The acquisition of linking theories: A Tolerance and Sufficiency Principle approach to deriving UTAH and rUTAH

Lisa Pearl & Jon Sprouse

Abstract

We investigate concrete acquisition theories for a derived approach to linking theory development, and explore to what extent two prominent linking theories in the syntactic literature – UTAH and rUTAH – can be derived from the data that English-learning children encounter. We leverage a conceptual acquisition framework that specifies key aspects of the child's acquisition task, including realistic child-directed input and a cognitively-motivated mechanism for inference (the sufficiency threshold, derived from the Tolerance and Sufficiency Principles). We find that rUTAH can be derived while UTAH can't, if children derive their linking theories from their input as specified here. We discuss the implications of these results for both syntactic theory and acquisition theory.

keywords: linking problem, UTAH, rUTAH, argument from acquisition, Sufficiency Principle, Tolerance Principle

1   Introduction

Consider the following sentence: "The little girl blicked the kitten on the stairs." Even if we, as adult speakers, don't know what "blick" means, we still have preferences about how to interpret this sentence. In particular, out of all the logically possible interpretations involving the little girl, the kitten, and the stairs, we prefer an interpretation where the little girl is doing something (blicking) to the kitten, and that event is happening on the stairs. The reason we as adults have this preferred interpretation is because we've solved the *linking problem*. That is, we have *linking theories* that link the thematic roles specified by a verb's lexical semantics to the syntactic argument positions specified by that verb's syntactic frame. Moreover, our linking theories are so well-developed that they can impose these links even when we don't know a verb's specific lexical semantics (as we see here with "blick").

Here, we ask to what extent two prominent linking theories in the syntactic literature, the Uniformity of Theta Assignment Hypothesis (**UTAH**) and the relativized Uniformity of Theta Assignment Hypothesis (**rUTAH**), can be derived from the kind of data that English-learning children encounter. We leverage a conceptual acquisition framework that specifies key aspects of the child's

acquisition task, including realistic child-directed input and a cognitively-motivated mechanism for inference that comes from the Tolerance and Sufficiency Principles (Yang, 2005, 2016, 2018).

The paper is organized as follows. We first discuss motivations for computationally investigating the derivation of linking theories. We then turn to our concrete investigation of derived approaches for linking theories. We begin by identifying possible acquisition targets, in the form of specific linking theory variants defined by UTAH and rUTAH. We then discuss how these acquisition targets impact the potential acquisition process. We subsequently review key acquisition modeling components and how they are implemented in the acquisition task of deriving linking theories. These components include (i) the learner's initial knowledge state that defines the hypothesis space for linking theories, (ii) the data the learner utilizes for acquisition, drawn from syntactically- and thematically-annotated English child-directed speech, and (iii) the inference process that leverages the Tolerance and Sufficiency Principles to yield the appropriate target knowledge state. Our results suggest that relativized approaches to linking theories like rUTAH are possible to derive from realistic English child-directed speech using the acquisition process we specify; this contrasts with fixed approaches like UTAH, where derivation fails. We conclude by discussing the implications of our results for both acquisition theory and syntactic theory.

2   Motivations

Linking theories are particularly interesting representations because they form the foundation of the interface between syntax and semantics. Linking theories are thus deeply embedded within theories of adult grammars, and consequently have been a central topic of investigation for theories of language acquisition. However, as two anonymous reviewers and Jeff Lidz observe, there's a robust acquisition literature arguing that linking theories must be innately specified (reviewed in detail below), and this literature is substantially larger than the literature exploring derived linking theories (e.g., Bowerman, 1988; Olguin & Tomasello, 1993; Goldberg, 1995; Akhtar & Tomasello, 1997; Dodson & Tomasello, 1998; Akhtar, 1999; Abbot-Smith, Lieven, & Tomasello, 2001; Goldberg, 2006; Boyd & Goldberg, 2011; Goldberg, 2013). Readers might therefore question whether linking theory knowledge needs to be derived at all, if its origin is already settled (as innate). In this section, we therefore review several key motivations for our investigation on the derivability of linking theories, recognizing the literature supporting innate linking theories and how the current study may contribute to that literature. We then turn to the details of our current study in section 3.

2.1   Investigating derivability in general

Here we briefly lay out our general beliefs about the value of investigating the derivability of representational components (like linking theories). First, from the perspective of representational theories (e.g., syntax), we believe it's useful to know which theoretical components are in principle derivable, and which don't seem to be; components that aren't derivable must be innate. Researchers can use this information to craft representational theories according to their overall goals (e.g., crafting a theory with minimal vs. more innate scaffolding). This reduces the possibil-

ity that theories will contain incongruous components (e.g., a representational theory intended to have minimal innate scaffolding whose components aren't derivable).

Second, from the perspective of acquisition theories, we believe it's useful to know which learning frameworks can derive which representational components from realistic child input (e.g., Pearl, 2017; Pearl, Ho, & Detrano, 2017). This information helps to define (and refine) the hypothesis space of potential representations that children navigate when acquiring language, as well as the target representation. This information also identifies which learning mechanisms (as implemented by specific learning frameworks) are effective for navigating a particular hypothesis space to identify a particular target representation.

Third, we take the view that proposals for innate scaffolding are special cases in acquisition theories – they're proposals of last resort because we can't (currently) observe innate knowledge directly. So, the general direction of investigation is to minimize or eliminate innate scaffolding proposals when (empirically) possible. Results that show us which representational components can be derived in principle – and which can't – contribute to that effort.

2.2   Syntactic bootstrapping and early knowledge of semantic-syntactic links

2.2.1   About syntactic bootstrapping

A leading theory for the acquisition of verb meaning is syntactic bootstrapping, first proposed by Landau and Gleitman (1985) and later elaborated by Gleitman (1990). Prior to this proposal, the dominant perspective on how children learned word meaning focused on strategies for inferring the correct meaning from real-world observation. An implicit assumption of this approach was that word meaning was learned prior to the acquisition of syntax (and compositional semantics). Landau and Gleitman highlighted serious issues with this assumption by focusing on verb meaning (as opposed to the prior literature's focus on noun meaning); they offered syntactic bootstrapping as an alternative approach.

In particular, Landau and Gleitman argued that there are certain pairs of verbs whose meanings have subtle differences, and these differences seem difficult to acquire just from the information in the real-world environment accompanying an utterance. One particularly relevant pair discussed by Gleitman (1990) is *chase* and *flee*. These verbs differ only in the perspective of the action, as realized in the subject position: in *The cat chased the lizard*, the subject (*the cat*) is the pursuer; in *The lizard fled the cat*, the subject (*the lizard*) is the evader. Importantly, the real-world context doesn't seem helpful for identifying this difference between *chase* and *flee*; all real-world contexts involving chasing also necessarily involve fleeing. For instance, the real-world context accompanying *The cat chased the lizard* involves a cat pursing a lizard and, necessarily, a lizard evading a cat. In this case, would *The cat VERBed the lizard* describe a pursuing action compatible with *chase* for the cat (i.e., if the *subject* is the AGENT), or an evading action compatible with *flee* for the lizard (i.e., if the *object* is the AGENT)?[1]

Still, Gleitman (1990) notes an observation by Steven Pinker (p.c.) that there can be some asymmetries in the accompanying real-world contexts across verbs; for instance, not all instances

---

[1]Here and throughout, we use SMALL CAPS to refer to thematic roles and *italics* to refer to syntactic positions.

of fleeing necessarily involve chasing: in *Lisa fled the city*, Lisa is fleeing, but the inanimate city isn't chasing. So, this utterance and situation would seem to pinpoint the evading perspective for *flee*, with the *subject* as the AGENT. With this information in hand, a child might then return to utterances like *The cat chased the lizard* with the knowledge that *chase* is unlikely to describe the evading action. However, corpus analyses by Gleitman (1990) suggest these utterances are relatively rare for *flee*, and so Gleitman suggested they might be too rare for children to reliably notice and leverage.

Gleitman (1990), reviewing work that ultimately appeared as Fisher, Hall, Rakowitz, and Gleitman (1994), instead suggests that the meanings of *chase* and *flee* could be reliably acquired if children already had some knowledge of the links between thematic roles and syntactic positions. Specifically, suppose children knew that the AGENT of the action links to the *subject* position; then, the real-world context, which uniquely indicates the pursuer and the evader, will also uniquely determine the meaning of the verb. For instance, let's return to the real-world context of a cat chasing a lizard and the lizard evading the cat. If children know the AGENT ↔ *subject* link, then *The cat VERBed the lizard* indicates the cat is the AGENT of VERB, and so VERB likely refers to the pursuing action (like *chase*); in *The lizard VERBed the cat*, the lizard is the AGENT of VERB, and so VERB likely refers to the evading action (like *flee*). In this way, prior knowledge of semantic-syntactic links (here, links between thematic roles and syntactic positions) could help children reliably learn verb pairs whose meanings differ solely in perspective, such as *chase/flee*, *buy/sell*, and *give/receive*.

More generally, syntactic bootstrapping inverted the worldview that had dominated acquisition studies since the earliest days of the field, by positing a role for syntax when children learn the meaning of individual words. That is, syntax isn't always learned after word meaning; in the case of verbs like these, syntax provides a way to identify word meaning. Because the process depends on children knowing semantic-syntactic links, the syntactic bootstrapping proposal has led to a large (and growing) empirical literature focused on (i) determining which types of semantic-syntactic links children would need to learn verbs, and (ii) precisely when in development that knowledge is available.[2]

2.2.2 Early linking knowledge

Importantly, there are many different types of semantic-syntactic links. Gleitman (1990) mentions some types (e.g., event meanings map to clauses, the AGENT links to the *subject* position). The types posited by the prominent linking theories we investigate here involve links between thematic roles (e.g., AGENT) and syntactic positions (e.g., *subject*). The empirical literature inspired by the syntactic bootstrapping proposal has uncovered at least two types of linking knowledge that children have access to early in development.

---

[2]We note for completeness that semantic bootstrapping, as first proposed by Grimshaw (1981) and elaborated by Pinker (1984, 1989), also proposes that children have access to semantic-syntactic links during early acquisition. In semantic bootstrapping, children use the meanings of individual words, combined with knowledge of semantic-syntax links, to learn complex syntax. We don't review the semantic bootstrapping literature here because its focus on the role of a single word as evidence during acquisition has generated less empirical literature on children's knowledge of semantic-syntactic links.

First, children can use the number of arguments selected by a verb to guide hypotheses about its meaning. A classic example is that children associate transitive constructions with causative meanings and intransitive constructions with non-causative meanings (e.g., L. Naigles, 1990; L. G. Naigles & Kako, 1993; Fisher et al., 1994; Fisher, 1996; L. Naigles, 1996; Fisher, 2002; Lidz, Gleitman, & Gleitman, 2003; Yuan & Fisher, 2009; Arunachalam & Waxman, 2010, 2011; Noble, Rowland, & Pine, 2011; Yuan, Fisher, & Snedeker, 2012). That is, *Lisa VERBed the penguin* is associated with Lisa causing something to happen to the penguin, while *Lisa VERBed* isn't. This finding suggests, at the very least, that children have an early bias toward a 1:1 mapping between thematic roles and syntactic arguments. So, if *Lisa* and *the penguin* are mentioned in the syntactic frame of VERB, then children expect the verb to have an event meaning involving two participants; if only *Lisa* is mentioned in the syntactic frame, then the event meaning will involve only one participant. This is a critical component of the prominent linking theories we investigate (as discussed more in section 6.3).

Second, children appear to know explicit links between thematic roles and specific syntactic arguments. Importantly for our purposes, this is the type of semantic-syntactic link we investigate the derivation of. So, it's useful to know the precise nature of the thematic-syntactic links children have access to, and how early this linking knowledge seems to be available. There are several key findings in the empirical literature (for deeper reviews, see Fisher, Jin, and Scott (2020) and Lidz (2020)). First, two-year-olds map intransitive verbs to AGENT-oriented or PATIENT-oriented meanings based on the subject's animacy (e.g., Bunger & Lidz, 2004, 2008). Second, two-year-olds map the *subject* of transitive verbs to AGENT and the *object* to PATIENT (e.g., Fisher et al., 1994; Gertner, Fisher, & Eisengart, 2006; Noble et al., 2011). Third, 16-month-olds show some evidence of mapping prepositional objects to INSTRUMENT roles for specific prepositions (e.g., *with*) (Lidz, White, & Baier, 2017). Taken together, these findings suggest that specific semantic-syntactic links (sometimes specific to a particular lexical item like *with*) are in place at early stages of language development.

2.2.3 What could still be derived and why we should try

Though we're personally sympathetic to the possibility that linking knowledge is innate, particularly given the empirical results mentioned above, we believe there are still reasons to investigate the derivability of linking theories like those that are prominent in the syntactic literature. First, Gleitman (1990) herself is explicit that the linking knowledge used for syntactic bootstrapping doesn't have to be innate; this knowledge just needs to be available before verb learning occurs. So, deriving linking knowledge is compatible with the syntactic bootstrapping approach; in particular, more rudimentary linking knowledge (e.g., lexically-specific or frame-specific) could be available earlier and more complex linking knowledge (e.g., constellations of links between thematic roles and syntactic positions irrespective of verb) could be derived later.

Related to this idea, the prominent linking theories we investigate here are composed of multiple components (see section 6.3 for more discussion on this point). It's therefore possible that some components are innate while others are derived. For instance, Fisher, Gertner, Scott, and Yuan (2010) propose a theory where children innately expect a 1:1 mapping between thematic roles and syntactic positions, but derive other semantic-syntactic mappings.

Third, as Noble et al. (2011) discuss in detail, the empirical results of the syntactic bootstrapping literature aren't always so clear-cut. If there is indeed some debate about the empirical facts concerning when children know certain types of linking knowledge, modeling investigations like this one can add new information. In particular, findings on the derivability of certain types of linking knowledge from children's input at different ages can be compared against empirical data on when children seem to have access to that linking knowledge.

Finally, we note that the argument for innateness of linking theories in the empirical literature is an "argument from earliness": the knowledge of linking theories is available too early to be derived. Arguments like this are strengthened by testing them. To test this argument for specific linking theories, we can attempt to construct learning approaches capable of deriving those linking theories in developmentally-plausible ways. The approach taken here is a typical first step in this process, considering if it's possible to derive specific linking theory knowledge from realistic child-directed input.

