

Dynamic Models of Simple Judgments: I. Properties of a Self-Regulating Accumulator Module

Douglas Vickers^{1,3} and Michael D. Lee²

This is the first of two papers comparing connectionist and traditional stochastic latency mechanisms with respect to their ability to account for simple judgments. In this paper, we show how the need to account for additional features of judgment has led to the formulation of progressively more sophisticated models. One of these, a self-regulating, generalized accumulator process, is treated in detail, and its simulated performance across a sample of tasks is described. Since an adaptive decision module of this kind possesses all the ingredients of intelligent behavior, it is eminently suited as a basic computing element in more complex networks.

KEY WORDS: connectionism; stochastic modeling; reaction time; discrimination; adaptation.

INTRODUCTION

This is the first of two papers, in which we examine both traditional and connectionist models for which response times are important measures of performance. In this first paper, we begin by summarizing the main features of simple perceptual judgments which have led to various developments in the structure of traditional stochastic latency models. We show how the need to account for additional features of the data has led to the formulation of progressively more sophisticated models. One of these, a self-regulating, generalized accumulator process, based on the work of

¹Department of Psychology, University of Adelaide, Adelaide, South Australia.

²Departments of Psychology and Electrical and Electronic Engineering, University of Adelaide, and the Cooperative Research Centre for Sensor Signal and Information Processing.

³Correspondence should be directed to Dr. D. Vickers, Psychology Department, University of Adelaide, Adelaide, South Australia 5005; e-mail: psyd-vic@psychology.adelaide.edu.au.

Vickers (1979), is described in some detail, and an indication given of its dynamic behavior across a small sample of empirical tasks. In our second paper, we consider the advantages afforded by the neural network modeling framework, and discuss ways in which such a framework might serve to extend the capabilities of a connectionist model, based on the interconnection of a number of self-regulating, generalized accumulator modules.

SIMPLE JUDGEMENTS

Psychophysical discrimination has typically been studied by instructing an observer to compare a stimulus of variable magnitude, v , with a second stimulus of constant or standard magnitude, s , and to indicate whether the variable appears to be greater or less than the standard, or equal to it, with respect to the dimension on which they vary. Research on simple discriminative judgements has focused on explaining variations in one (or more) of three dependent variables: the relative frequency with which the observer makes each alternative response to each class of stimulus, the time each response takes, and (less commonly) the degree of confidence expressed by the observer in each response. Attention has also focused on the effects of manipulating the following three independent variables.

Discriminability

The first—ostensibly objective—independent variable is that of *discriminability*. In the simplest cases, this is measured by the physical difference between v and s with respect to the relevant dimension on which judgements are made. (The qualification ‘ostensibly’ arises because the relationship between, for example, objective weight and perceived heaviness can only be inferred.) Theoretical models of judgement are usually applied by making simplifying assumptions about the internal representation of such stimuli.

In general, as the objective stimulus difference ($v-s$) is reduced by small, equal amounts, so the proportion of errors made by an observer, and the time taken to respond, both undergo a smooth increase, while the average confidence decreases. To account for the continuous increase in errors, it has traditionally been assumed that the sensory representation of a stimulus does not maintain a fixed value, but is subject to *noise* in the form of random fluctuations arising from a number of independent influences. For example, in the classical psychophysical model of Thurstone (1927a, 1927b), it was assumed that such fluctuations occurred from trial to trial, so that, over trials, the sensory effect could be represented by a

normal distribution. According to Thurstone, an observer's judgement depended on the momentary 'discriminal difference' ($V-S$) between these sensory representations on any given trial. From this, Thurstone (1928) concluded that the *psychometric function*, relating the probability of making a response of the form ' $V > S$ ' to the objective stimulus difference ($v-s$), should resemble the form of a cumulative normal ogive.

Other functions have also been suggested, such as the cumulative logistic (Bock & Jones, 1968). However, given the uncertainties associated with the relationship between physical and perceived measures of stimulus difference, the truncation effects on response probabilities at 0 and 100% accuracy, the inherent unreliability of psychological processes, and the close similarity between the cumulative normal and logistic functions, it is fair to say that either function provides a satisfactory description of the sigmoid, *nonlinear* relation found in most empirical data.

Bias

The second—more subjective—variable is *response bias*, or the comparative readiness of the observer to make each alternative response, independently of the stimulus information. The first systematic explanation of response bias was the signal detection theory (SDT) approach of Tanner and Swets (1954), and one of the clearest applications to discriminative judgements is that of Treisman and Watts (1966). According to these authors, the observer does not make a comparative judgement on the basis of sampling a discriminial difference which is greater (or less) than zero. Rather, the observer bases such decisions on whether the discriminial difference exceeds (or falls below) some criterion magnitude or cutoff. The position of this cutoff can be adjusted to reflect the expectation that one or the other stimulus will be presented. This conceptualization allows for the distinction between a sensitivity (or discriminative ability) parameter, d' , and a second parameter, β , representing changes in response bias. On this view, changes in sensitivity would correspond to increases or decreases in the steepness of the psychometric function, while changes in response bias would be indicated by shifts of the psychometric function along the dimension of ($v-s$) difference.

It may be questioned whether all of the presumed manipulations of response bias can be interpreted in terms of changes in a single parameter. In particular, Vickers (1985) has argued that it may be necessary to distinguish between situations in which the observer makes one response more slowly and *carefully* than the other and those in which the observer expects that one response is more *likely* than the other. In addition to this distinction (between *relative caution* and *expectation*), it may also be necessary to

distinguish a third situation, in which such an adjustment is induced *unconsciously* by an unannounced change (e.g., in relative frequencies) in the set of presented stimuli.

Caution

The interpretation of d' as an index of discriminative capacity implies that d' should remain unaffected by changes in an observer's attitude towards a discrimination task. Evidence against this is provided by Vickers and Packer (1982), who found that, under an accuracy set, observers produced more correct responses (for both stimulus classes), had longer response times, and made more confident judgements than under a set for speed. Such clear evidence for an improvement in accuracy for both responses implies a change in d' , rather than in β . The fact that this improvement follows a simple change in instructions implies the operation of a further attitudinal factor, which may be labelled *caution*. According to this hypothesis, observers can trade speed for accuracy, i.e., they can improve their discriminative performance by inspecting the stimulus for a longer time.

Such findings have necessitated a modification in the way in which sensory noise is thought to act on stimulus representations. The SDT approach assumes that, on any particular trial, the observer compares a single observation of the perceived difference in sensory effect with some predetermined cutoff. However, the ability to trade speed for accuracy implies that observers can improve the accuracy of their judgements by increasing the number of observations in the sample. Two main ways in which this might lead to increased accuracy in responding have been considered.

The simplest assumption, due to Crossman (1955), is that the observer takes a series of observations of the momentary discriminable differences and compares their average with that of some cutoff. If we suppose that each observation takes a constant time, and that observations are taken at a steady rate, then, we should expect that the effective discriminability of the stimuli, d' , should increase as a function of the square root of the number of observations taken (and, hence, of time). *Fixed sample* models of this kind have been put forward and discussed by a number of researchers (e.g., Green, Birdsall, & Tanner, 1957; Schouten & Bekker, 1967; Swets, Shipley, McKey, & Green, 1959; Taylor, Lindsay, & Forbes, 1967).

One situation, in which such models might apply, is where the time for which the stimulus is available is determined by the *experimenter*. In such cases, the observer has no obvious choice but to accumulate information until some external deadline signals the moment for its evaluation. In these cases, the probability of making a correct response, and the confi-

dence with which it is made, are both *direct* functions of the time for which the stimulus is available (Vickers, Burt, Smith, & Brown, 1985a).

