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Importance of Path Planning Variability: A Simulation Study

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Abstract

Individuals vary in the way they navigate through space. Some take novel shortcuts, while others rely on known routes to find their way around. We wondered how and why there is so much variation in the population. To address this, we first compared the trajectories of 368 human subjects navigating a virtual maze with simulated trajectories. The simulated trajectories were generated by strategy-based path planning algorithms from robotics. Based on the similarities between human trajectories and different strategy-based simulated trajectories, we found that there is a variation in the type of strategy individuals apply to navigate space, as well as variation within individuals on a trial-by-trial basis. Moreover, we observed variation within a trial when subjects occasionally switched the navigation strategies halfway through a trajectory. In these cases, subjects started with a route strategy, in which they followed a familiar path, and then switched to a survey strategy, in which they took shortcuts by considering the layout of the environment. Then we simulated a second set of trajectories using five different but comparable artificial maps. These trajectories produced the similar pattern of strategy variation within and between trials. Furthermore, we varied the relative cost, that is, the assumed mental effort or required timesteps to choose a learned route over alternative paths. When the learned route was relatively costly, the simulated agents tended to take shortcuts. Conversely, when the learned route was less costly, the simulated agents showed preference toward a route strategy. We suggest that cost

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or assumed mental effort may be the reason why in previous studies, subjects used survey knowledge when instructed to take the shortest path. We suggest that this variation we observe in humans may be beneficial for robotic swarms or collections of autonomous agents during information gathering.

Keywords: Adaptive behavior; Cognitive load; Computer simulations; Navigation; Neural networks; Path planning; Robotics

1. Introduction

Navigation strategies vary within and between organisms. It is thought that organisms ranging from honeybees to rats to humans utilize mental maps, beacons, and turn sequences to find their way around (Gallistel, 1990, 2017). People differ considerably in their ability to learn and navigate environments (Hegarty & Waller, 2005; Hegarty, Montello, Richardson, Ishikawa, & Lovelace, 2006; Ishikawa & Montello, 2006; Weisberg & Newcombe, 2016, 2018). Some rely more on survey knowledge where a mental representation of the physical environment is represented in an allocentric format, that is, a world-centered view that allows inferences about distances and directions between locations. Others rely more on an egocentric format where they use a series of movements to follow a set path or route-based navigation. Using survey knowledge allows for more flexible path planning. Through this mentalization, one can take shortcuts or plan paths through areas in the environment that have not previously been experienced. In contrast, familiar routes can be followed out of habit and may require less mental effort.

People use different types of knowledge to navigate (Chrastil & Warren, 2015). *Survey knowledge* contains metric information that includes distances and directions between locations. In some prior studies, survey knowledge was measured by pointing accuracy to endpoints within and between routes (Weisberg & Newcombe, 2016). In other studies and the present study, this knowledge enables flexible path planning resulting in shortcuts or planned trajectories over never experienced paths (Chrastil & Warren, 2014). *Route knowledge* consists of sequences of actions associated with places or decision points. Typically, the routes are set paths, familiar, and inflexible. Chrastil and Warren suggested that people may not need to acquire a globally consistent survey knowledge but a third type of knowledge, namely, labeled graph knowledge (Chrastil & Warren, 2015). *Graph knowledge* consists of a network of nodes linked by edges (i.e., graph), where the nodes are locations in the environment, and the edges are paths between those nodes. The nodes can be landmarks or decision points along a path. The edges can contain rough metric information (i.e., labeled graph knowledge) or be realized through non-metric topologies, but the metric information does not need to be globally consistent as a whole mental map. In the present study, we define topological strategies as using graph knowledge with landmarks as nodes and metric information between landmarks as edges (Chrastil & Warren, 2014). Note that these edges do not necessarily follow traversable paths but do influence which paths a person will take to reach a destination.

Boone et al. showed that there are differences in the human population on which knowledge people tend to use during navigation (Boone, Gong, & Hegarty, 2018; Boone, Maghen, &

Hegarty, 2019; Hegarty, He, Boone, & Chrastil, 2020). Men tended to demonstrate survey knowledge by taking more shortcuts than women, and women tended to use route knowledge by taking a learned route or at times taking the route in the reverse direction that it was learned (Boone et al., 2018). Although both females and males demonstrated route knowledge when told to find a location, they did have the capacity to use survey knowledge when instructed to take the shortest path (Boone et al., 2019). This suggests that many participants may have survey knowledge, but they might find it easier to take a learned route. It is an open question why this might be the case.

Rather than suggesting that some people are better at finding their way around than others, it might be the case that this population variation has benefits. For example, it has been shown that people and even birds are more adept at problem-solving in groups than individually (Laughlin, Hatch, Silver, & Boh, 2006; Liker & Bokony, 2009). The different skills or abilities brought by each individual can be complementary. Certainly, different organisms use different strategies related to their environmental niche. What about variation within organisms? Could this provide an advantage in overall navigation at the population level?

Understanding human variation may have advantages for robot navigation and possibly route planning for self-driving vehicles. In standard robotics navigation, algorithms are designed to find an optimal solution for navigational problems and use that repeatedly without variation. For instance, Simultaneous Localization and Mapping (SLAM) algorithms map novel environments while exploring them by keeping track of the robot's movements (Kohlbrecher, von Stryk, Meyer, & Klingauf, 2011; Mur-Artal, Montiel, & Tardos, 2015). These maps are then used to plan paths or routes. Although SLAM maps contain metric information, they rarely contain metadata related to the robot's motivation or behavioral state. Robot navigation and path planning are typically deterministic and strive to reduce variability (LaValle, 2011a, 2011b). In contrast, it has been shown that taking environmental cost and uncertainty into account can result in flexible navigation solutions (Hwu, Wang, Oros, & Krichmar, 2018; Xing, Zou, & Krichmar, 2020).

In the present paper, we explore the notion that not only is navigational variability the norm, but it may also provide advantages. In one set of computer simulations, we use a variety of path planning algorithms to model the variations in human navigation observed in prior studies (Boone et al., 2018, 2019; He, Boone, & Hegarty, 2020a, 2020b). Our method allowed us to quantify the different navigational strategies participants used during the task, such as applying survey, route, or graph knowledge. Similar to prior studies, we find variations between subjects, but we also find that subjects vary their strategies on a trial-by-trial basis. Furthermore, subjects sometimes switch from a route strategy to a survey strategy midway through a trajectory but rarely in the reverse order of survey then route.

In order to learn what determined the mix of strategies, we carried out a second set of computer simulations, in which we tested whether varying the cost of traversal in artificial environments can explain the variability within and between simulated agents. Cost is defined by the assumed mental effort to choose novel paths over familiar or learned routes and was implemented in the path planning algorithms by varying the number of timesteps to calculate different subpaths. Indeed, we find that lowering the cost of traversing alternative paths relative to familiar routes leads to shortcut taking; similar to when human subjects were instructed to take the shortest path (Boone et al., 2019). In general, such flexibility and variability may be advantageous for effectively exploring environments.

2. Methods

To examine path planning variation within and between subjects, we carried out two sets of computer simulations. In the first set, artificial trajectories were generated, using path planning algorithms from robotics that assumed different strategies use different knowledge (e.g., graph, survey, or route). Then we compared how similar the strategy-based trajectories were to human trajectories. The human trajectories were collected by Boone, He, and others in a virtual reality environment (Boone, 2019; Boone et al., 2019; He et al., 2020a; He et al., 2020b). The general approach to compare human trajectories with simulated trajectories was as follows: (a) Raw positional trajectories from human subjects were transformed into a grid map. (b) Different path planning strategies were simulated to see which strategy best fit an individual subject's behavior on a particular trial. Specifically, these strategies used either survey knowledge, route knowledge, or graph knowledge, as well as mixtures of these strategies, to plan paths. Using these path planning algorithms to model human navigation, we were able to show that there is variation between subjects and within subjects on a trial-by-trial basis.