2.3   The universality of the primary linking pattern

Another prominent argument for the innateness of linking theories comes from within the adult theoretical linguistics literature. The empirical observation is that there appears to be one dominant linking pattern (a **primary pattern**) that emerges for the vast majority of verbs in accusative languages: AGENT-like thematic roles tend to appear in syntactic *subject* position (AGENT ↔ *subject*), PATIENT-like thematic roles tend to appear in syntactic *object* position (PATIENT ↔ *object*), and INSTRUMENT/SOURCE/GOAL-like roles tend to appear in oblique syntactic positions such as *indirect object* or *object of a prepositional phrase* (INSTRUMENT/SOURCE/GOAL ↔ *oblique*). Deviations from this pattern tend to be constrained to a small set of semantically-restricted classes (e.g., psych verbs like *love* in English). This cross-linguistic uniformity can be directly explained if these patterns arise from innate knowledge of a linking theory (Fillmore, 1968; Carter, 1976; Baker, 1988; Perlmutter & Postal, 1984; Larson, 1990; Speas, 1990; Grimshaw, 1990). More specifically, it's not surprising to see uniformity in the linking pattern across human languages if humans innately know this linking pattern; that linking pattern would naturally be the default.

As with the empirical arguments from the acquisition literature, we're sympathetic to the argument that the universality of the primary pattern is best explained with innate knowledge. Yet, even in the face of this apparent universality, we again believe it's worthwhile to investigate the derivability of linking theories. Importantly, the universality has never, to our knowledge, been precisely quantified. More specifically, we know the primary pattern isn't perfectly universal because there are exceptions. How universal the primary pattern truly is could matter for the debate between innate, domain-specific scaffolding and other logically possible explanations (though we note that we're not aware of any concrete proposals for these alternative explanations). Still, a high degree of universality would be in line with innate, domain-specific scaffolding. In contrast, a lower degree of universality might be more compatible with other options that derive linking patterns from less-specific innate knowledge.

6

2.4   The choice of UTAH and rUTAH as test cases

For this study, we selected two prominent linking theories from the syntactic literature as case studies: the Uniformity of Theta Assignment Hypothesis (**UTAH**) and the relativized Uniformity of Theta Assignment Hypothesis (**rUTAH**). While we describe UTAH and rUTAH in detail in section 3, we want to first briefly motivate the selection of these two theories as case studies. We have three reasons.

First, UTAH and rUTAH are well-defined theories with broad empirical coverage of phenomena in adult grammars. Because they're well-defined, UTAH and rUTAH can be easily translated into target knowledge states for a developmental computational model. Because they have broad empirical coverage, UTAH and rUTAH are meant to apply to the verbs children would encounter in their input, and so are plausible target knowledge for children to learn. Though it would be useful to also explore concrete theories that have been proposed in the derived approach to linking theories, we couldn't find any with the same specificity and broad empirical coverage. So, we instead focus on UTAH and rUTAH.

Second, UTAH and rUTAH contain cognitively-plausible components that any linking theory (derived or innate) is likely to contain: thematic roles, syntactic positions, and the links between thematic roles and syntactic positions. So, we believe that any results here would be informative to researchers investigating other linking theories (or other specific implementations of UTAH and rUTAH).

Third, UTAH and rUTAH represent distinct locations in the space of possible linking theories – and are diametrically opposed along the fixed-relativized dimension, as discussed in more detail in section 3 below. So, we can presumably gain information about linking theories that differ along this dimension, even though we investigate just two linking theories.

We want to reiterate that we're exploring these two distinct linking theories with respect to their derivability from realistic child input because we're interested in making concrete the learning theory that needs to accompany each linking theory. That is, we ask if UTAH is derivable from realistic child input; this informs us about the innate scaffolding required for UTAH's acquisition. We ask the same for rUTAH, with the same goal. If, as we will see, one theory is derivable from realistic child input and the other isn't, this is a qualitative difference between the two theories. From this qualitative difference, we conclude that the first theory must have significant innate scaffolding for successful acquisition to occur (under the plausible learning assumptions we use) while the second doesn't have to.

2.5   Modeling choices and modeling conclusions

There are two primary benefits of developmental computational modeling. First, a model forces us to be explicit about all the components of the theory it encodes, both representations and learning mechanisms. Second, a model allows us to see complex interactions of those components which may not be clear without running the model. (See Pearl (in press) for more discussion of these benefits.) Importantly here, developmental computational modeling allows us to see whether UTAH and/or rUTAH (the linking theory representations) are derivable from realistic child input, under the plausible learning assumptions captured by the Tolerance and Sufficiency Principles (the

learning mechanisms). It's not obvious beforehand if they are or aren't.

The primary limitation of developmental computational modeling is the specificity of the conclusions that can be drawn. In particular, when modeling acquisition, all results are conditional on the specific learning assumptions in place. If the target knowledge is acquirable, we have an existence proof for how that process would work, given those learning assumptions. In contrast, if the target knowledge isn't acquirable, we can only say that it's not acquirable under those learning assumptions. This dependence on the learning assumptions means that it's very important to motivate any learning assumptions the model incorporates if we wish to have useful results. We describe the learning assumptions of our model in detail in section 4, but briefly mention here the three most prominent ones.

First, the model's input is realistic child-directed speech sampled from the CHILDES database of child-adult linguistic interactions (MacWhinney, 2000). This assures that the model is attempting to derive UTAH or rUTAH from the same type of data English children do. We note that there are known differences between child-directed and adult-directed speech at many levels of linguistic representation (see Ma, Golinkoff, Houston, and Hirsh-Pasek (2011) for a review of differences at the prosodic, lexical, and structural levels); so, it's preferable to use developmental model input based on known child-directed speech when available.

Second, the model's input includes accurate phrase structure information and thematic role information (Pearl & Sprouse, 2013b, 2019). This means we model an idealized child who has accurate representations of the syntactic percepts (i.e., the syntactic positions) and the semantic percepts (i.e., the thematic roles), and must derive the appropriate linking theory knowledge from this information. This idealization allows us to investigate if UTAH and rUTAH are derivable in the best-case scenario where the syntactic and thematic role information aren't noisy. If not, then it seems unlikely (though still possible) that the linking theory knowledge would be derivable with more child-like syntactic and semantic percepts. In contrast, if the linking theory knowledge is derivable, the next step would be to see if that's still true when the syntactic and semantic percepts are more rudimentary. We reiterate that this approach sidesteps the (potentially difficult) question of how both syntactic and thematic information could be learned without an innate linking theory, as the syntactic and semantic bootstrapping approaches assume. However, we feel this is similar in spirit to those bootstrapping approaches, which focus on the derivability of semantic knowledge while assuming accurate linking theory knowledge and syntactic percepts (syntactic bootstrapping) or the derivability of syntactic knowledge while assuming accurate linking theory knowledge and semantic percepts (semantic bootstrapping). Here, we focus on the derivability of linking theory knowledge while assuming accurate syntactic and semantic percepts.

Third, the model's inference mechanism is based on the Tolerance and Sufficiency Principles (Yang, 2005, 2016, 2018), which have several desirable properties (discussed in more detail in section 4.3.1). First, they are cognitively-grounded decision criteria; second, they seem well-suited to a learning problem where children must decide whether something is true in their language (i.e., linking theories like UTAH or rUTAH); third, they have been used in previous investigations that assess whether certain linguistic knowledge can be derived from realistic children's input (Yang, 2005; Legate & Yang, 2013; Schuler, Yang, & Newport, 2016; Yang, 2016; Pearl et al., 2017; Yang, 2017, 2018; Irani, 2019); fourth, they seem appropriate for investigating the derivability of

linking theories because researchers rarely (if ever) quantify what it means for the linking pattern captured by these theories to be "common" or "primary". As will be discussed in section 4.3.1, the Tolerance and Sufficiency Principles do exactly this: they quantify what it means for a pattern to be "common" enough. More specifically, these principles provide a threshold for determining when a pattern is sufficiently present in the data to be learned as a rule because the number of exceptions is low enough for the rule to tolerate them. That said, relying on these principles means our results are therefore true only for modeled children using this (plausible) inference mechanism; it's possible we might find different learning outcomes using different inference mechanisms.

We believe that our specific modeling choices are well-motivated and so will lead to informative results about the derivability of linking theories from children's input. However, our implementation is clearly only one possible way to investigate linking theory derivability; future work will be needed using models that make different, plausible modeling choices. If the same results are found, then we have further support for the learning process theories that accompany these different linking theory representations. To facilitate this future modeling work, we have also made the input to our models public.[3]

3   Linking theories as the target of acquisition

Linking theories must have (at least) three components: a specification of the thematic roles in the grammatical system, a specification of the syntactic positions in the grammatical system, and at least one principle that governs the mapping between thematic roles and syntactic positions. Here, we will decompose the two linking theories we use as case studies – UTAH and rUTAH – into their three components, and review how each accounts for the primary linking pattern.

3.1   UTAH

Thematic roles.   The UTAH linking theory assumes a finite number of thematic roles that are typically defined in terms of semantic features, although there's significant debate about what those features should be, and even whether such a specification is possible (Fillmore, 1968; Perlmutter & Postal, 1984; Jackendoff, 1987; Baker, 1988; Grimshaw, 1990; Speas, 1990; Dowty, 1991; Baker, 1997). For concreteness, here we follow the specific UTAH implementation from Baker (1997). Baker's implementation posits three fixed thematic macroroles (similar to Dowty's (1991) proto-roles), which we will indicate with small caps: AGENT, PATIENT, and OTHER. It's agnostic about the existence of finer-grained thematic roles at a semantic level. All it requires is that any finer-grained typology of thematic roles map to the three macroroles. For example, for Baker (1997), thematic roles that tend to involve internal causation (Levin & Rappaport Hovav, 1995) map to AGENT, roles that tend to involve external causation (Levin & Rappaport Hovav, 1995) map to PATIENT, and all other roles map to OTHER. Example (1) lists 13 common finer-grained thematic roles from the literature, and how they would map to the three macroroles in this implementation.

---

[3]The input sets described in section 4.2 can be found at https://github.com/lisapearl/linking-problem-code in the *input-representations* sub-directory.

(1)   Baker's (1997) three fixed macroroles and 13 common finer-grained thematic roles

     a.   AGENT: agent, causer, experiencer (when internally-caused), possessor
     b.   PATIENT: patient, theme, experiencer (when externally-caused), subject matter
     c.   OTHER: location, source, goal, benefactor, instrument

Syntactic positions.   Baker's (1997) formulation of UTAH similarly posits three syntactic positions, which are defined by specific syntactic features – again, with much debate about the details of the syntactic theory. For our purpose here, we can abstract away from many of these details. What matters is that there's regularity in the syntactic positions that can be mapped to regularity in the thematic roles. To that end, we'll simply call the syntactic positions in this implementation of UTAH *subject*, *object*, and *oblique* (such as the object of a prepositional phrase), and use italics to indicate that these are theory-specific labels. We don't intend to imply that subject, object, and oblique are theoretical primitives, but instead use these as cover terms that readers can substitute with any relevant syntactic analysis (e.g., specifier of TP for *subject*).

Linking principle.   Baker's (1997) formulation posits a linking principle that governs the mapping between thematic roles and syntactic positions: the AGENT role maps to the syntactic *subject* position, the PATIENT role maps to the syntactic *object* position, and the OTHER role maps to *oblique* positions.[4] That is, there are three links that together form a single 3-link theory.

Accounting for the primary linking pattern.   With these three components in place, Baker's UTAH implementation can account for the primary linking pattern. In sentences such as *Jack cut the pie with a knife*, the AGENT appears in *subject* position, the PATIENT appears in *object* position, and the OTHER (the instrument) appears in an *oblique* position. Exceptions to this pattern, as in the *The package arrived*, where a PATIENT appears in *subject* position, are handled by a derivational grammatical system that includes a movement operation. The NP *the package* enters the syntactic derivation in *object* position, in accordance with Baker's UTAH system, and then is moved to the *subject* position at a later point in the derivation. In this way, apparent exceptions to the primary pattern are only exceptions on the surface; at an early stage of the syntactic derivation, UTAH is indeed respected.

3.2   rUTAH

Thematic roles.   The mapping between thematic roles and syntactic positions in UTAH is *fixed* in the sense that each thematic role will map to the same syntactic position in every construction; in contrast, one of the defining features of rUTAH is that the mapping between thematic roles

---

[4]Though it's not typically discussed in the syntax literature, creating an explicit model using UTAH requires deciding whether the links are unidirectional or bidirectional. Here we assume bidirectional links (e.g., AGENT links to *subject* and *subject* links to AGENT). This seems most plausible given that links can be used both for production (where a link from thematic role to syntactic position is useful) and for comprehension (where a link from syntactic position to thematic role is useful). That said, readers who prefer a unidirectional version of UTAH can simply evaluate the portion of our results for the preferred directionality. See Baker (1997) for more discussion of the issue.

and syntactic positions is *relative* (hence the name – *relativized* Uniformity of Theta Assignment Hypothesis: Larson, 1988, 1990; Grimshaw, 1990; Speas, 1990). To achieve this, rUTAH first assumes that thematic roles are arranged in a hierarchy, such that certain thematic roles are "higher" or "lower" on the hierarchy than other roles. Example (2) lists 13 common finer-grained thematic roles in a hierarchy derived from Larson (1988, 1990). One interesting feature of the Larson hierarchy is that finer-grained roles need not be strictly ordered relative to one another. We indicate this by placing unordered roles in parentheses.

(2)   Hierarchy derived from Larson (1988, 1990):
        agent > causer > experiencer > possessor >
        subject matter > causee > theme > patient >
        (location, source, goal, benefactor, instrument)

Given this hierarchy, any given thematic role in a specific sentence can be relatively defined within that specific sentence as the HIGHEST, SECOND HIGHEST, THIRD HIGHEST, etc. To avoid the repetition of the word highest, we'll call these FIRST, SECOND, and THIRD here.

Syntactic positions. rUTAH similarly assumes a relative hierarchy for syntactic positions, often defined in structural terms (e.g., by c-command relations). For example, one common c-command-based hierarchy applied to the Baker-style syntactic positions would be *subject > object > oblique*. Here, we'll refer to the relative syntactic positions as *first-syn*, *second-syn*, and *third-syn*.

