Going Along or Going Independent? A Dynamic Analysis of Nonprofit Alliances

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November 17, 2017

Abstract

This paper investigates strategic alliances in the nonprofit sector in the form of franchising. Using a dynamic model of local public goods with endogenous affiliation and splitting, we show that local organizations may choose to affiliate with the national organization for faster capital accumulation. Temporary alliance occurs when a local organization strategically affiliates with the national organization only to break away after accumulating enough capital. Alliance is more likely to arise and persist when the local chapter is smaller, when the local chapter's mission is closer to the national organization's, when the national organization is more efficient in production, and when the local chapter is more patient. Moreover, regulation that requires the local chapter to be affiliated with the national organization would be welfare reducing when the local chapter is large, when the local and national missions differ substantially, and when production at the national organization is inefficient.

Keywords: nonprofit, strategic alliance, breakup, industry dynamics, capital accumulation.

JEL Classifications: D71, H41, L24, L31.

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1 Introduction

The nonprofit sector comprises a diverse range of not-for-profit and non-governmental organizations, such as art, health, and education nonprofits, labor unions, and business and professional associations. The nonprofit sector is an important part of the economy. It accounted for 9.2% of all wages and salaries paid in the U.S. in 2010 (Roeger et al., 2012), and contributed an estimated $905.9 billion to the U.S. economy in 2013 (McKeever, 2015). In the nonprofit sector, local organizations sometimes form strategic alliances in the form of a national or state-wide organization. These alliances typically emerge in the form of franchising. Examples of alliances formed in this fashion include American Cancer Society, American Heart Association, and California Alliance of Child and Family Services.

The opposite process, in which local chapters break away from the national organization and go it alone, also occurs periodically. A prominent recent example involves the national Alzheimer's Association in the U.S. From late 2015 to early 2016, several local chapters of the Alzheimer's Association, including those in Los Angeles, New York City, New Jersey, Texas, and San Diego, split from the national organization, citing the desire to focus exclusively on the needs of the local community (Gorman, 2016; Kandil, 2016). Similarly, in 2010, KCET, a non-commercial educational television station based in Los Angeles, broke away from the Public Broadcasting Service (PBS) to become an independent station, expressing an intention to shake off outside influence and put a stronger focus on the Los Angeles community (Hayden, 2010). In 2005, the Jacksonville regional office of the National Conference for Community and Justice broke away in order to retain all raised funds for the local community.[1]

While alliances and breakups in the for-profit sector have been the subject of many academic studies, such as franchising (for example see the extensive work on this topic by Francine Lafontaine and her coauthors, especially Lafontaine and Slade (2007), which provides a survey of this literature), research joint ventures (Caloghirou et al., 2003; Kamien and Zang, 1993; Suzumura, 1992; Kamien et al., 1992) and spinouts (Franco and Filson, 2006),

[1] http://www.bizjournals.com/jacksonville/stories/2005/09/19/daily31.html, accessed November 30, 2016.

	Type of good produced	Reliance on fund raising	Ease of access to capital	Managerial performance measures	Primary mode of operation
for-profits	focus on goods with a high private value	little-to-no fund raising	fairly open capital markets	profits as a measure of managerial performance	paid labor
nonprofits	focus on goods with a high social value	heavily reliant on fund raising	highly restrictive capital markets	no strong quantitative measure	volunteer labor

Table 1: Summary of the differences between for-profits and nonprofits

those in the nonprofit sector have received little attention in the literature.[2] There are several differences between for-profit ventures and nonprofit ventures that warrant studying nonprofits separately from their for-profit counterparts. Oster (1996) identifies five characteristics that distinguish nonprofits from for-profits: the type of product produced (nonprofits produce goods with high social value, for-profits produce goods with high private value), the use of fund raising as an income source, the difficulty in accessing capital, the absence of a quantitative measure of managerial performance (profits), and the dependence on volunteers.[3] The reliance on fund raising is tied directly to the nonprofits' relative difficulty in accessing capital. Unlike for-profits, nonprofits cannot issue equity and thus have fewer options to raise capital. This difference holds for access to capital markets in general (Hansmann, 1981). Therefore, for-profit firm has the incentive to operate if it expects future growth so long as price exceeds average variable cost, as it can take advantage of access to capital markets to temporarily fund its operations. Since nonprofits do not have such access they must operate at at least a break-even point or they cannot operate at all. These differences are summarized in Table 1.

Moreover, the welfare effects of increased competition in nonprofits producing public goods do not necessarily resemble that of for-profits producing private goods. For example, with nonprofits, increased competition does not necessarily imply increased welfare (through decreased deadweight loss). Furthermore, the conditions under which competition

[2] One can also think of strategic alliances in the context of political economy, such as Siqueira and Sandler (2001) and Johnson and Urpelainen (2012).

[3] The intersection between goods with high social value and goods with high private value is nonempty, so for-profit firms may provide both, but nonprofits are unique in that they will provide goods with high social value even if there is no private value, i.e. even if the good is not profitable.

is more/less efficient goes beyond economies of scale. Some of these factors include the balance of consumers, heterogeneity of consumers, and the degree of excludability of the public good produced. These differences require different modeling techniques.

This paper aims at filling the gap on nonprofit franchising in the literature by applying a dynamic model of local public goods to the class of nonprofit organizations that can be characterized by collaborative provision, whereby quasi-independent local chapters collaborate by way of pooling resources. Drawing on tools from differential game theory and numerical analysis, we use the model to understand: (1) when these alliances arise endogenously and how long they last, (2) if and when these alliances are beneficial to the local communities, and (3) if and when these alliances are beneficial to the national population.

There is a set of local organizations serving their respective local populations. The objective of each local organization is to provide services, henceforth referred to as treatment, to its corresponding population, where the preferred mode of operation (e.g. how the treatment is delivered) is determined by the characteristics of the population being served. We use the term treatment to describe the collection of all services that a nonprofit provides to its target population. In order to provide treatment, the local organizations must fundraise, which they do by exerting costly effort, for instance by manning phone banks or mailing letters to solicit donations. Local organizations can choose to be affiliated with a national (parent) organization, or act independently. When acting independently, a local organization takes the funds it receives at the current time and uses it to develop treatment, which accumulates over time. When affiliated, every affiliated chapter sends its funds to the parent organization who then aggregates the funds, which are transformed into treatment that the local chapters can provide to their populations.[4]

The benefits of affiliation and aggregation are two-fold. The first source of benefit is economies of scale, i.e., there may be efficiency gains if the parent organization is more efficient than a given local chapter. The second source of benefit is the non-rivalry nature of

[4] In the nonprofit sector, there is variation as to what proportion of the funds remains with the local chapter, so our assumption that all funds are aggregated at the national level is a simplifying assumption. The parsimonious specification of fund allocation in our model simplifies the analysis and facilitates the investigation of the local chapters' incentives and decisions in a dynamic setting. If we were to allow the partial retention of funds, then on the margin, affiliation is less valuable due to fewer contributions being aggregated and redistributed. Therefore, the results are qualitatively unchanged, though the thresholds for when and for how long to affiliate will be altered.

the public goods produced by the nonprofits. Specifically, in our model, the output generated by the national organization is non-rival (such as knowledge in the form of the discovery of risk factors for a disease and the development of treatment options and preventative techniques), in that it can be used by all affiliated local chapters, and the consumption by one chapter does not prevent simultaneous consumption by other chapters. By aggregating resources from all local chapters, the national organization produces an output that is much larger than any individual local chapter can produce on its own. This larger amount of output produced at the national level is then used by every affiliated local chapter. Due to this multiplication effect arising from the non-rivalry nature of the output, the aggregation is able to enlarge the benefit to the organization and to the society.

However, there are also costs associated with affiliation. In particular, the treatment is developed at the national level and is therefore not specifically targeted to the local community that the local chapter serves. Given the imperfect match between the local chapter and the national organization, there is a loss to the value of the treatment when it is applied to the local community. For example, with respect to a disease, a treatment plan or preventative technique developed with the nation's ethnic majority as the target population may be less effective when applied to a local community consisting primarily of an ethnic minority, due to biological and behavioral differences between those populations. Consequently, the value of treatment is discounted by the distance between the local and parent (global) missions. Moreover, the decentralized nature of the decision making process in the public goods setting induces both a static and dynamic free-rider problem (Fershtman and Nitzan, 1991), which is distinct from a for-profit setting.