A second set of computer simulations were carried out on artificially generated maps to investigate whether varying trajectory costs in the environment could explain the observed variation in the human population. In these simulations, the cost of traveling over locations in the environment was varied. The least costly path, as measured by the number of timesteps to calculate a path between locations, was compared to the path planners using specific knowledge. In this way, we were able to show that the cost of traversal, which we suggest is related to mental effort, can explain the variation in path planning strategies.

2.1. Data collection of human navigation trajectories

In the human navigation studies, 368 subjects learned the layout of a virtual maze environment by taking a fixed tour (see Fig. 1, middle) through the environment five times from a first-person perspective using a mouse and keyboard interface. There are 12 landmarks in the environment (see Fig. 1, right) and people were asked to remember the landmark locations. There were 20 trials in the test phase. On each trial, participants were transported to a landmark and were asked to find a goal landmark using the same mouse and keyboard interface. All subjects performed the same 20 trials, in which each trial had a specific start and goal location. Two hundred and six of these subjects were just told to go to the goal, and 162 were told to take the shortest path to the goal. These will be referred to in the remainder of the paper as "Goto Goal" and "Shortcut", respectively. Using the Cartesian coordinates of the environment (see Boone et al., 2019, fig. 13), a 13-by-13 grid world of the environment was created (see Fig. 1, left).

Raw trajectories from the subject data were converted into Cartesian coordinates that fit on the map shown in Fig. 1. This provided path coordinates for the 368 subjects when they

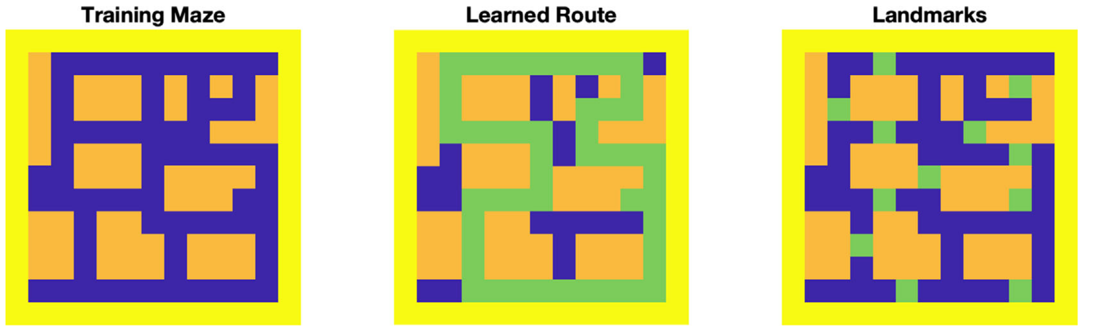

Fig 1. Maps of the virtual environment are used to collect human trajectories and to simulate the first set of strategy-based trajectory simulations. Left: Map is a 13 × 13 grid. Yellow denotes the border, orange denotes untraversable areas, and blue denotes traversable areas. Middle: Green denotes the learned route. Right: Green denotes landmark locations. The map was derived from (Boone et al., 2018, 2019).

navigated for the 20 trials during the test phase. The trials were annotated with the instructions Goto Goal or Shortcut.

2.2. *Strategy-based path planning algorithms to generate artificial trajectories*

Artificial trajectories were simulated using a suite of path planning algorithms based on the type of strategy people use to navigate the world (Chrastil & Warren, 2015). (a) *Survey*: Survey strategy uses survey knowledge that includes distances and directions between locations in an environment. In the present study, we assume the metric information of survey knowledge is globally consistent as a whole map, which is different from graph knowledge. Therefore, the survey strategy was indicated by high similarity between the observed paths and shortcuts that deviated from the learned route. (b) *Topological*: Topological strategy is based on using graph knowledge (i.e., a network of nodes linked by edges). In our simulations, the landmarks denote the nodes, and the edges are the Euclidean distances and directions between those nodes. In contrast with the survey knowledge, graph knowledge can only contain rough metric information (i.e., labeled graph knowledge) among local locations. In the present study, the topological strategy was indicated by high similarity between the observed paths to a path that followed a sequence of landmarks. Such a path may not be the shortest and may not follow the learned route. (c) *Route*: Route strategy was used relying on route knowledge, which consisted of a sequence of locations learned before. The learned route is shown in Fig. 1. Route strategy was indicated by high similarity between the observed path to the learned route.

To simulate the survey strategy, we used a spiking wavefront propagation path planner (Hwu et al., 2018). The spiking wavefront path planner will calculate the shortest traversable path between two points on a map. Any path planner used in robotics (e.g., A*, Djikstra) would work to simulate survey knowledge in this setup (for a review, see LaValle, 2011a, 2011b). Because the spiking wavefront planner is adaptive by taking into account the cost of traversal, it was chosen for these simulated experiments.

Briefly, the spiking wavefront propagation path planner works as follows. A neural network is created with neurons representing each location in the map. The neurons are connected to their four neighbors, representing movement in the North, South, East, and West directions. The weights between neurons are related to the cost of traversal. In the present study, the map in Fig. 1 had a weight value of 120 for the border (denoted in yellow), 100 for the untraversable locations (denoted in orange), and 1 for the open path (denoted in dark blue). These weights represent the cost of traversal, which is realized by delaying a signal propagating from one neuron to another according to the cost. The algorithm starts by generating a spike or action potential in the neuron representing the goal location. This action potential is propagated to its neighbors with a delay related to the connection weight. Once the action potential is received by the postsynaptic neuron, the postsynaptic neuron generates a spike and propagates this action potential to its neighbors. This propagation proceeds until the neuron at the start location spikes. At that point, the algorithm is halted. The path can be read out by working from the start neuron to the goal neuron, observing which neuron caused its neighbor to fire a spike. The result is the lowest cost path from start to goal. More details on the spiking wavefront propagation algorithm can be found in Hwu et al. (2018).

In the wavefront planner, similar to other path planning algorithms, the cost is reflected in how many timesteps it takes to move from one location neuron to another neuron. The optimal path between a starting neuron and a goal neuron is the path taking the least timesteps from start to finish. Obstacles and walls have very high costs, which causes the path planner to avoid these regions of the map. Typically, the shortest traversable metric path will take the least timesteps. However, the cost can also denote traversal difficulty (e.g., hills or loose gravel). In this case, the path with the least timesteps might cover a longer distance but travel over easier to traverse terrains. In the present paper, we suggest the cost could represent cognitive load. For example, the familiar learned route may take less mental effort to traverse even though it is longer than a shortcut.

To simulate topological strategy, the spiking wavefront path planner used landmarks from the starting location to the goal location to create a path. For each sub-path between landmarks, the landmark closest to the agent that was also toward the goal was chosen. The planner iterated through these sub-paths until the goal was reached.

To simulate route strategy, a path was planned from the starting location to the goal location on the learned route using the sequence of coordinates from the learned route. All 20 trials started and ended on locations situated on the learned route. As in the human studies, a reversed sequence of learned route coordinates could be used to plan paths.

We assumed that subjects might change their strategy midway through a trajectory. Therefore, we simulated a mixture of strategies to simulate the human data. For example, a subject might start on the route, and then halfway through realize they could take a shortcut for the remainder of the trajectory. Because trajectories were relatively short, we only switched strategies halfway through a trajectory.