Linking principle. rUTAH posits a linking principle that governs the mapping between the relativized thematic roles and the relativized syntactic positions: the FIRST thematic role maps to the *first-syn* syntactic position, the SECOND thematic role maps to the *second-syn* syntactic position, and so on.[5] So, as with UTAH, there are three links that together form a single 3-link theory. The difference is that in rUTAH, the links are defined over thematic roles and syntactic positions that are relative, rather than fixed.

Accounting for the primary linking pattern. One interesting feature of rUTAH is that, by implementing a relativized system, many of the apparent exceptions to the primary linking pattern cease to be exceptions. For example, the sentence *The package arrived* is an apparent exception to UTAH that requires a derivational grammar and a movement operation in Baker's (1997) system. But, under rUTAH, it's a paradigm example of the rUTAH mapping: the one and only thematic role, PATIENT, is the FIRST in the sentence, and it's mapped to the *first-syn* syntactic position, which is the *subject* position. There's no need for a movement operation (or, indeed, even a derivational grammar). This sentence is simply an example of the primary linking pattern. The fact that many of the exceptions to the linking patterns under UTAH become paradigmatic cases of the linking pattern under rUTAH will be particularly relevant for our acquisition models, as the Tolerance and Sufficiency Principles are directly concerned with the ratio of exceptions to paradigmatic cases for any hypothesized rule.

---

[5]As with the UTAH linking principle, we assume the rUTAH links are bidirectional.

3.3   UTAH and rUTAH for investigating derived acquisition approaches

As mentioned in the introduction, we focus on UTAH and rUTAH as case studies for modeling the acquisition of linking theories for several reasons. Again, the most important reason is that these theories are specified in fine enough detail to make the scope of the acquisition task clear. In particular, every acquisition theory for linking theories must include a specification of the thematic roles and syntactic positions in the system. These roles and positions then jointly contribute to a hypothesis space of potential links between roles and positions. Every acquisition theory must also include a bias to attend to links between roles and positions (i.e., the need to solve the linking problem must already be present in the child).

Moreover, every derived acquisition theory – that is, one involving derivation of the linking theory from the child's input data – must additionally include a procedure for generating explicit linking hypotheses to evaluate. We note that these hypotheses could be about basic links like AGENT → *subject* or the complex 3-link patterns that UTAH and rUTAH ultimately specify (see section 3.4 for more discussion). Similarly, every derived acquisition theory must specify a procedure for evaluating those hypotheses relative to the data. So, though concrete derived acquisition theories may make different choices in the details of each component (different roles, different positions, different hypothesis generation or evaluation procedures), the overall complexity of a derived acquisition theory, in terms of the number of components, is unlikely to vary too much.

Beyond clarifying the acquisition problem, UTAH and rUTAH have a number of properties that should make them of interest to researchers working in either innate or derived knowledge frameworks, as briefly mentioned in the introduction. We reiterate these properties here in light of the specific implementations of UTAH and rUTAH we investigate.

First, UTAH and rUTAH both involve complex linking patterns that are typically claimed to be innate; so, UTAH and rUTAH both offer the opportunity to explore if and how complex linking patterns could be learned from more general mechanisms. This can help determine how much innate scaffolding each linking theory requires for successful acquisition.

Second, UTAH and rUTAH both already include simplifying assumptions about the number of thematic roles and syntactic positions. These simplifying assumptions are cognitively plausible from the perspective of language development. That is, in order to map thematic roles to syntactic positions, children are likely to either (i) map the wide variety of fine-grained thematic roles in language to a small number of macroroles (as in UTAH), or (ii) view some roles as more salient than others, and order roles accordingly (as in rUTAH). Therefore, UTAH and rUTAH are likely to reveal information that is relevant to theorists exploring systems of differing complexity.

Third, UTAH and rUTAH are fairly far apart in the linking theory hypothesis space to the extent that fixed systems and relative systems are categorical opposites, and to the extent that UTAH and rUTAH are pure instantiations of these systems. That is, UTAH is fixed for all thematic roles and all syntactic positions; rUTAH is relative for all thematic roles and all syntactic positions. They thus define two poles in the linking theory hypothesis space, and will likely reveal information that is relevant to theorists exploring fixed systems, theorists exploring relative systems, and potentially even theorists exploring hybrid systems.

In summary, we believe that UTAH and rUTAH are excellent case studies for derived acquisition approaches because they're well-specified in the literature, cognitively plausible, and likely

to return useful information about whether the relevant linking knowledge can be derived from the input using a fixed vs. a relative system approach.

3.4   Linking theory options and acquisition implications

3.4.1   One 3-link theory means two acquisition stages

The linking principles described above for UTAH and rUTAH assume one 3-link theory. That is, this linking theory is a unit applicable to the verbs of the language, and it's made up of three individual links. So, if a verb doesn't follow any of the three links, it doesn't obey the linking theory.

To derive this linking theory, it seems that a child would need to go through two stages. First, she needs to derive the three individual links comprising the theory; second, she needs to derive the linking theory as a unit. How might children accomplish these two steps? One way is for children to assess the reliability of the linking hypotheses under consideration. Linking hypotheses that are reliable enough are maintained; linking hypotheses that aren't are discarded. In the first stage, a child could assess the reliability of individual links for the verbs of her language. If all goes well, this process would yield the appropriate three individual links out of all the logically possible ones; these individual links could then be composed into a single 3-link theory. In the second stage, a child could then assess the reliability of the 3-link theory for the verbs of her language.

3.4.2   Three 1-link theories means one acquisition stage

An alternative approach is to simply have three 1-link linking theories. That is, the individual links are themselves separable linking theories that apply to all the verbs of the language. So, for example, a verb might follow two individual links (e.g., AGENT ↔ *subject*, OTHER ↔ *oblique*) while not following a third (PATIENT ↔ *object*); in this case, the verb would obey two of the 1-link theories but not the third one.

Because the goal is three 1-link theories, there's no further need to create a more complex unit comprised of these individual links. So, to derive these 1-link linking theories, a child would need to go through only a single stage. She derives the three individual linking theories (i.e., the individual links), and she's finished. More specifically, she assesses the reliability of individual links against the verbs of her language. If all goes well, this process yields the three individual links of the appropriate 1-link linking theories. So, deriving three 1-link linking theories involves a simpler acquisition process than deriving one 3-link linking theory.

4   Acquisition components for deriving linking theories

The linking theory variant – whether one 3-link theory or three 1-link theories – serves as the target state for acquisition. This is the knowledge the child is attempting to derive from her input. To concretely investigate the acquisition process leading to that target state, we need to additionally specify other key acquisition components (Pearl & Sprouse, 2015, 2019; Pearl, in press): (i) the

initial state, comprising the knowledge, biases, and abilities the child begins with, (ii) the **acquisitional intake**, representing the data the child uses for updating her hypotheses, and (iii) the **inference** process, representing how the child updates her hypotheses on the basis of that acquisitional intake. We describe each in turn below, as they relate to deriving linking theories from an English child's input.

4.1 Initial state

As mentioned in section 3.3, a child trying to derive linking theories must already have a bias to attend to links between thematic roles and syntactic positions. Additionally, the child must begin already with certain knowledge: knowledge of the relevant thematic representations (fixed macroroles for UTAH or relativized roles for rUTAH), and knowledge of relevant syntactic positions (fixed syntactic positions for UTAH or relativized positions for rUTAH). With this in mind, the child can then define a hypothesis space of possible linking theories that connect those thematic representations (involving 3 roles) to those 3 syntactic positions.

Here, we further constrain the child's hypothesis space by allowing the child to have the following knowledge in her initial state:

- Links in the hypothesis space are unidirectional, and either go from role to position (e.g., AGENT → *subject*) or from position to role (e.g., *subject* → AGENT). We note that this allows a child to construct bidirectional links (e.g., AGENT → *subject* and *subject* → AGENT yield AGENT ↔ *subject*), but also allows for links to simply be unidirectional. So, if we assume unidirectional links are the child's hypothesis building blocks, the child has a larger hypothesis space of possible linking theories; these possible theories can include either unidirectional links or bidirectional links or both.

- Roles and positions can only participate in one link at a time. So, for example, AGENT and PATIENT can't both map to *subject* via the same unidirectional link, such as AGENT OR PATIENT → *subject*. Relatedly, *subject* and *object* can't both map to AGENT via the same unidirectional link, such as *subject or object* → AGENT.

These constraints lead to 18 individual links in the child's hypothesis space (3 positions x 3 roles x 2 directions). Children may then form 1-link or 3-link linking theories from these individual links.

In addition to the bias to attend to links and this knowledge defining the link types available, children also need whatever cognitive abilities are required to deploy this knowledge in real time, extract relevant information accurately enough from their input (such as the syntactic positions and thematic roles), and perform inference over that information to update their linking theory hypotheses. The exact nature of these cognitive abilities is outside the scope of this paper, but we note the necessity of these abilities for the acquisition process described here.

4.2 Data intake

We estimate English children's input from the child-directed speech data in the CHILDES Treebank (Pearl & Sprouse, 2013a, 2019), summarized in Table 1. This dataset contains realistic

samples of speech directed at American English children between one and five years old, annotated with linguistic and non-linguistic information. The portion of the CHILDES Treebank we used for this investigation involved ≈140K child-directed speech utterances from the BrownEve, BrownAdam, and Valian corpora (Brown, 1973; Valian, 1991) annotated with phrase structure information, animacy information, and the 13 mid-level thematic roles discussed in section 3. We divided these ≈140K utterances into age ranges based on the age of the child the speech was directed at: less than 3 years old (<3yrs), less than 4 years old (<4yrs), and less than 5 years old (<5yrs). We then constructed datasets representing the input to a child of a particular age.[6] We note that the datasets used as input for older children (e.g., <4yrs, representing a four-year-old child) include the data directed at younger children (e.g., <3yrs + data directed at children between the ages of three and four). This is because we assume that older children would learn from all the data they've heard up until that point.

We also note that it's currently unknown what age English children would acquire knowledge of the links implemented in UTAH/rUTAH (as opposed to more rudimentary linking knowledge). Because of this, we consider derivation of these linking theories at different ages (3, 4, and 5), as represented by these data intake sets.

Table 1: Child-directed speech data to three-year-old, four-year-old, and five-year-old English children. This includes the sources of these data in the CHILDES Treebank, the number of children the speech was directed at, the age range of the children the speech was directed at, the total number of utterances and words, the total number of verb types, and the number of verb types with 5 or more link instances in the dataset.

Dataset	Sources	children	ages	utterances	words	verbs	verbs >5
<3yrs	BrownEve, Valian	22	1;6-2;8	≈39.8K	≈197K	555	231
<4yrs	BrownEve, Valian, BrownAdam3to4	23	1;6-4;0	≈50.7K	≈254K	617	260
<5yrs	BrownEve, Valian BrownAdam3to4 BrownAdam4up	23	1;6-4;10	≈56.5K	≈285K	651	275

To derive linking theories, the relevant information for a verb's use are the thematic roles present and which syntactic position each role appears in. Therefore, we allow the modeled child's acquisitional intake to be the syntactic information corresponding to syntactic positions assumed by UTAH or rUTAH (*subject*, *first-syn*, etc.), the thematic information corresponding to the thematic roles assumed by UTAH and rUTAH (AGENT, FIRST, etc), and the syntactic positions the thematic roles appear in for each verb use, such as in (3).

---

[6]We extracted the verb lemmas by using python's WordNetLemmatizer package. The extracted lemmas were then manually checked by the first author, and child register verbs (e.g., *squoosh, squooshed, squooshing*) were resolved.

(3)   Examples of acquisitional intake

    a.   "The little girl kissed the kitten on the stairs."

        (i)   UTAH: *subject*=AGENT, *object*=PATIENT, *oblique*=OTHER

        (ii)   rUTAH: *first-syn*=FIRST, *second-syn*=SECOND, *third-syn*=THIRD

    b.   "The water is falling"

        (i)   UTAH: *subject*=PATIENT

        (ii)   rUTAH: *first-syn*=FIRST

To minimize data sparseness problems when assessing link reliability, we restrict our analyses – and therefore the child's acquisitional intake – to verbs that occur with at least 5 argument uses in the corpus. For example, consider a verb occurring in two utterances, one utterance with arguments in *subject* and *object* position (*She kissed the kitten*), and one utterance with arguments in *subject*, *object*, and *oblique object* position (*She kissed the penguin at the zoo*). This would yield 5 (2 + 3) total arguments across all utterances for this verb, and so this verb would be included in our analysis. Since each occurrence of an argument yields evidence for a link, we refer to an argument use of this kind as a "link instance" and we only include verbs with 5 or more link instances in our analyses.

From this corpus sample, we extrapolate the input that children of these ages encounter – in particular the quantity and distribution of verb link instances – using the procedure detailed in Appendix A. This procedure draws on Hart and Risley (1995, 2003) to estimate the amount of input children hear per hour, and Davis, Parker, and Montgomery (2004) to estimate how many hours per day children of different ages are awake. With this estimated child input, we can then apply the Tolerance and Sufficiency Principles to assess different linking hypotheses children could consider.

4.3   Inference

4.3.1   The Tolerance and Sufficiency Principles as decision criteria for reliability

Recall that the acquisition process we posited in section 3.4 involves the child assessing the reliability of different linking hypotheses on the basis of her input. One way to concretely model this process is to use the Tolerance and Sufficiency Principles (Yang, 2005, 2016, 2018). These principles together form a formal approach for determining when a child would choose to adopt a "rule", generalization, or default pattern to account for a set of items. Both principles are based on cognitive considerations of knowledge storage and retrieval in real time, incorporating how frequently individual items occur, the absolute ranking of items by frequency, and serial memory access. Importantly for our purposes, these principles are designed precisely for data where there are exceptions to the potential rule; the Tolerance Principle determines how many exceptions a rule can "tolerate" in the data before it's not worthwhile to have that rule at all; the Sufficiency Principle uses that threshold to determine how many rule-abiding items are "sufficient" in the data to justify having the rule. Therefore, the Sufficiency Principle, based on the threshold from the Tolerance Principle, provides a precise threshold for the number of sufficient items for a hypothesized rule or generalization, even if there are exceptions.