Conversely, a situation in which fixed sample models clearly do not apply is where the *observer* controls the time for which the stimulus is available. In such conditions, where the level of discriminability is held constant, the probability of a correct response, and its associated confidence, are both *inversely* related to the time taken (Vickers et al., 1985a; Vickers, Smith, Burt, & Brown, 1985b). The reversal of the relations between accuracy, confidence and the time for which a stimulus is inspected implies the operation of a quite different mechanism. This conclusion is strengthened by findings from the complementary set of experiments, in which the level of discriminability is varied unpredictably from trial to trial. In such cases, observers not only make more errors at the smaller stimulus differences, but also take longer and respond with lower confidence (Vickers, 1979). Such findings are inconsistent with the notion that observers decide beforehand to take a fixed number of observations.

RANDOM WALK AND ACCUMULATOR PROCESSES

The conclusion that observers can tailor the length of time for which they inspect a stimulus, without knowing beforehand the difficulty of the discrimination, requires a more sophisticated explanatory mechanism. One possibility, suggested in an early paper by Cartwright and Festinger (1943), is that the observer waits until the momentary fluctuations in sensory effect give rise to an observation of a certain minimum magnitude in favor of one response or the other. However, the finding that, with discriminability held constant, both the probability of a correct response, and the confidence with which responses are made, are inversely related to the time taken, is inconsistent with a memory-less process of this kind (Vickers, 1979).

Random Walk Models

One class of explanation, derived from statistical theory, and incorporating memory, is provided by *random walk* models, in which it is supposed that the momentary, (positive and negative) discriminial differences (or some function thereof) are summed over time until their sum attains a critical value (positive or negative), whereupon the process terminates, and the corresponding response is made. A model of this kind, based on the Sequential Probability Ratio Test of Wald (1947) was first proposed by Stone (1960) and developed by Laming (1968). By increasing the critical value which must be accrued, observers can improve their accuracy—at a

cost of requiring more observations (and taking longer to respond). In the case where the information accrued is the log-likelihood ratio of the stimulus alternatives, it can be shown that this process is statistically optimal, in that decisions at a certain level of accuracy will require, on average, a minimum number of observations (Wald & Wolfowitz, 1948).

Although mathematically elegant, this form of random walk model has a number of problems. The major difficulty is that it predicts that the times required to make a particular response correctly to a given stimulus should be the same as those required to make the alternative response (incorrectly). Empirical results do not conform to this simple pattern. Generally, when discriminations are difficult, and observers aim for accuracy, times for errors are longer than times for correct responses. Conversely, when discrimination is easy, and observers aim for speed, times for incorrect responses are similar to those for correct, and may sometimes be less (Vickers, 1980).

To account for the different relations between times for correct and incorrect responses, Link and Heath (1975) proposed an alternative random walk process, called Relative Judgement Theory (RJT). This version assumed that the random walk is driven by two distributions of increments, each derived from momentary differences between the sensory representation of the corresponding stimulus and some internal referent. Link and Heath showed that the relation between the predicted times for correct and incorrect responses depends upon the symmetry properties of the moment generating function of the distribution of increments to the walk. However, these authors did not explain how the moment generating function might vary with differences in the emphasis on speed or accuracy. Moreover, such a model does not apply to the results of experiments employing spatial arrays of randomly varying stimulus elements with known distributional characteristics (e.g., Smith and Vickers, 1988), or studies, such as that of Smith and Vickers (1989), in which subjects were presented with a rapid series of horizontal line segments of varying length, extending to the left or right of a central vertical line, representing zero. In this last experiment, line segments were generated by random sampling from one of two normal distributions of numbers. Positive numbers were represented by segments of proportionate length, extending to the right, and negative numbers by segments extending to the left. The task was to decide whether the distribution used to generate the segments on a particular trial had a mean which was positive or negative. Times for errors were significantly longer than those for correct responses, despite the fact that both distributions of sampled numbers were symmetric about zero. Such findings, which are typical of experiments using either spatial arrays or temporal

sequences of randomly varying elements (e.g., Pickett, 1967; Vickers et al., 1985a, 1985b), seem impossible to explain in terms of an RJT model.

An alternative form of random walk has been proposed by Ratcliff (1978). In Ratcliff's *diffusion* model, the assumption that sensory evidence is sampled discretely is replaced by the assumption that the evolution of the walk is continuous in time, so that this process constitutes a limiting case of the simple random walk (Cox & Miller 1965, ch. 5). While the diffusion model has the advantage of being (relatively) tractable, it also has a number of disadvantages. Firstly, the model has been used predominantly to account for the time required to recognize items in memory. Consequently, the input which drives the diffusion process is treated simply as a "bundle of information", which allows "some quantitative assessment of the extent to which certain (possibly featural) information is represented in the memory trace" (Ratcliff, 1978, p. 62). How such a general conception of "relatedness" would apply to the simple case of judging v and s to be equal is not clear. Secondly, no detailed account of a basis for confidence ratings has yet been developed. According to Ratcliff (1978, p. 96), "the only information available about the 'strength' of the item during the recognition process is in the comparison time". As a result, Ratcliff suggests that confidence may be assumed to be an inverse function of the time required to reach a decision. However, unless supplemented (in ways which are not obvious) by additional assumptions, this hypothesis does not account for experiments, such as that of Vickers and Packer (1982), in which observers made slow, accurate and highly confident responses in one set of trials and fast, erratic and less confident responses in the other. Thirdly, as pointed out by Proctor (1986), no comprehensive account is offered of the way in which parameter values might be adjusted in order to produce the pattern of estimated values obtained in fitting empirical data to the model.

Accumulator Models

A second main class of explanation is provided by *accumulator* models. The major difference between such models and the above random walk processes is that, instead of being accrued as a single signed total, evidence in an accumulator process is integrated in a separate total for each response. An alternative characterization of such models would be as 'parallel stochastic integrators' (cf Smith, 1995). Such a description underlines a difference in the inspiration for such models, which is more neurophysiological than statistical. For example, Vickers (1970) cited evidence in favor of unidirectional rate sensitivity in biological systems (Clynes, 1961; 1967), where specific neuronal units are sensitive to one direction of change only.

While it is possible, in principle, to extend random walk models to deal with multi-choice tasks, this is rarely attempted in practice, and such conceptualizations are difficult to translate into a neurophysiological realization. No such problem exists for the accumulator approach, since it is a simple matter to add as many more accumulators as there are responses. Again, although an accumulator model embodies a 'sub-optimal' decision process, it is not severely so, and it has a compensatory feature: even when the variable and standard are extremely similar, an accumulator process will never get 'hung up' indefinitely in reaching a decision.

Two main forms of accumulator model have been developed: the simple accumulator, or 'recruitment' process of La Berge (1962), in which unit increments are added to each total, and the generalized accumulator model, proposed by Vickers (1970; 1979), in which the increments are continuously varying. Evidence that the recruitment process gives a poor account of response time distributions has been summarized by Vickers (1979), and so we will focus solely on the generalized accumulator model in what follows. Expressions for predicted response probabilities, times and confidence are provided by Smith and Vickers (1988; 1989), and comprehensive empirical comparisons have been presented by Vickers (1979; 1980; 1985). A detailed comparison between random walk and accumulator models is given by Vickers and Smith (1985).

An advantage of the generalized accumulator process is that it correctly predicts that mean response times for errors, made with a high degree of caution, should take longer than those for correct responses, while times for correct and incorrect responses, made with an emphasis on speed, should be comparable. If it is assumed that response threshold values fluctuate, then this model can also accommodate the finding that errors may sometimes be faster (Vickers, 1979, p. 213).

