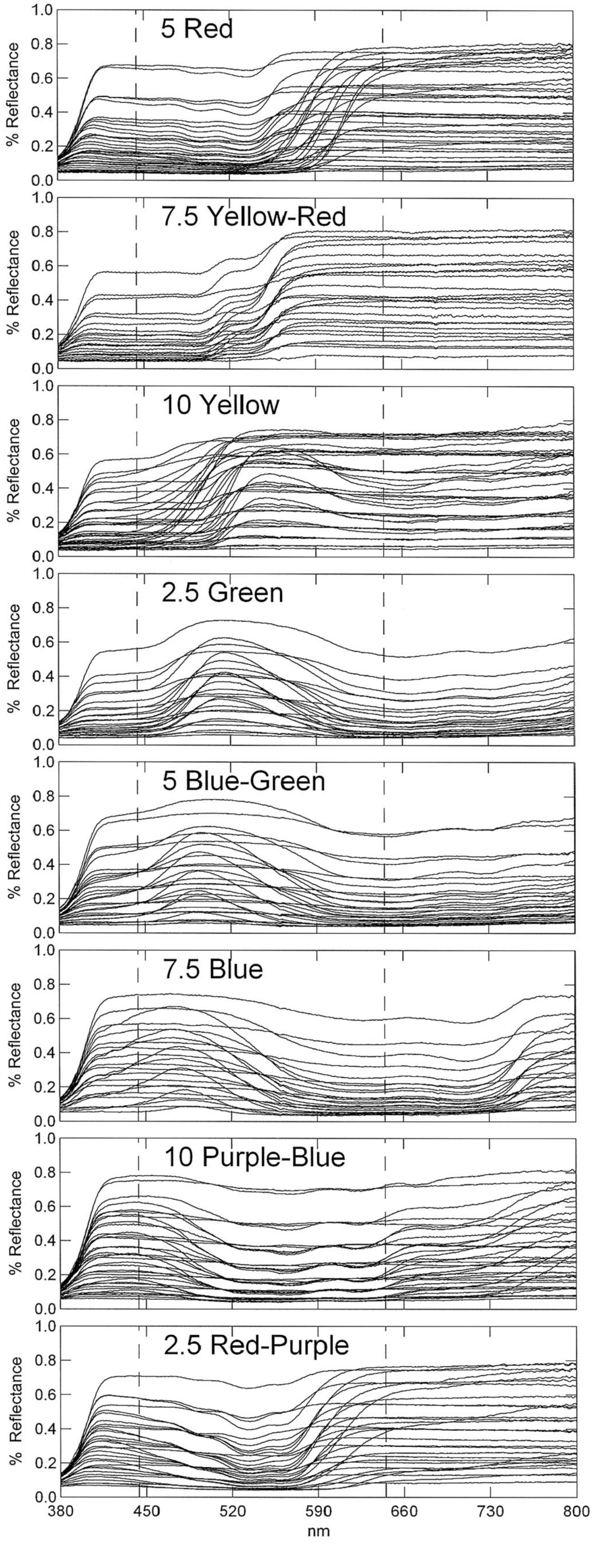

Fig. 3. The reflectance spectra of eight equally spaced pages of the Munsell atlas.

height of the spectra, whereas chroma varies with the "peakedness" of the curves, which is best seen in the panel for 2.5 green, where within each set of curves of equal means the spectra of various degrees of chroma display increasing peakedness.

Maloney (14) observed that, unlike spectral lights (which have a gap in the purple area), reflectance spectra vary in a smooth, continuous fashion around the color circle. Romney and Indow (ref. 15, p. 187) demonstrate the color circle very clearly with a diagram showing all 40 reflectance spectra with value 8 and chroma 6 that go continuously around the color circle. The Munsell system attempts to arrange the hues in equal perceptual steps and, therefore, the panels in Fig. 3 should be equally perceptually spaced. The physical differences between the panels appear to be about equally spaced, and the last panel and the first panel show the same degree of similarity as adjacent panels.