The local chapter has three options: (*i*) it can choose to never affiliate with the national organization, (*ii*) it can choose to affiliate indefinitely, or (*iii*) it can choose to affiliate for a finite time period to temporarily benefit from the parent organization's resources.

If a local chapter decides to break up with the parent organization, it need not leave empty handed due to the (impure) public goods nature of treatment. An example of the public goods that the local chapters would leave with is knowledge that is developed at the national level. For instance, consider national volunteer health organizations such as American Cancer Society and American Heart Association. A major output of such national organizations is knowledge, such as the discovery of risk factors for a disease and the development of treatment options and preventative techniques. Knowledge is non-excludable once it is learned, and a local chapter can still benefit from such knowledge even after they break away from the national organization. For example, Graham (2016) articulates how several of the local chapters of the National Alzheimer's Association split from the national organization due to disagreements on how resources should be allocated. The national organization was prioritizing scientific research and lobbying while the local chapters were more interested in direct ways to help their community members and families suffering from the disease. After splitting, these local organizations were able to take what they learned during their time affiliated with the national organization and apply it to fund and develop local programs such as the weekly "Memories in the Making" classes.

On the other hand, the national organization's output may also include things like pamphlets and handbooks, which may be trademarked and copyrighted and therefore excludable. So upon exiting, the local chapter can no longer use and/or redistribute these materials. These observations lead us to model the output as partially non-excludable, so that an exiting local chapter need not leave empty handed.

We refer to the regime under which the local chapter is free to make the affiliation/splitting decision as the endogenous alliance regime. This regime is compared to two others: the no alliance regime and the full alliance regime. Under the no (full) alliance regime, we impose the condition that the local chapter is never (permanently) affiliated.

By incorporating *strategic* and *temporary* affiliation into the analysis, the model in this paper is novel relative to existing studies on endogenous alliances and allows us to obtain several findings that are new to the literature. In particular, we find that endogenous alliance is longer when the local chapter is smaller, when the local chapter's mission is closer to the national organization's, when the national organization is more efficient in production, and when the local chapter is more patient. We also find that temporary alliance is more likely to occur when the public good is non-excludable, while more excludable goods lead to permanent alliances. Thus we find different behavior from those producing public, non-excludable goods (nonprofits) and those producing private, excludable goods (for-profits). Moreover, a regulation that requires the local chapter to be affiliated with the national organization would be welfare reducing from the social point of view when the local chapter is large, when the local and global missions differ substantially, and when production at the national organization is inefficient.

Related Literature. This paper is related to the economic literature in three areas: franchising, public/club goods, and firm behavior in the nonprofit sector.

For an excellent survey of the franchising literature, see Lafontaine and Slade (2007), which considers the subject of franchising in the broader context of empirical studies of vertical integration decisions. More recent work includes Zanarone (2009), Kosova and Lafontaine (2010), and Fan et al. (forthcoming), among others. Within the topic of franchise contract termination, existing studies have largely focused on termination and non-renewal decisions by franchisors (for example see Chapter 10 of Blair and Lafontaine (2005)), whereas terminations initiated by franchisees have not received much attention.

This paper extends this literature in two ways. First, one of our main focuses in this paper is the strategic decision of local chapters (franchisees) to affiliate with and later break away from a national organization (franchisor); such strategic temporary affiliation chosen by franchisees has not been formally analyzed in the franchising literature. Second, we study franchising in the context of nonprofits producing public goods (non-rival and non-excludable), which differs from and complements prior studies of this topic in the context of for-profits producing private goods (excludable and rival). For example, in our comparative statics analysis with respect to the degree of excludability, we find that the local chapter chooses temporary affiliation when the good is non-excludable, but chooses permanent affiliation when there is a high degree of excludability. This result suggests that the phenomenon of temporary affiliation is closely associated with the good being public. Therefore our model predicts that strategic affiliation followed by breaking away from the parent organization is more common in the nonprofit sector than in the for-profit sector, which is a new insight and is a testable prediction that can be brought to data for empirical examination.

For an overview of work on the provision of local public and club goods, see Cornes and Sandler (1996), Sandler and Tschirhart (1997), and Scotchmer (2002). Within the club goods literature, this paper relates to models of the endogenous formation and splitting of groups, including Jaramillo et al. (2003), Haimanko et al. (2004), Sacks (2017), and Carvalho and Sacks (2017), among others.[5]

The literature on firm behavior in the nonprofit sector has heavily focused on organizational structure (Ghatak and Mueller, 2011) and the differences between the nonprofit and for-profit sectors (Eisenstadt and Kennedy, 1981; Lakdawalla and Philipson, 2006). An important theme of this literature is that the behavioral differences between nonprofit and for-profit firms call for different analytical treatments for the two types, in both theoretical and empirical investigations. This paper provides such a treatment for the class of nonprofits characterized by decentralized collaboration. For a review of the literature see Malani et al. (2003) and Hansmann (2010).

This paper is also part of a small but growing literature that analyzes dynamic issues in the nonprofit sector, such as entry and exit (see, for example, Harrison and Laincz (2008) and Gayle et al. (forthcoming)). By studying the dynamic strategic alliances in the nonprofit sector, this paper complements the existing literature, and together these papers allow a more comprehensive understanding of the complex and interesting dynamics in the nonprofit sector.

The remainder of the paper is structured as follows. Section 2 outlines the theoretical model. Section 3 conducts analytical analysis to derive the equilibrium paths and utilities for both the local chapters and the parent organization. Section 4 establishes a numerical benchmark model and conducts comparative statics analysis on several key parameters of interest. Concluding remarks follow in Section 5.

2 Model

In this section we build a dynamic model of the formation and splitting of clubs (as local public goods) to study alliances and breakups of nonprofit organizations. While the model is written in the context of a nonprofit organization targeting a particular disease, such as Alzheimer's Association or American Cancer Society, the framework applies to many other types of nonprofit organizations as well.

[5] A more detailed literature review is given in Sacks (2017).

Our model embodies some key features of the nonprofit sector as discussed in the Introduction section. Here we describe these features and explain why we need a model for nonprofit franchises that is different from existing for-profit models.

First, in contrast to for-profits which generally produce goods with high private value, nonprofits generally produce goods with high social value, for which the usual market mechanism fails to deliver the socially optimal level of output (for example, see the market failure theory of the nonprofit sector discussed in Ott and Dicke (2015, pp. 107-109)). Consider, for example, nonprofit organizations that provide shelters for the homeless. The usual supply and demand model is not suitable for explaining the provision of such shelters, as the demand for them is absent: demand for a good comes from consumers who are willing and *able* to pay for the good, and in this case, due to their low income the homeless are unable to pay for the shelters. However, such shelters are valuable to not only the homeless, but also the society as a whole, due to positive externalities and social equality considerations. As a result, nonprofits enter the market and provide the shelters by pooling resources (donations of time and/or money) from volunteers.

Correspondingly, in our model, instead of letting the market outcome be determined by the usual supply and demand model, we model the local nonprofit organizations' costly efforts to solicit donations, and obtain their equilibrium output levels and affiliation decisions by solving their dynamic optimization problems in which they maximize the present discounted value of the benefits to the community that they serve, taking into consideration the cost and benefit of donation solicitation and the cost and benefit of affiliating with a national organization.

Second, the nonprofit sector primarily deals with public goods, which are non-rival and non-excludable. Our nonprofit alliance model embodies both the non-rivalry and the non-excludability features in the nonprofit sector, and is therefore different from the models developed for analyzing franchising in the for-profit sector, which primarily deals with private goods (rival and excludable).

Specifically, in our model, the output generated by the national organization is non-rival, in that it can be used by all affiliated local chapters, and the consumption by one chapter does not prevent simultaneous consumption by other chapters. This assumption is a reasonable approximation for many real-world examples in the nonprofit sector. For instance, consider nationwide volunteer health organizations such as American Cancer Society and American Heart Association. A major output of such organizations is knowledge, such as the discovery of risk factors for a disease and the development of treatment options and preventative techniques. Knowledge is a non-rival good, and all local chapters can benefit from it at the same time.

