What Common Ground Exists for Descriptive, Prescriptive, and Normative Utility Theories?

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Descriptive and normative modeling of decision making under risk and uncertainty have grown apart over the past decade. Psychological models attempt to accommodate the numerous violations of rationality axioms, including independence and transitivity. Meanwhile, normatively oriented decision analysts continue to insist on the applied usefulness of the subjective expected utility (SEU) model. As this gap has widened, two facts have remained largely unobserved. First, most people in real situations attempt to behave in accord with the most basic rationality principles, even though they are likely to fail in more complex situations. Second, the SEU model is likely to provide consistent and rational answers to decision problems within a given problem structure, but may not be invariant across structures. Thus, people may be more rational than the psychological literature gives them credit for, and applications of the SEU model may be susceptible to some violations of invariance principles.

This paper attempts to search out the common ground between the normative, descriptive, and prescriptive modeling by exploring three types of axioms concerning structural rationality, preference rationality, and quasi-rationality. Normatively the first two are mandatory and the last, suspect. Descriptively, all have been questioned, but often the inferences involved have confounded preference and structural rationality. We propose a prescriptive view that entails full compliance with preference rationality, modifications of structural rationality, and acceptance of quasi-rationality to the extent of granting a primary role to the status quo and the decomposition of decision problems into gains and losses.
(*Decision Analysis; Prescriptive Utility; Rank-Dependent Utility; Sign-Dependent Utility*)

1. Introduction

Almost 40 years of experience with the subjective expected utility (SEU) model (Fishburn 1982a; Savage 1954) has led to two firm conclusions. One, stressed primarily by psychologists, is that the model fails to describe behavior in many experimental and real-world situations. (For good surveys, see Camerer and Weber, 1993, and Schoemaker, 1982.) The other, stressed primarily by economists and decision analysts, is that the axioms of the SEU model are normatively appealing and that the model is useful in many applied situations.

Recall, the SEU model asserts the existence of a utility function over consequences and gambles[1] and a (subjective) probability distribution over partitions of a "universal" event such that preferences are ordered in the same way as the expected value of the utility function.

There is no doubt that SEU is descriptively wrong. In our view, the most damaging violations of the SEU

[1] We use, as a generic term, gamble to include both risky—a prescribed probability distribution—and uncertain—no prescribed distribution over events—alternatives.

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model so far encountered are that the decision context affects the apparent preference between two gambles and that some people may prefer something that is literally dominated by something else. (Detailed references are provided in §4.) The descriptive troubles of SEU do not stop there. SEU has been shown to be violated by status quo effects, by so-called isolation and cancellation effects, the purchase of probabilistic insurance and by an aversion against vague uncertainties (Ellsberg 1961; Kahneman and Tversky 1979). As a result of these descriptive failures of the SEU model, some theoreticians, in addition to the experimentalists, have begun to question the normative validity of the SEU model and have proposed generalizations (Fishburn 1988).

Even as the empirical evidence against the descriptive validity of SEU has mounted, the applied experience with the SEU model suggests both its acceptability by and value for decision makers. The SEU or, when the probabilities are prescribed, the EU model has been applied in many contexts, including medical decision problems (e.g., Pauker, 1976 and many other articles in the *Journal of Medical Decision Making*), investment and strategic business decisions (Hax and Wiig 1977), technology choice problems (Keeney et al. 1986), risk analysis (e.g., Merkhofer and Keeney 1987; and many articles in *Risk Analysis*). For summaries of applications, see Keeney and Raiffa (1976), von Winterfeldt and Edwards (1986) and Corner and Kirkwood (1991).

Very few decision analysts actually test the normative acceptability of the SEU model in the context of an applied problem. However, the general logic of the SEU model—separating the probability and utility part of a problem, encoding decision makers' preferences with utility functions over outcomes, encoding expert knowledge with probability distributions over consequences, and combining utilities and probabilities through expectation calculations—usually seems quite acceptable to most decision makers. Furthermore, the results of the modeling make sense to the decision makers, guide their thinking as well as action, and often provide new insights or generate dominating options. Recognizing the usefulness of decision analysis, some decision analysts have urged a return to the "old time religion" of SEU or, when applicable, EU theory (Edwards 1992, Howard 1992).

Recognizing the descriptive failure and the applied usefulness of the SEU model, this paper explores exactly what is the common ground held by both the experimentalists and the applied decision analysts. To do so, we find it useful to distinguish three major types of axioms: (1) structural rationality, which asserts the decision maker should be indifferent between formally equivalent descriptions of a gamble; (2) preference rationality, which is based on the principle that replacing something by something better is always desirable; and (3) quasi-rationality, which centers on partitioning consequences and gambles according to gains and losses. We will discuss three models plus data in the light of these axiom groups. Our findings are summarized in Table 1 of the conclusions section.

2. The Primitives for Binary Gambles

To make precise the various axioms of rationality and quasi-rationality, we need some notation. For much of the exposition, we restrict ourselves to simple binary gambles of the form "if event E occurs, you receive consequence x, and if its complement occurs, you receive consequence y." But what we have to say applies to general gambles with finitely many consequences.

Let $\mathcal{C}$ denote some set of pure consequences, for example all possible outcomes in a stock market venture. Let $\mathcal{E}$ denote an algebra of events, for example all possible events associated with changes in the stock market. A first-order gamble or lottery is an assignment of consequences to a finite partition of subevents. If the partition involves two events, the gamble is called binary.

Binary first order gambles can be notated in several ways. Suppose F is an event, such as a toss of a die, on which a gamble is based, and suppose E is a subevent of F. Then a binary gamble based on the partition $\{E, F\backslash E\}$ in which x is the consequence if E occurs and y when $\neg E = F\backslash E$ occurs can be denoted either $(x, E; y, F\backslash E)$ or $(x, E; y, \neg E)$. If the context makes clear what F is, one sometimes simply writes $(x, E; y)$ or uses the alternative mixture operator notation $x \circ_E y$. We will use both types of notation depending upon which we think will prove clearer. Second order gambles are generated by applying the operation $\circ_E$ to all first-order gambles as well as pure consequences, and higher-order gambles are defined recursively. Denote by $\mathcal{G}$ the set of pure consequences and all gambles recursively generated in this manner.

There is assumed to be a preference relation $\succsim$ over $\mathcal{G}$.

3. Structural Rationality

As was mentioned in the introduction, two types of rationality are distinguished, structural and preference. We take up the first here and the second in §4.

The question of what does and does not constitute rationality can be subtle. We take the not very courageous tack of saying that any property implied by the SEU representation is a form of rationality. Deeper arguments certainly are possible, but this is not really the place to go into them.