The intuition behind both principles is that the child is optimizing retrieval time. More specifi-cally, suppose a child is considering a rule that connects an item to some other information, such as a root connecting to its past tense form (Yang, 2005, 2016), a word connecting to its metrical stress pattern (Legate & Yang, 2013; Pearl et al., 2017), or thematic roles connecting to their syntactic positions (what we implement here). The potential rule compactly encodes some regularity – this is the pattern that several items in the dataset under consideration follow (e.g., default past tense morphology, a default stress pattern, or a default linking pattern).

When does it become useful to have a rule? One answer is that it's useful when having a rule makes the average retrieval time for any item in the dataset faster (Yang, 2005, 2016, 2018). That is, it's useful to have a past tense rule in order to retrieve a regular past tense form, it's useful to have a metrical stress rule to retrieve a predictable metrical stress pattern, and it's useful to have a linking rule (i.e., a linking theory) to retrieve a thematic role reliably associated with a syntactic position or a syntactic position reliably associated with a thematic role. However, if the past tense is too irregular, the metrical stress is too unpredictable, or the link is too unreliable, it's not useful to have the rule: retrieving the target information takes too long on average. Therefore, the child's decision about whether a rule should be adopted (i.e., past tense morphology viewed as regular, a metrical stress pattern viewed as predictable, or a link viewed as reliable) is based on its sufficiency threshold – a rule must have sufficient rule-abiding items for the child to decide to bother with it. Yang (2005, 2016, 2018) specifies this sufficiency threshold by considering how long it would take to access an item's target information with vs. without the rule. The retrieval process is assumed to involve serial search, which accords with current psycholinguistic data reviewed by Yang (2005, 2016).

The sufficiency threshold for adopting the rule is determined by a fairly complex equation that calculates the number of tolerable exceptions to that rule (this threshold comes directly from the Tolerance Principle – see Yang, 2005, 2016, 2018); however, this equation is well approximated by the much simpler equation $\frac{N}{ln(N)}$, where N is the number of items the rule could potentially apply to. That is, if there are $\frac{N}{ln(N)}$ or fewer exceptions in the set of items the rule could apply to, adopting the rule is useful in terms of retrieval time. In other words, the Sufficiency Principle requires a certain number of items that match a rule in order for that rule to be adopted as useful: that number is $N - \frac{N}{ln(N)}$. If there aren't that many items that match the rule, the rule isn't useful because adopting the rule slows down the average retrieval time.

Interestingly, this means that a potential rule needs to apply to a "super-majority" of items in order to be adopted. For example, a rule that could apply to 100 items allows only 21 exceptions (21%), and thus requires 79 items that match the rule under consideration. A rule that could apply to 1000 items allows only 144 exceptions (14.4%), and therefore requires 856 items that match the rule under consideration. A rule that could apply to 10000 items allows only 1085 exceptions (10.85%), and therefore requires 8915 items that match the rule under consideration. This has the practical effect of allowing only one option to be the rule (i.e., this disallows two or more "rules" for a set of items); this is because, by definition, only one option can ever hold a majority – let alone a super-majority.

So, to summarize, the Sufficiency Principle, incorporating the threshold derived from the Tol-erance Principle, provides a formal threshold for adopting a rule (the sufficiency threshold); this is

when a child would choose to view a certain pattern as dominant or reliable for a set of items and therefore its default pattern. These two principles have been used for investigating the acquisition of a default pattern or rule for a variety of linguistic knowledge types, including English past tense morphology (Yang, 2005, 2016), English noun pluralization (Yang, 2016), German noun pluralization (Yang, 2005, 2018), English nominalization (Yang, 2016), English metrical stress (Legate & Yang, 2013; Yang, 2015; Pearl et al., 2017), English *a*-adjective morphosyntax (Yang, 2015, 2016), English dative alternations (Yang, 2016, 2017, 2018), noun morphology in an artificial language (Schuler et al., 2016), and the development of causative use in English (Irani, 2019). Here, we'll use the sufficiency threshold for evaluating linking hypotheses at both the individual link level (1-link theories and the first stage of 3-link theories) and the multi-link level (the second stage of 3-link theories).[7]

4.3.2 Evaluating possible linking theories using the sufficiency threshold

Evaluating linking theories over all verbs.   Linking theories are meant to be generalizations about how the verbs of the language behave. Again, this is what allows inferences to novel verbs, like *blick* in *The little girl blicked the kitten on the stairs.* Though we, as speakers, haven't seen *blick* before, a linking theory that applies to all verbs of the language allows us to interpret this use of *blick* once we recognize it as a verb. With this in mind, a linking theory, whether a 1-link theory or a 3-link theory, can be evaluated by how many verb types obey it. If enough verb types obey the linking theory, a child using the sufficiency threshold would decide the linking theory is reliable enough for verbs of the language. Here, we can apply this to the English verb types in English children's input (from Table 1: <3yrs = 231, <4yrs = 260, <5yrs = 275). The Sufficiency Principle provides the sufficiency threshold ($N - \frac{N}{ln(N)}$) for how many verbs must obey the linking theory: <3yrs = 183, <4yrs = 209, <5yrs = 220. If at least that many individual verbs obey the linking theory, a child of that age could successfully derive the linking theory for English verbs from her input by using the sufficiency threshold.

   If the target state is three 1-link linking theories, the child goes through this evaluation once for 1-link patterns. If instead the target state is one 3-link linking theory, the child goes through this evaluation once for 1-link patterns, composes reliable 1-link patterns into possible 3-link patterns, and then goes through this evaluation again for the generated 3-link patterns.

Evaluating individual verbs.   How then can a child evaluate whether an individual verb obeys a particular linking theory? The intuition behind using the sufficiency threshold here is that the child is again assessing if having the linking theory aids the speed of production and comprehension of verbs and their arguments, this time over individual verbs. More specifically, the Sufficiency Principle can be used to determine if the the instances (i.e., the tokens) of this individual verb are reliable enough to support the linking pattern in question, with respect to this individual verb.

   Importantly, the verb tokens at the individual verb level are viewed through the lens of the different linking patterns under consideration, just as the verb types at the whole language level

---

[7]The evaluation process described in the next section can be run using the code and input sets available at https://github.com/lisapearl/linking-problem-code.

are. So, any individual verb has a linking pattern under consideration, and that linking pattern captures some of that verb's tokens (as demonstrated below in Tables 2 and 3). If the frequency of that linking pattern is sufficient, the child views the linking pattern as reliable enough at the individual verb level. In this way, applying the sufficiency threshold at the individual verb level mirrors applying the sufficiency threshold at the whole language level; in particular, there are linking patterns which occur with some frequency (individual verb = frequency of the verb's tokens obeying that linking pattern; whole language = frequency of the language's verb types obeying that linking pattern). If a linking pattern's frequency is sufficient, the pattern is deemed reliable for that level (individual verb or whole language).[8] For the individual verb level, we first describe the evaluation process for the 1-link patterns comprising 1-link theories and the individual links of 3-link theories; we then describe the evaluation process for the 3-link patterns comprising 3-link theories.

Table 2: Link instances across the syntactic positions of *subject*, *object* and *oblique* for the different thematic representations of UTAH, given example verbs and link instance counts from the <3yrs corpus. Instances compatible with the link under consideration are **bolded**. Sufficiency threshold analysis is shown, involving the total link instances N, the number of instances that obey the link under consideration, and the sufficiency threshold $N - \frac{N}{ln(N)}$. If the instances obeying the link are greater than or equal to the sufficiency threshold, the link is perceived as reliable.

	use			*break*			*belong*		
AGENT → *subj*	*subj*	*obj*	*obliq*	*subj*	*obj*	*obliq*	*subj*	*obj*	*obliq*
AGENT instances	**46**	0	0	**24**	0	0	**2**	0	12
Suff thresh	N	*# obey*	*thresh*	N	*# obey*	*thresh*	N	*# obey*	*thresh*
	46	46	33	24	24	16	14	2	8
Reliable?	Yes			Yes			No		
subj → AGENT	AGENT	PATIENT	OTHER	AGENT	PATIENT	OTHER	AGENT	PATIENT	OTHER
subj instances	**46**	0	0	**24**	46	0	**2**	25	0
Suff thresh	N	*# obey*	*thresh*	N	*# obey*	*thresh*	N	*# obey*	*thresh*
	46	46	33	70	24	53	27	2	18
Reliable?	Yes			No			No		

Table 2 demonstrates the evaluation process for 1-link patterns when assessing individual verbs. If the link goes from thematic role to syntactic position (e.g., AGENT → *subject*, as in the top part of Table 2), we consider all the links that link from that thematic role (i.e., we also include AGENT → *object* and AGENT → *oblique*). N is the set of link instances involving that thematic role (e.g.,

---

[8]We note that this approach of applying the sufficiency threshold to counts of tokens of an individual lexical item has also recently been used by Sneller, Fruehwald, and Yang (2019). In particular, the child perceives a probabilistic mixture of tokens in the input (i.e., different frequencies of two items) as a discrete mixture representing two variant rules. Using the same logic, a child here would perceive different frequencies of linking patterns for an individual verb as a discrete mixture representing different linking patterns for that verb. That said, if it turns out that the Tolerance and Sufficiency Principles only apply to counts of types for whatever reason, this will only affect the individual-verb-level analysis here. Interested readers could use the data here within a different learning framework to determine whether individual verbs obey the linking theories, and then use the sufficiency threshold to perform the analysis we use here over all verb types.

AGENT), and we count how many obey the link under consideration (AGENT → *subject*); if this number is greater than or equal to $N - \frac{N}{ln(N)}$, the link under consideration is reliable. In Table 2, we see that this leads to a three-year-old child considering the AGENT → *subject* link reliable for *use* and *break*, but not for *belong*.

Similarly, if the link goes from syntactic position to thematic role (e.g., *subject* → AGENT, as in the bottom part of Table 2), we consider all the links that link from that syntactic position (i.e., we also include *subject* → PATIENT and *subject* → OTHER). N is the set of link instances involving that syntactic position (e.g., *subject*), and we count how many obey the link under consideration (*subject* → AGENT); if this number is greater than or equal to $N - \frac{N}{ln(N)}$, the link under consideration is reliable. In Table 2, we see that this leads to a three-year-old child considering the *subject* → AGENT link reliable for *use*, but not for *break* and *belong*.

Table 3: Link instances across the syntactic positions of *subject*, *object* and *oblique* for the different thematic representations of UTAH, given example verbs and link instance counts from the <3yrs corpus. Instances compatible with UTAH's 3-link pattern are **bolded**. Sufficiency threshold analysis is shown, involving total link instances N, the number of instances that obey that 3-link pattern, and the sufficiency threshold $N - \frac{N}{ln(N)}$. If the instances obeying the 3-link pattern are greater than or equal to the sufficiency threshold, the 3-link pattern is perceived as reliable.

	use			*break*			*belong*		
	subj	*obj*	*obliq*	*subj*	*obj*	*obliq*	*subj*	*obj*	*obliq*
AGENT	**46**	0	0	**24**	0	0	**2**	0	12
PATIENT	0	**57**	1	46	**31**	0	25	**0**	0
OTHER	0	1	**4**	0	0	**1**	0	0	**12**
Suff thresh	N	*# obey*	*thresh*	N	*# obey*	*thresh*	N	*# obey*	*thresh*
	109	107	86	102	56	80	51	14	39
Reliable?	Yes			No			No		

Table 3 demonstrates the evaluation process for 3-link patterns when assessing individual verbs. Here, all link instances that obey the 3-link pattern are collapsed together, as are all the link instances that don't. That is, with a 3-link pattern in hand, the fine-grained details of where a specific thematic role (e.g., AGENT) appears don't matter. Instead, the child is considering whether each link instance (e.g., AGENT or PATIENT or OTHER) appears where it's expected to according to the 3-link theory (e.g., *subject* or *object* or *oblique*). N is then the entire set of link instances for the verb, and we count how many obey the 3-link pattern; if this number is greater than or equal to $N - \frac{N}{ln(N)}$, the 3-link pattern under consideration is reliable. In Table 3, we see that this leads to a three-year-old child considering UTAH's 3-link pattern reliable for *use*, but not for *break* and *belong*.

5 Results

5.1 1-link patterns

Tables 4 and 5 show sufficiency threshold analysis for individual links over the verb types in English children's input. We can see that the results are identical for all three ages, with the reliable links summarized in Table 6. For the UTAH-based links, two of the three appropriate role-to-position links are reliable and one of the appropriate position-to-role links is reliable. This contrasts with the rUTAH-based links, where all three appropriate role-to-position links are reliable and all three appropriate position-to-role links are reliable.

Table 4: Sufficiency threshold analysis of whether the individual links that use UTAH thematic representations and UTAH syntactic positions are reliable for English children's verbs at different ages. Verbs with 5 or more relevant link instances in the corpus are included in the analysis. The total number of verbs involving the link under consideration is shown (N), along with the number of verbs that obey the individual link and the sufficiency threshold ($N - \frac{N}{ln(N)}$). If the link is perceived as reliable for that age, the row is **bolded**.

age	role to position			N	# obey	thresh	position to role			N	# obey	thresh
<3	**AGENT**	$\rightarrow$	**subject**	**217**	**215**	**177**	subject	$\rightarrow$	AGENT	230	103	188
	PATIENT	$\rightarrow$	object	230	96	188	**object**	$\rightarrow$	**PATIENT**	**206**	**197**	**168**
	OTHER	$\rightarrow$	**oblique**	**162**	**147**	**131**	oblique	$\rightarrow$	OTHER	171	128	138
<4	**AGENT**	$\rightarrow$	**subject**	**245**	**242**	**201**	subject	$\rightarrow$	AGENT	259	123	213
	PATIENT	$\rightarrow$	object	259	110	213	**object**	$\rightarrow$	**PATIENT**	**238**	**224**	**195**
	OTHER	$\rightarrow$	**oblique**	**186**	**171**	**151**	oblique	$\rightarrow$	OTHER	200	145	163
<5	**AGENT**	$\rightarrow$	**subject**	**254**	**252**	**209**	subject	$\rightarrow$	AGENT	271	130	223
	PATIENT	$\rightarrow$	object	271	115	223	**object**	$\rightarrow$	**PATIENT**	**248**	**234**	**204**
	OTHER	$\rightarrow$	**oblique**	**197**	**179**	**160**	oblique	$\rightarrow$	OTHER	211	151	172

What does this mean for a child trying to derive either three 1-link theories or generate one 3-link pattern for further evaluation? At least four options seem plausible. The first and most conservative option is that the bidirectional links of a linking theory can only be successfully derived if both corresponding unidirectional links were perceived as reliable (e.g., AGENT $\leftrightarrow$ *subject* is derived only if both AGENT $\rightarrow$ *subject* and *subject* $\rightarrow$ AGENT were reliable). Under this approach of linking theory derivation, only rUTAH would enable the child to derive the appropriate three 1-link theories and generate the appropriate 3-link pattern for further evaluation. This is because UTAH only ever has reliable links in one direction for each thematic role and syntactic position, and so the child couldn't derive the appropriate 1-link theories or generate the appropriate 3-link pattern for UTAH.