Moreover, in our model, the output is partially non-excludable (in one comparative statics analysis we investigate the effect of varying the degree of excludability), so when a local chapter decides to break away from the national organization, it need not leave empty handed. For example, again consider nationwide volunteer health organizations such as American Cancer Society and American Heart Association. The knowledge that such organizations produce is non-excludable once it is learned, and a local chapter can still benefit from such knowledge even after they break away from the national organization. On the other hand, the national organization's output may also include things like pamphlets and handbooks, which may be trademarked and copyrighted and therefore excludable, and so upon exiting, the local chapter can no longer use and/or redistribute these materials. These observations are what lead us to model the output as partially non-excludable.

Furthermore, our comparative statics analysis with respect to the degree of excludability (presented in Subsection 4.2) shows that the local chapter's affiliation decision critically depends on how excludable the output is: the local chapter chooses temporary affiliation when the good is non-excludable, but chooses permanent affiliation when there is a high degree of excludability. Since non-excludability is one of the defining features of public goods as compared to private goods, this finding illustrates the importance of having a model for nonprofit franchises that is different from existing for-profit models, one that embodies the unique features of the non-profit sector.

2.1 Local and global missions

There is a nonprofit national (parent) organization made up of K independent local chapters, with the arbitrary chapter indexed by i. Time t flows continuously and is discounted by the idiosyncratic rate $\beta_i > 0$. If a chapter i is affiliated with the parent organization at time t, then $i \in p(t)$. Let $A_i(t)$ denote the affiliation decision for chapter i, where $A_i(t) = 1 \Leftrightarrow i \in p(t)$ and $A_i(t) = 0$ otherwise.

Each local chapter i is characterized by two features: its local mission $M_i \in \mathcal{M}$, where $\mathcal{M}$ is the set of missions, and the size of the population it serves, N_i. The parent organization is itself endowed with a global mission M_p. The chapters desire to serve the unique needs of their local communities, and therefore local missions are in general different from the global mission. For example, to better serve the large Vietnamese and Spanish communities residing in Orange County, California, the local Alzheimer's organization strives to advertise to those communities and recruit dedicated staff and volunteers who can linguistically and culturally relate (Kandil, 2016). However, such efforts may not be a priority for the national Alzheimer's Association.

We next define the match value between a local chapter and the parent organization. We let $(\mathcal{M}, 1-\mu)$ denote a metric space, normalized such that image$(1-\mu) = [0,1)$. Thus $\mu(M_i, M_p) \in (0,1]$ can be interpreted as the match value between a local chapter i and the parent organization. As discussed in the Introduction section, given the imperfect match between the local chapter and the national organization, when the output of the national organization is used by the local chapter to serve its community, the output's value is discounted by a factor of $\mu(M_i, M_p)$.

2.2 Donations

Each chapter exerts costly efforts to solicit donations. The marginal cost of soliciting x_i in donations is linear and given by ψx_i. Without loss of generality, we normalize fixed costs to be zero. Consequently, at time t, a local chapter who raises $x_i(t)$ incurs a total cost of $\frac{\psi}{2} x_i(t)^2$. Incorporating positive fixed costs would only affect the extensive margin and would serve to strengthen the nonprofits' incentives for temporary alliance. Specifically, if we interpret a fixed cost as a startup cost, and the nonprofits suffer extra capital (liquidity) constraints, then a nonprofit may not be able to initially survive on its own without the support of a national organization.[6] Thus those who already choose temporary alliance will

[6] As noted in Oster (1996), a significant difference between for-profits and nonprofits is the latter's limited access to capital markets.

still do so, those who choose permanent alliance will still do so, and some of those who choose to never affiliate will switch to temporary alliance.

We assume that all funds are aggregated at the national level, which is a simplifying assumption. In the nonprofit sector, there is variation as to what proportion of the funds remains with the local chapter. For example, in the context of nature conservation nonprofits, Taylor and Hammack (2007) mention that at Ducks Unlimited and the Rocky Mountain Elk Foundation, no funds stay at the local level, whereas the Quality Deer Management Association and Quail Unlimited share the proceeds between their national headquarters and local chapters, and at Pheasants Forever, all of the proceeds stay with the local chapter. Therefore, our assumption that the local chapters send all their funds to the parent organization can be viewed as a simplifying assumption and would provide a good fit for some, but not all, nonprofits. A more general approach would allow the possibility that local chapters can retain some of the funds, and ideally also vary the percentage of funds retained at the local level to assess how the results are affected. The parsimonious specification of fund allocation in our model simplifies the analysis and facilitates the investigation of the local chapters' incentives and decisions in a dynamic setting, and we therefore leave the more general approach for future research.

Aggregating across all affiliated local chapters, at each t the parent organization receives $\sum_{j \in p(t)} x_j(t)$. The national organization uses the resources it receives from the local chapters to develop treatment (here "treatment" is an umbrella term that encompasses education programs, treatment protocols, medical procedures, etc.) using transformation function $f(y) = \theta y$, $\theta > 0$. θ captures the national organization's efficiencies from economies of scale ($\theta > 1$) or inefficiencies from being a large organization ($\theta < 1$). The accumulation and depreciation of the treatment gives the state, i.e., the installed base.

Let $X_p(t)$ denote the installed base at time t and assume straight-line depreciation. Then

$$\dot{X}_p(t) = \theta \sum_{j \in p(t)} x_j(t) - \delta_p X_p(t), \tag{1}$$

where $\delta_p \in (0, 1]$ represents depreciation at the parent organization. We take a broad interpretation of depreciation to incorporate things such as employee/volunteer turnover, which requires resources be expended to train replacements, or new developments rendering old treatment obsolete.

2.3 Local benefits

In our model, the treatment is non-rival in the sense that consumption by one group does not prevent simultaneous consumption by other groups.

Correspondingly, each local chapters benefit from the installed base at the national level, weighted by both the match value between the local and global missions and the population served. The local benefits at time t for an affiliated local chapter take the form

$$u(A_i(t) = 1, x_i(t); N_i, M_i, M_p) = \mu\left(M_i, M_p\right) N_i X_p(t) - \frac{\psi}{2} x_i(t)^2.$$

When unaffiliated, the local organization loses benefits from agglomeration through two channels: the aggregation channel $\sum_{j \in p(t)} x_j\left(t\right)$ and the efficiency channel θ, but gains from being able to focus all efforts on the local mission. An unaffiliated local organization raises $x_i\left(t\right)$ at each t, and accumulates local installed base according to

$$\dot{X}_i(t) = x_i\left(t\right) - \delta_i X_i(t), \tag{2}$$

where $\delta_i \in (0, 1]$ represents depreciation at local organization i, which may vary across i and may or may not equal δ_p. The local benefits for an unaffiliated local organization take the form

$$u(A_i(t) = 0, x_i(t); N_i) = N_i X_i(t) - \frac{\psi}{2} x_i(t)^2.$$

Combining both cases, the local benefits are:

$$u(A_i(t), x_i(t); N_i, M_i, M_p) = \begin{cases} \mu\left(M_i, M_p\right) N_i X_p(t) - \frac{\psi}{2} x_i(t)^2 & \text{if } A_i(t) = 1 \\ N_i X_i(t) - \frac{\psi}{2} x_i(t)^2 & \text{if } A_i(t) = 0. \end{cases} \tag{3}$$

2.4 Breakups

Due to the (impure) public goods nature of treatment (as discussed in the Introduction section), if a chapter begins affiliated and later chooses to leave the parent organization, it need not leave empty handed, and may leave with some (or all) of the treatment. In particular, suppose that if chapter i leaves, then it leaves with $(1-\alpha)X_p(t)$, where $\alpha \in [0,1]$ denotes the (exogenous) degree of excludability. Furthermore, given the non-rival nature of the installed base, if chapter i leaves, the parent organization retains its entire installed base $X_p(t)$, but no longer receives contributions from the local chapters that exit.

2.5 Objective of a local organization

Each local organization i chooses a path of affiliation decisions, $(A_i(t))_{t=0}^{\infty}$, and a path of fund raising, $(x_i(t))_{t=0}^{\infty}$, to maximize the present discounted value of utility, given by

$$\int_s^{\infty} e^{-\beta_i(t-s)} u(A_i(t), x_i(t); N_i, M_i, M_p) dt$$

at each $s \in [0, \infty)$ subject to the dynamics specified in (1) and (2).