Trees and Normal Form

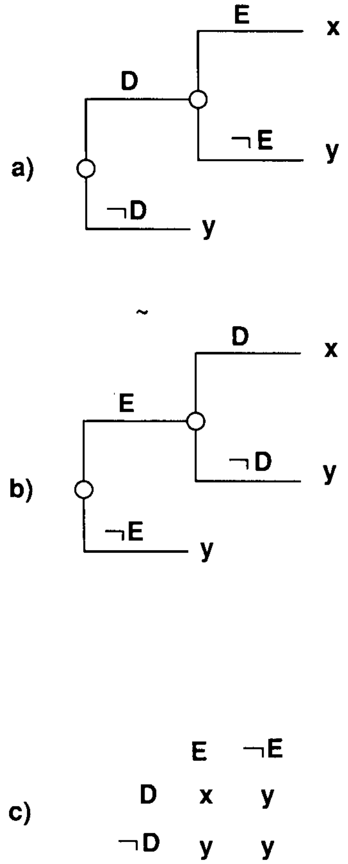
Both in theory and practice, the clearest way to represent first-order gambles is in the form of an event tree with consequences attached at the ends and probabilities, when they are prescribed, attached to the limbs. With prescribed probabilities of events and monetary consequences, this is obviously equivalent to providing a probability distribution. A higher-order gamble is a first-order gamble with at least one consequence that is itself a gamble. A first-order gamble may be formulated in many equivalent ways as higher order gambles. For example, the second order gamble $(x \circ_D y) \circ_E y$, in which the subevents D and E of F are independently realized, shown in Figure 1a should be treated as the same as the gamble $(x \circ_E y) \circ_D y$ shown in Figure 1b. The reason is that if we ignore the order of occurrence of the experiments that underlie events D and E, then both of these second-order gambles is equivalent to the first-order gamble in which x is the consequence if both D and E occur in separate realizations of the experiment and otherwise y is the consequence. Clearly, if the order in which the experiments occur matters to the subject or decision maker, these representations are no longer equivalent.

In risky contexts (i.e., with prescribed probabilities of events) the equivalence between a multistage gamble and its "multiplied down" normal form is often assumed. This asserted equivalence is composed of two further assumptions, namely, having two stages rather than one does not matter and probabilities multiply down in the usual fashion for independent events. Both assumptions can be questioned, certainly descriptively where the latter is not sustained (Bar-Hillel 1973, Starmer and Sugden 1991).

Such structural rationality assumptions are often referred to, in the risk literature, as "reduction of compound gambles" or "probabilistically equivalent" (Machina 1989) and, for uncertain consequences, as "accounting equivalences" (Luce 1988, 1990). As Tversky and Kahneman (1986) observed, these assumptions are often tacit, as when the choice domain is assumed to consist of probability distributions over money. Separating them from the assumptions about preference rationality is useful in gaining a better un-

Figure 1

An example of the structural condition of rationality known as event commutativity, Equation (3) below, that asserts that the gambles of panels (a) and (b) are seen as indifferent. Panel (c) shows them in tabular form.



derstanding of the failures of SEU. It is our general sense that the preference rationality axioms of §4 are much more likely to be descriptively valid than are many forms of structural rationality.

In some treatments of SEU, primarily by Savage (1954) and his followers, the entire problem is formulated in first-order terms by treating all alternatives as mappings from a fixed (large) set of states into a set of consequences. Applications of SEU are rarely cast this way. They usually take the form of a series of statements conditioned on one or another subevent occurring, in which case the issue of tree representations and equivalence arises.

There are somewhat similar, but very distinct, issues known as "consequentialism" (Hammond 1988a, b) and "dynamic consistency" (Machina 1989). Sarin (1992) and LaValle (1992) make related arguments. These authors consider decision trees in which the decision maker as well as chance makes decisions at various vertices other than the root. We do not treat such cases. Rather, we consider just those trees where, except for the root, only chance plays a role, which may be in a single or in multiple stages, and the question is whether the decision maker sees equivalent formulations as indifferent. Machina (1989) explicitly assumes all reductions of compound gambles and Hammond (1988a, b) assumes that when that is applicable and otherwise assumes what below we call universal accounting.

The Four Simplest Accounting Equivalences

Idempotence. Perhaps the most trivial of the structural rationality assumptions is that a gamble with identical consequences is indifferent to that sure consequence itself:

$$x \circ_E x \sim x. \qquad (1)$$

This assumption is so trivial both descriptively and normatively that it never has been tested.

Complementarity. The slightly more complex property of complementarity is also conceptually a trivial assumption:

$$x \circ_E y \sim y \circ_{\neg E} x, \qquad (2)$$

where $\neg E = F \backslash E$. The basic insight is that $\neg\neg E = E$. Neither its descriptive nor normative validity appears to be in question.

Event commutativity. We have already illustrated one version of the event commutativity assumption in Figure 1. For x and y that are both gains or both losses:

$$(x \circ_D y) \circ_E y \sim (x \circ_E y) \circ_D y. \qquad (3)$$

We are aware of just two empirical tests of this assumption. Ronen (1973) found evidence that subjects prefer to run the more probable event first. Brothers (1990), in a context of risk, used three different assessment procedures. The first involved choosing between the two sides, and only about one quarter of the subjects perceived the equivalence. The second used judged certainty equivalents and, on average, there was a slight tendency for subjects to favor the gamble with the higher probability of going on to the second stage. The third used choice-determined certainty equivalents, and event commutativity was well sustained.

From a normative perspective, event commutativity seems necessary: Figure 1c shows the two gambles in a matrix form indicating that they are identical. However, this equivalence is not so obvious when the sequence and timing of consequences matter to the decision maker. For example, when an early event is associated with a reward followed by an event with an equal sized loss, the net utility may be different from the case in which the event and outcomes occur in the opposite order.

Autodistributivity. This assumption is much like event commutativity except one event, E, is independently realized and there are three, not two, consequences:

$$(x \circ_E y) \circ_E z \sim (x \circ_E z) \circ_E (y \circ_E z). \qquad (4)$$

In other words, it does not matter whether the uncertainty about z is resolved vis-à-vis the whole gamble, $x \circ_E y$, or within each of its components. This version is called right autodistributivity; there is also a left-hand version which in the presence of complementarity, Equation (2), is equivalent to Equation (4).[2]

The key empirical issue is, again, whether the sequencing of compound gambles matters or not. As with

[2] Assuming idempotence, Equation (1), the single accounting equivalence $(x \circ_D y) \circ_E z \sim (x \circ_E z) \circ_D (y \circ_E z)$ includes both event commutativity, Equation (3), and right autodistributivity, Equation (4), as special cases.

event commutativity the only experiment that tested this assumption is Brothers' (1990). It was considerably less well sustained than event commutativity.