With some exceptions in the blue and purple panels, all of the spectra are similar outside the vertical dashed lines. These lines represent the shortest (444 nm) and longest (645 nm) wavelengths of monochromatic primaries used in the Stiles and Burch (11) 1959 experiments from which the prime colors were derived. It is obvious that the main information in reflectance spectra for distinguishing one color from another is contained within the boundaries of these lines. Research likely would reveal the optimal range to use when reconstructing color appearance from reflectance spectra. In the meantime, we will use the traditionally accepted range of 400–700 nm.

A final important characteristic of all of these spectral curves is that they are broad-band and characterized by a single well defined peak or depression. The import of this broad-band characteristic is that all ordinary surfaces reflect at least some light from all segments of the visual spectrum.

Our task in this article is to construct a model that will permit the calculation of the appearance of the Munsell painted surfaces (viewed individually in normal daylight on a neutral gray background) from the simulated prime color sensitivity curves shown in Fig. 1. This task may be viewed as asking how a color visual system might sense and transform physical reflectance spectra like those shown in Fig. 3 into the perceptual Munsell system (Fig. 2 *A–C*). The solution suggested in this paper is an opponent process model that specifies the details of implementing the transformation of the reflectance spectra into the Munsell color system by using simulated prime color curves.

Models

A Model for Transforming Reflectance Spectra into Munsell Color Locations. The first stage of our model is characterized by the following assumptions regarding the simulated prime color wavelength receptors: (*i*) there are three different types of simulated receptors that may be modeled with Gaussian sensitivity curves as shown in Fig. 1, and (*ii*) the output of each individual simulated receptor is independently adapted to approximately the mean level of ambient illumination from which variations in intensity can be calculated by a mechanism analogous to a common transistor circuit. These assumptions are similar to those made by Schrödinger (16) with respect to color-mixture curves derived from lights. He assumed independence among curves and assigned equal areas to each. Rinner and Gergenfurtner (17) present experimental evidence that such adaptation takes place in the human visual system within 25 ms. The assumption that each simulated receptor is independently adapted to a different segment of wavelengths should prove to be useful in modeling color constancy effects and eliminating the need to multiply the reflectance spectra by the spectra of the illumination.

In the second stage of the model, the signals from the individual simulated receptors are aggregated through opponent processes corresponding to the three separate axes that constitute the Cartesian coordinates of the Munsell color appearance