At the same time, differences in the nature of the experimental tasks, instructions and stimuli, combined with model flexibility, make it difficult to find response time orderings which cannot be explained by each model (albeit post hoc), using some combination of assumptions and parameter values (though see Vickers & Smith, 1985). Moreover, as Proctor (1986) has argued, it is unsatisfactory to claim post hoc fits to data in terms of elaborate configurations of parameter values, without also providing a set of hypotheses which constrain the ways in which these parameters can vary. For this reason, a more useful strategy of theory development is to consider how such models can be extended to apply to different tasks and to account for additional response measures, such as confidence. Since the accumulator model has been developed to give a satisfactory account of different tasks and measures, and has a comprehensive theoretical account of both confidence and criterion setting, we will focus on this model in what follows.

THREE-CATEGORY TASKS, SAME-DIFFERENT JUDGEMENTS, AND SIGNAL DETECTION

Although the recognition of invariance is arguably one of the most important achievements for an organism, little attention has been paid to explaining the process by which an observer can decide that two stimuli are *equal* with respect to some dimension. One reason is that it is difficult to develop a random walk process to deal with such decisions. The most obvious way, suggested by Nickerson (1969), is to suppose that the observer sets a time deadline for responding, such that, if neither a 'greater' nor a 'lesser' response threshold has been exceeded by the deadline, then an 'equal' response is made. However, as shown by Vickers (1975), one problem with this is that the empirical times for 'equal' and 'greater' (or 'lesser') responses depend upon the bias towards these respective responses, with times for 'equal' responses being either faster or slower than those for the other two response alternatives. Making the clock 'noisy' does not help, because this leads to the prediction that 'equal' responses are faster (or slower) than their comparative counterparts, at each level of stimulus discriminability. This is contradicted by data from three-category tasks, same-different judgements and signal detection (Vickers, 1979). Moreover, the notion that observers can naturally apply a deadline (as opposed to having to accumulate a predetermined amount of evidence before responding) seems implausible in view of the finding that observers in a discrimination study, employing backward masking, took *longest* to respond to those stimuli which were exposed for the *shortest* time (Vickers, Nettelbeck, & Willson, 1972).

To account for such tasks, the natural strategy, on an accumulator approach, is to suppose that the response threshold for the 'equal' accumulator should be compared with the sum of the (unsigned) totals of the momentary discriminial differences minus their modulus. Expressing the 'equal' response threshold in this way has the advantage that only the simplest processes of addition (excitation) and subtraction (inhibition) are assumed. In addition, the quantities tested in all three accumulators ('greater', 'lesser' and 'equal') are all commensurable, and hence are directly comparable. This formulation has the further attraction of being Bayesian in spirit, since evidence is directly accrued in favor of there being 'no difference', rather than this being a default conclusion from a failure to find some difference. Finally, any model which can account for performance in a unidimensional three-category task can also be modified to deal with signal detection and judgements of sameness and difference. All that is necessary is to assign different responses to the various decision outcomes.

CONFIDENCE

Rather than pursuing detailed quantitative comparisons between data and predictions, Vickers (1979) argued for an eclectic strategy of theory evolution. This involves developing a model to account simultaneously for multiple response measures, testing the conceptual validity of hypothesised parameters by comparing them with other measures of individual differences, and providing an account of the way in which parameters might be adjusted by the observer. For example, to account for confidence, all that is necessary in an accumulator model is to assume that the amount accrued for each alternative constitutes the *evidence* for that alternative. In addition, it is necessary to recognize that the question "How confident are you that A is the case?" implicitly involves some comparison "rather than B (or C, etc.)".

Given these assumptions, theoretical expressions for confidence in an accumulator model fall out naturally. On the *balance-of-evidence* hypothesis, proposed by Vickers (1979), the confidence with which a response is made in a two-choice discrimination is determined by the (actuarial) balance or difference between the amount of evidence accrued in favor of the response in question and that accrued in favor of the alternative. In multi-choice tasks, confidence in any one alternative is assumed to be determined by the arithmetic average of the confidence values for each of the component comparisons. That is, confidence in A (rather than B or C) is assumed to be equivalent to the average of the confidence in A (rather than B) and the confidence in A (rather than C).

As shown by Vickers (1979, ch. 6), the balance-of-evidence hypothesis gives a good qualitative account of the main features of confidence ratings in simple judgements. For example, it predicts the sigmoid relation generally found between confidence and discriminability, as discriminability is varied. With discriminability held constant, it predicts the inverse relation between confidence and response time, found when the observer controls the time for which stimulus information is available. Conversely, this hypothesis also predicts the finding that, when stimulus availability is limited by time, confidence is a direct function of the time for which the stimulus is made available (Vickers et al., 1985b). It predicts the apparent *underestimation* of accuracy shown by observers in unidimensional discrimination tasks, as well as the finding that confidence in errors is lower. As shown by Vickers and Packer (1982), the balance-of-evidence hypothesis also correctly predicts that confidence for responses made under an accuracy set is greater than that for responses made under a set for speed (contrary to traditionally accepted assumptions). More recently, Baranski and Petrusic (1994) have concluded that a balance-of-evidence hypothesis provides the

most promising account to date of the subjective calibration and resolution of confidence in perceptual judgements.

ADAPTATION AND SELF-REGULATION

Perhaps the weakest aspect of most of the stochastic latency mechanisms which have been developed to date is that they require the supposition of a *deus ex machina* to account for the way in which the various response parameters may be adjusted by the subject in response to changes in instructions, payoffs, or changes in the probability, range and discriminability of the stimulus alternatives. Various suggestions have been reviewed by Vickers (1985) and by Ratcliff (1987). According to Ratcliff (1987), "it is necessary to model the past history of a subject in order to account for criterion placement, and . . . this is a difficult task and one that is outside the scope of, yet is an important challenge to, current theory."

As pointed out by Vickers (1985), most approaches have attempted to account for observed effects in terms of some change in a single parameter of the hypothesized underlying decision process. For example, in the relative judgment model of Link and Heath (1975), a change in the bias towards one response rather than another is assumed to be indistinguishably reflected either as a change in the starting position of the random walk, relative to the two response thresholds, or as a shift of the response thresholds, relative to the starting point. This is the case whether such changes result from some knowledge that the two response alternatives are unequally likely or from a preference for making one response more carefully than the other.

In contrast, in the accumulator model, a change in the *expectation* that one stimulus will be more likely is represented by a shift in the starting point, while a change in the *relative caution* with which each response is made is represented by a change in the position(s) of the response threshold(s), with respect to zero. While these two representations give rise to equivalent predictions with respect to response probabilities and times, the pattern of predicted confidence measures is quite different in the two situations. Specifically, shifting the starting position towards threshold G should give rise to faster G responses, with an increased likelihood of making a G response (both correctly and incorrectly). However, G responses will also be made with *increased* confidence (since they benefit from a 'starting bonus', equivalent to the size of the shift in starting position). This bonus can be thought of as representing the observer's estimate of the prior odds of encountering one stimulus rather than the other, so that the confidence value for each trial constitutes an analogue of a Bayesian process of revision of opinion. In con-

trast, where the threshold G is shifted towards the starting position, the observer expects each stimulus to be equally likely, and there is no addition to the confidence evaluation for G responses. Indeed, the reduction of the G threshold will produce a *reduction* in the confidence with which G responses are made. As shown by Vickers (1985, pp. 85-90), empirical data from experiments manipulating expectancy and relative caution show exactly this pattern of response probabilities, times and confidence ratings.