Normatively, the same argument as with event commutativity of whether or not sequencing should matter applies here.

Universal Accounting Equivalences (Invariance)
In the presence of certain other axioms to be discussed in §4 plus some that ensure a rich structure, these four accounting equivalences imply SEU (see §5). From that linear form, it is easy to show that it follows that any two gambles that yield the same consequences under the same conditions, ignoring the order in which events are realized in a tree, are equivalent. Luce (1990) refers to this collection of properties as "universal accounting" and Tversky and Kahneman (1986) called it "invariance." It is clearly a normative requirement if all the decision maker cares about is the net consequence at the end of a tree. In the case where probabilities are prescribed, it becomes the reduction of compound gambles, which is implied by EU. Evidence against reduction is given by Bar-Hillel (1973) and Starmer and Sugden (1991).

As anyone who has attempted to teach SEU to undergraduates is aware, accounting equivalences are far from transparent. They require a good deal of discussion to make clear the nature of the equivalences. Thus, it is not surprising that these are one of the major casualties of theories that attempt to be descriptive.

4. Preference Rationality

We turn next to assumptions that state which changes in gambles are favored: replacing a gamble by a more preferred one (transitivity); replacing a consequence by a more preferred one (consequence monotonicity); making the best consequence more likely at the expense of the poorest one (event monotonicity). Let us make these explicit.

Transitivity
This simple property requires that for any three gambles or sure consequences f, g, and h

$$\text{if } f \gtrsim g \quad \text{and} \quad g \gtrsim h, \quad \text{then } f \gtrsim h. \tag{5}$$

Replacing g by f improves matters relative to h. Transitivity has been a cornerstone of almost all rational theories of choice with the exception of some recent generalized theories due to Fishburn (1982b) and Fishburn and LaValle (1988). Fishburn (1991) has argued in detail why it may not be normatively compelling, but his view certainly is not yet the consensus view.

The main empirical challenges[3] to the descriptive validity of transitivity are of three types. One is a failure of context invariance. Perhaps the sharpest evidence of this type is the study of MacCrimmon et al. (1980) in which a single pair of gambles is evaluated in two different contexts of three other gambles. When each set of five is rank ordered by preference, the common pair receives opposite evaluations by a substantial fraction of subjects. Studies by Loomes (1990) and Loomes and Sugden (1983, 1986) suggest that some violations of transitivity are due to such context effects. See also Mellers and Birnbaum (1982). This establishes that, descriptively, no theory that is based on separate evaluations of gambles can possibly work. In particular, the theories discussed here cannot, in this sense, be fully descriptive. For an example of a theory with decided context effects, see Luce et al. (1993).

A second challenge to transitivity is preference reversals. In this case one gamble is preferred to another, yet the preferred one is assigned a smaller certainty equivalent than is the less preferred. The effect is robust (Bostic et al. 1990; Goldstein and Einhorn 1987; Grether and Plott 1979; Hamm 1980; Holt 1986; Johnson et al. 1989; Karni and Safra 1987; Lichtenstein and Slovic 1971, 1973; Loomes 1990; Loomes and Sugden 1983; Mowen and Gentry 1980; Pommerehne et al. 1982; Reilly 1982; Safra et al. 1990; Schkade and Johnson 1989; Segal 1988; Slovic and Lichtenstein 1983; Tversky et al. 1988; Tversky et al. 1990; Waters and Collins 1984). Recent findings suggest that the preference reversal phenomenon may be due to response mode and bias effects rather than to intentional intransitivities (see Bostic et al. 1990; Tversky et al. 1990).

The third challenge to transitivity (Budescu and Weiss 1987, Montgomery 1977, Ranyard 1977, Tversky 1969) occurs when one parameter of the gamble is changed in such small amounts that the changes are ignored

[3] Although we will mention some of the major empirical phenomena that challenge these axioms, we do not attempt to go into great detail. Camerer and Weber (1993) give more a detailed discussion, especially about the impact of ambiguity of events.

until they cumulate. We are all familiar with this tactic, often used by real estate agents and car dealers. The phenomena is no doubt real, but one that a serious decision maker attempts to overcome.

From a normative perspective transitivity strikes us as a nonnegotiable feature of any utility model. (For a disagreeing perspective, see Fishburn, 1991.) Transitivity is a key feature of functional decision making as evidenced by the concern with money pumping and the "Dutch Book". Furthermore, it is required in practice to identify a sure winner among the options under consideration. Without the ability to identify a sure winner, why conduct an analysis in the first place?

Consequence Monotonicity[4]

This is simply the property that if a consequence of a gamble is replaced by a more preferred one, the modified gamble is preferred to the original one. In the binary case, the property may be stated formally as:

$$x \gtrsim y \quad \text{if and only if} \quad x \circ_E z \gtrsim y \circ_E z, \qquad (6)$$

where x and y are pure consequences. A similar monotonicity assumption can be formulated by creating gambles in which z is mixed in from the left, but assuming complementarity, Equation (2), it is unnecessary to do so.

The concept of consequence monotonicity is readily generalized to any finite gambles, and the generalization is assumed.

Substantial experimental evidence, the Allais paradox (Allais 1953, Allais and Hagen 1979, Kahneman and Tversky 1979, MacCrimmon 1968) and the certainty effect (Kahneman and Tversky 1979), have been interpreted as questioning the validity of the monotonicity assumption. However, as Luce (1990, 1992a) emphasized, the usual formulation of what is commonly called "independence" actually combines the fundamental monotonicity assumption with a reduction of compound gambles to normal form.

The few experiments that have tested monotonicity in its pure form have produced mixed results. Those using choice procedures (Brothers 1990, Kahneman and Tversky 1979, Keller 1985, Tversky and Kahneman 1986) have found few or no violations of monotonicity as such even though they do occur when second order gambles are reduced to equivalent first order ones and an accounting equivalence is assumed to hold. Those using judgment procedures such as buying and selling prices and attractiveness ratings (Mellers et al. 1992a, Mellers et al. 1992b) show a clear pattern of violations. Using binary gambles, some 50% to 60% of their subjects assigned larger certainty equivalents to dominated gambles than to the dominating ones when the probability is less than 0.15 of receiving a small positive value in the dominating gamble and 0 in the dominated one. (It does not work with zero replaced by a nonzero value.)

Like preference reversals, the difference is probably a response mode effect. Unpublished studies[5] indicate that when certainty equivalents are elicited using the choice procedure, PEST[6], that reduced preference reversals, the non-monotonicity vanishes. Thus, outcome monotonicity, to the extent that it has been tested in its pure choice form, also stands a chance of descriptive survival.