These assumptions led to 14 path planning algorithms that were used to simulate the human subject data and then used to simulate agent navigation in larger, varied environments (see

Table 1
Different strategies applied in the simulation

Strategy	Path Planner Description
Survey	Shortest path between a starting location and a goal. Uses the spiking wavefront path planner
Topological	Series of paths from the starting location to the goal location using landmarks as sub-paths. Uses the spiking wavefront path planner to plan paths between landmarks
Route	Plans a path using a sequence of coordinates from the learned route (see Fig. 1).
Route Reversed	Same as route strategy but uses the sequence of coordinates from the learned route in reverse order
Survey then Route	The first half of the trajectory uses the survey strategy, and the second half of the trajectory uses the route strategy. Note that the survey strategy may be slightly longer than 50% to ensure that the start of the route strategy is on the learned route
Survey then Reverse Route	The first half of the trajectory uses the survey strategy, and the second half of the trajectory uses the route strategy in reverse
Topological then Route	The first half of the trajectory uses the topological strategy, and the second half of the trajectory uses the route strategy
Topological then Reverse Route	The first half of the trajectory uses the topological strategy, and the second half of the trajectory uses the route strategy in reverse
Route then Survey	The first half of the trajectory uses the route strategy, and the second half of the trajectory uses the survey strategy
Reverse Route then Survey	The first half of the trajectory uses the route strategy in reverse order, and the second half of the trajectory uses the survey strategy
Route then Topological	The first half of the trajectory uses the route strategy, and the second half of the trajectory uses the topological strategy
Reverse Route then Topological	The first half of the trajectory uses the route strategy in reverse order, and the second half of the trajectory uses the topological strategy
Topological then Survey	The first half of the trajectory uses the topological strategy, and the second half of the trajectory uses the survey strategy
Survey then Topological	The first half of the trajectory uses the survey strategy, and the second half of the trajectory uses the topological strategy

Table 1). We omitted *Route* then *Route Reversed*, and vice versa, because this would lead to oscillations going forward and backward without reaching the goal.

2.3. Comparing human trajectories with artificial trajectories

We applied the 14 strategies given in Table 1 to each of the 20 trials. We used the Frechet distance metric to calculate how close the strategy resembled the path taken by each subject (Danziger, 2020; Eiter & Mannila, 1994). The Frechet distance between two curves in a metric space is a measure of the similarity between the curves. In our case, it is the distance between two sets of path coordinates. Frechet distance does not require that the two sets of points being compared have the same length.

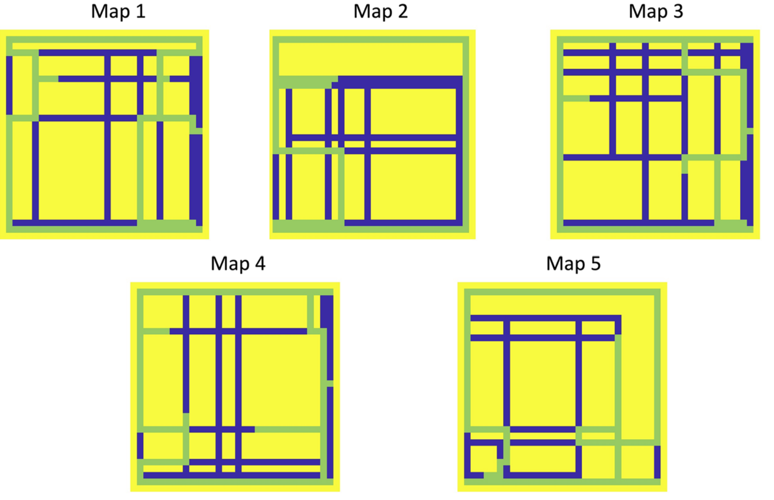

Fig 2. Maps used for simulated environments. The maps contain 32-by-32 grid locations. The yellow areas are untraversable. The green areas denote the learned route. The dark blue areas denote traversable open paths. The cost of the learned route is always one. The cost of the open path can vary from 1, 2, 4, 8, to 16.

2.4. Simulating trajectories on novel maps

To test whether these different path planning strategies scale and to provide a possible explanation for the variation in human behavior, we created five artificial maps that had 32-by-32 locations (see Fig. 2). Each map had an assumed learned route (shown in green in Fig. 2) and other traversable locations (shown in dark blue in Fig. 2). To reflect the assumption that familiar routes take less mental effort, the cost of traversing the learned route was set to one. The untraversable regions, which are shown in yellow, had an extremely high cost of 120, which mimics the walls or boundaries in an environment. To model the mental effort someone might take in trying paths that differ from the learned route, the cost of traversing alternative paths, shown in dark blue in Fig. 2, varied and were set to 1, 2, 4, 8, and 16. A value of 16 meant it was 16 times more costly to travel over the open path than over the learned route. In the spiking wavefront planner, this cost was applied by delaying the propagation of the wave by the cost. In this way, the best path may not be the shortest if traveling over the learned route is less costly. The learned route was randomly generated but tended to follow the perimeter of the map environment with forays into the center of the environment. Since it was not learned, there was no order associated with the learned route. Therefore, there was no preference for taking the route in the forward or reverse order. Thirteen landmarks were scattered across the environment. These were required to be on the traversable areas of the map. Sixteen trials were randomly generated using the landmarks as starting and goal locations. To provide complexity to the trajectories, trials could not be straight lines between landmarks.

2.5. Software and computation

All simulations and analyses were carried out using MATLAB (MathWorks) scripts on a 3GHz 6-Core Intel i5 iMac with 32GB of memory. The path planning algorithm and analysis scripts are available on GitHub:

- https://github.com/jkrichma/PathPlanning
- https://github.com/CarolHeChuanxiuyue/Variation-in-Human-Navigation-Strategies

3. Results

Simulations were carried out to emulate the variation in human navigation strategies with path planning algorithms. In the first set of simulations, we modeled the trajectories of subjects in the human navigation studies using combinations of path planning strategies. In the second set of simulations, we tested whether the cost of traversal on different maps could explain the variation observed in humans.

3.1. Reproducing human navigation strategies with path planning models

Each of the 368 subjects ran 20 trials, where each trial had different start and goal locations. To categorize human trajectories to show which strategy subjects were using on different trials, we ran all 14 path planning models (see Table 1) on each of these 20 trials and measured how closely each simulated trajectory resembled the subject trajectory using the Frechet distance metric.

For clarification, it is important to explain how we compare human trajectories with the simulated trajectories. For example, Fig. 3 shows a typical visualization of our results. The participant started their path near the top right corner of the maze (cyan square in Fig. 3a) and ended toward the bottom of the maze (green square in Fig. 3a). The lighter blue line represents the participant's actual path between the start and goal locations. Four different path planning models were applied with the same start and goal locations (Fig. 3c–f). Note how the simulated survey strategy closely matches the participant's trajectory (compare Fig. 3a with Fig. 3c). In contrast, note the difference between trajectories when comparing the subject's trajectory to the simulated route trajectories (compare Fig. 3a with Fig. 3e and f). Fig. 3b shows quantitatively how close each simulated path planning trajectory matches the actual trajectory for this start and goal location. In this case, the survey strategy (Sur) has the smallest Frechet distance, which suggests the subject took a shortcut between the start and goal locations. In Fig. 3g, we show the Frechet distance for all start and goal locations. The labels on the y-axis are the different trial numbers as given by the original human navigation studies (Boone, 2019; Boone et al., 2019; He et al., 2020a; He et al., 2020b). Each column depicts the different modeled path planning algorithm. Each cell in the heatmap denotes the Frechet distance for a given strategy and trial with cooler colors representing closer matches between the model and the participant's trajectories.