Such an analysis seems intuitively plausible. It is possible for someone to appreciate that it is important to be more careful about reaching one decision than another, while understanding that either decision is equally likely to be correct. Less obviously perhaps, it also appears necessary to distinguish between a *conscious adjustment* on the part of an observer (in response, for example, to instructions or to explicitly provided information), and an *unconscious adaptation*, induced by an unannounced or imperceptible change in the sequence of presented stimuli. (Subjective reports indicate that observers remain unconscious of even quite large changes in the relative frequencies of two alternative stimuli, provided that the decrease is gradual.) For example, data summarized by Vickers (1985) show that, when the probability of one stimulus is gradually decreased, then—paradoxically—observers become progressively *more likely* to make that response, to take *less time* to make it, and to make that response with *greater confidence* (with converse changes for the alternative response). As also shown by Vickers (1985), quite different effects are obtained when the observer is given explicit information regarding the relative likelihood of the two stimulus alternatives.

Such changes are the opposite to those predicted by the ideal observer hypothesis of signal detection theory, or indeed by any theory of 'rational' adjustment to changes in expectation. However, they become intelligible if we assume that the observer is unaware of the change, and that the primary effect is on a parameter of the decision process that is not normally thought of as under conscious control, namely, the *referent*, in terms of which sensory representations are classified as favoring one response or the other. As suggested by Vickers and Leary (1983), these data can be explained if it is supposed that the referent, like an adaptation level, is based on the cumulative average of all the sensory intensities experienced up to the trial(s) in question.

AN ADAPTIVE GENERALIZED ACCUMULATOR MODULE BASED ON A REFERENCE LEVEL OF CONFIDENCE

While no fully comprehensive, integrated theory of adaptation, adjustment and self-regulation has yet been successfully formulated, the above

account suggests that several important ingredients already exist. In particular, Vickers (1978; 1979) has proposed that an important source of cognitive control over simple judgements could be provided by comparing the confidence (or balance-of-evidence) in favor of each response with a desired target level of confidence for that response. Discrepancies between the target and the actual level obtained could then be accumulated in a second pair of control accumulators, with positive discrepancies (where the target exceeded the actual level) giving rise to proportionate increases in the response threshold for that response, and negative discrepancies giving rise to corresponding decreases. A representation of such a process in information-flow terms is shown in Figure 1.

In this model, it is assumed the subject operates with a *target level of confidence*, C_i , which is susceptible to conscious adjustment in response to instructions. On each trial, the actual confidence, c_i , in the response which eventuates is compared with C_i , and the amount of over- or underconfidence is accrued in the respective secondary, or control accumulator. Overconfidence, t_o , is accumulated in one accumulator and underconfidence, t_u , in the other. As soon as one of these totals reaches a predetermined threshold, K , this triggers an internal adjustment. If a critical amount of overconfidence has been accrued, then the threshold, k_i , in the primary accumulator is reduced. Conversely, if a critical amount of underconfidence is accrued, then the primary response threshold is increased. The amount by which the threshold is increased or decreased is proportional to the difference between the amounts of over- and underconfidence accrued at the time that one of these totals reaches the critical amount, K . (In other words, the internal adjustments are themselves proportional to the 'confidence' that an adjustment is appropriate.)

The *coefficient of proportionality*, x , serves to determine the coarseness of control exerted by the secondary accumulators. It does this in conjunction with the thresholds (K_i) assumed for these accumulators. For the present, these are simply assigned a uniform, moderate value. (However, in principle, any parameter of any accumulator process could be altered by any other accumulator process.) For different individuals, the coefficient of proportionality may vary between one (e.g., 0.2) that produces minimal, trial-to-trial adjustments in the primary thresholds and one that gives rise to intermittent, more dramatic changes (e.g., 2).

Figure 1 provides the algorithm for a dynamic process in which the control structure has a fractal character, with iterative processes on different time scales giving rise to external (overt) or internal (adaptive) responses. Since the system involves only addition and subtraction (as analogues of excitation and inhibition), the quantities accumulated are commensurable throughout. There is consequently no obvious limit to the

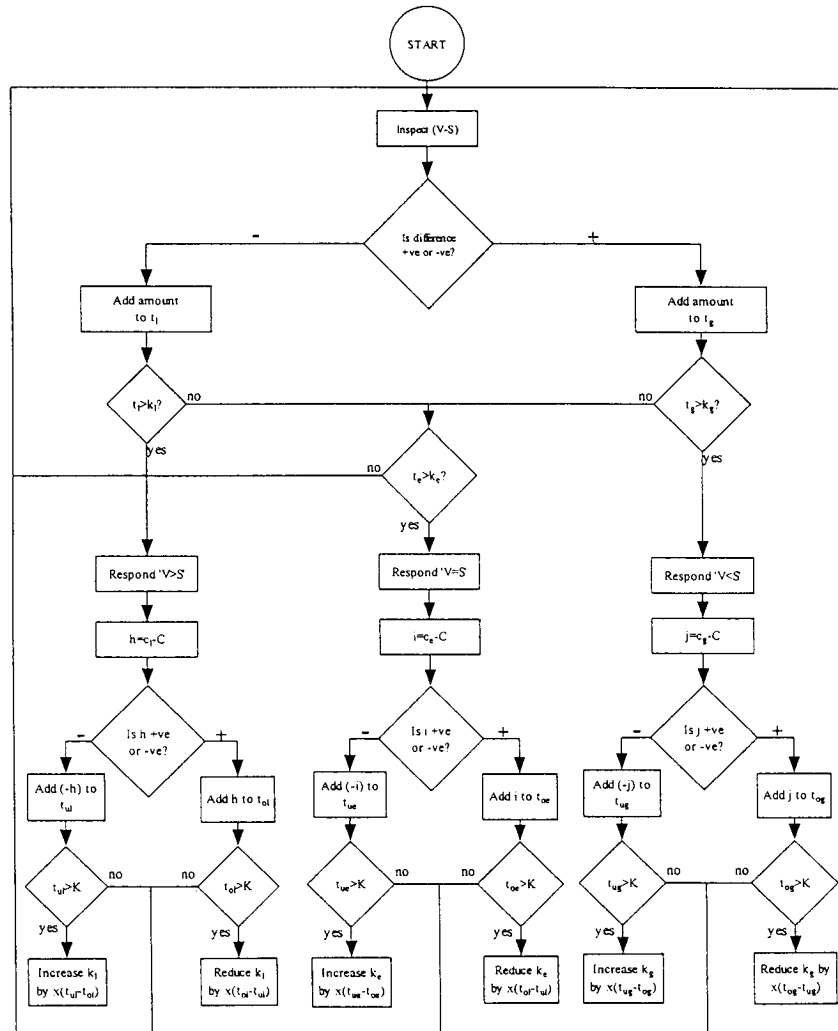


Fig. 1. A self-regulating accumulator module for a three-choice task. Values of target confidence, C , are here represented as equal, but can be supposed to differ in order to accommodate variations in relative caution for the three response alternatives.

potential of the module for rich connectivity with other modules. There is also no reason why the control structure of the module cannot be extended, with still higher-level accumulators controlling the parameters of those lower in the hierarchy. The limiting factor is the number of available accumulator units, since this rises as a function of 2^n , where n is the number of layers.

SELECTED PROPERTIES OF THE ADAPTIVE GENERALIZED ACCUMULATOR MODULE

Extensive computer simulations by Vickers (1978; 1979, pp. 201-237) have shown that the adaptive generalised accumulator module behaves as a stable, damped, negative-feedback control system, capable of incorporating variations in bias and caution, as well as making appropriate adjustments to response thresholds in answer to changes in relative probabilities, discriminabilities, sequential constraints and the range of difficulty in the sequence of presented stimuli. At the same time, the adaptive module is almost as simple as the static version. Changes in response bias are now interpreted in terms of changes in the relative values of the reference levels of confidence set for each response. Similarly, variations in overall caution depend upon whether the reference levels for all responses are set high or low. The reference levels of confidence are thus analogous to the response thresholds in the static model. Likewise, they are assumed to be susceptible to change by instructions or the manipulation of payoffs (as well as, perhaps, by other control processes).