From a normative perspective, consequence monotonicity, like transitivity, is an absolute necessity. Without it, one cannot separate the utility part of an analysis from its probability part. This separation, however, is necessary because decision makers provide inputs about the utility function to be used, whereas technical experts provide primary input about the probabilities. Consequence monotonicity justifies both the separation of these activities and their reintegration through an expectation model.

Event Monotonicity[7]

Consider three events C, D, and E, with C disjoint from both D and E, and consequences $x > y$. Event monotonicity requires that

$$x \circ_D y \gtrsim x \circ_E y \quad \text{if and only if} \quad x \circ_{C \cup D} y \gtrsim x \circ_{C \cup E} y. \qquad (7)$$

[4] Often this is called "dominance" but we avoid that term because its multiple uses in this domain make its meaning ambiguous.

[5] Work in progress by the authors.

[6] This is a modified up-down procedure in which choices between pure sums of money and the gamble are interspersed among many other choices and a computer program shifts the sum up when the gamble is selected and down when the money is chosen. The size of the shift is reduced by some factor, such as 2, each time the direction changes. It is described and used by Bostic et al. (1990).

[7] This has also been called "independence of a common consequence" and "cancellation" (Tversky and Kahneman 1986).

This assumption states, in effect, that increasing the likelihood of a desired consequence by the same amount does not alter the preference order. Note that this is a preference rationality axiom because it involves replacing y by the preferred x over the common event C in both gambles, and a factor that is common to two alternatives should be immaterial.

The most striking evidence against this normative hypothesis is the famous paradox of Ellsberg (1961), which was carefully explored experimentally by MacCrimmon and Larsson (1979) using medium-level business executives as subjects. Substantial violations were exhibited, and decision makers were typically unwilling to change their choices when confronted with them. Additional studies on this paradox are comprehensively summarized by Camerer and Weber (1993).

Normatively, the functional value of event monotonicity should be obvious. Without it, a decision maker could be induced to accept stochastically dominated gambles. From an operational perspective, this axiom is needed to assign probabilities to events.

In many presentations, especially by economists, the domain is limited to risky monetary alternatives, which are modeled as random variables. Within that framework, one treats an alternative as a cumulative distribution function $F(x)$, thereby basically building in all possible accounting equivalences. And a major property is first-order stochastic dominance: $F \gtrsim G$ if and only if, for each x, $F(x) \leq G(x)$. This formulation fails to distinguish clearly between consequence monotonicity and event monotonicity, which as we shall see is an important distinction.

5. The SEU Representation

Because it is so familiar, only a few words need be devoted to SEU. It asserts the existence of a utility function U over gambles and consequences and a finitely additive probability measure P over the entire family of events such that:

$$g \gtrsim h \quad \text{if and only if} \quad U(g) \geq U(h), \qquad (8a)$$

$$U(g, E; h, F\backslash E) = U(g)P(E|F) + U(h)[1 - P(E|F)], \qquad (8b)$$

where $P(E|F) = P(E \cap F)/P(F)$ is the conditional probability of E given that F occurs. This representation implies universal accounting equivalences, transitivity, monotonicity in consequences, and event monotonicity. Of the accounting equivalences, only four—idempotence, complementarity, event commutativity, and autodistributivity—are significant in the sense that in the presence of the other axioms and an adequate density of consequences and events, then the other accounting equivalences are consequences of these four.

The representation is readily generalized to finite gambles, i.e., assignments of consequences to finite event partitions, but we do not make it explicit.

6. Quasi-rationality

The SEU model makes reference neither to the role of the status quo among consequences nor to the fact that several consequences can be received at once. Indeed, when the consequences are amounts of money, the claim is made that the decision maker should be concerned only with final wealth. Thus, the status quo does not matter. In practice, final wealth is an elusive concept. (Do you know precisely—say within 5%—your current net worth?) Decision analysts usually structure consequences relative to the status quo and often treat costs, risks, and benefits as different attributes of the decision problem.

Within the SEU view, the sum of two sure consequences is indistinguishable from the joint receipt of both. But if the status quo matters, then there is no automatic meaning to the joint receipt of two consequences, sure money or gambles. Decision analysis sometimes must accommodate a decision makers's judgment that the joint receipt of two consequences is not equivalent to their sum.

To study the role of the status quo, joint receipts, and the separation of a decision problem into losses and gains, axioms need to be explored that are not implied by the standard SEU representation. We therefore refer to these axioms as quasi rational.

Status Quo, Gains, and Losses

The identification of the status quo element, labelled e, in the set of pure consequences dates back to Kahneman and Tversky's (1979) prospect theory. It is that consequence for which the decision maker feels neither a gain nor a loss. The role of the status quo has long been recognized, both in ordinary language and in the technical literature (Coombs 1975; Edwards 1962; Kahneman and Tversky 1979; Payne et al. 1980, 1981; Schoemaker 1982; Simon 1955), but aside from prospect theory, it has played little role in theoretical developments until recently. In particular, it is not a part of SEU.

The important technical feature of the status quo e is that it defines a singular point in the representation, and so the admissible transformations of the representation are restricted to multiplications by positive constants, rather than general linear transformations. In the language of measurement theory, such representations form a ratio, not an interval, scale.

Aggregation into Gains and Losses

Experience suggests that decision makers often restructure gambles with both gains and losses by treating such a first-order gamble as a second-order one that has three branches with gambles as their consequence: the subgamble of gains, the subgamble of losses, and the status quo. Let us make this formal.

For the present discussion, it is useful to go to more general gambles. Let the gamble g be based on a partition $\{E_i\}$ of the event F into n events and let the associated consequences be x_i. An example is shown in Figure 2a. Denote by $E(+)$ the union of all events associated with gains and by $g(+)$ the restriction of g to the subevent $E(+)$. Similarly $E(0)$ and $g(0)$ are, respectively, the union of subevents having e as the consequence and the restriction of g to $E(0)$. $E(-)$ and $g(-)$ correspond to the losses. (Of course, any of the three subevents may be vacuous.) Now create a second-order gamble g^2 by associating $g(j)$ with event $E(j)$, $j = +, 0, -$. This regrouping creates the sign-partitioned second-order gamble as illustrated in Figure 2b. It is plausible to treat g and the formally equivalent, second-order gamble g^2 as also equivalent (indifferent) in preference, i.e.,