The second option builds on the intuition that linking theories are expectations about the positional preferences of thematic roles (i.e., links from roles to positions), rather than the thematic role preferences of syntactic positions (i.e., links from positions to roles). Under this view, a child can derive the appropriate theory or derive the appropriate multi-link pattern if there's a unidirectional link from the appropriate thematic role to the appropriate syntactic position. This leads to the same

Table 5: Sufficiency threshold analysis of whether the individual links that use rUTAH thematic representations and rUTAH syntactic positions are reliable for English children's verbs at different ages. Verbs with 5 or more relevant link instances in the corpus are included in the analysis. The total number of verbs involving the link under consideration is shown (N), along with the number of verbs that obey the individual link and the sufficiency threshold ($N - \frac{N}{ln(N)}$). We note that all links are above the sufficiency threshold and so perceived as reliable.

age	role to position			N	# obey	thresh	position to role			N	# obey	thresh
<3	FIRST	→	first-syn	230	226	188	first-syn	→	FIRST	230	227	188
	SECOND	→	second-syn	207	198	169	second-syn	→	SECOND	208	197	170
	THIRD	→	third-syn	170	160	137	third-syn	→	THIRD	170	162	137
<4	FIRST	→	first-syn	259	254	213	first-syn	→	FIRST	259	254	213
	SECOND	→	second-syn	239	232	196	second-syn	→	SECOND	239	231	196
	THIRD	→	third-syn	200	188	163	third-syn	→	THIRD	200	191	163
<5	FIRST	→	first-syn	271	266	223	first-syn	→	FIRST	271	266	223
	SECOND	→	second-syn	248	240	204	second-syn	→	SECOND	248	239	204
	THIRD	→	third-syn	211	197	172	third-syn	→	THIRD	211	200	172

Table 6: The individual links that would be perceived by an English child using sufficiency threshold analysis as reliable, considering the verb types for the <3yrs, <4yrs, and <5yrs child-directed speech data.

system	role to position links			position to role links		
UTAH	AGENT	→	subject			
				object	→	PATIENT
	OTHER	→	oblique			
rUTAH	FIRST	→	first-syn	first-syn	→	FIRST
	SECOND	→	second-syn	second-syn	→	SECOND
	THIRD	→	third-syn	third-syn	→	THIRD

qualitative results as before: UTAH has only two of the three appropriate unidirectional links from role to position, while rUTAH has all three. So, as before, a child couldn't derive all three UTAH 1-link theories or the 3-link UTAH pattern for further evaluation. In contrast, she could do this for the three rUTAH 1-link theories and the 3-link rUTAH pattern.

The third option takes the opposite view, and builds on the intuition that linking theories are expectations about the the thematic role preferences of syntactic positions (i.e., links from positions to roles). Under this view, a child can derive the appropriate theory or derive the appropriate multi-link pattern if there's a unidirectional link from the appropriate syntactic position to the appropriate thematic role. This leads to the same qualitative results as before. In particular, UTAH has only one of the three appropriate unidirectional links from position to role, while rUTAH has all three. So, as before, a child couldn't derive all three UTAH 1-link theories or the 3-link UTAH pattern

for further evaluation. In contrast, she could do this for the three rUTAH 1-link theories and the 3-link rUTAH pattern.

The fourth and most liberal option is that the child considers any reliable unidirectional link between a thematic role and a syntactic position sufficient for deriving the corresponding 1-link theory or including the appropriate link in a 3-link pattern for further evaluation. Using this option, a child would be able to derive all three appropriate 1-link theories for both UTAH and rUTAH, and generate appropriate 3-link UTAH and rUTAH patterns for further evaluation. This is because at least one unidirectional link is viewed as reliable for each connection between a UTAH thematic role and UTAH syntactic position (i.e., AGENT ↔ *subject* has AGENT → *subject*; PATIENT ↔ *object* has *object* → PATIENT; OTHER ↔ *oblique* has OTHER → *oblique*). And, as before, rUTAH has reliable unidirectional links in both directions for all appropriate links.

While we don't know for certain how children would derive 1-link theories or the individual links that serve as building blocks for 3-link patterns, working through the plausible options highlights the learning assumptions needed to support derivation of different linking theory representations. In particular, if UTAH's linking theory is the correct target knowledge, then there's only one way a child could derive either the three 1-link theories or the building blocks for the 3-link pattern: she must use the relatively liberal option where only a single unidirectional link in either direction (role to position or position to role) is required to be reliable. In contrast, if rUTAH's linking theory is the correct target knowledge, a child could derive the three 1-link theories or the building blocks for the 3-link pattern in a number of ways – that is, the acquisition process for deriving rUTAH is more robust. This is presumably because rUTAH redefines many exceptions to the UTAH primary linking pattern as paradigmatic cases of the rUTAH primary linking pattern, and so it's easier for a child to view the rUTAH links as reliable from the input.

5.2   One 3-link theory

But what if we, as linguists, believe the correct target state is one 3-link theory? The previous analysis suggests that a child trying to derive UTAH may have a qualitatively harder time generating the appropriate 3-link pattern, compared with a child trying to derive rUTAH. However, let's suppose that the child has successfully generated the appropriate 3-link pattern for UTAH or rUTAH, and now needs to evaluate whether the 3-link pattern is reliable enough to derive the corresponding 3-link theory.

Recall from the evaluation process for 3-link patterns discussed in section 4.3.2 that sufficiency threshold analysis is done through the filter of the 3-link pattern: a link instance either obeys the 3-link pattern or it doesn't. This brings up a question about how to count instances of a 3-link pattern. Consider this use of the verb *pet*: *Lily pets the kitties*. If each link is considered an instance of the 3-link pattern, this use counts as two instances that obey the UTAH 3-link pattern: one for AGENT ↔ *subject* and one for PATIENT ↔ *object*. In contrast, if each verb use is considered an instance of the 3-link pattern, this use counts as a single instance that obeys the UTAH pattern, as all thematic roles are in their expected positions. Because it's unclear *a priori* which one a child would select, we show the results of both approaches (link-based and verb-use-based) to counting 3-link linking instances in Table 7 below.

Table 7: Sufficiency threshold analysis of whether the 3-link patterns for UTAH and rUTAH are perceived as reliable from the verb usage in English children's input at different ages. The total number of verbs with 5 or more link-based instances or verb-use-based instances in the corpus is shown, along with the number of verbs required to meet the sufficiency threshold and the number of verbs obeying the appropriate 3-link pattern for both UTAH and rUTAH. If the 3-link pattern is viewed as reliable for that age, the number obeying the 3-link pattern is **bolded**.

	age	total (N)	thresh	UTAH # obey	rUTAH # obey
	<3yrs	231	189	126	**229**
link-based	<4yrs	260	214	136	**253**
	<5yrs	275	227	142	**264**
	<3yrs	224	183	97	**220**
verb-use-based	<4yrs	255	209	108	**248**
	<5yrs	267	220	114	**255**

This analysis again suggests a qualitative difference between UTAH and rUTAH. Put simply, a child trying to derive one 3-link theory for UTAH won't be able to infer that the 3-link pattern is in fact reliable for all English verbs, no matter what age the child is and no matter how linking pattern instances are counted (by individual link or by verb use). The child's input before age three, four, and five never surpasses the required number of pattern-matching verbs to support this inference. In contrast, a child trying to derive one 3-link theory for rUTAH will always be able to infer that the 3-link pattern is reliable and therefore this pattern is good to have as a 3-link linking theory. This is because there are very few exceptional verbs when viewing the input through a rUTAH lens.

6   Discussion

We investigated how the linking theories specified by UTAH and rUTAH could be derived from English children's input, thereby providing an existence proof for derived approaches to linking theory development. More specifically, we specified an acquisition theory relying on a combination of linguistic knowledge and general-purpose learning mechanisms, evaluated it with respect to realistic samples of children's input, and discovered that rUTAH is derivable under these learning assumptions, but UTAH isn't. This result highlights a qualitative difference between UTAH and rUTAH when it comes to the learning process theory that must accompany each linking theory.

We first discuss the finding that UTAH is not derivable, and explore to what extent enriching our syntactic theory with movement might change those results. We then briefly consider the relative differences between UTAH and rUTAH, and to what extent our intuitions about the number of exceptions under each might (or might not) have predicted our results beforehand. We then discuss the different components of the proposed acquisition theory in more detail, speculating on their domain-specificity and innateness, and to what extent other linking theories might require the

same components. We then turn to the potential origins of the necessary syntactic and semantic percepts that our linking theory derivation process relies on. Finally, we discuss some possible future investigations that build on these findings.

6.1 The non-derivability of UTAH

Crucially, we found difficulties for the derivability of UTAH at both stages of the acquisition process: generating the complex 3-link pattern of UTAH from reliable individual links, and finding that 3-link pattern to be reliable enough. More specifically, our results suggest that UTAH is difficult to generate as a 3-link pattern because the input that English children receive doesn't support all three links in both directions. Generating the 3-link pattern requires the child to adopt the most liberal hypothesis generation procedure that we considered: assume a bidirectional link between thematic role and syntactic position if a unidirectional link is reliable enough in either direction. Then, even assuming the 3-link pattern can be generated, the 3-link version of UTAH is not derivable from children's input as a language-wide linking theory. This is because the surface forms of so many constructions are apparent exceptions to the 3-link linking theory.

A response to this result is to point out the innate form of UTAH typically assumed in the literature: that UTAH form leverages the existence of syntactic movement to reanalyze these exceptions as paradigmatic cases of the linking pattern. So, it's tempting to ask whether adding syntactic movement to the acquisition framework might change the derivability of UTAH by eliminating some (if not all) of the exceptions. But, as far as we can tell, movement can only be used to re-analyze potential exceptions if there's some knowledge *already in place* to tell the child which constructions involve movement. That is, for a sentence like *The package arrived*, where a PA-TIENT appears in surface *subject* position, the child would need to somehow know movement is responsible for the surface form of this sentence without already knowing that a PATIENT shouldn't appear in the *subject* position (i.e., the UTAH linking knowledge).

One possibility would be to leverage an unequivocal marker for movement, and then only build in innate knowledge of movement markers. That is, something about *The package arrived* would signal movement had occurred, and that something would be distinct from the fact that a PATIENT appeared in *subject* position. But, to the best of our knowledge, there aren't unequivocal markers for movement in many of these constructions, at least in English (this is a classic difference between A-movement and A'-movement).

Another possibility would be to freely allow movement for any analysis, even without direct evidence of movement. This would allow the child to reanalyze any exception as fitting the hypothesized linking pattern. So, for example, *The package arrived* could be reanalyzed as supporting the PATIENT $\leftrightarrow$ *object* link, where movement causes the PATIENT to appear in the surface *subject* position. Yet, this is clearly too powerful, as any given sentence could be taken as evidence in favor of any given linking theory: whenever a surface pattern doesn't conform to the hypothesized linking theory, a movement reanalysis would allow it to conform. For instance, *The package arrived* could be reanalyzed to support the PATIENT $\leftrightarrow$ *oblique* link, because movement (again) causes the PATIENT to appear in the surface *subject* position. To combat this problem of overapplying movement and therefore allowing any input instance to support any linking theory, we would need constraints on movement *already in place* to guide the child to apply movement in all and only the

25

correct constructions. That is, the child would already need to know she can't apply movement to allow *The package arrived* to support the PATIENT $\leftrightarrow$ *oblique* link. Instead, she would need to know that movement can only be applied for this instance to support the PATIENT $\leftrightarrow$ *object* link. Again, this would basically recreate an innate version of UTAH, with movement constraints (e.g., "only allow movement to support the PATIENT $\leftrightarrow$ *object* link") conspiring to create the underlying UTAH linking pattern (e.g., PATIENT $\leftrightarrow$ *object*).

Based on this reasoning, we tentatively conclude that the complex form of UTAH as typically proposed (a single 3-link theory) is unlikely to be derivable, at least given the acquisition approach investigated here. That said, it's potentially derivable as a set of three 1-link theories that differ in their directionality: AGENT $\rightarrow$ *subject*, *object* $\rightarrow$ PATIENT, OTHER $\rightarrow$ *oblique*. If we allow the child to assume that a reliable unidirectional link in either direction is enough to establish a link between thematic role and syntactic position, the child could then use these three links to construct something similar to UTAH. We leave it to future work to explore the consequences of decomposing UTAH into the three reliable unidirectional links uncovered here.

6.2   A note on the relative difference between UTAH and rUTAH

UTAH and rUTAH behave qualitatively differently in the acquisition framework developed here: UTAH isn't derivable, while rUTAH is. This qualitative difference is due to a quantitative difference: UTAH has relatively more surface exceptions to its primary pattern than rUTAH does. Of course, this quantitative difference isn't surprising. UTAH was designed to allow the links of the surface representations to vary after syntactic movement, while enforcing a uniform linking pattern at an underlying level of representation occurring before syntactic movement. So, the complexity of this representation was in the movement operations that yielded the surface forms, rather than in the syntactic and thematic components of the links. rUTAH, in contrast, was designed to enforce uniformity on the surface representations; but, this meant the complexity was in the thematic and syntactic components of the links, which are organized into hierarchies. Still, the upshot is that UTAH should have more surface exceptions than rUTAH.