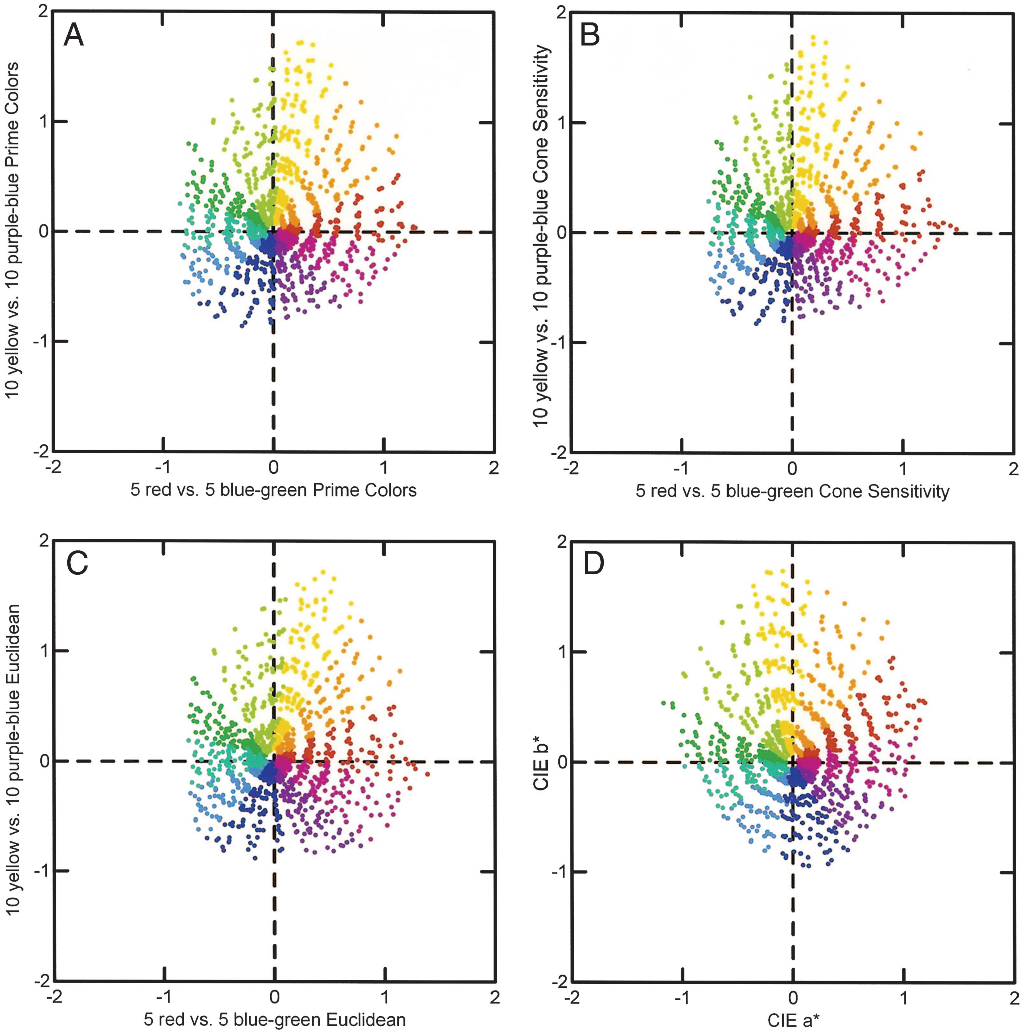

Fig. 4. Comparison of four models. (*A*) The prime color model. (*B*) The cone sensitivity model. (*C*) The Euclidean model. (*D*) The CIE L*a*b* model.

system. The first axis, the 5 red vs. 5 green-blue, is the difference between the response outputs (defined below) of the simulated long- and medium-wavelength receptors. The second axis, the 10 yellow vs. 10 blue-purple, is the difference between response outputs of the simulated short- and medium-wavelength receptors. The third axis, value, is the mean response output of the sum of one unit of the simulated long-wavelength receptor and two units of the simulated medium-wavelength receptor.

Application of the Model. The application of the model to the 1,269 reflectance spectra of the Munsell color atlas (13), the same set of reflectance data used in refs. 1, 15, and 18, is as follows: We represent the Munsell reflectance spectra as a matrix, $S_{1269\times301}$, measured on 1,269 chips at 1-nm steps from 400 nm to 700 nm. We represent the normalized absorbance spectra of the three receptors shown in Fig. 1 as a matrix, $Q_{301\times3}$. By simple matrix multiplication we obtain estimates of the receptor sums, $E_{1269\times3}$, as follows:

$$E_{1269\times3} = \{E_{\mathrm{l}}, E_{\mathrm{m}}, E_{\mathrm{s}}\} = S_{1269\times301}Q_{301\times3}, \quad [1]$$

where E_{l} is long wavelength, E_{m} is medium wavelength, and E_{s} is short wavelength. This operation is equivalent to summing the product of the chip reflectance spectra and each receptor value across all 301 wavelengths (400–700 nm).