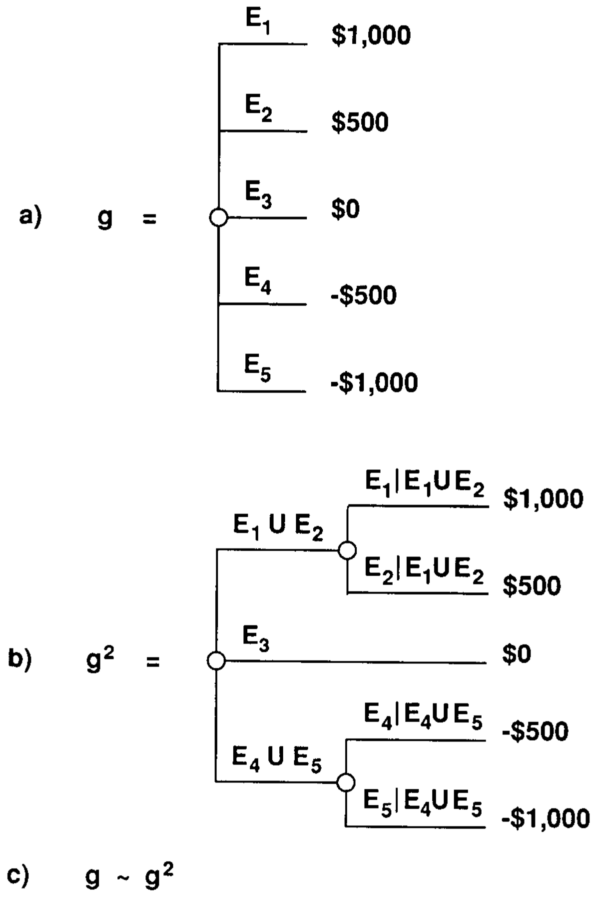
$$g^2 \sim g. \tag{9}$$

Note that we do not impose this rational equivalence for an arbitrary regrouping of the events,[8] but only for the very special grouping that treats the gains as one subgamble, the losses as a second, and the status quo as a third. This assumption (first stated explicitly in Luce and Fishburn, 1991) formulates, in a sense, the importance that decision makers place on the distinction between gains and losses; no other partitioning is comparable.

This aggregation assumption has not been studied empirically, and it is difficult to speculate meaningfully on whether subjects actually follow it. From a normative perspective, the issue is whether the grouping into three sets of events makes sense. In many applied situations it does—in fact, many applications separate an analysis of the "positive" part from that of the "negative" part,

Figure 2

An illustration of the aggregation assumption, Equation (9), that recasts any gamble into the formally equivalent two-stage gamble consisting of a three-outcome gamble whose consequences are the subgamble of gains, losses, and the status quo. The assumption, panel (c), is that these equivalent forms are perceived as indifferent.



[8] See §8 for the consequence of doing so.

e.g., cost-benefit or risk-benefit analyses. In some situations, however, such separation may be deemed unhelpful. For example, when examining choices among investment portfolios with both positive and negative consequences, one may want to maintain the ability to compare across and consider net gains and losses.

The aggregation assumption divides the theoretical as well as the applied problem into two components: an analysis of the three-outcome gambles of a single gain, a single loss, and the status quo, and analyses of gambles composed solely of gains or solely of losses.

Joint Receipt

The most unusual feature of some of the potentially descriptive properties to be described is the inclusion of a second operation $\oplus$ which is interpreted as the joint receipt of gambles and/or pure consequences (Luce 1991, Luce and Fishburn 1991). The symbol $g \oplus h$ is taken to mean the simultaneous joint receipt of independent gambles g and h. In terms of pure monetary consequences, some find it hard to think of this operation in any other way than as the numerical sum of the two consequences, for example, \$100 $\oplus$ \$1,000 would normally be thought of as equivalent to \$1,100. (We return in §7 to the consequences of this assumption.) However, for gambles or for a gamble and a sure consequence, the joint receipt operation has a less trivial interpretation: joint receipt of two gambles means playing them both simultaneously, but independently, and joint receipt of a gamble and a sure consequence means playing the one and receiving the other.

Thaler (1985) discussed joint receipt, or joint outcomes as he called it, and for money consequences he defined $x \oplus y$ to be *integrated* if it is evaluated as the sum $x + y$ and as *segregated* if each consequence is evaluated separately and the evaluations summed. Thus, if U denotes the utility function, $U(x \oplus y)$ is $U(x + y)$ in the integrated case and $U(x) + U(y)$ in the segregated case. Accepting Kahneman and Tversky's (1979) assumptions about the form of U, he argues from data that the operation is integrated when either both $x < 0$ and $y < 0$ or $xy < 0$ and $x + y > 0$; it is segregated otherwise, i.e., when either both $x > 0$ and $y > 0$ or $xy < 0$ and $x + y < 0$. Thaler and Johnson (1990) attempted to extend these ideas to the case of the joint receipt of a sum of money and a binary gamble of a gain and a loss. We do not attempt to use this more complex and probably more realistic assumption in the work below; rather, we assume that joint receipt is always segregated.

Monotonicity of Joint Receipts

An obvious rationality property of joint receipt is a version of monotonicity:

$$g \succsim h \quad \text{if and only if} \quad g \oplus z \succsim h \oplus z. \tag{10}$$

A similar assumption holds with the common consequence added from the left.

Although we are unaware of any empirical evidence favoring or contradicting this axiom, it is very hard to imagine descriptive violations in direct tests. Obviously this assumption is also an absolute must in any normative theory that includes joint receipts as a primitive.

The Interplay Between the Joint Receipts and Uncertainties

We now present two assumptions that refer explicitly to the decomposition of gambles into gains and losses. We hope that they may prove to be descriptive, but their descriptive validity has been only very partially tested. They are normatively controversial, but they seem to reflect the way reasonable people think and do business.

