# Equilibrium Effects of Superstition in the Housing Market

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**Abstract**

We investigate the interaction of product quality differentiation and consumer preference heterogeneity in durable goods markets, focusing on the effects of secondary market liquidity and consumer heterogeneity on equilibrium prices. We build an infinite-horizon dynamic model of the apartments housing market that captures the above features. Some apartments are considered lucky, and some consumers are superstitious. Lucky apartments are valued more highly than non-lucky ones only by superstitious consumers. Results show that the difference between the lucky apartment price and the non-lucky apartment price becomes smaller when the secondary market becomes less liquid and when consumers' preference heterogeneity becomes more persistent as opposed to time-varying.

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In many durable goods markets, products are differentiated in terms of their quality, and furthermore, consumers are heterogeneous in terms of the value they place on product quality. An example is the housing market in some Asian communities with a superstitious culture. In those markets, some apartments are considered lucky because their address numbers contain the so-called "lucky numbers" (see, for example, Shum, Sun & Ye (2014)). Among the consumers, some are superstitious and value lucky apartments more highly than non-lucky ones, while the others are non-superstitious and value lucky and non-lucky apartments equally.

Another example is the car market, in which new cars have a higher quality than used cars, but consumers differ in how they value a car's attribute of being new.[1] Some consumers are "new car lovers" whose valuation of a new car is much higher than that of a used car, while some other consumers are "used car lovers" whose valuation does not differ significantly between new and used cars.

In this article, we report the progress in an on-going project in which we investigate the interaction of the product differentiation and consumer heterogeneity described above. One focus is the effects of secondary market liquidity and consumers' preference shocks on the equilibrium prices.

In particular, we want to understand how the equilibrium prices change (1) when the secondary market becomes less liquid as transaction cost in the secondary market is heightened, and (2) when consumer heterogeneity becomes more persistent as the magnitude of their per-period random preference shocks is reduced.

# I Model

To investigate the above issues, we build a model of a durable goods market with product quality differentiation and consumer preference heterogeneity, using the apartments housing

[1] See Chen, Esteban & Shum (2013).

market as an example. Time is discrete. Consumers are infinitely-lived and forward-looking, and they incur transaction costs when selling the apartment that they own.

There are three goods indexed by $j = 0, 1, 2$, respectively, where $j = 1$ indicates a lucky apartment, $j = 2$ indicates a non-lucky apartment, and $j = 0$ indicates the outside good. There are two types of consumers indexed by $l = 1, 2$, respectively, where $l = 1$ indicates a superstitious consumer and $l = 2$ indicates a non-superstitious consumer.

For a non-superstitious consumer, the two types of apartments offer the same utility, whereas for a superstitious consumer, a lucky apartment offers a higher utility than a non-lucky apartment. Let $\alpha_j^l$ denote the per-period utility of good $j$ for a type $l$ consumer. Furthermore, let $\alpha_1 \equiv \alpha_1^1$ denote the utility that a superstitious consumer obtains from owning a lucky apartment, and let $\alpha_2 \equiv \alpha_1^2 = \alpha_2^1 = \alpha_2^2$ denote the utility when the consumer is non-superstitious and/or when the apartment is non-lucky, with $\alpha_1 > \alpha_2$. The outside good's utility $\alpha_0 \equiv \alpha_0^1 = \alpha_0^2$ is normalized to 0.

## A Consumers' problem

There is a continuum of infinitely-lived consumers with measure 1. Consumers are differentiated in two dimensions. First, consumers differ with respect to superstition. A measure $\lambda \in [0, 1]$ of consumers are superstitious ($l = 1$), and the remaining $1 - \lambda$ are non-superstitious ($l = 2$). A consumer's superstition is unchanging across time periods and represents a persistent component of preference heterogeneity across consumers.

Second, consumers also experience preference shocks that vary period-by-period. Let $\vec{\epsilon}_{it} \equiv (\epsilon_{ijt}, j = 0, 1, 2)$ be the vector of preference shocks of consumer $i$ in period $t$. These preference shocks represent time-varying horizontal differentiation among consumers. We assume these preference shocks are distributed type I extreme-value, i.i.d. across $(i, j, t)$.

The apartments are infinitely-lived and do not depreciate, and there is no new supply. The stocks of lucky and non-lucky apartments are therefore constant across time periods. Let $\theta_1$

and $\theta_2$ denote the stocks of lucky and non-lucky apartments, respectively, with $\theta_1 + \theta_2 \leq 1$. The apartments are fully utilized, i.e., no apartments are unoccupied in any period.

