Informational Aspects of Conflict*

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In war, the will is directed at an animate object that *reacts*.
Carl von Clausewitz, *On War*

1 Introduction

The problem of information seems integral to the analysis of conflict. Indeed, conventional wisdom in the field has long held that the outbreak of outright conflict can only be due to asymmetric information about relative power. (See, e.g., Blainey 1973.) If the two parties to a potential conflict agree on their respective probabilities of winning, and some resources are lost in case there is actual fighting, then there should be some peaceful solution that could be accepted by both.

Taken strictly, however, it is somewhat difficult to delineate a precise subfield studying information as it pertains to conflict. This is because *any* analysis of interaction using a game-theoretical approach is, at its heart, about the use

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of information. A conflict situation may be such that what is best for one party to do depends on what other parties plan to do, but if none of the participants *know* about this state of things—are *informed* about it—then from every individual point of view what we have is a non-strategic, single-person decision problem. Indeed, Robert Aumann, one of the founders of the discipline of game theory, has suggested that the field should fall under the broader rubric of *interactive epistemology*, as it fundamentally deals with rational actors reasoning about the reasoning of other rational actors. (See, e.g., Aumann 1999.)

In this chapter we therefore take a broad view of information. We apply it, however, to a narrowly defined class of conflict models. Specifically, we focus here on the literature that models conflict as a *contest*. We further narrow the focus by concentrating on 2-player contests. Within economics, there is a large literature on multi-player contests in the context of the study of rent-seeking. (See, e.g., Nitzan 1994 for a survey.) As armed conflict typically only involves two parties, it seems natural not to cover such models here. Finally, we do not study the tradeoff between aggressive and productive activities, as in such studies as, e.g., Skaperdas (1992), but instead take the value of winning a conflict as given.

We shall be especially interested in how manipulating information affects equilibrium efforts in contests.

In game theory, strategic situations are typically distinguished with regard to information along two different dimensions. One is the dichotomy of *perfect* versus *imperfect* information. An extensive form game, or game with a time dimension and sequential decisions, is said to be a game of perfect information if all players are at all times completely aware of exactly what has happened previously. If some action of some player is not observed by others, we instead have a game of imperfect information. The study of equilibrium along this informational dimension therefore deals with issues of the effects of *commitment* and observability of actions, something we discuss in Section 3, under the label of *strategic information*—information about what an opponent *does*.

A second distinct dichotomy is that of *complete* versus *incomplete* information. A game has complete information if every aspect of the game—i.e., who the players are, which actions they have available, and what their payoff functions are—is known to every player; and, indeed, not just known to every player, but *commonly known*, in the sense that everyone not only knows all the details of the game's specification, but also knows that everyone else knows them, and so on up to arbitrary levels of mutual knowledge. If such knowledge fails at some point, the game is one of incomplete information. For this reason we do not, strictly speaking, have a theory of games of incomplete information, as any situation where at least some players have a faulty understanding of what exactly is the nature of the situation would fall into this class. Instead, what is normally meant by a game of incomplete information is the smaller, more manageable class of games discussed by Harsanyi (1967–1968). Harsanyi suggested that many forms of incomplete information could be modeled as games that start with Nature drawing the *types* of the players, in the sense of their payoff functions, from commonly known distributions. Such games may thereupon be analyzed using a standard Bayesian approach.

The study of contests distinguished along this latter dimension therefore deals with issues of asymmetric information about players' utility from winning, their costs of producing effort, abilities, etc, and, notably, what private information about aggressive potential implies for the probability of outbreak of outright conflict instead of peaceful settlement—that is, we are here interested in the effects of a player's information about an opponent's *attributes*. These matters are discussed in Sections 4 and 5.

We conclude by drawing attention to some open questions worthy of further study.

2 Preliminaries

Except for Section 5, we shall be concerned here exclusively with two-player contests such that the probability of player $i \in \{1,2\}$ winning, given that efforts (which may be taken to be expenditures on arms, or the sizes of armies) are x_1

and x_2, is

$$p_i(x_1,x_2) := \begin{cases} x_i/(x_1+x_2) & \text{if } x_1+x_2>0 \\ 1/2 & \text{otherwise.} \end{cases}$$

This functional form for the *contest success function* (CSF), which gives rise to what is sometimes known as a "lottery contest," is familiar from the economic theory of rent seeking (Tullock 1980). Tullock (1975) used it for the study of litigation, but it has roots that go back at least to Haavelmo's (1954) discussion of international conflict. Hirshleifer (1989) discusses the empirical relevance of this form of the CSF, and Skaperdas (1996) provides an axiomatization.