Duplex Decomposition. Once a gamble is partitioned, using the aggregation axiom, Equation (9), into subgambles of gains, losses, and the status quo, the following question arises: How do subjects combine the three subgambles into an overall evaluation? The following strong assumption states that any sign-partitioned gamble is judged equivalent to the joint receipt of two simpler, *independent* gambles: one that pits the gains against the status quo, and the other that pits the losses against the status quo:

$$g^2 \sim [g(+) \circ_{E(+)} e] \oplus [g(-) \circ_{E(-)} e]. \tag{11}$$

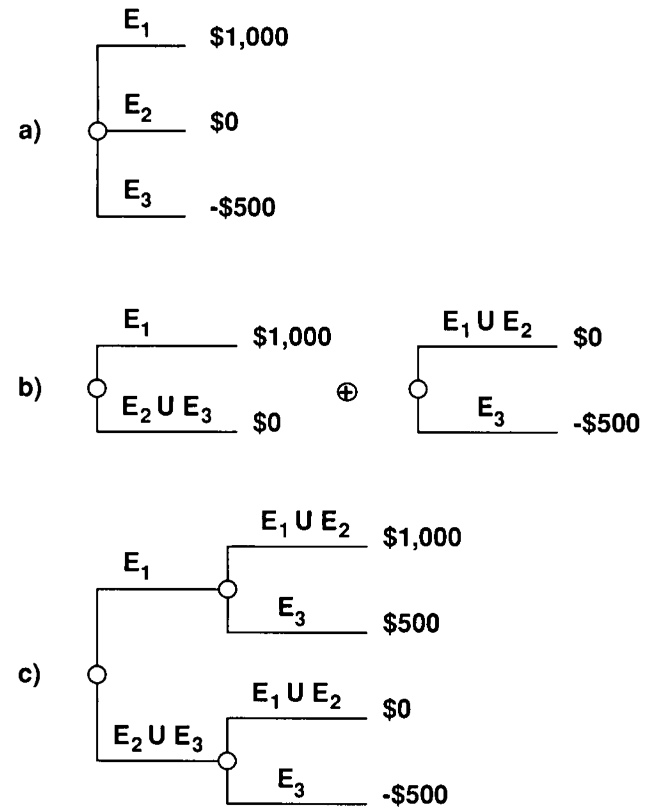
It is important to realize that the gambles on the right are run independently. Thus, it is entirely possible, for example, to receive $g(+) \oplus g(-)$ on the right but only one or the other on the left.

The joint receipt of the two binary gambles on the right side of the indifference equation was first referred to as a "duplex gamble" by Slovic (1967) who described the right side of Equation (10) as being ". . . as faithful an abstraction of real life decision situations . . ." as the left side. Using such duplex gambles, Slovic and Lichtenstein (1968) implicitly tested the duplex decomposition assumption for binary gambles, and within the noise level of their experiment they found that the sum of the selling prices of the duplex gambles was not significantly different from the selling price of the gamble giving rise to them. This is currently being investigated more fully.

From a normative perspective, this assumption introduces a major quasi-rationality as indicated in the following example. In Figure 3*a*, a simple three outcome gamble is shown. In Figure 3*b*, the same gamble is decomposed into its duplex form of the gains versus the status quo and the losses versus the status quo. Clearly the gamble in Figure 3*b* has a structure of possible outcomes and associated events significantly different from that of the original gamble, as shown in Figure 3*c*.

Figure 3

An illustration of the duplex decomposition assumption, Equation (11), in which a three-outcome gamble of a gain, a loss, and the status quo [panel (a)] is judged indifferent to the joint receipt of two independent binary gambles, one with the gain pitted against the status quo and the other with the loss pitted against the status quo [panel (b)]. Panel (c) shows the joint receipt of panel (b) as an equivalent two-stage process in which the chance experiment is run twice and the outcomes are as shown. It is clearly different from the original gamble of panel (a).



No rational argument exists for being indifferent between these gambles. Yet, there is something appealing about the "spirit" of this assumption. In its prescriptive interpretation, it says that people should separate the positive from the negative, analyze them separately, and combine them in some additive fashion.

Segregation. Consider a first-order gamble of just gains, and suppose the numbering of the events is from the one yielding the most preferred gain, x_1, to the one yielding the least preferred gain, x_m. Denote by $x \ominus x_m = y_i$ that element such that the joint receipt of it with x_m is equivalent to x_i, i.e., $y_i \oplus x_m \sim x_i$. Denote by g' the gamble that is generated from g by replacing each x_i by y_i, that is by "subtracting away" the least gain from each consequence. For gambles of all losses, a similar subtraction is made using the least loss.

In both cases, the segregation assertion is that:

$$g' \oplus x_m \sim g. \qquad (12)$$

This formalizes, in terms of $\oplus$, what Kahneman and Tversky (1979) described as the segregation phase of editing. They too assumed it is restricted to gambles of all gains or all losses. Segregation is a special case of the property that Pfanzagl (1959) called "consistency."

From a descriptive perspective the issue here is whether a sure thing is seen differently in the context of a gamble or by itself. This assumption has not been tested, and one can only speculate whether the two representations may be perceived as different. Normatively, such a difference could arise only if the framing causes a change in the status quo, which we assume does not occur.

7. The Rank- and Sign-Dependent Utility Representation

One approach to developing a (somewhat) descriptive theory is to retain all that one can of the structural and preference rationality—we have argued for idempotence, complementarity, event commutativity, transitivity, and consequence monotonicity—and, one way or another, introduce additional structure through properties that attempt to be frankly descriptive. Two of the latter were just described. We ask: What representation is consistent with these properties? One such is the rank- and sign-dependent utility (RSDU) model of Luce (1991) for binary gambles [see Equation (14) below] and of Luce and Fishburn (1991) for general first-order ones, which we do not write down explicitly here.

Like SEU, RSDU theory leads to a weighted average representation, but it has two distinguishing features.

First, it explicitly acknowledges the status quo and formulates a dependency of both utilities for consequences and of the weights assigned to events on whether the consequences are gains or losses relative to the status quo. In part, these are formulated using joint receipt.[9]

Second, the weights that are associated to events in the RSDU representation are not, in general, probabilities, and furthermore their values depend upon their position in the rank order induced by the preferences for the consequences that arise in the gamble. Conditions that force them to be probabilities are described in §8.

The Representation for Joint Receipt

Given a suitable set of axioms for the joint receipt operation $\oplus$ (they are too complex to repeat here; see Luce and Fishburn, 1991), one can derive that there exists a utility function for pure consequences and gambles such that

$$g \succsim h \quad \text{iff} \quad U(g) \geq U(h), \tag{13a}$$

$$U(g \oplus h) = \begin{cases} A(+)U(g) + B(+)U(h) \\ \qquad + C(+)U(g)U(h) & \text{if } g \succsim e, h \succsim e, \\ A(+)U(g) + B(-)U(h) & \text{if } g \succsim e \succsim h, \\ A(-)U(g) + B(+)U(h) & \text{if } h \succsim e \succsim g, \\ A(-)U(g) + B(-)U(h) \\ \qquad + C(-)U(g)U(h) & \text{if } e \succsim g, e \succsim h; \end{cases} \tag{13b}$$

$$U(e) = 0. \tag{13c}$$

Here g and h are either sure consequences or binary gambles, U is the real valued utility function, $A(+)$ and $B(+)$ are positive constants that weight the utilities U for positive consequences, and $A(-)$ and $B(-)$ are weights for negative consequences, $C(+) > 0$ defines an interaction between g and h in the positive quadrant, and $C(-) < 0$ does the same in the negative quadrant. Observe that the $A(s)$ and $B(s)$, $s = +, -$, are dimensionless whereas $C(s)$ must have the dimension $1/U$.