Each consumer owns a single good in each period. Let $s_{it} \in \{0, 1, 2\}$ denote the good owned by consumer $i$ at the start of period $t$, and let $B^l_{jt}$ denote the measure of type $l$ consumers who own good $j$ at the start of period $t$. Accordingly, the matrix $\mathbf{B_t} \equiv (B^l_{jt},\ j = 0, 1, 2 \text{ and } l = 1, 2)$ summarizes apartment holdings by consumer types at the start of period $t$.

The two-element vector $\vec{B}_t \equiv (B^1_{1t}, B^2_{2t})$ consists of the stock of lucky apartments owned by superstitious consumers and the stock of non-lucky apartments owned by non-superstitious consumers. We can capture the industry state using $\vec{B}_t$, as the rest of the elements in $\mathbf{B_t}$ are pinned down once we know $B^1_{1t}$ and $B^2_{2t}$:

$$\begin{cases} B^2_{1t} = \theta_1 - B^1_{1t}, \\ B^1_{2t} = \theta_2 - B^2_{2t}, \\ B^1_{0t} = \lambda - B^1_{1t} - B^1_{2t} = \lambda - B^1_{1t} - (\theta_2 - B^2_{2t}), \\ B^2_{0t} = (1-\lambda) - B^2_{1t} - B^2_{2t} = (1-\lambda) - (\theta_1 - B^1_{1t}) - B^2_{2t}. \end{cases} \tag{1}$$

The industry state space is then $\Omega \equiv \{(B^1_{1t}, B^2_{2t}) | 0 \leq B^1_{1t} \leq \min\{\lambda, \theta_1\}, 0 \leq B^2_{2t} \leq \min\{1-\lambda, \theta_2\}, \theta_2 - \lambda \leq B^2_{2t} - B^1_{1t} \leq (1-\lambda) - \theta_1\}$. The restrictions on $B^1_{1t}$ and $B^2_{2t}$ are given by the requirement that all elements in $\mathbf{B_t}$ are non-negative.

Let $p_{jt}$ be the price of good $j$ in period $t$ and let $\vec{p}_t = (p_{0t}, p_{1t}, p_{2t})$ be the price vector. $p_{0t} = 0$ for all $t$. Let $k_j$ denote the transaction cost incurred when selling good $j$, with $k_1 = k_2 = k$ and $k_0 = 0$, i.e., the transaction cost $k$ is incurred if and only if the consumer sells an apartment.

Consumer $i$ derives the following utility in period $t$. If she keeps her current good, $s_{it}$, she derives utility $\alpha^{l_i}_{s_{it}} + \epsilon_{is_{it}t}$. If she sells her current good and purchases good $j$ instead, $j \neq s_{it}$, her utility is $\alpha^{l_i}_j + p_{s_{it}t} - p_{jt} - k_{s_{it}} + \epsilon_{ijt}$.

We can compactly express consumer $i$'s utility flow in period $t$ as

$$u(a_{it}, \vec{p}_t, s_{it}, \vec{\epsilon}_{it}; l_i) = \underbrace{\alpha_{a_{it}}^{l_i} + \mathbf{1}_{a_{it} \neq s_{it}} \cdot (p_{s_{it}t} - p_{a_{it}t} - k_{s_{it}})}_{\equiv \tilde{u}(a_{it}, \vec{p}_t, s_{it}; l_i)} + \epsilon_{ia_{it}t},$$

where $a_{it} \in \{0, 1, 2\}$ denotes $i$'s consumption choice in $t$.

We now consider the dynamic optimization problem of each consumer $i$ given $(\vec{p}_t, \vec{B}_t, s_{it}, \vec{\epsilon}_{it})$, the variables that affect the consumer's choice. Let $P(\cdot)$ denote the price function which maps from the industry state to the prices, i.e., $\vec{p}_t = P(\vec{B}_t)$, and let $L(\cdot)$ denote the law of motion of the industry state, i.e., $\vec{B}_{t+1} = L(\vec{B}_t)$. Consumers observe the state and prices in the current period and form expectations of the state and prices in the next period, and in equilibrium those expectations are correct.