Given this relative difference, it's tempting to conclude that the qualitative difference in derivability via the Tolerance and Sufficiency Principles is also expected – more surface exceptions means it should be harder to derive UTAH using these principles than it is to derive rUTAH. Importantly, harder isn't the same as impossible. In fact, there are three possible qualitative patterns if it's harder to derive UTAH than rUTAH: (i) both UTAH and rUTAH are derivable (i.e., both reach the sufficiency threshold), (ii) neither UTAH and rUTAH are derivable (i.e., neither reach the sufficiency threshold), and (iii) rUTAH is derivable while UTAH isn't. We had no intuitions prior to this modeling study which of these qualitative patterns it would be, because that required the analysis we did to determine the thresholds for each verb in the child-directed input, and the thresholds for all the verbs collectively at each age. To our minds, the only *a priori* conclusion to draw based on the relative difference in surface exceptions is that we shouldn't see UTAH being derivable this way while rUTAH isn't.

Because UTAH did end up having too many surface exceptions to be derivable under the Tolerance and Sufficiency Principles, it's interesting to consider how prevalent the primary pattern of UTAH actually is. That is, UTAH is typically described in the literature as having a primary pat-

tern that is substantially more frequent than the exceptions – so much so that the primary pattern is encoded as the only pattern in the theory, with apparent exceptions arising due to movement. The quantitative threshold determined by the Tolerance and Sufficiency Principles is one (cognitively-grounded) way to evaluate how much more frequent the primary pattern is than the exceptions. And, by that metric, the primary pattern is perhaps *not* as frequent as it's made out to be in the UTAH literature (at least in English child-directed speech) – there are far more apparent exceptions than is typically acknowledged.

In contrast, the primary pattern for rUTAH really is substantially more frequent than its exceptions. We don't know the motivations of the theorists who developed rUTAH, but one motivation could have been an intuition that fixed approaches like UTAH might lead to too many exceptions by some cognitive metric; in contrast, a relative approach like rUTAH might not. This is precisely what we've found here, using analysis of child-directed speech. Future work might use the same analysis over adult-directed language to determine if the primary patterns for both UTAH and rUTAH are sufficiently present. We discuss this idea in more detail in section 6.5.

6.3   The components necessary for deriving linking theories

One significant conclusion from our investigation is that there are several major components necessary for deriving linking theories, whether they are a set of 1-link theories or a single 3-link theory. Based on the acquisition theory specified here, we propose that any theory of how children derive linking theories will require the main components in Table 8. This component specification will hopefully allow future researchers to reference these same main components, and so facilitate comparisons across different learning approaches. Additionally, Table 8 lists our specific implementations of these main components.

For each implementation of the main components, we can ask whether that implementation is likely to be domain-specific (to language) or domain-general, and whether that implementation is likely to be innately-specified or derived during the acquisition process. Our goal is to determine if any of the component implementations are likely to be simultaneously language-specific and innate, as this component type figures most prominently in the debates between innate and derived approaches to linking theories (as well as many other aspects of language acquisition).

Looking first at the thematic system, our implementation involves either three fixed macro-roles (when deriving UTAH) or a relative hierarchy of roles (when deriving rUTAH). Thematic roles are based on non-linguistic concepts of event participants. Because of this, they are likely to be domain-general (though they may contribute to language differently than other cognitive domains) and innate.

Looking next to syntactic structure, our implementation involves either fixed syntactic positions or a relative hierarchy based on c-command relations. In contrast with thematic roles, syntactic positions are likely domain-specific and, at least in their final form, derived from prior language experience. We note that we remain agnostic as to whether innate, domain-specific knowledge is required to derive these syntactic positions.

Turning to biases about links, our implementation involves four key pieces. First is the bias to look for links between roles and positions, which appears to be domain-specific, as we know of no equivalent in other domains. One possibility is that this bias is simply innate. Another

Table 8: Proposed main components that are required to derive linking theories, and the implementations used here (indented beneath), along with their likely categorization according to domain-specificity and innateness according to current knowledge. The rightmost column indicates whether the derivation of these components relies on other innate components. Component implementations that might currently be considered domain-specific and innate, or that are derived from other components that are potentially domain-specific and innate, are **bolded**. Where known, potential domain-specific and innate components are explicitly listed, such as the Tolerance Principle (TolP) and Suffiency Principle (SuffP).

component and our implementation	domain specificity or generality	innate or derived	derived from innate components
a thematic system			
fixed macro-roles	domain-general	innate	
relative hierarchy of roles	domain-general	innate	
syntactic structure			
fixed syntactic positions	domain-specific	derived	(possibly)
relative hierarchy (c-command)	domain-specific	derived	(possibly)
biases about links			
look for links	domain-specific	either	(possibly)
look for 1-to-1 links	domain-specific	derived	mutual exclusivity
look for a single reliable link	domain-specific	derived	TolP & SuffP
the ability to track links	domain-general	innate	
a procedure to generate linking patterns			
generate 1-link patterns	either	innate	
generate 3-link pattern	either	innate	
a procedure to evaluate linking patterns			
sufficiency threshold	domain-specific	derived	TolP & SuffP

possibility is that this bias is a specific instantiation of a more general bias to look for correlations between active representations in any single cognitive domain (e.g., active representations in the visual domain, the spatial domain, the social cognition domain, the language domain, etc.). The question then is how to formulate that bias in such a way as to yield the links we want (e.g., between thematic roles and syntactic positions), while not yielding links that we don't want (i.e., between thematic roles and anything else active during language processing). Such fine-tuning likely requires innate knowledge, though the status of that innate knowledge (i.e., whether it's domain-specific or domain-general) is currently unknown. We note that we believe any theory of deriving linking theories will need to use this bias to look for links between roles and positions in order to get started. So, this bias is likely true of all theory implementations.

The next link bias in our implementation is to look for a 1-to-1 mapping between roles and positions. That is, the modeled child only considered links that involved a single syntactic position

or thematic role (e.g., AGENT → *subject*), rather than allowing disjunctive options that involved multiple syntactic positions or multiple thematic roles (e.g., AGENT OR PATIENT → *subject*; PATIENT → *subject or object*). As this bias applies to links, which are language-specific, we list this bias as domain-specific. In terms of its origin, this bias might be thought of as similar to (and thus derived from) the mutual exclusivity bias that young children often show during early word learning (Markman & Wachtel, 1988; Golinkoff, Hirsh-Pasek, Bailey, & Wenger, 1992), where they assume each word refers to a distinct referent. It remains an open question what the origins of the mutual exclusivity bias are (Clark, 1988; Markman & Wachtel, 1988; Golinkoff, Mervis, & Hirsh-Pasek, 1994; Markman, Wasow, & Hansen, 2003; Frank, Goodman, & Tenenbaum, 2009), in particular whether they are innate and/or language-specific. However, if the 1-to-1 bias is indeed derived from it, the 1-to-1 bias would therefore be domain-specific and derived.

The third link bias in our implementation is that the child must assume there is only a single reliable link per role or position (i.e., AGENT is linked to only one of the available options, *subject* is linked to only one of the available options, etc.). This domain-specific knowledge derives from the Tolerance and Sufficiency Principles (as discussed in section 4.3.1), which we might plausibly take to be domain-general because of their reliance on item storage and retrieval, irrespective of what cognitive domain the item comes from. However, Yang (2016) notes that a core component on which these principles are based (the Elsewhere Condition) comes from studies of language processing – so, it's possible that this component is in fact language-specific. The Tolerance and Sufficiency Principles themselves are likely innate, as it's unclear how a child would learn to optimize item retrieval with respect to item access time.

The final link bias in our implementation is the ability to track links. This bias is likely derived from the innate domain-general ability to track frequencies (Saffran, Aslin, & Newport, 1996; Xu & Tenenbaum, 2007; Smith & Yu, 2008; Denison, Reed, & Xu, 2011; Denison, Bonawitz, Gopnik, & Griffiths, 2013; Stahl, Romberg, Roseberry, Golinkoff, & Hirsh-Pasek, 2014; Yurovsky, Case, & Frank, 2017); this frequency-tracking ability is then applied to linking patterns as cognitive objects.

Turning to the procedures for generating linking patterns, our implementation generated either 1-link patterns or 3-link patterns. The procedures that we postulated were all domain-specific because they only apply to linking patterns. However, it's currently unknown if general-purpose mechanisms of explicit hypothesis generation (see Perfors, 2012 for discussion) would suffice to generate a set of reasonable linking patterns. If so, these procedures could be an example of a domain-general procedure applied to the domain-specific hypothesis space of linking theories. Therefore we list it as "either". At our current level of understanding, the hypothesis generation procedures would also likely need to be innate, as it's unclear how to break this sort of hypothesis generation down into learnable components.

Turning finally to procedures for evaluating linking patterns, our implementation was the sufficiency threshold. This threshold is applied to linking patterns, which are domain-specific cognitive objects, and so the threshold is domain-specific in our implementation. However, as discussed in section 4.3.1, the sufficiency threshold is derived from the Tolerance and Sufficiency Principles. As we previously noted, these principles are likely innate, but at least one building block these principles use may be either language-specific or domain-general.

Taken together, there are two component implementations that are potentially both domain-specific and innate, given our current level of understanding, both of which are about the procedures for generating linking patterns. The remaining component implementations are likely to be either domain-general or derived or both. However, as noted above, there are several component implementations that may rely on innate, domain-specific building blocks: the implementations of the syntactic structure (whether fixed or relative), three of the link biases (look for links, look for 1-to-1 links, look for a single reliable link), and the procedure for evaluating linking patterns that was based on the sufficiency threshold. It's always possible that future work may find a way to reduce the number of domain-specific and innate components to zero. For now, our implementation for how to derive linking theories seems to potentially require two, with an additional six that may involve innate, domain-specific building blocks.

Given that this component list rests on the specific implementation we propose here for deriving linking theories, it's reasonable to wonder if the overall complexity of the system could have been simplified by making different specific choices for each component (other thematic systems, other syntactic systems, other linking pattern generation and evaluation procedures, etc). There are surely other implementation choices for each of the components in our acquisition theory, but we don't believe that different choices would substantially lessen the complexity of the system. This is because we attempted to choose the simplest available options that are both cognitively plausible and theoretically motivated.

In particular, we tested both fixed and relative thematic systems. We chose thematic systems and syntactic systems that only have three roles. We explored how to generate and evaluate both a set of 1-link theories and a single 3-link theory compatible with current UTAH and rUTAH specifications. In short, it's not obvious how a substantial amount of complexity could be removed from the system – the linking problem seems to simply be a problem with a certain amount of inherent complexity. This is likely why the dominant linking theories in the syntactic literature both appear to contain the same amount of complexity, but shift that complexity between movement operations in UTAH and a relativized hierarchy of roles and positions in rUTAH.

As mentioned in section 3, we tested UTAH and rUTAH because we believe these are ideal case studies for exploring the bounds of the learning problem associated with the acquisition of linking theories. Therefore, we also believe that the main components listed above should extend to the acquisition of any linking theory that can be stated with enough specificity, even if the precise implementation of the acquisition process differs from the one proposed here. If there are linking theories that diverge substantially from the UTAH/rUTAH systems in form or content, the next step to evaluate them with respect to acquisition will be to formulate those theories in enough detail such that we can apply the acquisition approach demonstrated here. This involves the implementation of the main components in Table 8 and evaluation on child-directed speech data like those contained in the CHILDES Treebank.

6.4   How the necessary semantic and syntactic percepts might arise

The derivation approach we explored here for linking theories assumes that children have reliable semantic and syntactic percepts of the information in their input, in the form of the thematic roles and syntactic positions. Yet, as discussed in our motivations, several prominent acquisition

approaches, such as syntactic bootstrapping, traditionally assume that linking theory knowledge is already present. So, how could children achieve these reliable semantic and syntactic percepts without already having linking theory knowledge?

One promising avenue is for children to initially rely on rudimentary thematic and syntactic percepts, which are refined into more adult-like percepts over time. As an example of this approach, Perkins and colleagues (Perkins, Feldman, & Lidz, 2017; Perkins, 2019) demonstrate how children can leverage rudimentary syntactic percepts, and an awareness that they sometimes misperceive syntactic information, to classify whether verbs are transitive, intransitive, or alternating; this transitivity verb class information then allows children to infer whether the verb should have a direct object syntactic position in general.

More specifically, under this approach, children track how often the verb seems to have a direct object immediately after it. For example, this allows children to recognize that *hug* has a direct object in *Lily hugged Jack*, but doesn't allow children to recognize that *hug* has a direct object in *Who did Lily hug?* In this way, children's syntactic percept is rudimentary because it's noisy (e.g., *wh*-questions are missed, and in fact may be parsed as if the verb has no direct object). Yet, Perkins and colleagues (Perkins et al., 2017; Perkins, 2019) demonstrate that a modeled child, who assumes misparsing occurs, can still learn that verbs like *hug* should have a direct object in general by learning to filter out some of the perceived syntactic noise. Thus, the rudimentary syntactic percept that misparses *Who did Lily hug?* can be refined over time into a more adult-like syntactic percept that recognizes the direct object in that *wh*-question. Importantly, the process described by Perkins and colleagues relies only on syntactic information (i.e., the perceived distribution of direct objects immediately after verbs); so, neither semantic percepts nor linking theory knowledge is needed to refine the rudimentary syntactic percept into a more adult-like syntactic percept.

In a similar way, we could imagine that rudimentary semantic percepts might be refined into more adult-like semantic percepts over time. We're currently unaware of concrete proposals demonstrating this process for semantic percepts, but it's possible the general approach of Perkins and colleagues could work. In particular, a child might have a rudimentary semantic percept for certain types of verbs (e.g., verbs relating to actions that are hard to observe like *hear*, or unobservable mental states like *forget* and *love*). So, rather than more adult-like percepts of *Jack hears/forgets/loves Lily*, with *Jack* as the EXPERIENCER of hearing, loving, or forgetting, and Lily as the SUBJECT MATTER, children might have misperceptions of these thematic roles (e.g., Lily with no thematic role at all, as she's not affected by Jack's activity.) That is, children's rudimentary semantic percept could lead to a misperception of the thematic role information in their input. If children are aware that they can misperceive this information, they may be able to effectively filter their semantic information, classify verbs appropriately (e.g., *hear*, *forget*, and *love* as subject-experiencer verbs), and so develop a more adult-like semantic percept over time (e.g., Lily receiving a thematic role in the object position for these verbs). As with the syntactic percept refinement, it's possible this process could occur with only semantic information, rather than relying on syntactic percepts or linking theory knowledge.