Note that Equation (13) satisfies monotonicity of joint receipts, Equation (10).

The simplest way to interpret this representation is in terms of sure things. It says that when two sure things are of opposite signs, then the utility of the joint receipt of both is equal to a weighted sum of their individual utilities. If, on the other hand, the two sure things are of the same sign, the utility of the joint receipt involves both the sum of the component utilities and an interaction term, which is positive when both are positive and negative when both are negative. Clearly, it is possible for the interaction terms $C(+)$ and $C(-)$ to be zero, allowing for purely additive utility functions for joint receipts.

In what follows, we assume for simplicity the purely additive case, i.e., $A(s) = B(s) = 1$ and $C(s) = 0$. Note that if, for all pure sums of money x and y, we assume $x \oplus y = x + y$, then U is forced to be proportional to money. In this connection, however, see the discussion in §6 of Thaler's (1985) and Thaler and Johnson's (1990) ideas on integrated and segregated joint receipt.

The Representation for Binary Gambles

The portions of Equation (13) involving the interaction terms for gains and losses separately are closely interlocked with the rank-dependent representation in the same domains. Specifically, one is able to show, for a very rich set of events, the following rank- and sign dependent representation: For all consequences x, y there exist weighting functions $W_s(E)$ spanning the unit interval $(0, 1)$, $s = +$ for gains and $s = -$ for losses such that

[9] Wakker and Tversky (unpublished) show that one can axiomatize the RSDU representation without invoking such an operation, but introducing it both makes formal some of the informal discussion in Kahneman and Tversky (1979) and renders very natural both the rank dependence of gains and losses and provides a reason why the weights over gains and losses do not add to 1.

$$U(x, E; y, F\backslash E) = \begin{cases} W_+(E|F)U(x) + [1 - W_+(E|F)]U(y) & \text{if } x \gtrsim y \gtrsim e \\ W_+(E|F)U(x) + W_-(E\backslash F|F)U(y) & \text{if } x > e > y \\ W_-(E|F)U(x) + [1 - W_-(E|F)]U(y) & \text{if } e \gtrsim x > y, \end{cases} \quad (14)$$

where $W_s(E|F) = W_s(E)/W_s(F)$. Note that the weight W_+ is assigned to the event when the consequence assigned to that is a gain and W_- when it is a loss.

The cases where $y > x$ are obtained from Equation (14) by using complementarity, Equation (2).

One of the striking features of the model is that the weights across gains and losses fail, in general, to add to 1, i.e.,

$$W_+(E|F) + W_-(F\backslash E|F) \neq 1.$$

Axioms of Binary RSD Utility

Of the structural accounting equivalences, binary RSDU Equation (14), implies idempotence, complementarity, and event commutativity. It does not imply right autodistributivity or any of the more complex accounting equivalences.

Of the preference rationality axioms, RSDU implies transitivity and monotonicity of consequences. It does not imply event monotonicity.

Of the quasi-rational axioms, it implies duplex decomposition.

Extensions to Multiple Consequences

The theory must, of course, cover more complex gambles than binary ones, even iterated binary ones. For an exact description, the reader should consult the technical papers. But the idea is simple. Complex alternatives are decomposed as suggested above (Equation 9) into a binary gamble of gains and losses. Each of these in turn is shown, primarily using segregation, to have a weighted average representation like that of SEU except that the weights depend not only on the underlying event but on the rank order position among the consequences of the consequence attached to that event. This is known as a rank-dependent utility representation.

As we show in §8 the addition of autodistributivity to the RSDU representation eliminates the rank dependence. To the extent that event commutativity but not autodistributivity is sustained in data, RSDU is more descriptively accurate than SEU.

Relations to Other Representations

The RSDU representation is closely related to two general developments. First, when restricted to risky gambles it exhibits rank- and sign-dependence of the same type as postulated in Kahneman and Tversky's (1979) prospect theory and it is substantially the same as Tversky and Kahneman's (1992) generalization with cumulative representation of weights. In the latter paper, the representation is fit to data with considerable success. Second, in the domain of all gains or of all losses, it exhibits a rank-dependent form that is closely related to the rank-dependent representations studied by Chew et al. (1987), Gilboa (1987), Luce (1988), Quiggin (1982), Schmeidler (1989), Segal (1987a, b; 1989) Wakker (1989, 1991, 1993) and Yaari (1987).

8. The Sign-Dependent Utility Representation

The failure of event monotonicity is certainly a major point of departure between normative and descriptive theories. Its failure affects not the representation as a weighted average, but keeps the weights from behaving like probabilities. In particular, if Equation (7) fails, so does finite additivity. Indeed, as Luce and Narens (1985) showed, event monotonicity in Equation (14) is equivalent to: if $D \cap F = E \cap F = \varnothing$,

$$W_s(D) \geq W_s(E) \quad \text{if and only if} \quad W_s(D \cup F) \geq W_s(E \cup F). \quad (15)$$

Clearly, this is implied by finite additivity, but does not imply it (see Chapter 5 of Krantz et al. 1971).

Although, for the most part, normative theorists and applied decision analysts are quite flexible about ways to formulate the underlying structure and the utility aspect of a problem, they tend to be much less flexible about the probability aspect. Rank-dependent weights may well be important in describing behavior, but it is unlikely that they contribute much to enhancing the normative usefulness of decision analysis. Rank dependence creates two problems in normative decision analysis: (1) probabilities often are elicited from experts who do not know what consequences attach to the events and therefore cannot conditionalize their estimates on ranks; and (2) rank weights do not behave like probabilities, and, in particular, they are not in general additive across disjoint events.

Thus, normatively, we need to impose some additional structural rationality assumption(s) to force the weights (separately for gains and losses) to be finitely additive, which as is easily seen is equivalent to eliminating rank dependence. The resulting model is called sign-dependent utility (SDU).

Luce (1992b) proved that if the same aggregation principle as was used to partition gambles into gains and losses, Equation (9), is assumed for any binary partition of the gains and losses separately, then finite additivity follows. Basically, this is the assumption that we are free to restructure the gambles of gains, and separately gambles of losses, in any way we please. This is radical axiomatic overkill because, as is shown in the Appendix, autodistributivity along with a rank-dependent representation is sufficient to force finite additivity and so SEU.

SDU is a *possible* prescriptive theory. It behaves like SEU in the domains of gains and losses separately, in particular, the weights behave like probabilities in each domain. However, the special role of the status quo makes prescriptive uses of SDU different in at least two ways from uses of SEU.