The response outputs of the receptors is defined as the cube root of $E_{1269\times3}$. The cube root function has been found to be a convenient approximation for fitting reflectance spectra to the perceptual Munsell system (1), to analyze cone sensitivities and lateral geniculate nucleus responses (19), and is used in the CIE 1976 L*a*b* equations (3). The Cartesian coordinates for the three axes posited above may be obtained from the following equations:

$$\text{Yellow vs. blue-purple axis} = \sqrt[3]{E_{\mathrm{m}}} - \sqrt[3]{E_{\mathrm{s}}}, \quad [2]$$

$$\text{Red vs. green-blue axis} = \sqrt[3]{E_{\mathrm{l}}} - \sqrt[3]{E_{\mathrm{m}}}, \quad [3]$$

$$\text{Value axis} = (\sqrt[3]{E_{\mathrm{l}}} + (2\times\sqrt[3]{E_{\mathrm{m}}}))/3. \quad [4]$$

With no further adjustment (other than rescaling), these axes define locations in the Munsell perceptual color space derived from the simulated prime color sensitivity functions. The results of the calculations, without any rescaling, are plotted in Fig. 2 *D–F* for comparison with the conceptual Munsell color system plotted in Fig. 2 *A–C*. The model specifies that the calculations should place the reflectance spectra according to the perceptual structure of the color samples from which they were obtained. As can be seen, the overall pattern is remarkably similar to the conceptual Munsell structure. The achromatic point is the zero intersection point of the red vs. green-blue and yellow vs.

Table 1. Stewart–Love redundancy index among four models

From	To	Index
Prime colors	L*a*b*	0.9932
Cone sensitivity	L*a*b*	0.9893
Euclidean	L*a*b*	0.9906
Prime colors	Euclidean	0.9893
Cone sensitivity	Euclidean	0.9870
L*a*b*	Euclidean	0.9906
Prime colors	Cone sensitivity	0.9996
L*a*b*	Cone sensitivity	0.9997
Euclidean	Cone sensitivity	0.9983
L*a*b*	Prime colors	0.9989
Cone sensitivity	Prime colors	0.9993
Euclidean	Prime colors	0.9983
Mean		0.9945

blue-purple axes. Fig. 2 shows that the model derived from the prime color functions corresponds closely to the perceptual structure represented by the Munsell system.

An advantage of the model is that it suggests a solution to the problem of global color constancy. Color constancy is the well known perceptual phenomenon in which objects retain a constant color appearance under fairly large changes in the spectral composition of the illumination. By "global color constancy," we mean the situation in which the illumination at the retina is in the photopic range and the saccadic range of the eyes is limited to the field of interest. Although the spectral composition of the source may change slowly over time, the viewing field, including the test color chip and its neutral background, are illuminated by the same source during a given comparison. The specification in our model that the simulated receptors sensitive to different wavelengths adapt independently provides a mechanism that maintains color constancy over a large spectral range of changes in illumination. If, for example, we doubled the amount of long wavelength illumination (say, with a fully diffused red light) uniformly for the color chip and its neutral background, then as the simulated long-wavelength receptor operates independently, it would adjust to the new mean level, maintaining a constant apparent color of the test color chip.

A Comparison with Other Models. A natural question is how the prime color model performs relative to other models. We compare three models with the prime color model shown in Fig. 2 *D–F*. The first, the one recently proposed by D'Andrade and Romney (1), will be called the cone sensitivity model. The cone sensitivity curves constitute the current standard for color vision and are derived from linear combinations of the Stiles and Burch (11) color-matching functions. (These are the same color-matching functions used by Hornæs *et al.* (10) to derive the prime colors.) For the cone sensitivity model, we recalculated the D'Andrade and Romney model (without D65 illumination) by using the reflectance spectra for the total 1,269 Munsell color sample (D'Andrade and Romney used a different sample). The cone sensitivity model involves taking the cube root of the cone sums and then adjusting for opponent process weights. A final empirical estimation of nine parameters is required to fit the model to the conceptual Munsell system.