More generally, under certain learning assumptions, it's possible a child could use rudimentary percepts to learn more adult-like percepts. The adult-like semantic and syntactic percepts are the information pieces needed for the linking theory derivation approach we investigate here. How-

ever, it's also possible that our derivation approach could operate over more rudimentary percepts (particularly if some sort of input filter is in place to remove some of the perceived noise); we discuss this interesting possibility below as future work.

6.5 Future investigations

While we have provided an existence proof for a derived approach to linking theory development using the Tolerance and Sufficiency Principles, there of course remain several interesting questions. First, this approach was evaluated over American English input to children. In future work, we can ask whether the same results obtain over other languages and dialects.

Another interesting question concerns making the inference process more realistic. Here, we assumed the child could keep track of all the relevant link pattern counts and make a single decision based on the sufficiency threshold at the end. That is, we assumed a batch learning process. However, we know that children's acquisition is incremental, with them processing information as they encounter it. So, it would be useful to see if the sufficiency threshold process we proposed here would yield the same results if children's input were processed incrementally (e.g., see Wang and Mintz (2008) for translating a batch inference process for syntactic categorization to an incremental inference process). The process itself would be fundamentally the same: the child would track the relevant link pattern counts from her input. So, for example, at one time, the child might have encountered 30 uses of a verb, where 24 of them obey the link pattern under consideration. Later on, she might have encountered 50 more uses, where 35 of them obey the link pattern under consideration – this would lead to 30+50=80 uses total, with 24+35=59 obeying the link pattern under consideration. The child's decision about link pattern reliability could be made as often as she liked – for example, after the first time point in the above example where 30 verb uses had been encountered and also after the second time point where 80 verb uses had been encountered. In the example above, assessing reliability at the first time point would cause the child to think the link is reliable (30-$\frac{30}{ln(30)}$=22, and 24 uses obey the link pattern); however, assessing reliability at the second time point would cause the child to think the link is unreliable (80-$\frac{80}{ln(80)}$=62, and only 59 uses obey the link pattern). So, a child's assessment might well fluctuate over time. In this way, at any point in development, we could see if the modeled child would view a given link pattern as reliable, and so use it to derive a linking theory. This kind of incremental inference would allow us to observe the trajectory of linking theory derivation, rather than a snapshot at the end of three, four, or five years old.[9]

A related interesting question concerns the syntactic and semantic percepts used to build the links of the potential linking theory patterns. As noted in the previous section, we assumed reliable extraction of both the syntactic positions and the thematic roles for any data point in the child's input. Yet, children likely have more rudimentary syntactic and semantic percepts earlier in language development. If linking theory derivation occurs during this earlier time, then that

---

[9]We note that this particular incremental approach also requires the child to have some sort of stopping mechanism for assessing links, unless we think children continuously reevaluate the links that underlie linking theories; in that case, it would only be the reliability of their data that prevents constant fluctuation. A sample stopping mechanism might be something like the child noticing a set amount of time has passed without the link reliability changing (either becoming reliable or ceasing to be reliable).

derivation process would need to succeed when relying on those more rudimentary percepts. That is, a more realistic model might rely on more child-like approximations of the syntactic positions and thematic roles in any data point, and still try to derive UTAH and rUTAH.

For instance, suppose the child's rudimentary syntactic percepts were like those suggested by Perkins and colleagues (Perkins et al., 2017; Perkins, 2019): the child parses direct objects appearing immediately after the verb as direct objects (e.g., *Jack hugged Lily*), but not when the direct object is in a different position (e.g., *Who did Jack hug?*). The learning mechanism suggested by Perkins and colleagues allows the child to filter out data viewed as misparsed – in this case, perhaps the *wh*-question would be ignored because the child recognizes she can't parse it properly. So, the effect is that the syntactic data the child learns from are a subset of the available syntactic data: she learns from the data she's capable of parsing at that point. As also mentioned in the previous section, a semantic analogue of this process might involve the child having rudimentary semantic percepts (e.g., in *Jack loved Lily*, not recognizing the thematic role of *Lily* because Lily's not observably affected by Jack's loving action). Similar to the situation with the rudimentary syntactic percepts, a child using the learning mechanism of Perkins and colleagues might filter out data viewed as misperceived – in this case, perhaps ignoring semantic percept data with *love*.

Taken together, an early linking theory derivation process that relies on filtered rudimentary syntactic percepts and filtered rudimentary semantic percepts might therefore end up with a "cleaner" data intake, comprised of data that the child recognizes she can parse properly (e.g., transitive sentence frames of observable action verbs). So, the linking theory derivation process here might operate quite well over this earlier data intake; moreover, it's possible that fewer apparent exceptions might arise because of this filtered subset. For instance, UTAH links might be easier to derive if the data intake is filtered this way. If linking theory knowledge can in fact be derived from child-like syntactic and semantic percepts, this could explain how very young children show some linking theory knowledge. Deriving links from child-like percepts could also explain why that early linking theory knowledge may be less abstract than adult linking theory knowledge, and depend on semantic properties like animacy or lexical items like *with* (Bunger & Lidz, 2004, 2008; Lidz et al., 2017).

Relatedly, if we assume continual refinement of syntactic percepts, semantic percepts, and linking theory knowledge, it's plausible that children update their linking theories over time, on the basis of the syntactic and semantic percepts they have at any particular point. For instance, English two-year-olds seem to subcategorize links by animacy in intransitive frames like *X is VERBing* (i.e., $subject_{+animate} \rightarrow$ AGENT, $subject_{-animate} \rightarrow$ PATIENT). This could be due to the syntactic and semantic percepts they learn from before age two. However, by age three, they may have refined the syntactic and semantic percepts they learn from, and try to identify a single *subject* link that abstracts away from animacy (e.g., *subject* $\rightarrow$ FIRST). Their previous linking knowledge may show up as a prior bias towards a *subject* link that connects to certain thematic roles and not others (e.g., FIRST and SECOND, but not THIRD; AGENT and PATIENT, but not OTHER). A future developmental computational model could implement this developmental possibility more concretely, and determine if some model implementation can capture current empirical data about what children know when.

Another interesting avenue is to explore other learning approaches for deriving linking theories

from realistic children's input. As mentioned, we provided an existence proof using the Tolerance and Sufficiency Principles as the core learning principles, but there are many other learning approaches that may be cognitively plausible (e.g., Bayesian inference (Perfors, Tenenbaum, Griffiths, & Xu, 2011), variational learning (Yang, 2002, 2004, 2012)). Future work can thus see if these other learning approaches are able to derive UTAH and rUTAH from children's input and, if so, whether rUTAH is derivable while UTAH isn't.

A final interesting question relates to the universality of the primary linking pattern, a key empirical motivation for theorists to propose that linking theory knowledge is innate. As noted in our discussion of motivations, the assumed universality of the primary linking pattern has yet to be precisely quantified. The approach we take here, of tracking and reasoning about the link-obeying data vs. the exceptional data, could be one way to determine if a given language obeys the primary linking pattern. More specifically, within a language (like English), researchers could track how often realistic adult-directed data obey the primary linking pattern; the Tolerance and Sufficiency Principles could be used to determine if the ratio of link-obeying instances to exceptions is above the threshold necessary to classify the language as obeying the primary linking pattern. Then, if we're able to do this across many languages, we would have a sense of how universal it is for languages to obey the primary linking pattern (at least, according to the Tolerance and Sufficiency Principles). This general approach to quantification could be extended to other linking theories and other thresholds to provide a firmer foundation for debates about the universality of the primary linking pattern.

7 Conclusion

We built concrete acquisition theories for UTAH and rUTAH that assumed a derived – rather than innate – approach to the development of linking theory knowledge. Our goal was to explore the complexity of the acquisition problem created by linking theories, and to do so using well-specified linking theories that occupy relatively distinct positions within the hypothesis space of possible linking theories. We leveraged a conceptual acquisition framework that specified key aspects of the child's acquisition task: the initial state, data intake, inference mechanism, and target knowledge state. The initial state involved minimal domain-specific, prior knowledge and incorporated cognitively-plausible learning abilities; the data intake was based on linguistically-annotated realistic child-directed input from the CHILDES Treebank (Pearl & Sprouse, 2013a, 2019) and empirically-based estimates of data quantity; the inference mechanism relied on Yang (2005, 2016)'s cognitively-motivated Tolerance and Sufficiency Principles; and the target linking knowledge states were specified by UTAH and rUTAH. Using this framework, we found that UTAH is difficult to derive from English children's input, whether as a single 3-link theory or a set of three 1-link theories. In contrast, we found that rUTAH is easy to derive as either a single 3-link theory or a set of three 1-link theories. Moreover, these results hold for English children's input at ages three, four, and five – it doesn't matter the age (in this age range) that children attempt to derive these linking theories. Our results suggest a qualitative learnability difference for UTAH and rUTAH (and not just a quantitative difference in the number of exceptions) based on the acquisition theories specified here. These results in turn suggest that the learning theory

accompanying the UTAH representation requires more innate scaffolding than the learning theory accompanying the rUTAH representation, because UTAH can't be derived this way while rUTAH can.

Beyond our concrete results, our acquisition framework also highlights the components necessary for deriving linking theories, some of which may be both domain-specific and innate given our current understanding of child language acquisition. These components include a bias to look for links between thematic roles and syntactic positions, a bias to look for 1-to-1 links, and the procedure for generating linking patterns to evaluate. An interesting open question is whether a way can be found to derive these components from other, more fundamental, components. Finally, our results suggest that the sufficiency threshold, derived from the Tolerance and Sufficiency Principles, is a useful, cognitively-grounded evaluation procedure for deriving linking theories from children's input. It is our hope that these results will spur future research both into the comparison-via-acquisition of other linking theories, and into the syntactic consequences of the learnability of UTAH and rUTAH.

Acknowledgements

References

Abbot-Smith, K., Lieven, E., & Tomasello, M. (2001). What pre-school children do and do not do with ungram- matical word orders. *Cognitive Development*, *16*(2), 679–692.

Akhtar, N. (1999). Acquiring basic word order: evidence for data-driven learning of syntactic structure. *Journal of Child Language*, *26*, 339–356.

Akhtar, N., & Tomasello, M. (1997). Young children's productivity with word order and verb morphology. *Developmental Psychology*, *33*, 952–965.

Arunachalam, S., & Waxman, S. R. (2010). Meaning from syntax: Evidence from 2-year-olds. *Cognition*, *114*, 442–446.

Arunachalam, S., & Waxman, S. R. (2011). Grammatical form and semantic context in verb learning. *Language Learning and Development*, *7*, 169–184.

Baker, M. C. (1988). *Incorporation: A theory of grammatical function changing*. Chicago, IL: University of Chicago Press.

Baker, M. C. (1997). Thematic roles and syntactic structure. In *Elements of grammar* (pp. 73–137). Springer.

Bergelson, E., & Swingley, D. (2012). At 6–9 months, human infants know the meanings of many common nouns. *Proceedings of the National Academy of Sciences*, *109*(9), 3253–3258.

Bowerman, M. (1988). The 'No Negative Evidence' problem: How do children avoid constructing an overly general grammar? In J. Hawkins (Ed.), *Explaining language universals* (pp. 73–101). Oxford, England: Blackwell.

Boyd, J., & Goldberg, A. (2011). Learning what not to say: The role of statistical preemption and categorization in a-adjective production. *Language*, *87*(1), 55–83.

Brown, R. (1973). *A first language: The early stages*. Cambridge, MA: Harvard University Press.

Bunger, A., & Lidz, J. (2004). Syntactic bootstrapping and the internal structure of causative events. In *Proceedings of the annual boston university conference on language development* (Vol. 28, pp. 74–85).

Bunger, A., & Lidz, J. (2008). Thematic relations as a cue to verb class: 2-year-olds distinguish unaccusatives from unergatives. In (Vol. 14, p. 4).

Carter, R. (1976). Some linking regularities. In B. Levin & C. Tenny (Eds.), *On Linking: Papers by Richard Carter*. Cambridge, MA: Center for Cognitive Science, MIT.

Clark, E. V. (1988). On the logic of contrast. *Journal of Child Language*, *15*(2), 317–335.

Davis, K. F., Parker, K. P., & Montgomery, G. L. (2004). Sleep in infants and young children: Part one: normal sleep. *Journal of Pediatric Health Care*, *18*(2), 65–71.

Denison, S., Bonawitz, E., Gopnik, A., & Griffiths, T. L. (2013). Rational variability in children's causal inferences: The sampling hypothesis. *Cognition*, *126*(2), 285–300.

Denison, S., Reed, C., & Xu, F. (2011). The emergence of probabilistic reasoning in very young infants. In *Proceedings of the Annual Meeting of the Cognitive Science Society* (Vol. 33).

Dodson, K., & Tomasello, M. (1998). Acquiring the transitive construction in english: The role of animacy and pronouns. *Journal of Child Language*, *25*, 605–622.

Dowty, D. (1991). Thematic proto-roles and argument selection. *Language*, 547–619.

Fillmore, C. J. (1968). The case for case. In E. Bach & R. Harms (Eds.), *Universals in Linguistic Theory*. New York: Holt, Rinehart, & Winston, Inc.

Fisher, C. (1996). Structural limits on verb mapping: The role of analogy in children's interpretation of sentences. *Cognitive Psychology*, *31*, 41–81.

Fisher, C. (2002). Structural limits on verb mapping: the role of abstract structure in 2.5-year-olds' interpretations of novel verbs. *Developmental Science*, *5*(1), 55–64.

Fisher, C., Gertner, Y., Scott, R. M., & Yuan, S. (2010). Syntactic bootstrapping. *Wiley Interdisciplinary Reviews: Cognitive Science*, *1*(2), 143–149.

Fisher, C., Hall, D. G., Rakowitz, S., & Gleitman, L. (1994). When it is better to receive than to give: Syntactic and conceptual constraints on vocabulary growth. *Lingua*, *92*, 333–375.

Fisher, C., Jin, K.-s., & Scott, R. M. (2020). The developmental origins of syntactic bootstrapping. *Topics in Cognitive Science*, *12*, 48–77.

Frank, M. C., Goodman, N. D., & Tenenbaum, J. B. (2009). Using speakers' referential intentions to model early cross-situational word learning. *Psychological Science*, *20*(5), 578–585.

Gertner, Y., Fisher, C., & Eisengart, J. (2006). Learning words and rules: Abstract knowledge of word order in early sentence comprehension. *Psychological Science*, *17*(8), 684–691.