First, any formal decision analysis based on SDU must be viewed as conditional on the definition of the status quo and so of gains and losses relative to it. Changes in these definitions may require a reassessment of the utility function derived using, e.g., Equation (14). For example, if the uncertain consequences of an investment decision are expressed in terms of net gains or loss (subtracting out the invested amount) rather than total gains or losses (including the invested amount), SDU may well not give rise to the same utility function. This dependence of utility functions on the framing of the status quo is well known among both experimentalists (see, e.g., Schoemaker 1990) and practitioners (see, e.g., von Winterfeldt and Edwards 1986). We see such dependence in prescriptive settings as a strength, not a weakness, of SDU. Furthermore, SDU helps to identify powerful and unacceptable conditions under which these dependencies disappear (e.g., $x \oplus y \sim x + y$ for same sign *and* mixed domains).[10]

Second, SDU requires that gambles with mixed consequences be evaluated in three steps: Decomposition into gains and losses, separate evaluation of each domain, and then reaggregation using Equation (14). Clearly, this creates no problem in contexts, such as risk-benefit and cost-benefit analyses, where gambles are naturally decomposed in this way. In contexts where gambles with mixed consequences, such as investment and legal decisions, are the norm, the three-step procedure of SDU may seem cumbersome compared to the one-step procedure of SEU. This procedural disadvantage may, however, be balanced by the existence of new and simple methods for constructing utility functions based on joint receipt and, of course, on the strong assumption of utility independence of the status quo that was discussed above.

9. Conclusions

Table 1 summarizes our analysis. The rows show the three groupings of properties and the columns the three models, SEU, RSDU, and SDU along with a summary of the empirical evidence. We are suggesting that SEU represents the normative ideal, RSDU is an attempt to build a somewhat descriptive theory, and SDU embodies a compromise of the normative and descriptive to accommodate the prescriptive practice of dealing with gains and losses separately. The difference between SDU and RSDU is that the weights are finitely additive—probabilities—which causes the rank dependence to evaporate.

The table makes clear that the common ground for all three theories consists of the three simplest accounting equivalences—idempotence, complementarity, and event commutativity—as well as the two basic prefer-

[10] Two reviewers of this article pointed out that many decision analysts do, in fact, acknowledge the role of the status quo. But instead of assigning it the special role that it has in SDU, they use it merely as a convenient reference point for defining consequences in the SEU model. This is fine so long as one either makes the additional assumption that the utility function recovered does not depend on that choice or, like, SDU that the results of the analysis are restricted to the gain or loss domain over which the function was elicited.

Table 1 Summary of Properties and Models

Property	Normative = SEU	Prescriptive = SDU	Descriptive RSDU	Descriptive Data
ACCOUNTING EQUIVALENCES				
Idempotence (Equation 1)	Yes	Yes	Yes	Probably[1]
Complementarity (Equation 2)	Yes	Yes	Yes	Probably[1]
Event Commutativity (Equation 3)	Yes	Yes	Yes	Inconsistent
Universal	Yes[2]	GLS[3]	No[4]	No
PREFERENCE RATIONALITY				
Transitivity	Yes	Yes	Yes	Mostly
Consequence Monotonicity	Yes	Yes	Yes	Yes[5]
Event Monotonicity	Yes	GLS	No	No
QUASI-RATIONALITY				
Separation Gains, Losses	No	Yes	Yes	Yes
Duplex Decomposition	No	Yes	Yes	Maybe[6]
Segregation	No	Irrelevant	Yes	Not Studied

[1] Not studied.
[2] SEU implies universal accounting. Only one equivalence in addition to Equations (1) to (3) is needed to prove SEU.
[3] GLS = holds for gains and for losses separately.
[4] None beyond Equations (1) to (3) hold.
[5] Only when studied in isolation from accounting equivalences. Allais paradox shows joint violation of consequence monotonicity and an accounting equivalence.
[6] This has been tested in only one study and it was supported.

ence rationality conditions—transitivity and consequence monotonicity. It is entirely possible that RSDU fails as a descriptive theory on the issue of event commutativity.

When we look for differences we see that SEU and SDU agree completely in the domains of gains alone and of losses alone, but come totally apart over the quasi-rational axiom that is the link across gains and losses. RSDU and SDU agree on that link, but differ with respect to the weights being probabilities and so whether or not rank dependence is exhibited separately in gains and in losses.

The main issue with the prescriptive value of SDU is whether the status quo dependency is acceptable. The status quo is real in the sense that subjects in experimental tasks as well as managers facing real world decisions, almost without exception, phrase their problems in terms of deviations from the status quo, not in terms of some ultimate net utility or total wealth.

SDU theory attempts to accommodate this view formally, but at the cost of introducing controversial assumptions about disaggregating and aggregating gains and losses in uncertain alternatives. How severe this cost is depends on the decision problem. It may well be the case that in some decision contexts, e.g., investment decisions, the implications of the quasi-rational assumptions are unacceptable, whereas in other contexts, e.g., risk-benefits decisions, SDU may be quite acceptable.[11]

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Appendix

We assume complementarity, right autodistributivity, and rank dependence, Equations (2), (4), and (14). Let F be an arbitrary event and let $\{E, \neg E\}$ be an arbitrary partition of it. For simplicity, write $W(E) = W_s(E|F)$. Assuming that x, y are in the domain corresponding to $s = +$ or $-$, by Equation (14):

$$U(x \circ_E y) = \begin{cases} U(x)W(E) + U(y)[1 - W(E)] & \text{if } x \gtrsim y, \\ U(x)[1 - W(\neg E)] + U(y)W(\neg E) & \text{if } x < y. \end{cases}$$

Now, select $x > z > y$ such that $x \circ_E y > z$. Then

$$U[(x \circ_E y) \circ_E z] = U(x \circ_E y)W(E) + U(z)[1 - W(E)],$$

$$\begin{aligned} U[(x \circ_E z) \circ_E (y \circ_E z)] &= U(x \circ_E z)W(E) + U(y \circ_E z)[1 - W(E)] \\ &= U(x)W(E)W(E) + U(z)[1 - W(E)]W(E) \\ &\quad + U(y)[1 - W(\neg E)][1 - W(E)] + U(z)W(\neg E)[1 - W(E)]. \end{aligned}$$

By Equation (4) these are equal. Since x, y, z can be varied, their coefficients must match. In particular, that of $U(z)$ is:

$$1 - W(E) = [1 - W(E)]W(E) + W(\neg E)[1 - W(E)],$$

whence $1 = W(E) + W(F \backslash E)$, which, as is easily seen, is finite additivity of W_s.

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