As a benchmark, we shall start out by considering a simultaneous-move contest in which all aspects of the game are commonly known among the players. Suppose the players value winning at $v_1 > 0$ and $v_2 > 0$, respectively,[1] that effort comes at unit cost and that both players are risk neutral. The payoff function of player i is then

$$u_i(x_1,x_2) := p_i(x_1,x_2)v_i - x_i. \tag{1}$$

Note that there cannot be an equilibrium in which neither player expends positive effort, since given that the other player expends zero effort, you could win with probability one in return for an arbitrarily small effort. Hence if the players make their effort decisions simultaneously and independently, the best reply of player i given the effort of player $j \neq i$ is given by the first order condition

$$\frac{\partial u_i}{\partial x_i} = \frac{x_j}{(x_1+x_2)^2} v_i - 1 = 0.$$

That is, player i's *best reply function* is

$$x_i = \sqrt{x_j v_i} - x_j.$$

Since in equilibrium both players must be playing best replies, we must have that

$$x_1 = \frac{v_1^2 v_2}{(v_1+v_2)^2}$$

[1] We shall speak of differences in valuations throughout, but the same framework can also be used to address differential effort costs, or a combination of both. Suppose, more generally, that player i values winning at v_i and that one unit of effort costs him c_i. Then maximizing $p_i v_i - c_i x_i$ is, of course, equivalent to maximizing $p_i(v_i/c_i) - x_i$.

and

$$x_2 = \frac{v_1 v_2^2}{(v_1+v_2)^2}.$$

In equilibrium, the expected payoff of player i is therefore

$$u_i^C := \frac{v_i^3}{(v_1+v_2)^2}.$$

Notice that player 1's equilibrium probability of winning is

$$p_1(x_1,x_2) = \frac{x_1}{x_1+x_2} = \frac{v_1}{v_1+v_2}.$$

The player with the higher valuation therefore has a greater probability of winning in equilibrium. Dixit (1987) calls this player the *favorite*, the other the *underdog*.

While not surprisingly equilibrium efforts are increasing in the valuations of the players, perhaps more interesting is the fact that the greater is the difference in valuation of winning between the players, the lower is aggregate equilibrium effort. To see this, let $v_1 = v - \delta$ and $v_2 = v + \delta$ for some v and some $\delta > 0$. Aggregate equilibrium expenditure is then

$$x_1 + x_2 = \frac{v_1^2 v_2 + v_1 v_2^2}{(v_1+v_2)^2} = ((v-\delta)+(v+\delta))\frac{(v-\delta)(v+\delta)}{((v-\delta)+(v+\delta))^2} = \frac{v^2-\delta^2}{2v},$$

which clearly declines in δ. (Also see Konrad 2009.) Intuitively, the greater incentive to win of the player with the higher valuation makes it more costly for the low-valuation player to participate.

3 Strategic information

3.1 Commitment

In a single-person decision problem, or game against Nature, having more information when you select your action is always beneficial. The more precise is your information about whether there will be sunshine or rain later in the day, the more likely is your decision to carry an umbrella or not to be optimal.

In a strategic interaction this is not necessarily so. Having more information can hurt a player, as this fact may be exploited by others. A familiar example from economics is the Stackelberg model of competing firms, one of which makes its observable and irrevocable supply decision before the other firm makes its choice. The firm that goes second is better informed, since it makes its choice knowing what the first firm has done. Nevertheless, anticipating a rational response on the part of the firm that goes second, the first firm can improve its profit relative to equilibrium of a simultaneous-move model, and that of the second firm decreases. In the same spirit, sequential decision-making and commitment in contests is studied by, e.g., Dixit (1987), Baik and Shogren (1992), and Morgan (2003).

Now assume that player 1 makes its irrevocable effort decision first and that this decision is observed by player 2 before player 2 makes its decision. We use backward induction to find a subgame perfect equilibrium. If player 1 knows player 2 to be rational, then it knows that for any x_1, the response of player 2 is given by the best-reply condition (1) as

$$x_2 = \begin{cases} \sqrt{x_1 v_2} - x_1 & \text{if } x_1 < v_2 \\ 0 & \text{otherwise.} \end{cases}$$

The problem facing the first-mover in a sequential contest is therefore *de facto* that of selecting his favorite point on the second-mover's best-reply function.

Hence at an equilibrium in which player 2 exerts positive effort, player 1's payoff function, taking into account player 2's rational response, is in effect

$$u_1 = \frac{x_1}{x_1 + x_2} v_1 - x_1 = \frac{x_1}{x_1 + (\sqrt{x_1 v_2} - x_1)} v_1 - x_1 = \sqrt{x_1} \frac{v_1}{\sqrt{v_2}} - x_1,$$

which is maximized at

$$x_1 = \frac{v_1^2}{4 v_2},$$

with player 2's effort at

$$x_2 = \frac{2 v_1 v_2 - v_1^2}{4 v_2}.$$

If we have $v_1 \geq 2 v_2$, however, in equilibrium player 1's effort is $x_1 = v_2$ and player 2's is zero.

Hence if we have $v_1 < v_2$, aggregate effort in the unique subgame perfect equilibrium is

$$x_1 + x_2 = \frac{v_1^2}{4v_2} + \frac{2v_1v_2 - v_1^2}{4v_2} = \frac{v_1}{2} < \frac{v_1^2 v_2 + v_1 v_2^2}{(v_1+v_2)^2},$$

i.e., aggregate effort is lower under sequential play than under simultaneous play. If we have $v_1 = v_2$, the outcome is the same as under simultaneous decisions. Finally, if we have $v_1 > v_2$, sequential play involves greater aggregate effort than simultaneous play—even though it may be the case that player 2 exerts no effort at all. In Dixit's terminology, a favorite who is able to commit will be more aggressive than otherwise, an underdog less aggressive.