3 Equilibrium Analysis

For tractability, we make the following normalizations and assumptions. Set $K = 2$, where we interpret $i = 1$ as a local chapter and $i = 2$ as the collection of the remaining chapters, and $N_1 + N_2 = 1$, where N_1 is the proportion of the population served by chapter 1 and N_2 is the proportion served by the remaining chapters. We assume that chapter 2 is affiliated with the parent organization ($A_2(t) = 1$) for all t, and normalize $X_2(t) = X_p(t)$, $\mu(M_2, M_p) = 1$ and $\delta_2 = \delta_p$. Thus in this section, we study the conditions under which chapter 1 (the local chapter) is willing to affiliate on the extensive margin, and conditional on affiliating, the length of the affiliation (intensive margin), taking chapter 2's affiliation with the national organization as given. This assumption and the linear-state structure of the game are simplifying assumptions that allow us to both highlight and isolate the effects of the local chapters behavior, since that is the primary focus of this paper.

Given that we (i) normalize $\mu(M_2, M_p) = 1$ and (ii) require chapter 2 to remain affiliated for all t, we can offer two alternate interpretations of chapter 2. Firstly, we can view chapter 2 as the parent organization, which due to its better fit may have a greater return to fundraising than the corresponding local chapter (fundraising is a key place where economies of scale can occur and benefit the nonprofits). Secondly, we can view chapter 2 as the collection of other local chapters and the parent organization. In either interpretation, the resulting equilibrium, welfare, and comparative static properties are identical. We could also allow different marginal costs ψ for chapters 1 and 2, which would affect contributions vis-à-vis free-riding on the intensive margin, but leaves the extensive margins qualitatively unaffected.

We consider three regimes. In the endogenous alliance regime, the local chapter strategically chooses whether or not to affiliate, along with how long to remain affiliated. This corresponds to the model that we describe above. In the full alliance regime, chapter 1 is required to affiliate with the national organization and remain indefinitely (for example due to regulations in the industry). For comparison purposes, we also consider the no alliance regime, in which chapter 1 never joins the national organization.

3.1 No Alliance Regime

We first consider the no alliance regime. Under this regime, the local chapter never affiliates with the parent organization, acting independently at all times ($A_1(t) = 0$ for all t). However, chapters 1 and 2 are still strategic in their fund raising decisions. Chapter i's objective is given by the series of problems

$$\max_{(x_i(t))_{t=s}^{\infty}} \int_s^{\infty} e^{-\beta_i(t-s)} u\left(0, x_i(t); N_i, M_i, M_p\right) dt,$$

which by Pontryagin's maximum principle, can be represented by

$$\max_{x_i(t)} \mu\left(M_i, M_p\right) N_i X_i(t) - \frac{\psi}{2} x_i(t)^2 + \lambda_i(t)\left[(1 + \mathbf{1}\{i = 2\}(\theta - 1)) x_i(t) - \delta_i X_i(t)\right],$$

where $\mathbf{1}\{\cdot\}$ is an indicator function equaling unity if the argument is true and zero otherwise. The first-order and adjoint conditions to this problem are

$$-\psi x_i^{NA}(t) + (1 + \mathbf{1}\{i = 2\}(\theta - 1))\lambda_i(t) = 0$$
$$\beta_i\lambda_i(t) - \dot{\lambda}_i(t) = \mu\left(M_i, M_p\right) N_i - \delta_i\lambda_i(t).$$

The solutions to this system are given by

$$x_1^{NA}(t) = \frac{N_1}{\psi(\delta_1 + \beta_1)}, \quad \text{if } i = 1 \tag{4}$$
$$x_2^{NA}(t) = \frac{\theta N_2}{\psi(\delta_p + \beta_2)}, \quad \text{if } i = 2. \tag{5}$$

Substituting (4) into (2) and (5) into (1) and solving the differential equations using the initial condition $X_i(0) = 0$ yields the treatments

$$X_1^{NA}(t) = \frac{N_1}{\psi\delta_1(\beta_1 + \delta_1)}\left(1 - e^{-\delta_1 t}\right) \tag{6}$$
$$X_p^{NA}(t) = \frac{\theta N_2}{\psi\delta_p(\beta_2 + \delta_p)}\left(1 - e^{-\delta_p t}\right). \tag{7}$$

We denote the present discounted value function under the no alliance regime by

$$V_i^{NA} \equiv \int_0^\infty e^{-\beta_i t} u\left(0, x_i^{NA}(t); N_i, M_i, M_p\right) dt, \tag{8}$$

subject to $X_i(t) = X_i^{NA}(t)$. This present discounted value function will be used in subsequent sections for welfare comparisons.

3.2 Full Alliance Regime

We now consider the full alliance regime. Under this regime, the local chapter is affiliated with the parent organization, and remains so indefinitely, i.e., $A_1(t) = 1$ for all t. As before, the chapters are still strategic in their fundraising decisions. Chapter i's objective is given by the series of problems

$$\max_{(x_i(t))_{t=s}^{\infty}} \int_s^{\infty} e^{-\beta_i(t-s)} u\left(1, x_i(t); N_i, M_i, M_p\right) dt,$$

which can be rewritten as

$$\max_{x_i(t)} \mu\left(M_i, M_p\right) N_i X_p(t) - \frac{\psi}{2} x_i(t)^2 + \lambda_i(t)\left[\theta\left(x_1(t) + x_2(t)\right) - \delta_p X_p(t)\right],$$

with first-order and adjoint conditions

$$\begin{aligned} -\psi x_i^{FA}(t) + \theta \lambda_i(t) &= 0 \\ \beta_i \lambda_i(t) - \dot{\lambda}_i(t) &= \mu\left(M_i, M_p\right) N_i - \delta_p \lambda_i(t). \end{aligned}$$

The solution to this system is given by

$$x_i^{FA}(t) = \frac{\theta \mu\left(M_i, M_p\right) N_i}{\psi\left(\beta_i + \delta_p\right)}. \tag{9}$$

Substituting (9) into (1) and solving the corresponding differential equation using $X_p(0) = 0$ yields

$$X_p^{FA}(t) = \theta\left(\frac{\mu\left(M_1, M_p\right) N_1}{\beta_1 + \delta_p} + \frac{N_2}{\beta_2 + \delta_p}\right)\left(1 - e^{-\delta_p t}\right). \tag{10}$$

We denote the present discounted value function under the full alliance regime for chapter i by

$$V_i^{FA} \equiv \int_0^{\infty} e^{-\beta_i t} u\left(1, x_i^{FA}(t); N_i, M_i, M_p\right) dt, \tag{11}$$

subject to $X_p(t) = X_p^{FA}(t)$.

3.3 Endogenous Alliance Regime

We analyze the endogenous alliance regime in three stages. Firstly we presuppose that the length of the affiliation period is nonzero, finite, and given by the unknown time T', which is found by maximizing the present discounted value of utility. Secondly we use this value T' to find the equilibrium contributions post-exit, which are used to derive the pre-exit contributions. Lastly we find the present discounted value playing this strategy and compare it to the two possible deviations: $T = 0$ (no affiliation) and $T = \infty$ (full affiliation).[7] If there is no profitable deviation, then the equilibrium affiliation length T^* is given by $T^* = T'$. If there is a profitable deviation, then $T^* = 0$ if the most profitable deviation involves no affiliation and $T^* = \infty$ if the most profitable deviation involves full affiliation. The endogenous regime is thus characterized by $T^* \in \{0, T', \infty\}$.