As outlined so far, the adaptive module is capable of generating detailed predictions for response probabilities, times and confidence ratings in detection, discrimination and three-category tasks under a very wide range of conditions. What follows, therefore, is a necessarily selective characterization of some of its main properties.

Effects of Exposure to a Series of Discriminations of Randomly Varying Difficulty

When presented with a stationary series of discriminations of randomly varying difficulty, as in the psychophysical method of constant stimuli, the overall behavior of the adaptive generalized accumulator module resembles that of the static version, and response probabilities again show the classic sigmoid relation to stimulus difference.

For example, Figure 2 (top) shows data for response probabilities obtained in a computer simulation of 51,000 trials, in which the module was presented with a normal distribution of positive and negative magnitudes, representing ($V-S$) differences. A total of 51 different values of the mean $M(V-S)$, with a standard deviation of 1, and ranging from -2.5 up to $+2.5$, in steps of 0.1, were presented in random order, as in the psychophysical method of stimulus differences. Thus, there were approximately 1,000 simulations at each value of $M(V-S)$. The value of the secondary threshold, K , was held constant at 3 for all control accumulators. The coefficient of pro-

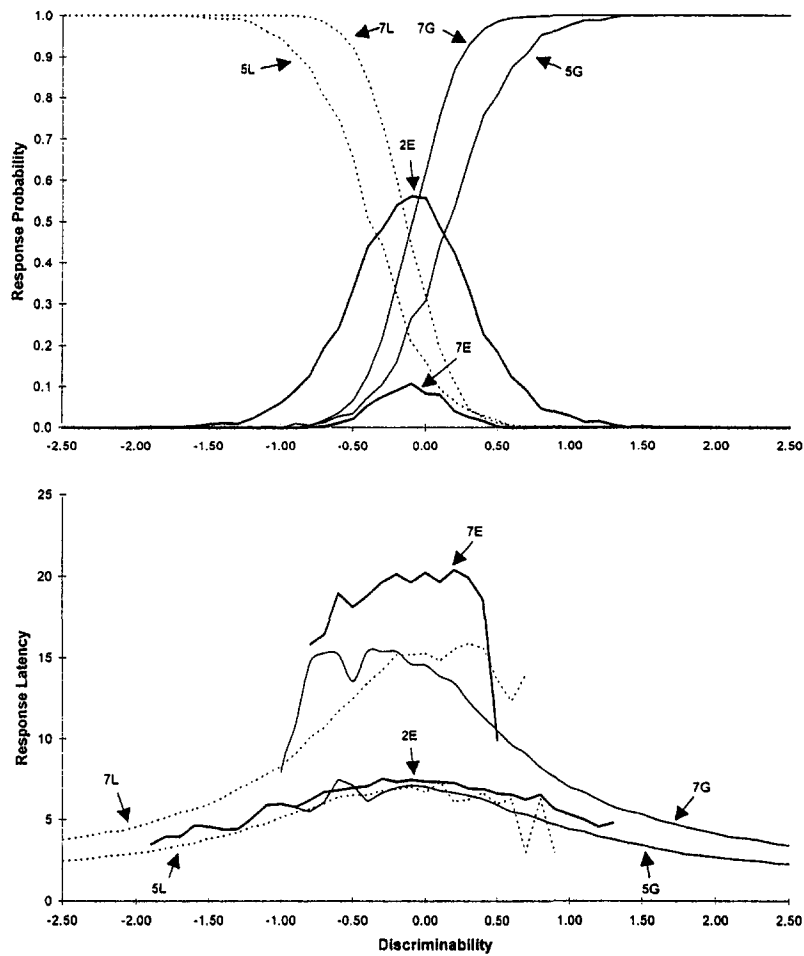


Fig. 2. Patterns of response probabilities (top) and latencies (bottom) produced by computer simulation of an adaptive accumulator module for three-category judgment, operating on a range of stimulus differences, with a mean which varies randomly from trial to trial, as in the method of constant stimuli.

portionality, x , was set at 0.5, which appears to provide quite frequent adjustments of moderate extent.

Figure 2 (top) shows how the probability of each of the three response alternatives varies with discriminability under two sets of values for the target levels of confidence (L, E, and G) for 'greater than', 'equal to', and 'lesser than' responses, respectively. Under the high value (7L, 7E, and 7G), the target values for confidence are all set to 7, while, under the low value

(5L, 2E, and 5G), target values for the 'greater than' and 'less than' responses are set to 5, while that for the 'equal to' response is 2. The data shown thus represent performance by an observer who is highly cautious overall, in comparison with that of one who is concentrating on speed, rather than accuracy, particularly with respect to the 'equal' response. Meanwhile, Figure 2 (bottom) shows the corresponding patterns of response times. Qualitatively, at least, these simulations capture the essential features of the empirical data for three-category judgments presented by Vickers (1975) and reviewed by Vickers (1979, ch. 4).

As in the static version, the steepness of the psychometric function for the 'greater' and 'lesser' responses varies with the reference level(s) of confidence set for each response, so that the model is capable of appropriately reflecting changes in response bias and caution, whether it is operating in two-category, signal detection, or three-category mode. However, the steepness of this function also varies inversely with the coefficient of proportionality, x , and the value of the threshold, K , assumed for the control accumulators, with these effects being greater with lower values of the reference level for confidence, C . That is, when the model makes frequent fine adjustments to the primary thresholds, accuracy is high, particularly when the target level for confidence is also high. Conversely, when adjustments are coarser and more intermittent, accuracy falls, particularly when the target level for confidence is low. Thus, the performance of the adaptive module is not only determined by variations in response bias and caution, but also by the control characteristics of the system.

If it is assumed that the provision of external feedback has the effect of exaggerating the threshold adjustments produced by internal feedback, then this might explain the otherwise surprising finding by McNicol (1975) that, whenever external feedback was provided, detectability measures were *reduced*. At this stage, however, it is not envisaged that the threshold values for the secondary (control) accumulators should be susceptible to conscious manipulation. Like the coefficient of proportionality, it is assumed that these are determined by *organismic* factors (such as limitations in short term memory capacity), that characterize the adaptive style of each individual subject. Some evidence consonant with this has recently been provided by a study of the discrimination of relative frequency by Vickers and Preiss (reported in Vickers, Neubauer, Jensen, Deary and Caryl, in press). These authors found that psychometric measures of intelligence were inversely related to individual differences in values of the correlation dimension, calculated from subjects' reaction time sequences. The correlation dimension quantifies the complexity of the trajectory of a time series, embedded in n dimensional space, with high values being symptomatic of an unconstrained, or random, series (Kaplan & Glass, 1995). In this context, Vickers et al. interpreted the

correlation dimension as varying inversely with the degree of constraint (or quality of control), exercised by subjects over their primary response thresholds. A similar explanation could account for the recurrent finding that performance in psychometric tests of intellectual ability tends to be negatively correlated with the number of errors made, as well as with both the mean and standard deviation of response latencies in a wide range of reaction time tasks (Nettelbeck, 1987; Brody, 1992).