In equilibrium under sequential decisions, with player 1 going first, player 1's expected utility is therefore

$$u_1^S := \frac{v_1^2}{4v_2}.$$

Since we have that

$$u_1^S = \frac{v_1^2}{4v_2} \geq \frac{v_1^3}{(v_1+v_2)^2} = u_1^C$$

for all v_1 and v_2, the player who goes first always does at least as well as in the simultaneous-move game.

Now consider endogenous timing of contest effort commitment. Specifically, let there be two time periods, and allow each player to decide independently whether to commit effort in the first or the second period. If both players commit in the first period, or both commit only in the second period, then play is as in the simultaneous-move equilibrium. If one player commits in the first period and the other in the second period, then play is as in the unique subgame perfect equilibrium of the sequential-move game. Baik and Shogren (1992) show that in equilibrium the underdog commits in the first period and the favorite in the second, leading to lower aggregate equilibrium effort than under simultaneous moves.

That viewing the problem of commitment as crucially an informational one is not just gratuitous is brought home by the fact that commitment that is not *observed* might as well not have happened. In a memorable scene in Stanley Kubrick's 1964 film *Dr Strangelove* it is revealed that the Russians have installed a "Doomsday Device" that responds automatically with a counter-attack when it detects that missiles have been launched against the Soviet Union, thus ensuring mutual annihilation. In the spirit of military secrecy, however, the Russians have neglected to tell anybody about it, and now it is too late.

Bagwell (1995) pointed out that even the *slightest* uncertainty about whether a commitment has been correctly observed is enough to undermine any commitment effects. (Nevertheless, van Damme and Hurkens 1997 argue that one of the mixed-strategy equilibria of Bagwell's example, which converges to the unique subgame perfect outcome of the sequential game with perfect observability when the probability of correct observation approaches one, is more reasonable than the other.) Morgan and Várdy (2007) make a similar point in the explicit context of contests. Consider, as we have done, a 2-player sequential contest. Suppose the player who goes second can learn about the effort of the leader by paying a fee, which can be arbitrarily small. It cannot be the case that the second-mover ever pays the fee in a subgame perfect equilibrium, since in any equilibrium his choice of effort must be optimal *given* the effort of the leader—that is, the concept of Nash equilibrium already implies knowledge of the first player's action, so that it cannot be rational to pay to learn about it. This observation obviously generalizes beyond contest games. Whether the result that in equilibrium one would never pay to learn what the opponent is doing is an interesting insight or instead points to a fundamental logical flaw in the very concept of equilibrium itself is beyond the scope of the present discussion.

3.2 Delegation

Schelling (1960) pointed out that the potential benefits of commitment may also be attained through strategic delegation, when a player hires an agent to play the game on his behalf, provided, of course, that the delegation contract is observable and cannot be secretly renegotiated. Delegation in contests is studied by, among others, Baik and Kim (1997), Wärneryd (2000), and Konrad *et al* (2004).

If the effort of the agent is observable and verifiable, a first-best contract can be written that specifies the agent's remuneration as a function directly of his effort. Since the best a player could hope to achieve through delegation is his favorite point on the opponent's best-reply function, the optimal contract therefore specifies the effort associated with this point and pays the agent the smallest sum compatible with his rational participation if he exerts this effort, and nothing otherwise. If efforts are not verifiable, however, the agent has to be provided with an incentive to nevertheless exert effort. As we shall see, such a moral-hazard problem in the agency relation may have the effect of lowering equilibrium efforts and making both parties better off in a two-sided delegation equilibrium—even though both players are now also paying for the services of agents.

We illustrate this by considering the following example. Start by assuming only player 1 is able to hire an agent, who will exert effort x_1^a in the contest on the player's behalf. The agent is exactly like player 1, risk neutral and with access to the same conflict technology as his principal. Assume a contract between the player and his agent takes the form of a simple contingent fee w_1, to be paid to the agent by the player in case the contest is won. In case the contest is lost, the agent is paid nothing. Again, we use backward induction to find a subgame perfect equilibrium, starting with the game between player 1's agent and player 2, given that some contract specifying w_1 has already been signed, cannot be secretly renegotiated, and has been observed by player 2. The agent's payoff function is then

$$u_1^a(x_1^a, x_2) := p_i(x_1^a, x_2)w_1 - x_1^a,$$

whereas player 2's payoff function is as before. Hence in equilibrium in the contest, the agent will expend

$$x_1^a = \frac{w_1^2 v_2}{(w_1 + v_2)^2}$$

and player 2 will expend

$$x_2 = \frac{w_1 v_2^2}{(w_1 + v_2)^2}.$$

The agent therefore wins the contest on behalf of player 1 with probability $w_1/(w_1 + v_2)$.