The Bellman equation for consumer $i$'s dynamic decision problem is then

$$V(\vec{p}_t, \vec{B}_t, s_{it}, \vec{\epsilon}_{it}; l_i) = \max_{a_{it}} \left[ u(a_{it}, \vec{p}_t, s_{it}, \vec{\epsilon}_{it}; l_i) + \beta \tilde{V} \left( P(L(\vec{B}_t)), L(\vec{B}_t), a_{it}; l_i \right) \right], \quad (2)$$

where $\beta \in (0, 1)$ is consumers' discount factor, and

$$\begin{aligned} \tilde{V}(\vec{p}_t, \vec{B}_t, s_{it}; l_i) &\equiv E_{\vec{\epsilon}} V(\vec{p}_t, \vec{B}_t, s_{it}, \vec{\epsilon}_{it}; l_i) \\ &= \log \left\{ \sum_{h=0,1,2} \exp \left( \tilde{u}(h, \vec{p}_t, s_{it}; l_i) + \beta \tilde{V} \left( P(L(\vec{B}_t)), L(\vec{B}_t), h; l_i \right) \right) \right\} \end{aligned} \quad (3)$$

is the expected value function before consumer $i$'s shock is observed, with the latter substitution following from the assumption that the $\epsilon$'s are distributed type I extreme-value.

Accordingly, the choice probability of good $j$ by a consumer who currently owns good $s$ and who is of type $l$ takes the multinomial logit form

$$q_j(\vec{p}_t, \vec{B}_t, s; l) = \frac{\exp \left( \tilde{u}(j, \vec{p}_t, s; l) + \beta \tilde{V} \left( P(L(\vec{B}_t)), L(\vec{B}_t), j; l \right) \right)}{\sum_{h=0,1,2} \exp \left( \tilde{u}(h, \vec{p}_t, s; l) + \beta \tilde{V} \left( P(L(\vec{B}_t)), L(\vec{B}_t), h; l \right) \right)}. \quad (4)$$

## B Aggregate demand functions

We next aggregate up the choices of all consumers to obtain the aggregate demand function for each type of apartments in period $t$.

Let $Q^D_{jt}$ denote the quantity demanded of type $j$ apartments in period $t$ and let $Q^{Dl}_{jt}$ denote the quantity demanded of type $j$ apartments by type $l$ consumers in period $t$. The aggregate demand functions are then

$$Q^D_{jt} = \sum_{l=1,2} Q^{Dl}_{jt} = \sum_{l=1,2} \left( \sum_{h=0,1,2} \left( B^l_{ht} \cdot q_j(\vec{p}_t, \vec{B}_t, h, ; l) \right) \right). \tag{5}$$

### C Equilibrium and steady state

The equilibrium price function $P(\cdot)$, the equilibrium law of motion $L(\cdot)$, and consumers' equilibrium expected value function $\tilde{V}(\cdot)$ satisfy the following conditions for every state $\vec{B}_t$:

(i) *Market clearance:* At the equilibrium prices $P(\vec{B}_t)$, quantity demanded equals quantity supplied for each type of apartments, i.e., $Q^D_{jt} = \theta_j$ for $j = 1, 2$.
(ii) *Consistency of the law of motion:* The industry state evolves according to $(B^1_{1,t+1}, B^2_{2,t+1}) = L\left((B^1_{1t}, B^2_{2t})\right) = (Q^{D1}_{1t}, Q^{D2}_{2t})$, i.e., the next-period apartment holdings based on the equilibrium law of motion $L(\cdot)$ equal the relevant quantities demanded in the current period.
(iii) *Consistency of consumers' expected value function:* Consumers' equilibrium expected value function $\tilde{V}(\cdot)$ satisfies the recursive equation (3).

We compute the equilibrium in the model by solving a system of equations consisting of the above conditions. Given the equilibrium law of motion $L(\cdot)$, the steady state of the model $(B^{1ss}_{1t}, B^{2ss}_{2t})$ satisfies the condition $L\left((B^{1ss}_{1t}, B^{2ss}_{2t})\right) = (B^{1ss}_{1t}, B^{2ss}_{2t})$, which allows us to compute the steady state of the model. The results we report below are steady state results.

## II Preliminary results and next steps

In our baseline specification, we consider the following parameter values. We set $\alpha_1 = 2$ and $\alpha_2 = 1.6$, so that for superstitious consumers, a lucky apartment offers 25% higher utility than a non-lucky apartment. We set $\lambda = 0.1$, so that 10% of the consumer population is superstitious. We set $\theta_1 = 0.2$ and $\theta_2 = 0.6$, so that 80% (= $(0.2+0.6)/1$) of the consumers own apartments, and one quarter of the apartments are lucky apartments.