Suppose that the affiliation period is nonzero and finite, and given by T'. The post-exit problem is

$$\max_{(x_i(t))_{t=s}^{\infty}} \int_s^{\infty} e^{-\beta_i(t-s)} u\left(A_i(t), x_i(t); N_i, M_i, M_p\right) dt$$

subject to $X_i\left(T'\right) = (1 - \mathbf{1}\{i = 1\}\alpha) X_p\left(T'\right)$, for all $s \geq T'$, where $A_1(t) = 0$ for all $t \geq T'$ and $A_2(t) = 1$ for all $t \geq T'$. Note that this problem is analogous to that of Section 3.1, so the solutions are given by (4) and (5). The treatments evolve according to

$$X_1\left(t > T'\right) = \frac{N_1}{\psi\delta_1(\beta_1 + \delta_1)} + \left[(1-\alpha)X_p\left(T'\right)) - \frac{N_1}{\psi\delta_1(\beta_1 + \delta_1)}\right] e^{-\delta_p(t-T')} \tag{12}$$

$$X_p\left(t > T'\right) = \frac{\theta N_2}{\psi\delta_p(\beta_2 + \delta_p)} + \left[X_p\left(T'\right) - \frac{\theta N_2}{\psi\delta_p(\beta_2 + \delta_p)}\right] e^{-\delta_p(t-T')}. \tag{13}$$

The pre-exit contributions are then found by solving the series of problems

$$\max_{(x_i(t))_{t=s}^{T'}} \int_s^{T'} e^{-\beta_i(t-s)} u\left(1, x_i(t); N_i, M_i, M_p\right) dt + \int_{T'}^{\infty} e^{-\beta_i t} u\left(A_i(t), x_i^{NA}(t); N_i, M_i, M_p\right) dt$$

for each $s \in [0, T')$. Utilizing Pontryagin's maximum principle and the finite-horizon transver-

[7] Note that there can be no profitable deviation to an interior $T \neq T'$, as this difference would imply that T' was not the value that maximized the present discounted value of utility.

sality condition

$$\lambda_i\left(T'\right) = \frac{\partial}{\partial X_p\left(T'\right)}\left(e^{\beta_i T'}\int_{T'}^{\infty} e^{-\beta_i t} u\left(A_i(t), x_i^{NA}(t); N_i, M_i, M_p\right) dt\right),$$

it follows that, conditional on exiting at an interior time T',

$$x_1^{EA}\left(t \leq T'\right) = \frac{\theta\mu\left(M_1, M_p\right) N_1}{\psi(\beta_1+\delta_p)} + \left[(1-\alpha)\frac{N_1}{\psi(\beta_1+\delta_1)} - \frac{\theta\mu\left(M_1, M_p\right) N_1}{\psi(\beta_1+\delta_p)}\right] e^{-(\beta_1+\delta_p)(T'-t)} \tag{14}$$

$$x_2^{EA}\left(t \leq T'\right) = \frac{\theta N_2}{\psi(\beta_2+\delta_p)}. \tag{15}$$

Substituting (14) and (15) into (1) and solving the corresponding differential equation yields

$$X_p^{EA}\left(t \leq T'\right) = a\left(1-e^{-\delta_p t}\right) + be^{-(\beta_1+\delta_p)T'}\left(e^{(\beta_1+\delta_p)t} - e^{-\delta_p t}\right), \tag{16}$$

where

$$a = \frac{\theta^2}{\psi\delta_p}\left[\frac{\mu\left(M_1, M_p\right) N_1}{\beta_1+\delta_p} + \frac{N_2}{\beta_2+\delta_p}\right], \quad b = \frac{\theta}{\psi(\beta_2+2\delta_p)}\left[\frac{(1-\alpha)N_1}{\beta_1+\delta_1} - \frac{\theta\mu\left(M_1, M_p\right) N_1}{\beta_1+\delta_p}\right].$$

Using (14)-(16), T' is given by the solution to

$$\max_T \lim_{s\to T}\int_s^T e^{-\beta_1(t-s)} u\left(1, x_1^{EA}(t); N_1, M_1, M_p\right) dt + \int_T^{\infty} e^{-\beta_1 t} u\left(0, x_1^{EA}(t); N_1, M_1, M_p\right) dt.$$

This value can only be computed numerically. Let

$$V' \equiv \int_0^{\infty} u\left(A_i(t), x_i^{EA}(t); N_1, M_1, M_p\right) dt$$

conditional on exit occurring at T'. The two possible unilateral deviations involve leaving at time $T=0$ and contributing according to (4) or never leaving and contributing according to (9). Thus

$$T^* = \begin{cases} 0 & \text{if } \max\left\{V_1^{NA}, V_1^{FA}, V'\right\} = V_1^{NA} \\ T' & \text{if } \max\left\{V_1^{NA}, V_1^{FA}, V'\right\} = V_1^{FA} \\ \infty & \text{if } \max\left\{V_1^{NA}, V_1^{FA}, V'\right\} = V'. \end{cases} \tag{17}$$

Finally, let

$$V_i^{EA} = \int_0^\infty e^{-\beta_i t} u\left(A_i(t), x_i^{EA}(t); N_i, M_i, M_p\right) dt \tag{18}$$

conditional on exit occurring at T^*.

T^* is only available numerically, and moreover, the utilities are such that sufficient conditions to characterize both T^* and the associated comparative statics and welfare implications are analytically unavailable. We therefore employ numerical methods to learn about the properties of T^*.

4 Results

In this section, we first present the results from a benchmark parameterization to better understand endogenous alliance and breakup. We then discuss a set of comparative statics results, which shed light on how various aspects of the model affect the equilibrium outcome.

Table 2: Parameters of Interest

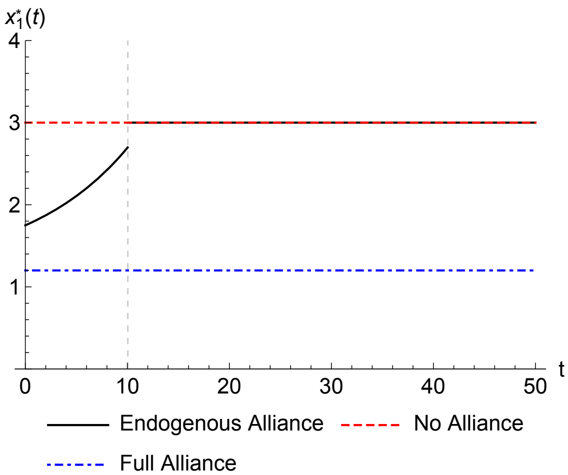
Parameter	Range	Benchmark Value	Interpretation
N_1	$(0,1)$	0.30	Proportion of the population served by chapter 1.
μ_1	$(0,1]$	0.40	Match value between chapter 1 and the national organization.
θ	$\mathbb{R}_+$	1	Efficiency/inefficiency at the national organization.
β_1	$\mathbb{R}_+$	0.05	Discount rate for chapter 1.
β_2	$\mathbb{R}_+$	0.05	Discount rate for chapter 2.
α	$[0,1]$	0.10	Degree of excludability.
ψ	$\mathbb{R}_+$	1	Rate of change of the marginal cost of soliciting donations.
δ_1	$(0,1]$	0.05	Depreciation rate of installed base for chapter 1.
δ_p	$(0,1]$	0.05	Depreciation rate of installed base for the national organization.

4.1 Benchmark

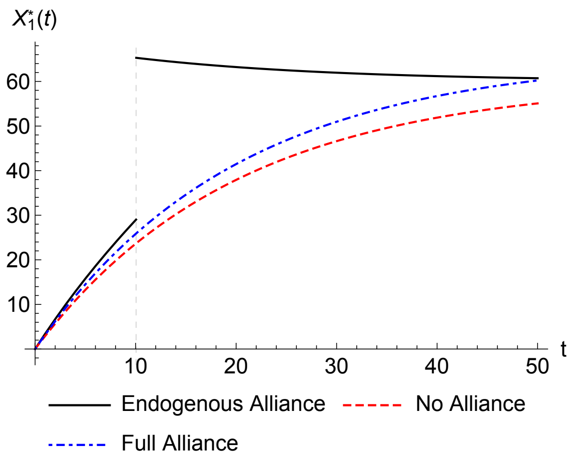
For the benchmark, we consider the parameterization given in Table 2. We set N_1 to 0.3, so that the local chapter has 30% of the whole population. We set μ_1 (i.e. $\mu(M_1, M_p)$), the match value between the local mission and the global mission, to 0.4, capturing the misalignment between the local chapter and the national organization's objectives. We set θ to 1, corresponding to the case in which the national organization's efficiencies from economies of scale and its inefficiencies from being a large organization cancel out each other. We set the two chapters' discount rates, β_1 and β_2, to 0.05, approximately corresponding to a yearly interest rate of 5%. We set α, the degree of excludability with respect to the installed base, to 0.1, which implies that when a local chapter breaks away from the national organization, it is allowed to leave with 90% of the (non-rival) installed base accumulated at the national organization. ψ, the rate of change of the marginal cost of soliciting donations, is set to 1. Finally, the rates of installed base depreciation, δ_1 and δ_p, are set to 0.05. To summarize, the benchmark parameterization is $(N_1, \mu_1, \theta, \beta_1, \beta_2, \alpha, \psi, \delta_1, \delta_p) = (0.3, 0.4, 1, 0.05, 0.05, 0.1, 1, 0.05, 0.05)$.