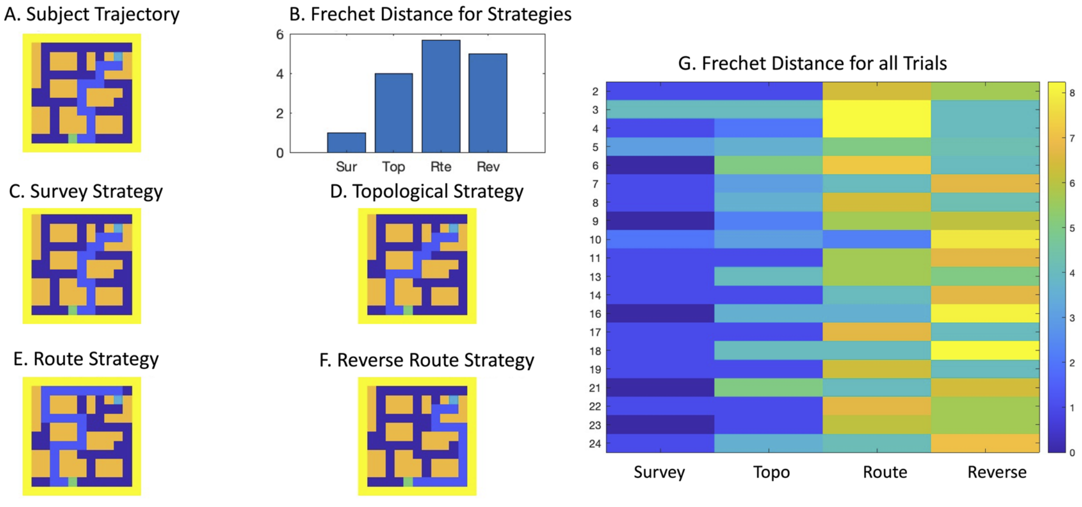

Fig 3. Representative example of a subject that always used a survey strategy across 20 trials. (a) Subject trajectory is shown in light blue for Trial 13. The start is marked in cyan and the goal is marked in green. (b) Frechet distance for Trial 13. The y-axis shows the Frechet distance between the subject trajectory and different path planners. Sur = survey, Top = topological, Rte = route, and Rev = reverse route. (c) Simulated trajectory for Trial 13 using survey path planner. (d) Simulated trajectory for Trial 13 using Topological path planner. (e) Route trajectory path planner, which follows the learned route, for Trial 13. (f) Reverse route trajectory path planner, which follows the learned route in reverse, for Trial 13. (g) Heatmap showing the Frechet distance for four strategies in all 20 trials. The rows are trials, and the columns are strategies. The colorbar to the right of the heatmap denotes the Frechet distance value.

Due to individual differences in strategy use, different subject trajectories are aligned with different simulated trajectories generated by the path planners. For example, Fig. 3 shows a representative example of a subject that preferred to use survey strategy on all trials (see Fig. 3b and g). Fig. 3c–f show the different strategies on Trial 13. Note how close the survey path planner in Fig. 3c resembles the subject trajectory in Fig. 3a. The heatmap in Fig. 3g show that the trajectories calculated by the survey path planning algorithm most resembled this subject's trajectories on all trials, as denoted by the smaller Frechet distances (cooler colors) in the Survey column of Fig. 3g, compared to the larger Frechet distances (warmer colors) in the topological, route and reverse route strategy columns.

In contrast, Fig. 4 shows a representative example of a subject that preferred to use route knowledge. Note how in this case, the route path planning algorithm almost identically matches the trajectory of this subject (compare Fig. 4a to e). In all 20 trials, the Frechet distance between the route path planning algorithm and the subject's trajectory was the smallest (Fig. 4g).

We hypothesized that subjects may mix strategies within a given trial. Therefore, we tested which strategies given in Table 1 best predicted the subject's navigation behavior. The boxplots in Fig. 5 show the distributions of Frechet distances for the strategy that best predicted a trajectory. The box on the left shows the Frechet distances when only the four pure strategies

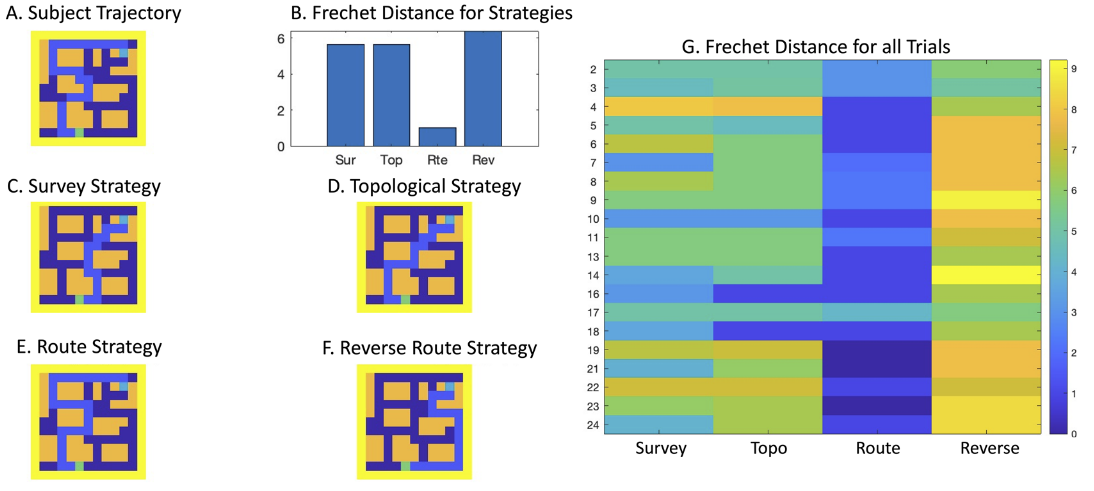

Fig 4. Representative example of a subject that always used route strategy across 20 trials. (a) Subject trajectory is shown in light blue for Trial 13. The start is marked in cyan and the goal is marked in green. (b) Frechet distance for Trial 13. The y-axis shows the Frechet distance between the subject trajectory and the different path planners. Sur = survey, Top = topological, Rte = route, and Rev = reverse route. (c) Simulated trajectory for Trial 13 using survey path planner. (d) Simulated trajectory for Trial 13 using topological path planner. (e) Route trajectory, which follows the learned route, for Trial 13. (f) Reverse route trajectory, which follows the learned route in reverse, for Trial 13. (g) Heatmap showing the Frechet distance for four strategies in all 20 trials.