From the point of view of the signing of the contract with the agent, player 1's expected payoff is then

$$u_1(w_1) = \frac{w_1}{w_1 + v_2}(v_1 - w_1), \tag{2}$$

which is maximized at

$$w_1 = \sqrt{v_1 v_2 + v_2^2} - v_2. \tag{3}$$

It now seems not unreasonable to conjecture that incentives to delegate obtain in a similar fashion as with direct commitment; i.e., the underdog may wish to delegate and the favorite not. Unfortunately, the present framework does not allow this issue to be investigated in an analytically tractable manner. We shall therefore only consider a special case—that of identical valuations. As will be recalled from the discussion of direct commitment above, in this case there is no incentive to commit. Strategic delegation is nevertheless not without interest under these conditions.

Suppose, then, that we have $v_1 = v_2 = v$. First consider a player 1 who is the only one to delegate. From (2) and (3) we see that we must have

$$w_1 = v(\sqrt{2} - 1)$$

and hence

$$u_1 = (3 - 2\sqrt{2})v < \frac{v}{4};$$

i.e., the delegating player is strictly worse off than under direct and simultaneous play. The non-delegating player 2, however, is strictly better off, with an expected equilibrium payoff of $v/2$. Hence no player has an incentive to delegate.

But suppose both players are *required* to be represented by agents—as is sometimes the case, for instance, in litigation. The equilibrium choices of contingent fees w_1 and w_2 must now satisfy

$$w_1 = \sqrt{v w_2 + w_2^2} - w_2$$

and

$$w_2 = \sqrt{v w_1 + w_1^2} - w_1.$$

Hence we must have

$$w_1 = w_2 = \frac{v}{3}.$$

As it happens, a contingent fee of one-third of what is recovered for the client by the attorney is a standard feature of lawyer-client contracts in US civil-justice cases, although perhaps not too much should be made of this coincidence. From (2) we have that equilibrium expected utility is

$$u_1 = u_2 = \frac{v}{3} > \frac{v}{4}.$$

When both players are required to delegate, they therefore both end up better off than if they had played directly. Even though they are both paying agents, the moral hazard problem in the agency relations induces lower equilibrium efforts than under direct play.

4 Non-strategic information

4.1 Independent valuations

Suppose a player's valuation is either v_L or v_H, with both *ex ante* equally likely and $0 < v_L < v_H$. A player knows his own valuation with certainty, but not that of the other. Denoting by $x_j(v_L)$ and $x_j(v_H)$ the opponent's expenditure when he is of low and high type, respectively, Player i's expected payoff is

$$u_i(v_i, x_i, x_{jL}, x_j(v_H)) := \frac{1}{2} p_i(x_i, x_j(v_L)) v_i + \frac{1}{2} p_i(x_i, x_j(v_H)) v_i - x_i.$$

Assuming an interior equilibium exists, the first-order condition for a best reply expenditure x_i is then

$$\frac{\partial u_i(v_i, x_i, x_j(v_L), x_j(v_H))}{\partial x_i} = \frac{1}{2}\frac{x_j(v_L)}{(x_i + x_j(v_L))^2} v_i + \frac{1}{2}\frac{x_j(v_H)}{(x_i + x_j(v_H))^2} v_i - 1 = 0.$$

By symmetry, in equilibrium we must have $x_1(v_L) = x_2(v_L) = x_L$ and $x_1(v_H) = x_2(v_H) = x_H$. Hence we need only solve two equations, one for each type, to find that

$$x_L = \frac{v_L^2 + 6v_L v_H + v_H^2}{8(v_L + v_H)^2} v_L$$

and

$$x_H = \frac{v_L^2 + 6v_L v_H + v_H^2}{8(v_L + v_H)^2} v_H.$$

This example belongs to a small class of such problems that are analytically solvable—in particular, the assumption that both values are equally likely is necessary. Malueg and Yates (2004) study a slightly more general model.

Defining

$$b := \frac{v_L^2 + 6v_L v_H + v_H^2}{8(v_L + v_H)^2},$$

and noting that $x_L = b v_L$ and $x_H = b v_H$, we can now consider expected aggregate effort in this contest. With probability 1/4, the players both have low valuation and hence expend in aggregate $2b v_L$. With probability 1/4, both have the high valuation, in which case a total of $2b v_H$ is expended. Finally, with probability 1/2, one player has the low valuation and the other the high valuation, so that total effort is $b(v_L + v_H)$. That is, aggregate expected equilibrium effort when valuations are private information is

$$X^P := \frac{1}{4}2b v_L + \frac{1}{4}2b v_H + \frac{1}{2}b(v_L + v_H) = b(v_L + v_H).$$

For comparison, consider a contest where the valuations are identically distributed *ex ante* but commonly known among the players at the time of play. With probability 1/4, such a contest will have two players with the low valuation, so we know from Section 2 that total effort will be $v_L/2$ in equilibrium. With probability 1/4, both are of the high type and hence expend in total $v_H/2$.