Figure 1 shows how the key equilibrium outcome variables in this model evolve over time under the benchmark parameterization. Each figure plots three time series: the solid line indicates the Endogenous Alliance regime, the dashed line indicates the No Alliance regime, and the dash-dot line indicates the Full Alliance regime. The vertical line in each figure, corresponding to $t \approx 10$, indicates the time at which the local chapter breaks away from the alliance under the Endogenous Alliance regime.

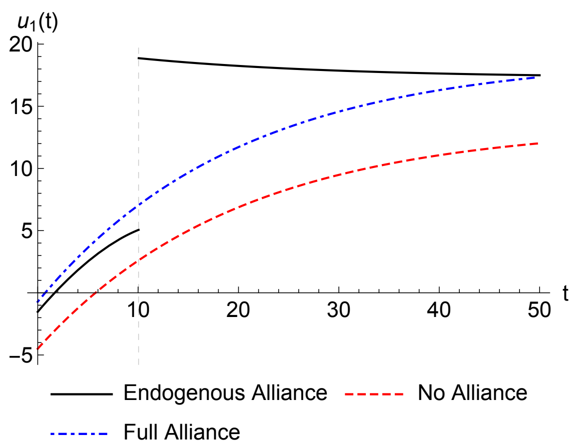
Donations at the Local Chapter. Figure 1(a) plots for $x_1^*(t)$, the level of instantaneous donations at the local chapter. The figure shows that the local donations are much higher when the local chapter is not affiliated with the national organization: under the No Alliance regime, $x_1^*(t)$ is constant at 3, whereas under the Full Alliance regime, $x_1^*(t)$ is constant at 1.2. This difference results from the misalignment between the local mission and the global mission, which makes local fundraising under the Full Alliance regime less effective in increasing local benefits and hence less desirable from the local chapter's point of view.



(a) Local chapter fundraising.



(b) Local chapter installed base.



(c) Local chapter benefits.

Figure 1: Characteristics of the benchmark model.

Under the Endogenous Alliance regime, local donations increase from 1.75 at $t = 0$ to 2.69 at $t \approx 10$ when the local chapter splits from the national organization. The reason for this increase is as follows. The marginal benefit of contributing at time t is a function of the discounted post-split value. Before the split, as t increases, the benefits are discounted less because $e^{-\beta_1(T^*-t)}$ is an increasing function of t. Thus the marginal benefit is increasing in t, while the marginal cost is constant in t, hence $x_1^*(t)$ is increasing on the interval $[0, T^*]$. After the split, local donations increase to the No Alliance level and stay constant thereafter.

Local Installed Base. Figure 1(b) shows the evolution of the local installed base, $X_1^*(t)$, over time. When the local chapter is affiliated with the national organization, we take the matched value adjusted installed base, $\mu_1 X_p(t)$, to be the effective local installed base. A clear pattern emerges: at any point in time, the local installed base is the lowest under No Alliance, and the highest under Endogenous Alliance.

Under No Alliance, even though the level of local donations are high (as shown in Figure 1), the local chapter fails to benefit from donations at the national level. Therefore $X_1^*(t)$ increases at a lower rate under No Alliance than under Full Alliance. However, Endogenous Alliance allows the local chapter to attain an even higher $X_1^*(t)$ than under Full Alliance, for the following two reasons. First, as discussed above, pre-split local donations are higher under Endogenous Alliance than under Full Alliance, resulting in a higher $X_1^*(t)$ pre-split. Second, after the split, the local chapter is able to focus exclusively on the needs of the local community, thereby substantially increasing the effective local installed base (subject to the excludability of the installed base). Specifically, at the time of the split, the effective local installed base increases by a factor of 9/4, from $\mu_1 X_p(t) = 0.4X_p(t)$ to $(1-\alpha)X_p(t) = 0.9X_p(t)$.

Local Benefits. Figure 1(c) shows the evolution of the local benefits, $u_1(t)$, over time. Throughout, local benefits are higher under Full Alliance than under No Alliance, as the former allows the local chapter to take advantage of the larger installed base at the national level. Compared to Full Alliance, Endogenous Alliance results in lower pre-split local benefits but much higher post-split local benefits. In other words, under Endogenous Alliance, the local chapter sacrifices some short-term benefits (due to the misalignment in the missions when being affiliated with the national organization) in exchange for higher benefits post-split.

Under all three regimes, the local benefits are initially negative, because in the early periods the cost of soliciting donations are larger than the benefits derived from a small installed base. However, the three regimes differ substantially in terms of how long it takes for the local benefits to turn positive. Specifically, local benefits turn positive at $t = 5.75$ under No Alliance, at $t = 1.69$ under Endogenous Alliance, and at $t = 0.75$ under Endogenous Alliance. Therefore, it is much easier for the local chapter to achieve positive local benefits under either Endogenous Alliance or Full Alliance than under No Alliance. Moreover, the amount of loss that the local chapter has to suffer before achieving positive local benefits is also much larger without alliance. Under No Alliance, the local chapter accumulates a discounted loss in utility of 11.25 before local benefits turn positive. That number decreases to 1.24 under Endogenous Alliance, and to 0.26 under Full Alliance.

The above discussion points to two important advantages that the local chapter gets when it chooses temporary alliance. First, by being affiliated with the national organization initially and later breaking away from it, the local chapter is able to enjoy a higher level of benefits, thanks to the initial fast accumulation of installed base and the subsequent focus on local needs post-split—the best of both worlds. Second, being affiliated with the national organization, at least initially, allows the local chapter to achieve positive local benefits much faster and at a much smaller startup cost. This is especially important given the liquidity constraints that a local chapter often faces at early stages.

4.2 Comparative Statics

We now turn to the comparative statics results with respect to the key parameters in the model. Specifically, we vary the following parameters one at a time, and assess how the equilibrium outcome changes in response: size of the local chapter N_1, match value with the national organization μ_1, production efficiency at the national organization θ, forward-looking behavior of the local chapter β_1, and excludability of treatment α.

Size of the local chapter. Table 3 reports the comparative statics results with respect to the size of the local chapter, N_1. For each value of $N_1 \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$, the table reports T^*, the length of the alliance, V_1, the present discounted value (PDV) of chapter 1 benefits, and V_{1+2}, the PDV of chapter 1 and chapter 2 benefits combined, for each of the three regimes.

We see that under the Endogenous Alliance regime, as N_1 increases, the local chapter is less likely to affiliate with the national organization. When $N_1 = 0.1$, the local chapter chooses to affiliate with the national organization forever, whereas when $N_1 = 0.9$, the local chapter chooses to be independent forever. For the values in between, i.e. when $N_1 \in \{0.3, 0.5, 0.7\}$, the local chapter chooses to affiliate with the national organization initially and break away later, and the length of such endogenous alliance decreases with N_1, from $T^* = 10.05$ at $N_1 = 0.3$ to $T^* = 5.50$ at $N_1 = 0.7$.

The reason for the above pattern lies in the tradeoff that the local chapter faces when it chooses whether to affiliate with the national organization. By joining the alliance, the local chapter benefits from faster accumulation of the installed base, however this benefit diminishes as the local chapter becomes larger relative to the national organization. At the same time, affiliation with the national organization comes with a set of restrictions that make the local chapter unable to fully focus on the local needs (the misalignment of local and global missions), and this downside does not diminish as the local chapter becomes larger. Therefore, as N_1 increases, it is more likely that the downside associated with alliance outweighs the benefit, resulting in earlier termination of the alliance or even no alliance.

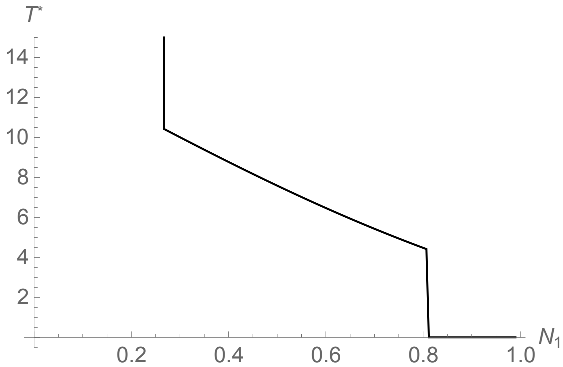
This pattern in our results is consistent with evidence from the nonprofit sector. Revisiting an example mentioned in the Introduction section, the local chapters that broke away from the Alzheimer's Association in the U.S. (including the chapters in Los Angeles, New York City, and San Diego) were among the association's largest chapters.