Times for Correct and Incorrect Responses

As mentioned earlier, an important factor in differentiating between random walk and accumulator models has been the finding, in tasks employing difficult discriminations, that times for errors are generally longer than for correct responses. This relationship also characterises the behavior of simulations of the adaptive module, particularly when the control parameters, x and K , are both low and the reference level for confidence, C , is high. However, when both x and K are high, times for errors are slightly *lower* than for correct responses, particularly at high discriminability levels and with low values of C . This result is consistent with the finding of faster times for errors in a number of choice reaction time experiments employing highly discriminable stimuli and emphasizing speed rather than accuracy (Vickers, 1979; 1980). In terms of the adaptive module, coarse, intermittent adjustments of the primary response thresholds give rise to longer runs of trials in which lower, biased thresholds have an increased likelihood of resulting in faster responses that are less likely to be correct. This will be particularly likely when discriminations are relatively easy (requiring only low thresholds) and experimental conditions favor fast responding, with low values of the confidence reference level, C .

Effects of a Progressive Sequence of Stimulus Differences

The above effects are obtained when the adaptive module is presented with a stationary series of discriminations of randomly varying difficulty, as in the method of constant stimuli—a psychophysical method expressly designed to minimize any adjustment or adaptation on the part of the subject. More striking dynamic effects are to be expected whenever the simulated model is presented with a series of discriminations that is constrained in some way. For example, in the method of limits, the subject is presented with an ascending or descending series of stimulus differences between the variable and the standard, and reports, at each step in the series, whether the variable appears 'less than', 'equal to', or 'greater than' the standard.

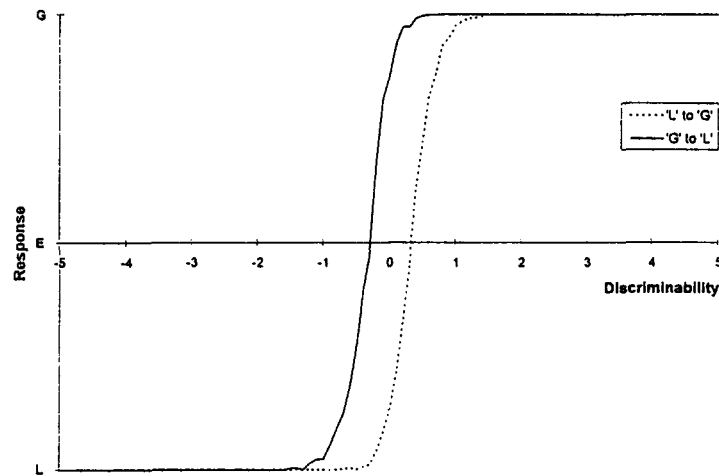


Fig. 3. The proportion of simulated subjects who responded V is 'greater than' S at each stimulus difference when the adaptive accumulator module is tested with ascending and descending series of stimulus differences, as in the method of limits.

The method of limits is well-known for producing so-called errors of 'habituation' and 'anticipation.' The former is the tendency to persist in responding 'lesser' beyond the point of objective equality in a series of ascending (or 'greater' in a descending) series of trials. The second is the converse tendency to change response too soon.

The method of limits in psychophysics resembles the feeding of a ramp input into a dynamical system, and is a recognized technique for revealing a tendency to overshoot or undershoot on the part of the system. Vickers (1979, pp. 220-227) has presented detailed results obtained by this method when the adaptive module is simulated, operating in two-category mode. For example, in an ascending series of trials, the variable, v , initially has a value substantially less than the standard, s . This value is then increased, in small steps, through the point of objective equality, up to a point where v is substantially greater than s . The simulations show that, when tested on an ascending series, the adaptive module begins by lowering the threshold for 'lesser' responses of the form ' $V < S$ '. As v approaches equality with s , thresholds for both 'lesser' and 'greater' responses are raised. Finally, past this point, the threshold for 'greater' responses is decreased, while that for 'lesser' responses remains high.

As shown by these simulations, the probability of making a 'greater' (or 'lesser') response is sensitive to the starting point of the series as well as the size of the steps by which v is increased. Whether these factors lead

to errors of habituation or anticipation depends also on the control parameters, x and K , as well as on the reference level, C . The effects of step size are most marked when the adaptive process makes frequent fine adjustments to the thresholds. When the module operates with low values of C (for both responses), it tends to produce errors of habituation. Conversely, with high values of C , errors of anticipation are more likely.

As an illustration, Figure 3 shows how the three-category version of the adaptive module responds when presented with a progressively increasing or decreasing series of discriminabilities (values of $M(V-S)$). As before, the value of K is set to 3, the value of x is 0.5, while that of the target confidences for the 'lesser', 'equal' and 'greater' responses are all fixed at the comparatively low value of 2. The figure shows how the proportion of 500 simulated subjects responding 'greater than' varies, according to whether the stimulus difference is being increased or decreased.

The point at which the adaptive module produces 50% of 'greater' responses is delayed on ascending trials, and persists on descending trials, beyond the point of objective equality. The error of habituation, shown by the module in this instance, is not due to any differential bias towards one or the other response (although it could be enhanced or cancelled by assuming different values of C for the two responses). Rather, it is a property of the dynamics of the self-regulating module. More generally, this differential response to increases and decreases, termed *hysteresis*, is a recognised signature of the operation of a nonlinear dynamical system (Van Geert, 1994).

Effects of a Step Change in the Prior Probabilities of Alternative Responses

A second type of input, designed to reveal instability in a dynamical system, is provided by a step change in discriminability or in the prior probability that one or the other response will be correct. With the exception of a handful of studies discussed by Vickers (1985), however, most psychophysical experiments assume static models of the underlying judgment processes, and few experiments have explicitly studied responses to abrupt changes in stimulus input. A possible exception is provided by studies of vigilance phenomena, where practice trials for detecting signals are followed by a protracted test session in which the objective probability of a signal is dramatically reduced. Typically, such experiments show a decline in the proportion of signals correctly detected, as well as in the proportion of false alarms. Accompanying these changes, an increase in the time for correct 'signal' responses and a decrease in the time for 'correct rejections' have also been reported (Davies & Tune, 1970).