Finally, with probability 1/2 one player will have the low valuation and the other the high valuation, in which case total effort is $(v_L^2 v_H + v_L v_H^2)/(v_L + v_H)^2$. That is, *ex ante* aggregate expected equilibrium effort in this perfect information case is

$$X := \frac{1}{4}\frac{v_L}{2} + \frac{1}{4}\frac{v_H}{2} + \frac{1}{2}\frac{v_L^2 v_H + v_L v_H^2}{(v_L + v_H)^2} =$$
$$\frac{(v_L + v_H)^3 + 4 v_L v_H (v_L + v_H)}{8(v_L + v_H)^2} = b(v_L + v_H) = X^P.$$

Thus expected aggregate effort is the same regardless of whether information is private or not. This may be thought to be an artifact of the risk neutrality assumption. But then again, as we shall see next, it does not carry over to the common-value case.

Not very much is known at this point about the case when players' valuations are drawn from different distributions.[2] Hurley and Shogren (1998a, 1998b) consider other special cases. Fey (2008) proves existence of equilibrium in a model where the type of each player is drawn from a uniform distribution.

4.2 Common value conflicts

Consider now a two-player contest where *ex post* both players would value the prize the same, but at least one player is uninformed of this value. To simplify, suppose there are just two possible values of the prize, v_H and v_L, where we have $0 < v_L < v_H$. Let the probability of v_H be q. We assume one player, player I, is informed about the actual realization of the value, whereas the other, player U, knows only the prior distribution. Alternatively, we need not assume the informed player is *perfectly* informed. Suppose instead he receives a signal about the realized value, such that the signal can take one of two values. We could

[2]That is, not very much is known about independent valuations in *this particular class* of contest models. In the contest literature, another popular model is the contest in which the player who exerts the greatest effort wins with probability one. This type of contest is also known as a *first-price all-pay auction,* and the case of independent valuations is quite well understood in this context. (See, e.g., Krishna and Morgan 1997.)

then instead take v_L and v_H to be the conditional expectations of the actual value, given the two possible values of the signal, with q characterizing the distribution of the conditional expectation.

Let x_U be the effort of the uninformed player, and $x_I(v_L)$ and $x_I(v_H)$ the efforts of the informed player when he has observed the value to be v_L and v_H, respectively. The best-reply function of the informed player is then

$$x_I(v) = \begin{cases} \sqrt{x_U v} - x_U & \text{if } x_U < v \\ 0 & \text{otherwise.} \end{cases}$$

There can be no equilibrium such that $x_I(y_H) = x_I(y_L) = 0$. If such an equilibrium existed, it would have to be the case that $x_U \geq v_H$. But the uninformed player would not be playing a best reply, since he could lower his effort and still win with probability one in both states of the world, as the informed player would always be exerting zero effort. Therefore there is no equilibrium where the informed player never exerts positive effort.

Consider next the uninformed player. The rational uninformed player must take the opponent's privileged informational situation into account. In a standard, first-price, common-value auction, if everyone bids as if the object were worth its expectation given only their own information, then a player's winning indicates that he has very likely overestimated the value, since everyone else's estimates are lower. This phenomenon, which does not appear in equilibrium, is known as the "winner's curse" in auction theory. An analogue of it is, of course, a concern in the present setting, and therefore the uninformed player must discount his estimation of the prize below its *ex ante* expectation.

There are two possible types of equilibria. In the one we consider first, both informed types are active. That is, we have $x_I(v_L) > 0$ and $x_I(v_H) > 0$. The uninformed player's expected payoff function is

$$u_U(x_U, x_I(vL), x_I(v_H)) = (1-q)\frac{x_U}{x_U + x_I(v_L)} v_L + q\frac{x_U}{x_U + x_I(v_H)} v_H - x_U.$$

The first order condition for a best reply effort is therefore

$$(1-q)\frac{x_I(v_L)}{(x_U + x_I(v_L))^2} v_L + q\frac{x_I(v_H)}{(x_U + x_I(v_H))^2} v_H - 1 =$$

$$(1-q)\frac{1}{\sqrt{x_U}}\sqrt{v_L}+q\frac{1}{\sqrt{x_U}}\sqrt{v_H}-2=0,$$

so the equilibrium effort of the uninformed player is

$$x_U=\frac{(q\sqrt{v_H}+(1-q)\sqrt{v_L})^2}{4}.$$

In effect, in equilibrium the uninformed player acts as if estimating the value of the prize, not at its expectation, but at the square of the expectation of its square root.

The equilibrium efforts of the two types of the informed player may now, of course, be readily computed. But we shall be interested only in one feature of these efforts. Notice that, from the informed player's best-reply function, his expected effort may be written as

$$(1-q)(\sqrt{x_U v_L}-x_U)+q(\sqrt{x_U v_H}-x_U)=\sqrt{x_U}((1-q)\sqrt{v_L}+q\sqrt{v_H})-x_U=$$
$$\sqrt{x_U}2\sqrt{x_U}-x_U=x_U,$$

since from the expression for the equilibrium value of x_U we have that $(1-q)\sqrt{v_L}+q\sqrt{v_H}=2\sqrt{x_U}$. That is, in equilibrium the two players exert the same expected effort. Hence aggregate expected effort in this type of equilibrium is

$$2x_U=\frac{(q\sqrt{v_H}+(1-q)\sqrt{v_L})^2}{2}.$$

Compare this outcome with the two relevant symmetric information scenarios. Consider, first, a situation in which both players are uninformed. Both being risk neutral, from the discussion in Section 2 we know that in equilibrium they would each exert effort $((1-q)v_L+qv_H)/4$, so that aggregate effort would be

$$\frac{(1-q)v_L+qv_H}{2}.$$

If both were informed, then when the value was v_L, they would each in equilibrium exert effort $v_L/4$, and when the value was v_H they would each exert effort $v_H/4$. Hence the *ex ante* expected aggregate effort would be the same under this scenario as when both are uninformed.