We next turn to the local and global benefits (V_1 and V_{1+2}, respectively). Under Endogenous Alliance, the local chapter is unrestricted in its affiliation choice, and can choose no alliance or full alliance if it wants to. Therefore, V_1 is always largest under Endogenous Alliance among the three regimes. As discussed above, the benefit from being affiliated with the national organization diminishes as the local chapter becomes larger. Consequently, for the local chapter, Full Alliance is better than No Alliance when N_1 is small ($N_1 \in \{0.1, 0.3\}$),

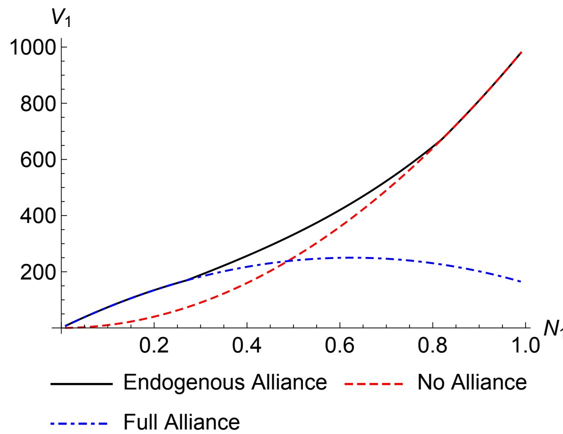
Table 3: Size of the local chapter

N_1	No Alliance			Endogenous Alliance			Full Alliance		
	T^*	V_1^{NA}	V_{1+2}^{NA}	T^*	V_1^{EA}	V_{1+2}^{EA}	T^*	V_1^{FA}	V_{1+2}^{FA}
0.1	0	10	820	∞	73.6	955.6	∞	73.6	955.6
0.3	0	90	580	10.05	188.8	795.3	∞	182.4	840.4
0.5	0	250	500	7.65	332.4	700.1	∞	240	690
0.7	0	490	580	5.50	523	691.8	∞	246.4	504.4
0.9	0	810	820	0	810	820	∞	201.6	283.6

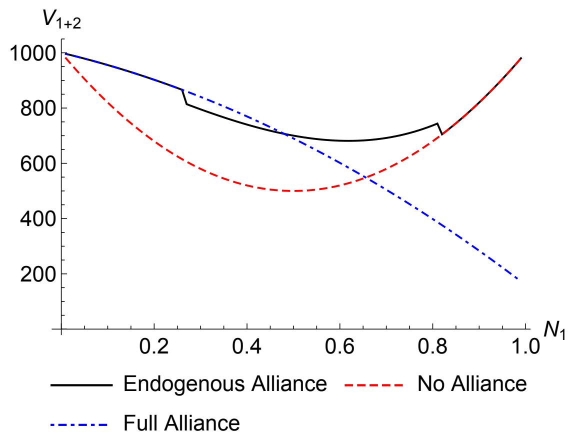
Notes: T^* corresponds to the length of the alliance, beginning at $t = 0$. $V_{1+2}^z = V_1^z + V_2^z$ for $z \in \{NA, EA, FA\}$. Highlighted values correspond to the efficient regime(s); the same for subsequent tables.



(a) Length of affiliation, Endogenous Alliance.



(b) Local chapter benefits.



(c) Welfare.

Figure 2: Comparative statics for N_1.

while No Alliance is better than Full Alliance when N_1 is large ($N_1 \in \{0.5, 0.7, 0.9\}$).

As for global benefits, the regime that maximizes V_{1+2} changes from Full Alliance to Endogenous Alliance to No Alliance as N_1 increases. In particular, when the local chapter is large ($N_1 \in \{0.5, 0.7, 0.9\}$), a regulation that requires the local chapter to be affiliated with the national organization (the Full Alliance regime) would be socially inefficient as it would reduce V_{1+2} compared to the Endogenous Alliance regime. The reason is that the imposed alliance leads to a large reduction in chapter 1's benefits, and this loss outweighs the benefits that chapter 2 obtains from the alliance.

Figure 2 plots the results from a broader set of parameter values, $N_1 \in \{0.01, 0.02, ..., 0.99\}$. Figure 2(a) shows that when $N_1 < 0.27$, the local chapter chooses to affiliate with the national organization forever. When $N_1 > 0.82$, the local chapter chooses to stay independent forever. In-between those two cutoff values, the local chapter chooses temporary alliance and the length of alliance decreases with N_1. Figure 2(b) shows that the local chapter's benefits are always maximized under the Endogenous Alliance regime. With respect to the local chapter's benefits, the Full Alliance regime dominates the No Alliance regime when $N_1 < 0.49$, and the opposite is true when $N_1 \geq 0.49$. Finally, Figure 2(c) illustrates that requiring Full Alliance can be welfare reducing. Specifically, when $N_1 \geq 0.48$, the Full Alliance regime is strictly dominated by the Endogenous Alliance regime in terms of global benefits V_{1+2}.

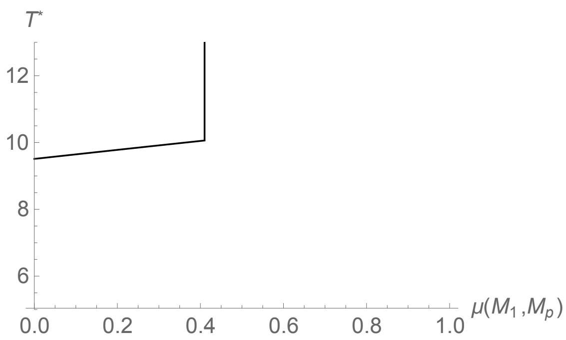
Local chapter's match value. Table 4 reports the comparative statics results with respect to the local chapter's match value, $\mu(M_1, M_p)$. As $\mu(M_1, M_p)$ increases, the local chapter is more likely to affiliate with the national organization. For $\mu(M_1, M_p) \in \{0.2, 0.4\}$, affiliation is temporary, with the length of the affiliation period increasing from $T^* = 9.78$ to $T^* = 10.05$ as $\mu(M_1, M_p)$ increases from 0.2 to 0.4. For $\mu(M_1, M_p) \in \{0.6, 0.8\}$, the local chapter chooses to affiliate with the parent organization permanently.

When choosing to affiliate when $\mu(M_1, M_p)$ is small, the local chapter is trading off a poor match value for economies of scale. With the remaining parameters set at the benchmark values given in Table 2, the benefits from economies of scale are large enough that it is never a best response for the local chapter to choose not to affiliate at all. If $\mu(M_1, M_p)$ is small, then the local chapter initially benefits from economies of scale; however, diminishing marginal returns in the development of treatment decreases the marginal benefit of affiliation. Eventually, the local chapter's best response is to un-affiliate, sacrificing $\alpha X_p(T^*)$ in exchange for the ability to utilize treatment in the manner that serves the local population best. Once

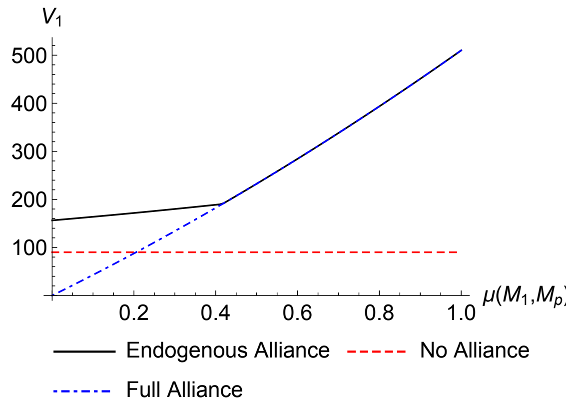
Table 4: Match Value with the national organization

μ_1	No Alliance			Endogenous Alliance			Full Alliance		
	T^*	V_1^{NA}	V_{1+2}^{NA}	T^*	V_1^{EA}	V_{1+2}^{EA}	T^*	V_1^{FA}	V_{1+2}^{FA}
0.2	0	90	580	9.78	171.7	763.9	∞	87.6	661.6
0.4	0	90	580	10.05	188.8	795.3	∞	182.4	840.4
0.6	0	90	580	∞	284.4	1026.4	∞	284.4	1026.4
0.8	0	90	580	∞	393.6	1219.6	∞	393.6	1219.6

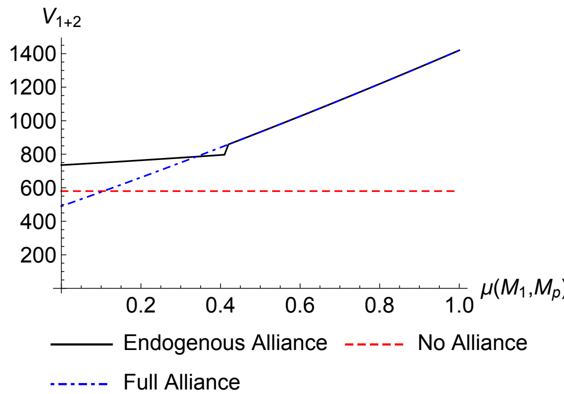
Notes: $\mu_1 \equiv \mu(M_1, M_p)$.