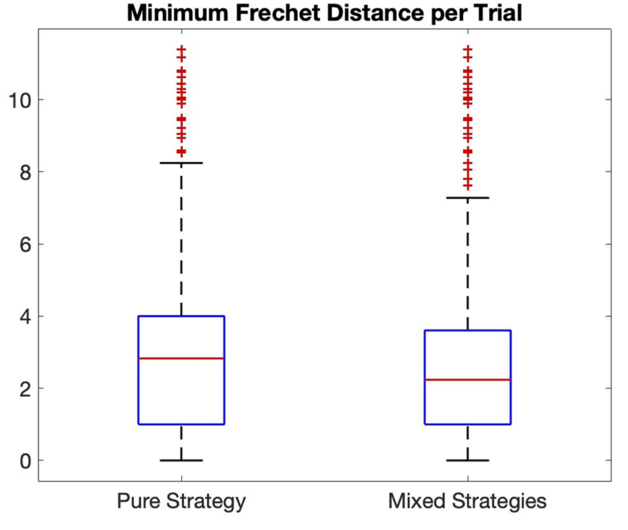

Fig 5. Minimum Frechet distance for the strategy that best predicts the subject's trajectory for all subjects and all trials. The boxplot on the left is the prediction when using the same strategy throughout a given trial. The boxplot on the right is the prediction when the subject's switched strategies midway through a trajectory. On each box, the red line is the median, the blue lower and upper edges of the box represent the 25th and 75th percentiles, respectively, the whiskers extend to extreme datapoints that are not considered outliers, and the outliers are denoted with red plus signs.

(survey, topological, route, and reverse route) were considered. The box on the right shows the distribution of Frechet distances when all 14 strategies in Table 1 were considered. The minimum Frechet distance when using a pure strategy (i.e., always stick with the same strategy in a given trial) to predict trajectories (*Median* = 2.8) was significantly larger than when including a mixture of strategies (*Median* = 2.2, Wilcoxon signed-rank test, $V = 442270$, $p < .001$). This suggests that subjects occasionally mix strategies within a trial.

Subjects would occasionally start on a route, and then switch to a survey strategy. Fig. 6 shows a histogram of which of the 14 path planners best predicted the subject's behavior. On 7.99% of the Goto Goal trials and 9.2% of the Shortcut trials, subjects started using route knowledge (either forward or reverse) and then switched to using survey knowledge. For example, see the representative subject in Fig. 7, who during Trial 3, started on the learned route and then about midway through their trajectory took a shortcut. Interestingly, subjects did not tend to start using survey strategy or topological strategy and then switch to the learned route.

In Boone et al. (2019), it was shown that participants had the capacity to use survey strategy when instructed to take the shortest path to a goal location. We wondered whether we could observe this result in the path planner predictions. Fig. 6a shows the best fitting path planner algorithm for those participants who were told to go to the goal location. Note the large percentage of trials where subjects used the route strategy. Fig. 6b shows the best-fitting path planner algorithm for participants who were told to take the shortest path. Note the decrease in route strategies in this case.

A chi-square test of independence showed a significant relationship between the instructions and the strategy used ($\chi^2(13) = 146.37$, $p < .001$). As shown in Fig. 6c, Goto Goal condition was positively associated with using route strategy (Pearson residuals = 6.23) but negatively associated with using survey strategy (Pearson residuals = –4.03). In contrast, Shortcut condition was positively associated with using survey strategy use (Pearson residuals = 4.54) but negatively associated with using the route strategy (Pearson residuals = –7.03). Fig. 6d shows that the route strategy used in Shortcut condition (33.72%), as well as Goto Goal condition (26.51%), and the survey strategy used in Shortcut condition (14.10%), as well as the Goto Goal condition (11.09%), contributed most to the Chi-square score. In agreement with Boone et al. (2019), these simulations suggest that subjects could take shortcuts if instructed to do so.

Given that subjects use a range of strategies for navigating these environments, we considered whether they stick to the same strategy in all 20 trials, similar to the subjects in Figs. 3 and 4 or if they alter strategies from trial to trial as depicted in Fig. 7. Looking across all subjects and all trials, it appeared that most subjects were similar to that shown in Fig. 7, where depending on the trial, the same subject might apply a different navigation strategy.

To measure how much a subject varied their strategy from trial to trial, we constructed a metric of strategy variation:

$$S_i = \sum_{j=1}^{20} I_{ij} = \begin{cases} 1 \; \textit{if strategy i used on trial j} \\ 0 \; \textit{otherwise} \end{cases} \tag{1}$$

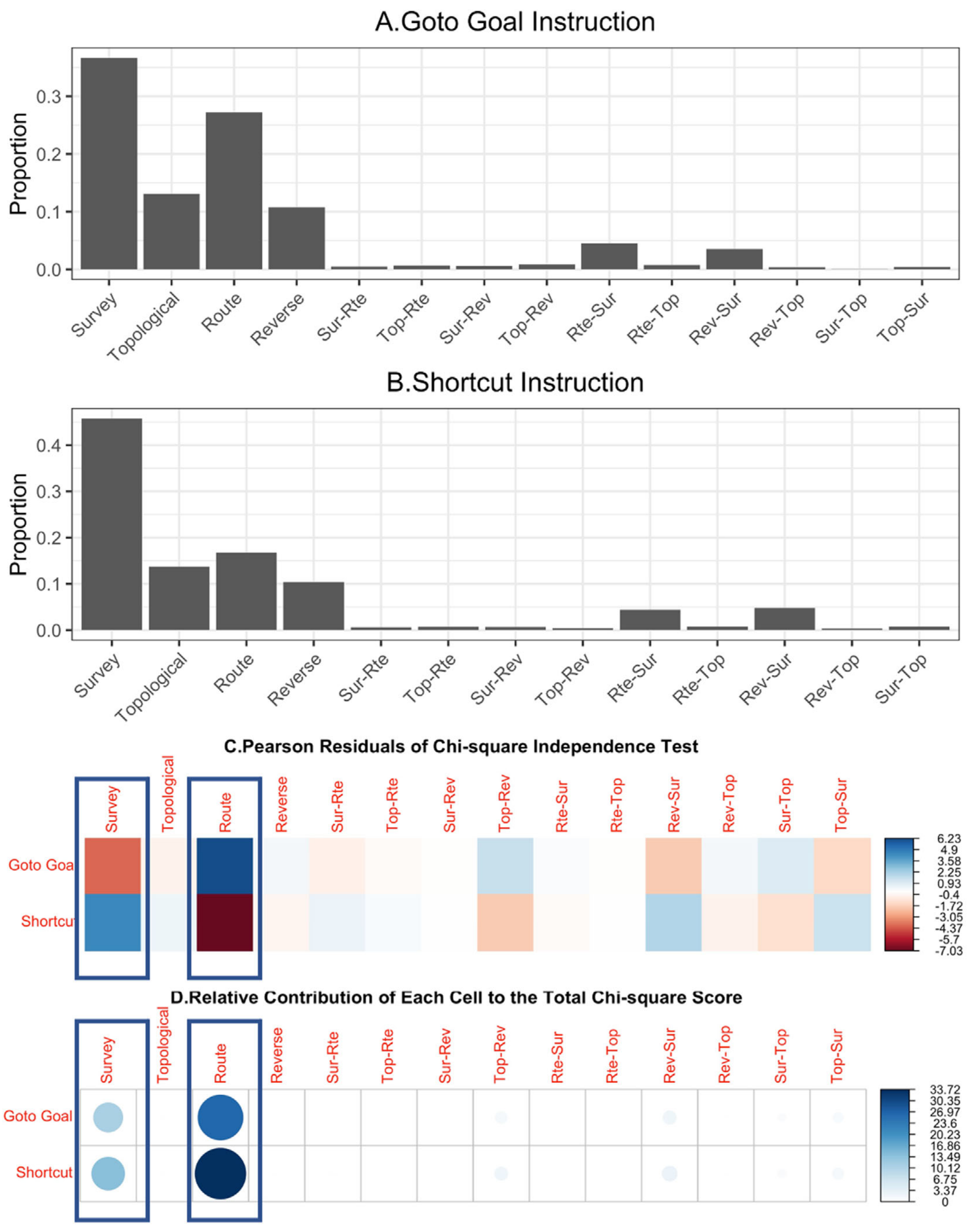

Fig 6. Strategies used under different conditions. (a) Proportion of the strategy that best predicts a participant's behavior when they were instructed to go to the goal location ($n = 206$). (b) Proportion of the strategy that best predicts a participant's behavior when they were instructed to take the shortest path to the goal location ($n = 162$). Sur = survey, Top = topological, Rte = route, Rev = reverse route. The remaining 10 labels represent combined

strategies. For example, SurRte represents survey strategy on the first half of the trajectory then route strategy on the second half on the trajectory. (c) Pearson residuals for each strategy using the Chi-square independent test. The Positive residuals are in blue. Positive values in cells specify a positive association between the instructions given and the specific strategy choice. (d) The relative contribution of each strategy to the total Chi-square score. A larger and darker circle indicates a more contributing cell to the Chi-square score.