Since the square root is a strictly concave function, and by Jensen's inequality the expectation of the square root of a stochastic variable is strictly less than the square root of the expectation of the variable, we see that aggregate expected effort under asymmetric information is strictly lower than under either symmetric information scenario.

Since we must have $x_U < v_L$ for this to be an equilibrium, as otherwise the informed player would not exert any effort when the value was v_L, we must have that

$$q < \frac{\sqrt{v_L/v_H}}{1-\sqrt{v_L/v_H}} =: \hat{q}.$$

In the second type of equilibrium, only the highest informed type is active; i.e., we have $x_I(v_L) = 0$. The uninformed player's first-order condition then reduces to

$$q\frac{x_I(v_H)}{(x_U + x_I(v_H))^2} v_H - 1 = q\frac{1}{\sqrt{x_U}}\sqrt{v_H} - 1 - q = 0,$$

so we have that

$$x_U = \left(\frac{q}{1+q}\right)^2 v_H.$$

In order for this to be consistent with the lowest type expending nothing, we must have $x_U \geq v_L$, i.e., that

$$q \geq \hat{q}.$$

Again we note that both players exert the same effort in expectation, as we have that

$$\mathrm{E}x_I = q(\sqrt{x_U v_H} - x_U) = \frac{q^2}{1+q} v_H - q x_U = (1+q)x_U - q x_U = x_U.$$

We also have that

$$2x_U = 2\left(\frac{q}{1+q}\right)^2 v_H < \frac{q v_H}{2} < \frac{(1-q)v_L + q v_H}{2},$$

so that again expected aggregate effort is strictly less than under either symmetric information scenario. As this result generalizes we conclude that, unlike in the independent valuations model, with common values asymmetric information tends to lower aggregate effort.

By the strict convexity in x_I of the relevant part of p_U, the uninformed player's probability of winning, we have that

$$(1-q)p_U(x_U, x_I(v_L)) + q p_U(x_U, x_I(v_H)) > \frac{x_U}{x_U + \mathrm{E}x_I} = \frac{x_U}{2x_U} = \frac{1}{2}.$$

That is, the uninformed player's expected probability of winning is, somewhat surprisingly, strictly greater than that of the informed player. As an example, note that in the second type of equilibrium, the uninformed player's probability of winning is

$$1-q+q\frac{x_U}{x_U + x_I(v_H)} = 1-q+\frac{q^2}{1+q} > 1/2.$$

Let $q \to 0$ and $v_L/v_H \to 0$ in such a fashion that the condition for the existence of equilibrium is satisfied. Then the probability of the uninformed player winning approaches 1.

It can be shown also in more general settings that the uninformed player always wins with a strictly higher probability than does the informed player. This notwithstanding, an uninformed player is always strictly worse off than an informed player. There is therefore an incentive to acquire information. Consider two initially uninformed players who each have the option of acquiring information about the realization of the prize prior to entering the contest. In equilibrium, both would acquire information, even though in *ex ante* terms nothing is gained by this, as they go from one symmetric information scenario to an equivalent other.

More general results about 2-player common-value contests with asymmetric information are proved in Wärneryd (2003).

It should be noted that the crucial feature of the common-value contest under asymmetric information is, perhaps paradoxically, not that the valuations are exactly the same, but rather that, in effect, at least one player does not know its own type. The common-value contest is therefore not a special case of the independent valuations model. The independent valuations model is a model of a situation with asymmetric information about *attributes of the players*, i.e., their payoff or utility functions. The common-value model is a model of a situation with asymmetric information about an *attribute of the prize*. In fact, we can easily allow the players' *ex post* valuations to differ. For instance, let y be an uncertain attribute of the prize, and $v_i(y)$ the valuation of player i.

5 Asymmetric information and conflict outbreak

So far, we have made no distinction between conflict efforts—in the sense of, e.g., arms investments—and actual aggression. And, indeed, in static situations of complete information about players' attributes, there would seem to be no reason for actual conflict ever to break out. Given that the players understand what their equilibrium probabilities of winning would be, they could simply share whatever they are in conflict over according to those probabilities, and avoid actually fighting. (See, e.g., Skaperdas 1992 for an argument to this effect.) In particular if there is some cost associated with fighting itself, such a solution would be strictly efficient. In the real world, of course, wars do break out and people do take each other to court. We therefore need a theory of conflict outbreak.

Blainey (1973) influentially argued that war, if it is costly, can occur instead of peaceful settlement only because states overestimate their probability of winning. In this Section we investigate this issue formally by means of a simple example, based on the more general discussion found in Bester and Wärneryd (2006). The results apply, of course, not only to war, but also, for example, to the analysis of pre-trial bargaining in litigation.