We then examine how the prices change as we vary the transaction cost $k$ and the variance of consumers' preference shocks $Var(\epsilon)$. Table 1 reports the ratio $p_2/p_1$ for all combinations of the following $k$ and $Var(\epsilon)$ values: $k \in \{0, 1, 2, 3, 4\}$, and $Var(\epsilon) \in \{\pi^2/6, (7/8) \times \pi^2/6, (6/8) \times \pi^2/6, (5/8) \times \pi^2/6, (4/8) \times \pi^2/6\}$.[2]

We find that as $k$ is increased so that the secondary market becomes less liquid, the difference between the lucky apartment price ($p_1$) and the non-lucky apartment price ($p_2$) becomes smaller, i.e., $p_2/p_1$ becomes closer to 1. For example, with $Var(\epsilon) = \pi^2/6$, an increase of $k$ from 0 to 4 results in the price ratio increasing from 30.5% to 90.7%.

The price difference also becomes smaller as $Var(\epsilon)$ is decreased so that consumers' preference heterogeneity becomes more persistent. For example, with $k = 0$, a reduction of $Var(\epsilon)$ from $\pi^2/6$ to $(4/8) \times \pi^2/6$ results in the price ratio increasing from 30.5% to 50.0%. In the table, the price ratio is the highest at 96.5% when $k$ is the highest at $k = 4$ and $Var(\epsilon)$ is the lowest at $Var(\epsilon) = (4/8) \times \pi^2/6$.

The intuition for the above results is as follows. First, when $k$ is increased so that the secondary market becomes less liquid and shrinks in size, the price premium for lucky apartments due to the resale motive—which relies on trading in the secondary market—diminishes, resulting in a smaller difference between the prices of the two types of apartments.

Second, when $Var(\epsilon)$ is decreased so that consumers' preference heterogeneity becomes more persistent as opposed to time-varying, in the steady state a larger proportion of superstitious consumers will already own lucky apartments, given that there are more lucky apartments than superstitious consumers. Consequently, the lucky apartments traded in the market will increasingly be sold to non-superstitious consumers, who value the two types of apartments equally. As a result, the difference between the prices of lucky and non-lucky apartments

[2] When the scale parameter of the type I extreme value distribution for $\epsilon$ is normalized to 1, $Var(\epsilon) = \pi^2/6$.

decreases.

In ongoing research, we are exploring different specifications of the model (including alternative distributional assumptions on $\epsilon$) as well as calibration and estimation of the model using real-world data, in order to better understand the interesting dynamics resulting from the interaction of product quality differentiation and consumer preference heterogeneity in durable goods industries.

## References

**Chen, Jiawei, Susanna Esteban, and Matthew Shum.** 2013. "When do secondary markets harm firms?" *American Economic Review*, 103(7): 2911–34.

**Shum, Matthew, Wei Sun, and Guangliang Ye.** 2014. "Superstition and "lucky" apartments: Evidence from transaction-level data." *Journal of Comparative Economics*, 42(1): 109–117.

Table 1. Ratio of $p_2/p_1$ for different combinations of $k$ and $Var(\varepsilon)$

$\alpha_1 = 2$, $\alpha_2 = 1.6$, $\lambda = 0.1$, $\theta_1 = 0.2$, $\theta_2 = 0.6$

| | Transaction cost $k$ | | | | |
|---|---|---|---|---|---|
| | 0 | 1 | 2 | 3 | 4 |
| $Var(\varepsilon) = \pi^2/6$ | 30.5% | 55.8% | 75.4% | 85.5% | 90.7% |
| $Var(\varepsilon) = (7/8)*\pi^2/6$ | 34.8% | 60.7% | 78.7% | 87.6% | 92.0% |
| $Var(\varepsilon) = (6/8)*\pi^2/6$ | 39.4% | 65.6% | 81.9% | 89.6% | 93.5% |
| $Var(\varepsilon) = (5/8)*\pi^2/6$ | 44.5% | 70.7% | 85.1% | 91.5% | 95.1% |
| $Var(\varepsilon) = (4/8)*\pi^2/6$ | 50.0% | 76.0% | 88.3% | 93.7% | 96.5% |