$$C = 1 - \frac{\sum_{i=1}^{14} \begin{cases} 1 \; if \; S_i \neq 0 \\ 0 \; otherwise \end{cases}}{14} \tag{2}$$

$$P = \frac{\max(S)}{20} \tag{3}$$

$$SV = \frac{C + P}{2} \tag{4}$$

where S_i represents the number of times a participant used strategy i across the 20 trials, I indicates if the strategy i was used on trial j. C represents to what extent different strategies were used. For example, if Survey was used on all trials, the value of C would be $(1 - \frac{1}{14})$ or 0.93, whereas if 12 different strategies were used across the 20 trials, the value of C would be $(1 - \frac{12}{14})$ or 0.14. p represents how often the preferred strategy is used. Where *max(S)* is

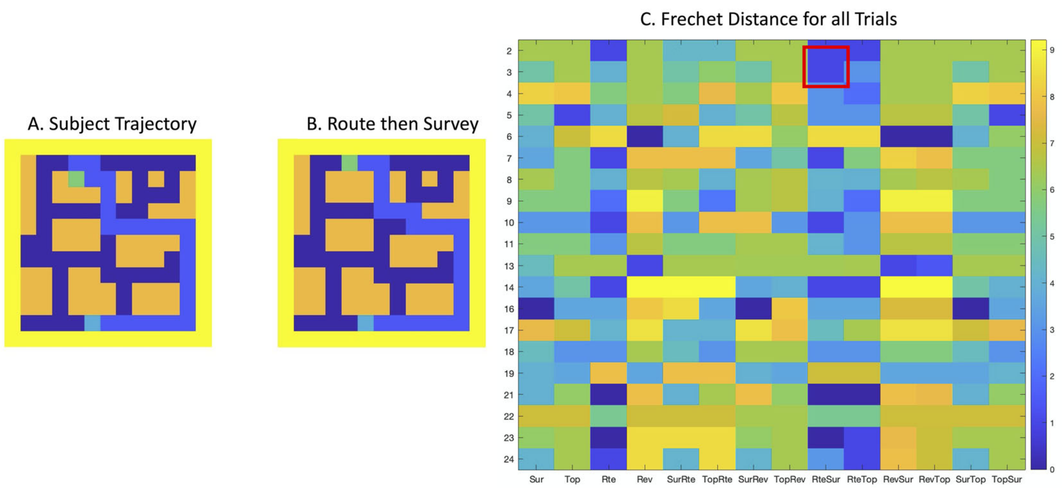

Fig 7. Representative subject that used mixed strategies. (a) Subject's trajectory on Trial 3. Cyan denotes the start location and green denotes the goal location. (b) Subject's behavior was best predicted by starting on the learned route and then taking a shortcut by switching to the survey strategy. (c) Heatmap of the Frechet distance between all trials and all 14 path planners. The rows are trials, and the columns are strategies. The colorbar to the right of the heatmap denotes the Frechet distance value. The red outline is for Trial 3, which is shown in (a) and (b).

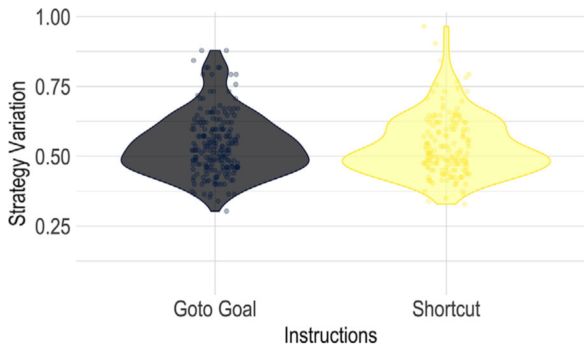

Fig 8. Strategy variation between trials for all subjects. The violin plots show the distribution of strategic variation (*SV* in Eq. 4). Two hundred and six subjects were instructed to go to the goal location; 162 subjects were instructed to take the shortest path to the goal location.

how often the preferred (i.e., maximum) strategy is used across all 20 trials. For example, if the survey strategy was used on all trials, the value of p would be $(\frac{20}{20})$ or 1. If the survey strategy was the most commonly used strategy but was only used twice, the value of p would be $(\frac{2}{20})$ or .10. *SV* is the overall metric for strategy variation where a value close to 0 means the subject varied their strategy from trial to trial and a value close to 1 means the subject always used the same strategy.

Subjects did tend to vary their strategy from trial to trial. The violin plots in Fig. 8 shows the *SV* values (Eq. 4) calculated for all subjects by different instructions conditions. Mean of *SV* is 0.54, which is significantly smaller than 0.965 ($t(367) = -74.06$, $p < .001$), which is theoretically largest value of *SV*, indicating no variation across 20 trials. Two sample t tests shows that strategy variation in Goto Goal condition ($M = 0.54$) is not significantly different from that in Shorctut condition ($M = 0.54$), $t(366) = 0.49$, $p = .63$, two-tailed. In general, subjects tended to vary their strategy somewhat from trial to trial, regardless of the instructions. This suggests that environmental differences or different difficulty of a given trial may impact the preferred navigation strategy.

Taken together, these simulations suggest that not only do subjects have varying preferred navigation strategies, but also subjects vary their navigation strategies both within and between trials. In the next section, we use another set of simulations to investigate why this might occur.

3.2. Strategy variation due to cost of traversal

To test whether the cost of traversing over an environment might impact a subject's navigation strategy, we generated maps where the cost ratio of the traversable paths to learned route varied (see Fig. 2). As discussed in the Methods section (see Sections 2.2 and 2.4), cost reflected the mental effort to travel over a path, where it was assumed that the learned route took the least effort and had a cost of 1, and the other potential paths had a cost that varied. Five maps were used and there were 16 trials per map. As described in the Methods

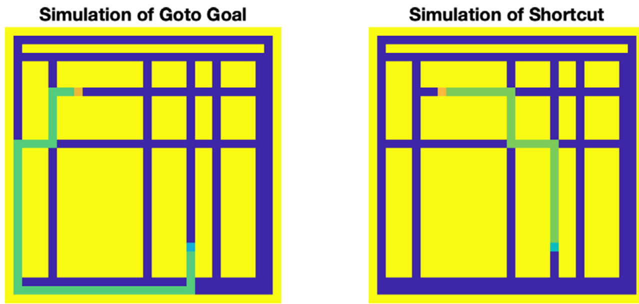

Fig 9. Simulation of navigation strategies with different cost maps. The green pixels denote the path. The cyan pixel is the start location, and the orange pixel is the goal location. The left side shows the path the survey path planner takes when the learned route is 16 times less costly than the other traversable areas. The right side shows the path taken when the learned route is two times less costly than the other traversable areas.

section, the cost ratio of traversable path to the learned route ranged from 16:1, 8:1, 4:1, 2:1, to 1:1. For example, in the 16:1 case, the learned route will take 16 times less timesteps than other paths. So the agent will tend to choose the learned route. However, in the 1:1 case, all paths take the same number of timesteps to move from one location to another. Therefore, the path planner will choose the shortest path. These ratios were sufficient to capture the range of behaviors observed in the human data.