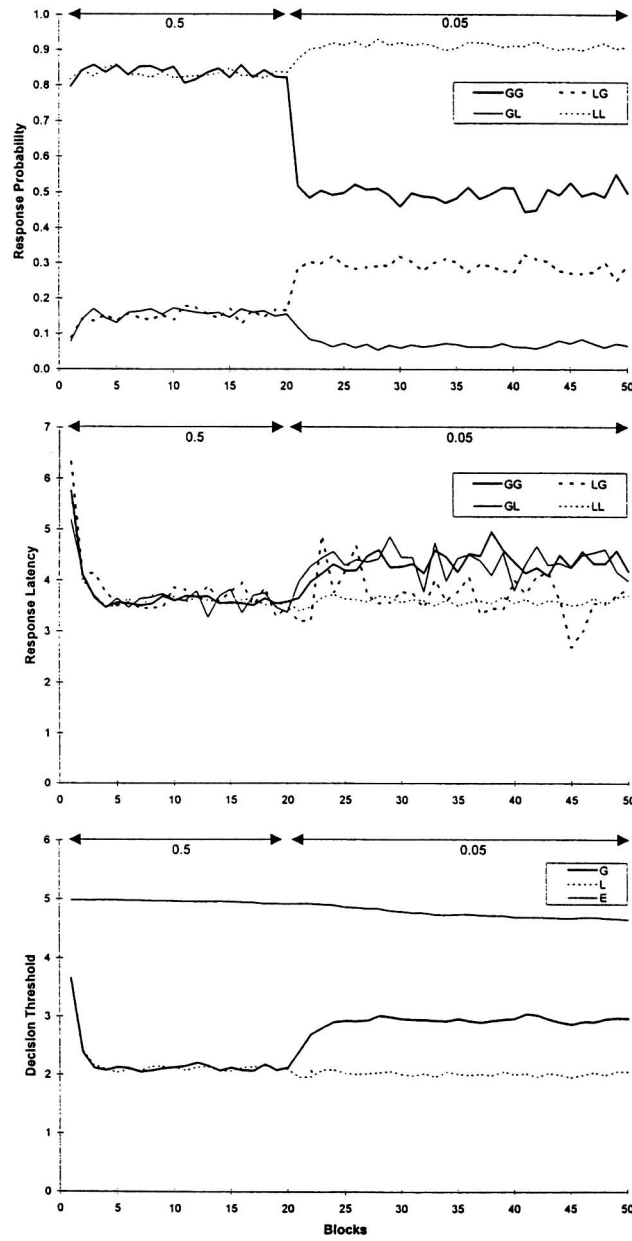


Fig. 4. Patterns of response probabilities, latencies, and response threshold values produced by computer simulation of an adaptive accumulator module for three-category judgment, operating on a stimulus difference of ± 0.5 , when the probability of an objectively 'greater than' stimulus undergoes a step reduction from 0.5 in early blocks of trials down to 0.05 in later blocks. The data shown represent averages over 200 simulated subjects. In this case, the target level of confidence for all three responses was set to 2.

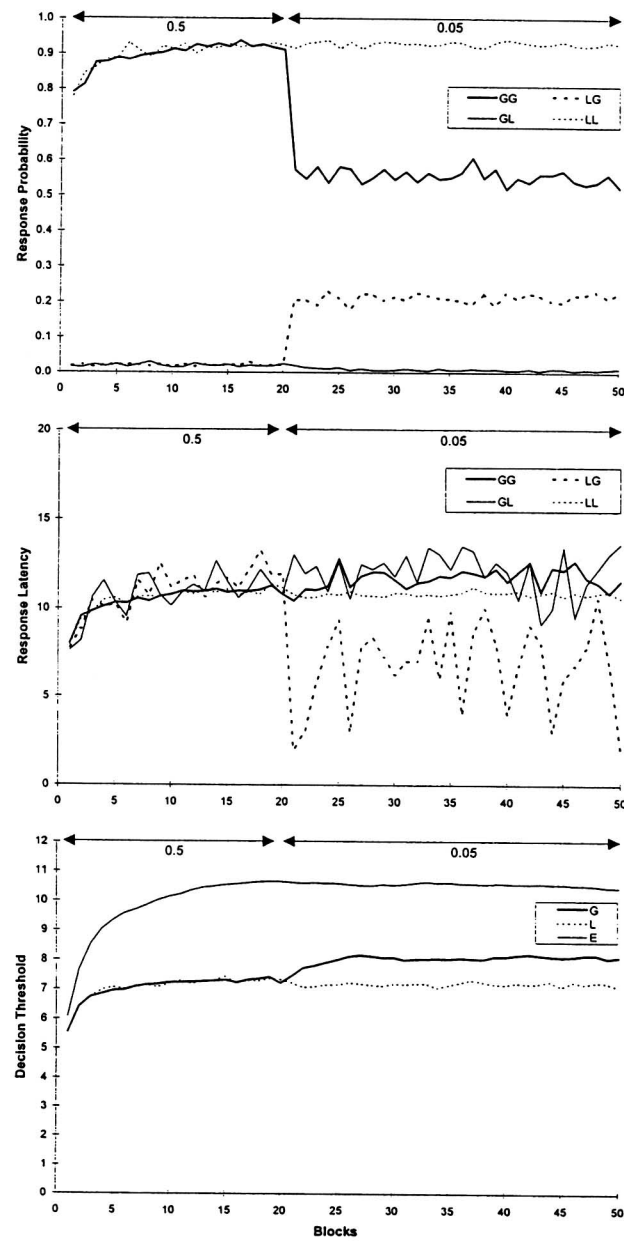


Fig. 5. Patterns of response probabilities, latencies, and response threshold values produced by computer simulation of an adaptive accumulator module for three-category judgment, operating on a stimulus difference of ± 0.5 , when the probability of an objectively 'greater than' stimulus undergoes a step reduction from 0.5 in early blocks of trials down to 0.05 in later blocks. The data shown represent averages over 200 simulated subjects. In this case, the target level of confidence for all three responses was set to 5.

Such changes can be easily mimicked by the adaptive module. For example, Figures 4 and 5 show the patterns of changes in primary response thresholds, response probabilities, and latencies when the objective probability of a 'greater than' stimulus is reduced abruptly from 0.5 to 0.05 over successive blocks of 10 trials. In each case, the value of $M_{(V-S)}$ is ± 0.5 , K is set to 3, and $x = 0.5$, while the initial value of the primary response thresholds (k_i , k_e , and k_g) was set to 5. The only difference between the Figures is in the value of the target level of confidence. In Figure 4, this is set to 2 for all three responses, while, in Figure 5, the value is set to 5. Comparing the two Figures, we can see that the initial adaptations in latencies and thresholds are in opposite directions. However, the response to the step reduction in the probability of a 'greater than' response is similar in both cases: the threshold for 'greater' responses increases, the probability of a correct 'greater' response declines, while the relative time required increases somewhat. Converse changes tend to occur for the 'lesser' response.

The above changes are qualitatively similar to empirical findings, as well as to those we would expect on the 'ideal-observer' hypothesis in the theory of signal detection. However, as argued above, in the discussion of adaptation and self-regulation, it is also necessary in practice to take account of other factors, such as unconscious shifts in subjective referents (in terms of which individual observations of the sensory input are classified as favoring one or the other response alternative). It is also necessary to consider the extent to which the subject is aware of any changes in the prior probabilities of alternative responses, since this may mean that other parameters of the decision process, such as the starting point of the accumulative process, may be adjusted.

CONCLUSION

If the proposed adaptive generalized accumulator module is supplemented by the supposition that the reference level is based on a cumulative average of sensory intensities, and that conscious expectation is represented by shifts in the starting point of the accumulation process, then it may be argued that we have the basis for a self-regulating, adaptive and adjustable decision module, which is capable of responding intelligently to any manipulation of the stimulus input and of displaying (albeit in miniature) all the essential ingredients of intelligent behavior: discrimination, recognition of identity, memory and adaptation. Since the information on which decisions and adjustments are based is commensurable throughout the system, any part of any one module can communicate with any part of any other.

This means that there is virtually unlimited flexibility in the way in which different modules can be configured into a more complex network.

At the same time, this flexibility and predictive richness presents two major challenges for further research. The first concerns the problem of analyzing the behavior of the adaptive module in ways that combine generality with quantitative testability. While the dynamic response to changes in experimental conditions seems likely to continue to pose problems for parameter estimation and model testing, there seem to be better prospects for characterizing the behavior of such adaptive systems under stable conditions. It is here that the tools of nonlinear dynamical analysis appear to hold promise, and current work is pursuing this.

The second challenge is to determine the extent to which the adaptive module can be linked together with other similar modules, as a 'cognitive tile' or basic computing element, to accomplish more complex tasks. This will be taken up in our second paper (Vickers & Lee, in press), in which we will focus on the behavior of a parallel array of such modules, which we term a Parallel Adaptive Generalized Accumulator Network, or PAGAN.