The second model, which we call the Euclidean model, is a modification of the Euclidean representation of the reflectance spectra presented in Romney and Indow (18). Romney and Indow demonstrated that a singular value decomposition of a matrix such as $S_{1269\times301}$ from our Eq. **1** results in a Euclidean representation of the physical relation among Munsell chips. For the comparison presented here, we modify their procedure by taking the cube root of each value in the matrix before calculating the singular value decomposition. Such a model is based solely on the physical reflectance spectra.

Table 2. Stewart–Love redundancy index from models to conceptual Munsell system

From	To	Index
Prime colors	Munsell	0.9810
L*a*b*	Munsell	0.9832
Cone sensitivity	Munsell	0.9784
Euclidean	Munsell	0.9769
Mean		0.9799

The third model is the CIE L*a*b* color (3) system, an internationally recognized standard that is qualitatively similar to the Munsell color appearance system. L*a*b* coordinates were reported along with the Munsell reflectance spectra in the web site referred to earlier. Because these calculations were made in Finland, they provide a source of color appearance data independent of the results computed above. The L*a*b* system involves extensive computations based on the Stiles and Burch (11) color-matching functions and a cube root transformation. In contrast to the other models, the CIE specification for L*a*b* multiplies the reflectance spectra by a standard illuminant (D65 or A).

Including the model derived from the prime colors, we have four sets of 1,269 3D coordinates representing the color appearance calculations involving the Munsell color chips. Fig. 4 presents a visual representation of the four sets of chromaticity coordinates for qualitative examination. The prime color model and the L*a*b* coordinates are plotted in their natural orientations (the L*a*b* coordinates were originally reported on a scale of 1–100 and have been divided by 45 for ease of comparison). The cone sensitivity and Euclidean models were rotated and rescaled to be in the same orientation as the prime color model. In examining the figure, the slightly different orientation of the L*a*b* plot needs to be taken into account.

In addition to the qualitative results shown in Fig. 4, we present a quantitative index of similarity among the models. The measure we use for the similarity between any two sets of variables is the Stewart–Love redundancy index (20), a measure of the proportion of variance accounted for in one set of variables by another set (a multivariate version of r^2 between two individual variables). Because the Stewart–Love index is invariant under linear transformations, it is unaffected by the rotations made to the cone sensitivity and Euclidean models in the plots of Fig. 4. Because the index is an asymmetric measure, there are 12 pairwise comparisons among the four sets of coordinates. The results of this calculation (Table 1) converge on a single configuration. The fact that the four models give comparable results provides critical confirmation of valid measurement and description. Of interest is the fact that the Euclidean model of the cube root of the reflectance spectra provides virtually identical information as the models based on psychophysically derived color matching functions. On the whole, the mean value of 0.9945 for the redundancy index seems a reasonable summary.

Table 2 contains a summary of how well each of the four models predicts the Munsell conceptual model according to the Stewart–Love redundancy index. These values, although quite high, are all lower than any value in Table 1. We interpret the results to indicate that the models are more similar to each other than to the Munsell conceptual system. We assume that the difference between the four models and the conceptual Munsell system is due to a variety of errors in the implementation and manufacturing of the chips. The fact that the four models are virtually identical qualitatively, including the details of the compression of the blue chroma levels visible in Fig. 2*E* and Fig. 4, is evidence that the chips vary systematically from the ideal Munsell. D'Andrade and Romney (1) noted the same qualitative deviations and cited evidence that the compression was probably an error in the implementation of the Munsell color atlas.

Summary

The economy of the opponent process implied by the prime color model as expressed in Eqs. **1–4** is an advantage over opponent process models dependent on a larger number of estimated parameters and more extensive algebraic manipulations. The mathematical derivation of prime colors from the color matching functions of Stiles and Burch (11) provides peak locations in the visible spectra for simulating receptors that allow a model for the color appearance of single-color chips on a neutral background as represented in the perceptually based Munsell color system. The model corresponds closely to three comparison models but is more economical and provides a possible mechanism for color constancy of single-color chips on a neutral background.

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