We used the survey path planner to generate simulated agent trajectories on these five maps with different costs of not following the learned route. As expected, in the 1:1 case, the preferred strategy was always a survey strategy since there was no cost advantage for using the learned route. The 8:1 and 4:1 case showed a variation of strategies somewhere between the 16:1 and 2:1 cases. Therefore, we further analyzed the more extreme 16:1 and 2:1 cases. As with the human subject data, the Frechet distance metric was used to measure which path planner most closely resembled the simulated agents (i.e., the survey path planner on the maps with different costs of taking learned route).

Fig. 9 shows a representative example of a simulation trial where the path varied due to differences in the cost of traversing a location. On the left side of Fig. 9, the survey path planner was applied to the 16:1 map. That is, it was 16 times less costly to take the learned route than other traversable paths. In this case, the trajectory most resembled a route strategy of a subject that was instructed to go to the goal (compare the light green trajectory in Fig. 9 with the light green learned route in Fig. 2). On the right side of Fig. 9, the same survey path planner was applied to the map where the learned route was only two times less costly than the other paths. In this case, the trajectory resembled a survey strategy where the subject takes a novel shortcut. Similar differences were observed on the different maps and different trials when using the same planner but different ratios of traversal costs.

The simulations showed similar variation in preferred strategies to that of the human subjects. Fig. 10 shows the distributions of preferred strategies (i.e., minimum Frechet distance) taken for the 16:1 map (Fig. 10a) and the 2:1 map (Fig. 10b). Comparing these distributions to the subject data depicted in Fig. 6, when the learned route was less costly (Goto Goal condition), the total proportion of taking Rte or Rev strategies (31.25%) is not significantly

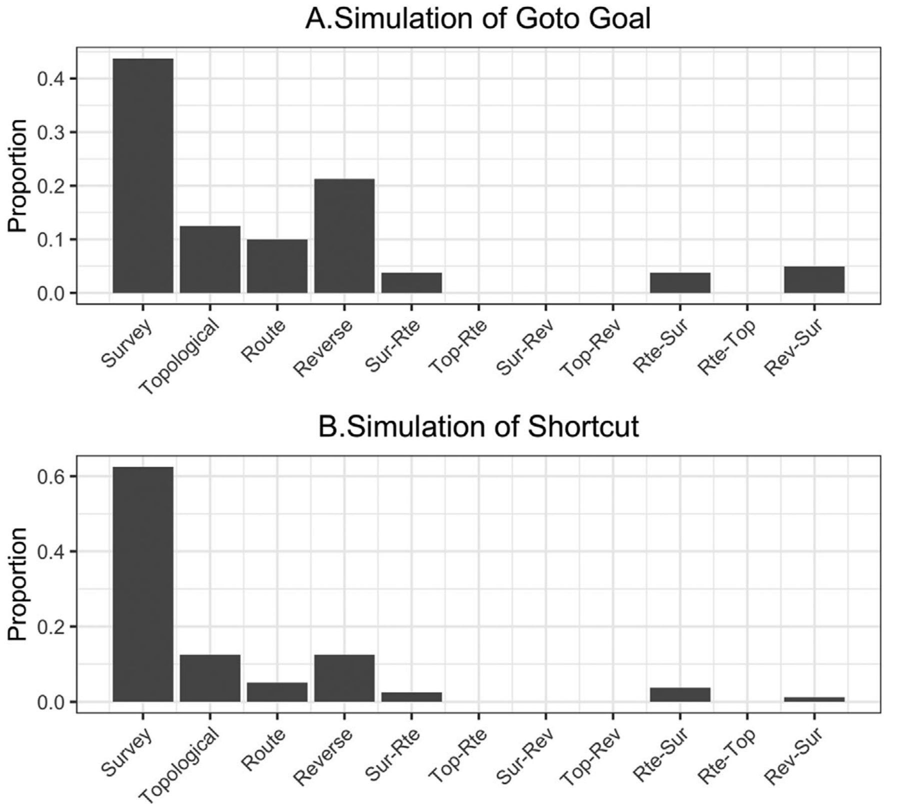

Fig 10. Simulations of preferred navigation strategies. (a) Goto Goal instructions were simulated using the 16:1 map (the number of total trials is 80). (b) Shortcut instructions were simulated using the 2:1 map (the number of total trials is 80).

different from the total proportion of taking Rte or Rev strategies by human participants (37.98%), χ^2 (1) = 1.24, $p = .27$. Conversely, when the learned route was only slightly less costly than other traversable regions (Shortcut condition), the total proportion of taking Rte or Rev strategies (17.50%) dropped and is not significantly different from the total proportion of taking Rte or Rev strategies by human participants (21.31%), χ^2 (1) = .47, $p = .49$. Note that in these simulations, the agent does not have knowledge of route direction. Therefore, route (Rte) and reverse route (Rev) can be considered together. It is also of interest that the mixed strategies of route then survey (RteSur and RevSur) in the simulations (Goto Goal: 8.75 %; Shortcut: 5.00%) showed similar proportions to those in the human subject data (Goto Goal: 7.99 %; Shortcut: 9.17%), Goto Goal: χ^2 (1) = .002, p = .97; Shortcut: χ^2 (1) = 1.18, $p = .28$.

4. Discussion

The present paper suggests that the variation observed in human navigation is both between subjects and on a trial-by-trial basis for an individual and that this variability might be explained by taking the cost of traversing an environment into consideration. This has implications for both cognitive science and autonomous navigation systems.

We took a unique approach to understanding this variation by using path planning algorithms from robotics to quantify when and where people apply route, survey, or graph knowledge to solve navigation tasks. Our path planning algorithms were able to replicate human navigation data from prior studies. As observed in human studies, we showed that people are more likely to take a survey strategy when instructed to take the shortest path than when told simply to get to the target location (Boone et al., 2019). Our approach allowed us to further quantify the variation. In addition to the reported variation in prior studies, we found that individual subjects may switch strategies between trials or even within trials.

In order to learn what determined the mix of strategies, we carried out a second set of simulations where we varied the cost of taking a given route relative to other potential paths, showing that when the cost of taking a shortcut relative to a familiar route was low, simulated agents were more likely to take a shortcut by using survey knowledge. Cost can be environmental and physical. For instance, it is more difficult to travel over loose gravel than a smooth road. Cost could also be cognitive. We suggest cost could be cognitive in that it may take more mental effort to choose a novel shortcut than a familiar route, but there may be situations where it is advantageous to take the shortest path. We show how this might be realized by varying the cost heuristic of a path planner used in robotics.