Gleitman, L. (1990). The structural sources of verb meanings. *Language Acquisition*, *1*(1), 3–55.

Goldberg, A. E. (1995). *Constructions: A construction grammar approach to argument structure*. University of Chicago Press.

Goldberg, A. E. (2006). *Constructions at work: The nature of generalization in language*. Oxford University Press on Demand.

Goldberg, A. E. (2013). Argument structure constructions versus lexical rules or derivational verb templates. *Mind & Language*, *28*(4), 435–465.

Golinkoff, R. M., Hirsh-Pasek, K., Bailey, L. M., & Wenger, N. R. (1992). Young children and adults use lexical principles to learn new nouns. *Developmental Psychology*, *28*(1), 99.

Golinkoff, R. M., Mervis, C. B., & Hirsh-Pasek, K. (1994). Early object labels: The case for a developmental lexical principles framework. *Journal of Child Language*, *21*(1), 125–155.

Grimshaw, J. (1981). Form, function, and the language acquisition device. In C. L. Baker & J. J. McCarthy (Eds.), *The logical problem of language acquisition.* MIT Press.

Grimshaw, J. (1990). *Argument structure.* Cambridge, MA: The MIT Press.

Hart, B., & Risley, T. (1995). *Meaningful differences in the everyday experience of young american children*. Baltimore, MD: P.H. Brookes.

Hart, B., & Risley, T. R. (2003). The early catastrophe: The 30 million word gap by age 3. *American Educator*, *27*(1), 4–9.

Irani, A. (2019). How Children Learn to Disappear Causative Errors. In M. M. Brown & B. Dailey (Eds.), *Proceedings of the 43rd Boston University Conference on Language Developmen* (pp. 298–310). Somerville, MA: Cascadilla Press.

Jackendoff, R. (1987). The status of thematic relations in linguistic theory. *Linguistic inquiry*, *18*(3), 369–411.

Landau, B., & Gleitman, L. R. (1985). *Language and experience: Evidence from the blind child*. Harvard University Press.

Larson, R. K. (1988). On the double object construction. *Linguistic Inquiry*, *19*(3), 335–391.

Larson, R. K. (1990). Double objects revisited: Reply to Jackendoff. *Linguistic Inquiry*, *21*(4), 589–632.

Legate, J., & Yang, C. (2013). Assessing Child and Adult Grammar. In R. Berwick & M. Piatelli-Palmarini (Eds.), *Rich Languages from Poor Inputs* (pp. 168–182). Oxford, UK: Oxford University Press.

Levin, B., & Rappaport Hovav, M. R. (1995). *Unaccusativity: At the syntax-lexical semantics interface* (Vol. 26). MIT press.

Lidz, J. (2020). Learning, memory, and syntactic bootstrapping: A meditation. *Topics in Cognitive Science*, *12*, 78–90.

Lidz, J., Gleitman, H., & Gleitman, L. (2003). Understanding how input matters: verb learning and the footprint of universal grammar. *Cognition*, *87*, 151–178.

Lidz, J., White, A. S., & Baier, R. (2017). The role of incremental parsing in syntactically conditioned word learning. *Cognitive Psychology*, *97*, 62–78.

Ma, W., Golinkoff, R. M., Houston, D. M., & Hirsh-Pasek, K. (2011). Word learning in infant- and adult-directed speech. *Language Learning and Development*, *7*(3), 185–201.

MacWhinney, B. (2000). *The CHILDES Project: Tools for Analyzing Talk*. Mahwah, NJ: Lawrence Erlbaum Associates.

Markman, E. M., & Wachtel, G. F. (1988). Children's use of mutual exclusivity to constrain the meanings of words. *Cognitive Psychology*, *20*(2), 121–157.

Markman, E. M., Wasow, J. L., & Hansen, M. B. (2003). Use of the mutual exclusivity assumption by young word learners. *Cognitive Psychology*, *47*(3), 241–275.

Naigles, L. (1990). Children use syntax to learn verb meanings. *Journal of child language*, *17*(02), 357–374.

Naigles, L. (1996). The use of multiple frames in verb learning via syntactic bootstrapping. *Cognition*, *58*, 221–251.

Naigles, L. G., & Kako, E. T. (1993). First contact in verb acquisition: Defining a role for syntax. *Child development*, *64*(6), 1665–1687.

Noble, C. H., Rowland, C. F., & Pine, J. M. (2011). Comprehension of Argument Structure and Semantic Roles: Evidence from English-Learning Children and the Forced-Choice Pointing Paradigm. *Cognitive Science*, *35*, 963–982.

Olguin, R., & Tomasello, M. (1993). Twenty-five-month-old children do not have a grammatical category of verb. *Cognitive Development*, *8*, 245–272.

Pearl, L. (2017). Evaluation, use, and refinement of knowledge representations through acquisition modeling. *Language Acquisition*, *24*(2), 126–147.

Pearl, L. (in press). Modeling syntactic acquisition. In J. Sprouse (Ed.), *Oxford handbook of experimental syntax*. Oxford University Press.

Pearl, L., Ho, T., & Detrano, Z. (2017). An argument from acquisition: Comparing English metrical stress representations by how learnable they are from child-directed speech. *Language Acquisition*, *24*, 307–342.

Pearl, L., & Sprouse, J. (2013a). Computational Models of Acquisition for Islands. In J. Sprouse & N. Hornstein (Eds.), *Experimental Syntax and Islands Effects* (pp. 109–131). Cambridge: Cambridge University Press.

Pearl, L., & Sprouse, J. (2013b). Syntactic islands and learning biases: Combining experimental syntax and computational modeling to investigate the language acquisition problem. *Language Acquisition*, *20*, 19–64.

Pearl, L., & Sprouse, J. (2015). Computational modeling for language acquisition: A tutorial with syntactic islands. *Journal of Speech, Language, and Hearing Research*, *58*, 740–753.

Pearl, L., & Sprouse, J. (2019). Comparing solutions to the linking problem using an integrated quantitative framework of language acquisition. *Language*.

Perfors, A. (2012). Bayesian models of cognition: What's built in after all? *Philosophy Compass*, *7*(2), 127–138.

Perfors, A., Tenenbaum, J., Griffiths, T., & Xu, F. (2011). A tutorial introduction to Bayesian models of cognitive development. *Cognition*, *120*(3), 302–321.

Perkins, L. (2019). *How Grammars Grow: Argument Structure and the Acquisition of Non-Basic Syntax"* (Unpublished doctoral dissertation). University of Maryland, College Park.

Perkins, L., Feldman, N., & Lidz, J. (2017). Learning an input filter for argument structure acquisition. In *Proceedings of the 7th Workshop on Cognitive Modeling and Computational Linguistics (CMCL 2017)* (pp. 11–19).

Perlmutter, D. M., & Postal, P. M. (1984). Impersonal passives and some relational laws. *Studies in Relational Grammar*, *2*, 126–170.

Pinker, S. (1984). *Language learnability and language development.* Cambridge, MA: Harvard

University Press.

Pinker, S. (1989). *Learnability and cognition: the acquisition of argument structure.* Cambridge, MA: MIT Press.

Saffran, J. R., Aslin, R. N., & Newport, E. L. (1996). Statistical learning by 8-month-old infants. *Science*, *274*(5294), 1926–1928.

Schuler, K., Yang, C., & Newport, E. (2016). Testing the Tolerance Principle: Children form productive rules when it is more computationally efficient to do so. In *The 38th Cognitive Society Annual Meeting, Philadelphia, PA*.

Scott, R. M., & Fisher, C. (2009). Two-year-olds use distributional cues to interpret transitivity-alternating verbs. *Language and cognitive processes*, *24*(6), 777–803.

Smith, L., & Yu, C. (2008). Infants rapidly learn word-referent mappings via cross-situational statistics. *Cognition*, *106*(3), 1558–1568.

Sneller, B., Fruehwald, J., & Yang, C. (2019). Using the Tolerance Principle to predict phonological change. *Language Variation and Change*, *31*(1), 1–20.

Speas, M. (1990). *Phrase structure in natural language* (Vol. 21). Springer Science & Business Media.

Stahl, A. E., Romberg, A. R., Roseberry, S., Golinkoff, R. M., & Hirsh-Pasek, K. (2014). Infants segment continuous events using transitional probabilities. *Child Development*, *85*(5), 1821–1826.

Thiessen, E. D., & Saffran, J. R. (2003). When cues collide: use of stress and statistical cues to word boundaries by 7-to 9-month-old infants. *Developmental Psychology*, *39*(4), 706.

Valian, V. (1991). Syntactic subjects in the early speech of American and Italian children. *Cognition*, *40*(1), 21–81.

Wang, H., & Mintz, T. (2008). A dynamic learning model for categorizing words using frames. In H. Chan, H. Jacob, & E. Kapia (Eds.), *Proceedings of the 32nd annual Boston University Conference on Language Development [BUCLD 32]* (pp. 525–536). Somerville, MA: Cascadilla Press.

Xu, F., & Tenenbaum, J. (2007). Word Learning as Bayesian Inference. *Psychological Review*, *114*(2), 245-272.

Yang, C. (2002). *Knowledge and Learning in Natural Language*. Oxford, UK: Oxford University Press.

Yang, C. (2004). Universal grammar, statistics or both? *Trends in Cognitive Science*, *8*(10), 451–456.

Yang, C. (2005). On productivity. *Yearbook of Language Variation*, *5*, 333–370.

Yang, C. (2012). Computational models of syntactic acquisition. *WIREs Cognitive Science*, *3*, 205–213.

Yang, C. (2015). Negative knowledge from positive evidence. *Language*, *91*(4), 938–953.

Yang, C. (2016). *The price of linguistic productivity: How children learn to break the rules of language*. MIT Press.

Yang, C. (2017). Rage against the machine: Evaluation metrics in the 21st century. *Language Acquisition*, *24*(2), 100–125.

Yang, C. (2018). A User's Guide to the Tolerance Principle. *lingbuzz*, lingbuzz/004146.

Yuan, S., & Fisher, C. (2009). "really? she blicked the baby?" two-year-olds learn combinatorial facts about verbs by listening. *Psychological Science*, *20*(5), 619–626.

Yuan, S., Fisher, C., & Snedeker, J. (2012). Counting the nouns: Simple structural cues to verb meaning. *Child Development*, *83*(4), 1382–1399.

Yurovsky, D., Case, S., & Frank, M. C. (2017). Preschoolers flexibly adapt to linguistic input in a noisy channel. *Psychological Science*, *28*(1), 132–140.

A   Calculating the number of verb links children hear

At the individual verb level, a child using the sufficiency threshold would consider the verb links that obey and disobey the pattern under consideration (1-link or 3-link). This means that to calculate the sufficiency threshold appropriately, we need to extrapolate from our corpus input sample to the true count of verb link instances for each individual verb. If we assume our corpus distribution of verb link instances reflects the true distribution that children hear, we simply need to estimate the true number of link instances children hear. We counted the individual verb link instances based on the utterances in our corpus sample that American English children hear at different ages (in particular, by age three, four, and five). So, if we can calculate the total utterances children hear by these ages, we can calculate the necessary multiplier for the verb link instance counts in our corpus. Then, we can multiply any individual verb's link instances by this multiplier and apply the sufficiency threshold.

To estimate the total number of utterances children hear by different ages, we draw on Hart and Risley (1995, 2003), who find that professional class parents spoke an average of 487 utterances per hour to their children ages 13-36 months. We also draw on Davis et al. (2004), which provides average total daily sleep hours for children; we subtract sleeping hours from total hours per day (24) to calculate the waking hours during which children hear input from their caretakers.

We also assume that children need certain linguistic knowledge and abilities in place before they can reliably extract the syntactic positions that the arguments appear in. In particular, they need to be able to segment the speech stream (potentially available at 7 months: Thiessen & Saffran, 2003), identify the meaning of word forms appearing in certain positions (potentially available at 6 months: Bergelson & Swingley, 2012), and recognize enough syntactic structure to reliably identify syntactic positions (potentially available at 28 months: L. Naigles, 1990; L. G. Naigles & Kako, 1993; Scott & Fisher, 2009; Yuan & Fisher, 2009). We note that the learning process investigated here assumes reliable syntactic percepts of argument positions like *subject*, *object*, and *oblique*. So, we take 28 months to be the lower age bound for this learning process. Given this, we calculate children's waking hours starting at 28 months, when they would hear utterances and potentially be able to reliably extract the relevant verb argument information (i.e., syntactic position and thematic role of the argument).

Based on Davis et al. (2004), children sleep for approximately 13 hours/day at 2 (24-35 months), 12 hours/day at 3 (36-47 months), and 11.5 hours/day at 4 (48-59 months). Therefore, we can calculate their waking hours and total utterances heard, as in Table 9.

Table 9: Calculating the total utterances children hear by ages three (36 months), four (48 months), and five (60 months), for the purposes of learning linking theories. These calculations are based on waking hours per day (waking), total waking hours, and children hearing 487 utterances per waking hour. Cumulative utterances heard by age three (<3yrs = <36 months), four (<4yrs = <48 months), and five (<5yrs = <60 months) are shown.

age	age range	waking	total waking hours	total utt	cumulative utt
<3yrs	28 - 35 months	11	11 hrs/day * 365 days/yr * 8/12 = 2676.67	1303537	1303537
<4yrs	36 - 47 months	12	12 hrs/day * 365 days/yr = 4380	2133060	3436597
<5yrs	48 - 59 months	12.5	12.5 hrs/day * 365 days/yr = 4562.5	2221938	5658535

   With these total utterance estimates, we can then extrapolate from our corpus samples by multiplying the individual verb utterance counts by an appropriate constant. This constant is calculated for each dataset in Table 10. As mentioned in the beginning, individual verb links were derived from the utterances in our corpus; so, this same multiplier can be used to estimate the true counts of verb links that children of different ages would have heard in their input (i.e., corpus count * multiplier = true count). We then apply the sufficiency threshold to these counts.

Table 10: Calculating the multiplier constant for each dataset, based on the number of utterances in the corpus sample and the number of utterances children would have heard by that age.

dataset	# utt	# utt heard	multiplier
<3yrs	39772	1303537	1303537/39772 = 32.775
<4yrs	50737	3436597	3436957/50737 = 67.741
<5yrs	56461	5658535	5658535/56461 = 100.220