Consider two players who can each be either of the low strength type, $\underline{t}$, or the high strength type, $\overline{t}$, with $\underline{t} < \overline{t}$. We assume the players' types are independent draws from the same distribution, with the probability of the low type equal to $q \in (0,1)$. The relative strengths of the players determine their probabilities of winning an outright conflict, according to

$$p_i(t_1,t_2) := \begin{cases} 0 & \text{if } t_i < t_j \\ 1/2 & \text{if } t_i = t_j \\ 1 & \text{if } t_i > t_j. \end{cases}$$

The problem concerns the division of a cake of fixed size 1. In case of outright conflict, some of the cake is destroyed and only $\theta < 1$ remains. Hence reaching a peaceful agreement will always be efficient. Nevertheless, as we shall see, peaceful agreements may be fundamentally impossible if the strengths of the players are their private information.

To put this as starkly as possible, we assume the players can sign binding agreements with a third-party mediator. This, naturally, is unlikely to be true of international relations in reality. But it should be stressed that this approach encompasses also all the contracts that the players could enforce themselves; the point will be that *even* with extensive opportunities for commitment, a peaceful solution may not always be attainable.

A contract will here be taken to be an incentive-compatible and individually rational mechanism. By appeal to the relevation principle (Myerson 1979) we restrict attention to direct mechanisms, i.e., mechanisms where the only thing the players do is report their types to the mediator or mechanism designer.

A mechanism will be a pair of functions, β and π, which take as input the type reports of the players, where β is the share of the cake received by player 1 in case conflict does not occur (and hence player 2 gets $1-\beta$), and π is the probability of conflict. We allow a mechanism to specify a probability of outright conflict precisely since we wish to argue that if conflict is not destructive enough, then there may be no incentive-compatible and individually rational mechanism that always assigns probability zero to conflict.

A mechanism is incentive compatible if no type of any player has an incentive to report his type falsely, given that the opponent reports his truthfully. Hence the mechanism (β,π) is incentive compatible for the low type of player 1 if we have that

$$q((1-\pi(\underline{t},\underline{t}))\beta(\underline{t},\underline{t}) + \pi(\underline{t},\underline{t})(1/2)\theta) + (1-q)(1-\pi(\underline{t},\overline{t}))\beta(\underline{t},\overline{t}) \geq$$

$$q((1-\pi(\overline{t},\underline{t}))\beta(\overline{t},\underline{t})+\pi(\overline{t},\underline{t})(1/2)\theta)+(1-q)(1-\pi(\overline{t},\overline{t}))\beta(\overline{t},\overline{t}).$$

Similarly, the mechanism is incentive compatible for the high type of player 1 if we have that

$$q((1-\pi(\overline{t},\underline{t}))\beta(\overline{t},\underline{t})+\pi(\overline{t},\underline{t})\theta)+(1-q)((1-\pi(\overline{t},\overline{t}))\beta(\overline{t},\overline{t})+\pi(\overline{t},\overline{t})(1/2)\theta)\geq$$
$$q((1-\pi(\underline{t},\underline{t}))\beta(\underline{t},\underline{t})+\pi(\underline{t},\underline{t})\theta)+(1-q)((1-\pi(\underline{t},\overline{t}))\beta(\underline{t},\overline{t})+\pi(\underline{t},\overline{t})(1/2)\theta).$$

A corresponding set of conditions must, of course, hold for the two types of player 2. Now suppose the mechanism is peaceful, in the sense of having $\pi(t_1,t_2)=0$ for all (t_1,t_2). From player 1's incentive-compatibility constraints, we then have that

$$q\beta(\underline{t},\underline{t})+(1-q)\beta(\underline{t},\overline{t})\geq q\beta(\overline{t},\underline{t})+(1-q)\beta(\overline{t},\overline{t})$$

and

$$q\beta(\overline{t},\underline{t})+(1-q)\beta(\overline{t},\overline{t})\geq q\beta(\underline{t},\underline{t})+(1-q)\beta(\underline{t},\overline{t}).$$

We must therefore have that

$$q\beta(\underline{t},\underline{t})+(1-q)\beta(\underline{t},\overline{t})=q\beta(\overline{t},\underline{t})+(1-q)\beta(\overline{t},\overline{t}),$$

i.e., that both types of player 1 have the same expected payoff under the mechanism. The simple explanation for this is, of course, that if you are never called upon to actually fight, the cost of lying about your type is zero. If one type received a greater share of the cake under the mechanism, the other type would then have a strict incentive to claim to be the first type also.

Let V_1 be this common expectation of the types of player 1. Since both types have the same expectation, the *ex ante* expectation of player 1 is also equal to V_1; i.e., we have that

$$V_1=q(q\beta(\underline{t},\underline{t})+(1-q)\beta(\underline{t},\overline{t}))+(1-q)(q\beta(\overline{t},\underline{t})+(1-q)\beta(\overline{t},\overline{t})).$$

Consideration of the corresponding conditions for player 2 shows that we must have $V_2=1-V_1$.