Although it has been suggested that people vary in their ability to navigate and some tend to demonstrate route or survey knowledge (Boone et al., 2018, 2019; Chrastil & Warren, 2015; Weisberg & Newcombe, 2018), we found that people varied their strategy on a trial-by-trial basis (see Figs. 7 and 8). Furthermore, we found that subjects can change strategies within a trial (see Figs. 6 and 7). It was interesting that within a trial approximately 8% to 9% of the time, a subject would start on the learned route and then switch to an allocentric strategy by using survey knowledge to take shortcuts. This suggests that at some point, they felt confident enough to leave the route and take a shortcut. Moreover, the opposite mixed strategy rarely occurred. That is, subjects did not start a trajectory using survey knowledge, and then switch to the route once connecting with the route. It may suggest that participants use less mental effort (low cost) by initially following a familiar route but then realize that with a little more mental effort (higher cost on an alternate path) they can reach their destination quicker by applying survey knowledge. Previous research on hierarchical structure of environmental knowledge has shown the mental map of the whole environment contain many sub-regions, and the mental map, as a part, is contained in larger regions (McNamara, 2013). The present results suggest that the strategy people use to go across the sub-regions may differ from the strategy used for navigating within a sub-region. Wiener and Mallot have also shown one useful wayfinding heuristic called fine-to-coarse heuristic, where people plan a route to the region containing the goal and only once inside that region, they will determine the subsequent specific route (Wiener & Mallot, 2003). It has been recently shown that once freely navigating bats choose a shortcut using vector navigation, they do not vary their path as they approach a goal (Harten, Katz, Goldshtein, Handel, & Yovel, 2020; Toledo et al., 2020).

The cost of traversing portions of the environment might be able to explain the observed variability. Previous research on executive control and decision-making has demonstrated that people tend to conserve cognitive effort, which is referred to as the law of less work or demand avoidance (Kool, McGuire, Rosen, & Botvinick, 2010). He and Hegarty have demonstrated that different people have various preferences in putting mental efforts in solving navigational problems. All these add variability in navigational strategies within and between different individuals (He & Hegarty, 2020). In a set of simulations, we were able replicate the variability within and between subjects by varying the ratio between the cost of taking a learned route and the cost of taking shortcuts. When the cost of taking the learned route was relatively low, the simulated agents utilized a route strategy fairly often (see Fig. 10a). However, when the cost of taking other potential paths was low relative to the learned route (e.g., the 2:1 map condition), the simulated agent utilized a survey strategy (see Fig. 10b). For the path planning algorithm, it means that although it may take half as many timesteps to calculate a path over the familiar route, there are cases where it takes less time to compute an alternate shorter path. From a human navigation point of view, it might be that although it takes less mental effort to follow a well-traveled route, a little more effort might lead one across a novel, shorter route. In the case of Boone et al. (2019), an instruction to take the shortest path was able to tip this balance in favor of survey knowledge suggesting that subjects had this capability but did not express it unless told to do so. We suggest that the instruction forced participants to make the mental effort to think of alternatives.

Although, we have suggested that cognitive load might be the reason why people use different strategies for navigating, there could be other reasons why subjects might choose to use one type of strategy over another. One alternative may be related to confidence in the current navigation strategy (Oess, Krichmar, & Rohrbein, 2017). If subjects were unsure about their allocentric position within the environment, they might rely on a strategy utilizing a familiar route. As they become more confident with their allocentric frame of reference, they might switch to a survey strategy. We observed this strategy shift in subjects and were able to simulate it with our path planning algorithms. Another reason may be environmental. The availability and salience of different cues, as well as the complexity of the layout, influence the quality of acquired environmental knowledge and how people apply the knowledge (Carlson, Hölscher, Shipley, & Dalton, 2010; Chai & Jacobs, 2010; He et al., 2020a; He, Han, Churaman, & Brown, 2020). In a series of robot navigation tasks, variations in the cost of traversal (e.g., how many timesteps the path planner considered traversing over uneven terrain) could shift the robot from taking a direct but bumpy and hilly path from one location to another to taking a longer but smoother path on a paved sidewalk (Hwu et al., 2018). In the future, we plan to test these alternatives by varying the uncertainty of the environment, as well as the physical characteristics of the environment.

This population variability may be explained from the perspective of evolution. Multiple memory systems may have evolved to provide redundancy and fault tolerance (Sherry & Schacter, 1987). Degeneracy, which is defined as having multiple means to achieve the same outcome, may play an important role in navigational variation (Edelman & Gally, 2001; Gallistel, 1990). For example, different roles in hunting and gathering, mating competition, as well as fertility-and-parental-care, all may lead to a wide range of variations in tackling risks and costs of navigation (Cashdan & Gaulin, 2016; Gagnon et al., 2018; Gagnon, Cashdan, Stefanucci, & Creem-Regehr, 2016). In general, varying navigation strategies in a population might lead to increased adaptability in response to environmental change or uncertainty.

Variations in neuromodulatory signaling may explain population variability during navigation. For example, depending on dopaminergic or serotonergic activation from the orbitofrontal cortex or the medial prefrontal cortex, rodents may shift from putting more effort to get a larger, but more difficult to obtain a reward, to taking an easier but less rewarding choice (Rudebeck, Walton, Smyth, Bannerman, & Rushworth, 2006). Natural variations in the dopaminergic catechol-O-methyltransferase gene can result in risk-taking variability (Roussos, Giakoumaki, Pavlakis, & Bitsios, 2008). This might have implications for navigation as well. Whereas a shortcut might be a risky endeavor, following a known route might be the safer option. Similarly, there are natural variations in the serotoninergic system that may affect behavior (Caspi, Hariri, Holmes, Uher, & Moffitt, 2010). For example, recent optogenetic experiments with mice showed that altering the level of serotonin could vary how long an animal would wait for a reward (Miyazaki et al., 2018). In human studies, individual differences in risk-taking traits, as well as spatial anxiety experienced during navigation, have been demonstrated to be influential in human navigation behaviors (He & Hegarty, 2020; Lawton & Kallai, 2002; Pingel, 2012). People who are more likely to avoid taking risks and experience more spatial anxiety tend to use route strategy more often than survey strategy.

We recently applied the idea that serotonin levels affect patience to a robot navigation task (Xing et al., 2020). The robot was tasked to find a set of global positioning system (GPS) waypoints in an outdoor park. If the simulated serotonin levels were high, the robot patiently searched until it found the waypoints. However, when the simulated serotonin levels were low, the robot gave up searching for hard-to-find waypoints, due to difficult terrain or weak GPS signals, and took a shortcut to another waypoint. Such variability could be advantageous in self-driving vehicles. For example, being assertive at a four-way stop might break a deadlock, or impatience due to changing traffic patterns may result in re-routing the vehicle's path.

Recently, a neurobiologically inspired model was proposed to explain when an agent may shift navigation strategies (Edvardsen, Bicanski, & Burgess, 2020). It was suggested that the place cell system supported topological navigation, and the grid cell system supported vector navigation. They suggested that vector navigation, similar to a survey strategy, was used until the agent got stuck. Once stuck, a hippocampal replay of place cells was invoked to choose an alternative route. Interestingly, this is different than what was observed in our analysis and simulation of human subjects. Subjects analyzed in the present study tended to start with the route strategy and then occasionally switched to a survey strategy. The opposite order was rarely observed. However, the two environmental conditions are different. In the case of Edvardsen et al., the agent began using survey knowledge but was subsequently blocked by an unexpected obstacle. In future work, it will be of interest to see if the algorithms in the present paper can also demonstrate this remapping given similar environmental constraints.

We suggest that populational variability may confer advantages for information gathering by biological organisms and artificial systems. Heterogenous teams of agents have been used to solve a wide range of problems (Engelbrecht, 2010; Hara, Shiraga, & Takahama, 2012; Valle, Venayagamoorthy, Mohagheghi, Hernandez, & Harley, 2008). In a heterogeneous robotic swarm, certain tasks can be solved efficiently through cooperation and functional specialization (Dorigo et al., 2013). Therefore, in future studies, it will be important to take this variability into consideration as a benefit rather than a liability.

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