REFERENCES

- Baranski, J.V. & Petrusic, W.M. (1994). The calibration and resolution of confidence in perceptual judgments. *Perception and Psychophysics*, 55, 412-428.
- Bock, D.R., & Jones, L.V. (1968). *The measurement and prediction of judgment and choice*. San Francisco: Holden-Day.
- Brody, N. (1992). *Intelligence*. San Diego, CA: Academic Press.
- Cartwright, D. & Festinger, L. (1943). A quantitative theory of decision. *Psychological Review*, 50, 595-621.
- Clynes, M.E. (1961). Unidirectional rate sensitivity: A biocybernetic law of reflex and humoral systems as physiologic channels of control and communication. *Annals of the New York Academy of Science*, 92, 9946-9969.
- Clynes, M.E. (1967). Implications of rein control in perceptual and conceptual organization. *Annals of the New York Academy of Science*, 156, 629-670.
- Cox, D.R. & Miller, H.D. (1965). *The theory of stochastic processes*. London: Methuen.
- Crossman, E.R.F.W. (1955). The measurement of discriminability. *Quarterly Journal of Experimental Psychology*, 7, 176-195.
- Davies, D.R., & Tune, G.S. (1970). *Human vigilance performance*. London: Staples Press.
- Green, D.M., Birdsall, T.G., & Tanner, W.P. (1957). Signal detection as a function of signal intensity and duration. *Journal of the Acoustical Society of America*, 29, 523-531.
- Kaplan, D., & Glass, L. (1995). *Understanding nonlinear dynamics*. New York: Springer Verlag.
- La Berge, D. (1962). A recruitment theory of simple behaviour. *Psychometrika*, 27, 375-396.
- Laming, D.R.J. (1968). *Information theory of choice-reaction times*. London: Academic Press.
- Link, S.W. & Heath, R.A. (1975). A sequential theory of psychological discrimination. *Psychometrika*, 40, 77-105.
- McNicol, D. (1975). Feedback as source of information and as a source of noise in absolute judgments of loudness. *Journal of Experimental Psychology: Human Perception and Performance*, 104, 175-182.
- Nettelbeck, T. (1987). Inspection time and intelligence. In P.A. Vernon (Ed.), *Speed of information processing and intelligence*, (pp. 295-346). Norwood, NJ: Ablex.

- Nickerson, R.S. (1969). "Same-different" response times: A model and a preliminary test. In W.G. Koster (Ed.), *Attention and performance II*, *Acta Psychologica*, 30, 257-275.
- Pickett, R.M. (1967). Response latency in a pattern perception situation. *Acta Psychologica*, 27, 160-169.
- Proctor, R.W. (1986). Response bias, criteria settings, and the fast-same phenomenon: A reply to Ratcliff. *Psychological Review*, 88, 291-326.
- Ratcliff, R. (1978). A theory of memory retrieval. *Psychological Review*, 85, 59-108.
- Ratcliff, R. (1987). More on the speed and accuracy of positive and negative responses. *Psychological Review*, 88, 552-572.
- Schouten, J.F. & Bekker, J.A.M. (1967). Reaction time and accuracy. *Acta Psychologica*, 27, 143-153.
- Smith, P.L. (1995). Psychophysically principled models of visual simple reaction time. *Psychological Review*, 102, 567-593.
- Smith, P.L. & Vickers, D. (1988). The accumulator model of two-choice discrimination. *Journal of Mathematical Psychology*, 32, 135-168.
- Smith, P.L. & Vickers, D. (1989). Modeling evidence accumulation with partial loss in expanded judgment. *Journal of Experimental Psychology: Human Perception and Performance*, 15 (4), 797-815.
- Stone, M. (1960). Models for reaction time. *Psychometrika* 25, 251-260.
- Swets, J.A., Shipley, E.F., McKey, M.J., & Green, D.M. (1959). Multiple observations of signals in noise. *Journal of the Acoustical Society of America*, 31, 514-521.
- Tanner, W.P. & Swets, J.A. (1954). A decision-making theory of visual detection. *Psychological Review*, 61, 401-409.
- Taylor, M.M., Lindsay, P.H., & Forbes, S.M. (1967). Quantification of shared capacity processing in auditory and visual discrimination. *Acta Psychologica*, 27, 223-229.
- Thurstone, L.L. (1927a). Psychophysical analysis. *American Journal of Psychology*, 38, 368-389.
- Thurstone, L.L. (1927b). A law of comparative judgment. *Psychological Review*, 34, 273-286.
- Thurstone, L.L. (1928). The phi-gamma hypothesis. *Journal of Experimental Psychology*, 11, 293-305.
- Treisman, M. & Watts, T.R. (1966). Relation between signal detectability theory and the traditional procedures for measuring sensory thresholds: Estimating d' from results given by the method of constant stimuli. *Psychological Bulletin*, 66, 438-454.
- Van Geert, P. (1994). *Dynamic systems of development*. New York: Harvester Wheatsheaf.
- Vickers, D. (1970). Evidence for an accumulator model of psychophysical discrimination. *Ergonomics*, 13, 37-58.
- Vickers, D. (1975). Where Angell feared to tread: Response time and frequency in three-category discrimination. In P.M.A. Rabbitt & S. Dornic (Eds.), *Attention and Performance V* (pp. 455-469). London: Academic Press.
- Vickers, D. (1978). An adaptive module for simple judgments. In: J. Requin (Ed.), *Attention and Performance VII*, (pp. 599-618). Hillsdale, NJ: Lawrence Erlbaum.
- Vickers, D. (1979). *Decision processes in visual perception*. New York: Academic Press.
- Vickers, D. (1980). Discrimination. In A.T. Welford (Ed.), *Reaction times*, (pp. 25-72). New York: Academic Press.
- Vickers, D. (1985). Antagonistic influences on performance change in detection and discrimination tasks. In: G. d'Ydewalle (Ed.), *Cognition, information processing and motivation*, (pp. 79-115). Amsterdam: North-Holland.
- Vickers, D., Burt, J., Smith, P.L., & Brown, M. (1985a). Experimental paradigms emphasising state or process limitations: I. Effects on speed-accuracy tradeoffs. *Acta Psychologica*, 59, 129-161.
- Vickers, D. & Leary, J.N. (1983). Criterion control in signal detection. *Human Factors*, 25, 283-296.
- Vickers, D., & Lee, M.D. (in press). Dynamic models of simple judgments: II. Properties of a Parallel, Adaptive, Generalized Accumulator Network (PAGAN) model for multi-choice tasks. *Nonlinear Dynamics, Psychology, and Life Sciences*.
- Vickers, D., Nettelbeck, T., & Willson, R.J. (1972). Perceptual indices of performance: The measurement of 'inspection time' and 'noise' in the visual system. *Perception*, 1, 263-295.

- Vickers, D. & Packer, J.S. (1982). Effects of alternating set for speed or accuracy on response time, accuracy, and confidence in a unidimensional discrimination task. *Acta Psychologica*, 50, 179-197.
- Vickers, D., Neubauer, I.J., Jensen, A.R., Deary, I.J., & Caryl, P.G. (in press). Information processing and intelligence. In: Mervielde, I., Deary, I.J., De Fruyt, F., & Ostendorf, F., *Personality Psychology in Europe*, 7. Ghent, Belgium: Tilburg University Press.
- Vickers, D. & Smith, P.L. (1985). Accumulator and random-walk models of psychophysical discrimination: A counter-evaluation. *Perception*, 14, 471-497.
- Vickers, D., Smith, P.L., Burt, J., & Brown, M. (1985b). Experimental paradigms emphasising state or process limitations: II. Effects on confidence. *Acta Psychologica*, 59, 163-193.
- Wald, A. (1947). *Sequential analysis*. New York: Wiley.
- Wald, A. & Wolfowitz, J. (1948). Optimum character of the sequential probability ratio test. *Annals of Mathematical Statistics*, 19, 326-339.