Next consider the individual rationality constraints. In order to voluntarily consent to be bound by the mechanism, each type of each player must expect at least the payoff he would get in an outright conflict. In particular, it must hold for the highest type of player 1 that

$$V_1 \geq q\theta + (1-q)(1/2)\theta$$

and for the highest type of player 2 that

$$V_2 \geq q\theta + (1-q)(1/2)\theta.$$

Summing these two inequalities, and utilizing that $V_1 + V_2 = 1$, we see that a peaceful mechanism only exists if we have that

$$\theta \leq \hat{\theta} := \frac{1}{1+q}.$$

The problem is that the highest type of a player may think itself likely to win an outright conflict. If we are then dealing with players who are both of the highest type, there may be no way of splitting the cake peacefully that is compatible with their overestimations of the outside option, provided enough would remain of the cake in case of conflict. One might think that this suggests that peace is less likely the likelier are the players to be of the high type. The truth is just the opposite, as we can see from the fact that $\hat{\theta}$ is declining in q, the probability that a given player is of the low type. It is precisely when the opponent is *ex ante* very likely to be of the low type that the high type of a player may think the outside option conflict is more attractive.

Related results are discussed by Powell (1996) and Brito and Intriligator (1985). The latter allow for endogenous arms investments; the private information instead concerns the parties' payoff functions. What results is therefore a signaling game.

Fearon (1995) is critical of rationalizations of war that are built exclusively on mutual misperceptions of the probability of winning (as might seem, at first glance, to be the case with the model of the present Section):

> [T]he states know that there is some true probability p that one state would win in a military contest. . . . [I]t could be that the states

> have conflicting estimates of the likelihood of victory, and if both sides are optimistic about their chances this can obscure the bargaining range. But even if the states have private and conflicting estimates of what would happen in a war, if they are rational, they should know that there can be only one true probability that one or the other will prevail (perhaps different from their own estimate). Thus rational states should know that there must in fact exist a set of agreements all prefer to a fight.

The parties, Fearon argues, would seem to have an incentive to communicate and share their information, since they would know that this must uncover agreements that both would prefer to outright conflict. Private information *by itself* cannot be a barrier to reaching agreement; only together with obstructions to the communication of such information, or incentives to misrepresent information, can private information prevent peace. This argument appears to fail to take into account that the process of information revelation in bargaining *itself* may endogenously create incentives to lie, as the parties connive to improve their relative position. In the setting discussed in this Section we have, of course, incorporated the potential exchange of information directly into the model, and shown that incentives to misrepresent may arise directly out of the effect of one's announcements on one's bargaining position.

Fey and Ramsay (2007) note that once two states face each other on the battlefield, at the very least, they should realize that something is wrong, since they could not both be willing to fight if they both had the correct perception of the probability of prevailing. This is, of course, an application of Aumann's (1976) more general result that two players who have access to the same information cannot "agree to disagree," as it cannot be common knowledge among them that their posteriors are different if they have the same prior.

One might think that information-gathering through espionage might facilitate reaching peaceful agreements. Bernard (2008) notes that, in the absence of the possibility of signing binding agreements, espionage may actually raise the probability of conflict, as any power imbalances will now be detected with certainty.

Finally, Garfinkel and Skaperdas (2000) is an example of a literature that argues that costly outright conflict can occur also in the absence of informational problems. Their story is a dynamic one, in which a player may find it in his interest to attack today if this improves his position tomorrow, even if peaceful settlement would be efficient.

6 Suggestions for future research

As should be evident already from this brief and necessarily non-exhaustive survey, many topics in the analysis of information as it relates to conflict remain wide open. For instance, apart from a handful of special cases, next to nothing is known about probabilistic contest models with independent valuations.

In all the models studied in this chapter, we have assumed that the parties involved are unitary actors, who make decisions on their own to maximize their individual, well-defined objective functions. Yet in most applications to international conflict it does not seem reasonable to view states as players in this fashion. Defense and aggression decisions by a nation are the outcomes of political processes, involving many actors with at least potentially conflicting interests. While the politico-economic literature contains models that take this into account—for instance, by assuming that defense policy is set by a politician who must have voter support—little appears to have been done that relates to the specifically informational issues that arise. Thus we might ask, for instance, if voters delegating conflict decisions to a politician has any special, perhaps even counter-intuitive, implications for conflict efforts in international equilibrium.

An area we have barely touched on here is that of information acquisition and information revelation in conflict, for the simple reason that little formal analysis of these issues appears to be available. Yet all nations keep military secrets, and all nations employ intelligence services. Why? A naive reading of sources such as Fearon (1995) would seem to suggest that, on the contrary, it would be in the interest of states to be as open as possible about their military capabilities, as this would seem to make peaceful agreements easier to reach. But this is, of course, an argument from an entirely static model. In a dynamic perspective, something we have not discussed here, other possibilities may arise. A nation might want to shroud its current capability in mystery as it attempts to increase it for the future. But then again a currently powerful player would seem to have an incentive to reveal itself as such, so a policy of non-disclosure could only signal weakness. The resultant signaling game is another example of something that should prove a fertile field for further study.

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