(a) Length of affiliation, Endogenous Alliance.



(b) Local chapter benefits.



(c) Welfare.

Figure 3: Comparative statics for $\mu(M_1, M_p)$.

$\mu(M_1, M_p)$ becomes large enough, the benefits from economies of scale are so valuable that permanent affiliation is the most profitable strategy because the losses from being forced to apply the treatment according to the global mission as opposed to the local mission are small.

Turning to welfare, we see that either the Endogenous Alliance regime or the Full Alliance regime maximizes total welfare, depending on the value of $\mu(M_1, M_p)$. When $\mu(M_1, M_p) = 0.2$, welfare is maximized under the Endogenous Alliance regime. Under the Full Alliance regime, the local chapter's benefits are small because the treatment is discounted by eighty percent. Also, by (9), fundraising efforts are lower when $\mu(M_1, M_p)$ is small under the Full Alliance regime, which depresses the development of treatment and global benefits. By (14), contributions are greater under the Endogenous Alliance regime, which increases development of treatment, benefiting both the local chapter and the global organization. When $\mu(M_1, M_p) \in \{0.4, 0.6, 0.8\}$, welfare is maximized under the Full Alliance regime. The reason lies in the relationship between local chapter contributions and $\mu(M_1, M_p)$. Recall from equation 9 that under the Full Alliance regime, the local chapter increases its fundraising efforts as $\mu(M_1, M_p)$ increases. Thus both the local chapter's benefits and the parent organization's benefits are increasing in $\mu(M_1, M_p)$, giving rise to the pattern that we see.

Figure 3 plots the information in Table 4 for a broader set of parameter values $\mu(M_1, M_p) \in \{0, 0.01, 0.02, \ldots, 1\}$. The figure reaffirms the results discussed above. Figure 3(a) shows that when $\mu(M_1, M_p) < 0.41$, the local chapter chooses to affiliate for a finite time horizon, and when $\mu(M_1, M_p) \geq 0.41$, the chapter chooses to affiliate indefinitely. Figure 3(b) shows that for all values of $\mu(M_1, M_p) \in \{0, 0.01, 0.02, \ldots, 1\}$, from the local chapter's point of view, the Endogenous Alliance regime is strictly preferred to the No Alliance regime, and is weakly preferred to the Full Alliance regime. Figure 3(c) shows that for small values of $\mu(M_1, M_p)$, the Endogenous Alliance regime maximizes welfare, while for larger values, the Full Alliance regime maximizes welfare. For $\mu(M_1, M_p) < 0.34$, the Endogenous Alliance regime is efficient. For $0.34 \leq \mu(M_1, M_p) < 0.41$, the Full Alliance regime is strictly efficient. Lastly, the two regimes are equally efficient for all $\mu(M_1, M_p) \geq 0.41$.

Production efficiency at the national organization. Table 5 reports the comparative statics results with respect to the production efficiency at the national organization, θ. Under the Endogenous Alliance regime, small values of θ induce no affiliation, medium values of θ induce temporary affiliation, and large values of θ induce permanent affiliation. Specifically, no affiliation occurs when $\theta \in \{0.25, 0.5\}$, temporary affiliation occurs when $\theta \in \{0.75, 1\}$, and permanent affiliation occurs when $\theta = 1.25$. When temporary affiliation occurs, the length of the affiliation period increases from $T^* = 7.99$ when $\theta = 0.75$ to $T^* = 10.05$ when $\theta = 1$.

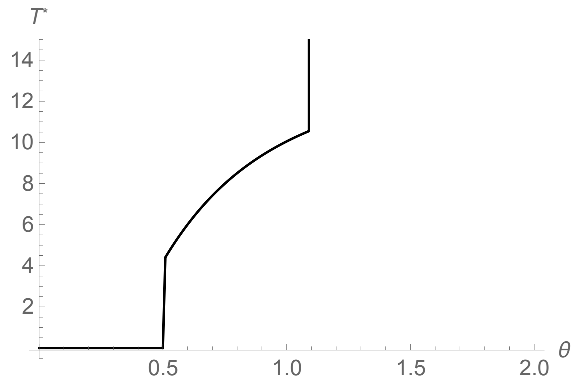
Recall that θ is interpreted as follows: \$1 in fundraising is converted to θ units of treatment. Thus a small θ indicates an inefficient parent organization, and therefore for small θ ($\theta \in \{0.25, 0.5\}$), the local chapter chooses not to affiliate under the Endogenous Alliance regime. Not only is the match value imperfect, so losses occur due to differences in the local and global missions, but deadweight loss also occurs in the conversion of funds to treatment. Rather than sending its donations to an inefficient parent organization, the local chapter can keep its funds local, allocate the resources more efficiently, and develop treatment targeted to the local population. Nonetheless, affiliation may occur even when there are inefficiencies in the parent organization: at $\theta = 0.75$, the benefits from aggregation outweigh the losses due to inefficiencies via θ and the imperfect match value $\mu(M_1, M_p)$, and the local chapter chooses temporary alliance. The local chapter continues to choose temporary alliance when θ is increased to 1. When the parent organization exhibits production efficiencies ($\theta = 1.25$), the efficiency gains coupled with the aggregation benefits induce the local chapter to choose to affiliate permanently.

Together, $\mu(M_1, M_p)$ and θ illustrate the underlying tradeoff in the tension between the efficiencies induced by economies of scale (which in this context refers to θ and the aggregation effect) and the costs of economies of scale (an imperfect match value). When unaffiliated, $\mu(M_1, M_1) = 1$, while $\mu(M_1, M_p) < 1$ under affiliation. In order to gain efficiencies from economies of scale, the local chapter must sacrifice its freedom to serve the local population in the manner it would prefer.

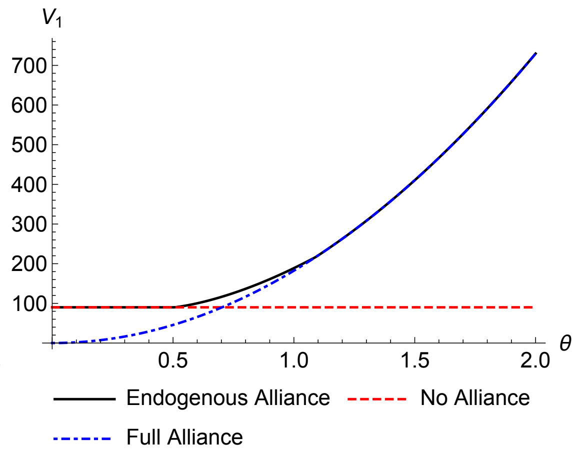
While it is no surprise that Endogenous Alliance maximizes welfare when the parent organization is inefficient ($\theta \in \{0.25, 0.5, 0.75\}$), it is important to discuss this result in the context of nonprofit entities. Inefficiencies can be interpreted in two ways: (i) the parent organization exhibits diseconomies of scale, i.e. it is more expensive for the larger organization to develop treatment than the corresponding local chapters, or (ii) the parent organization is wasteful. For example, the parent organization could be using the funds to support ad-

Table 5: Production efficiency at the national organization

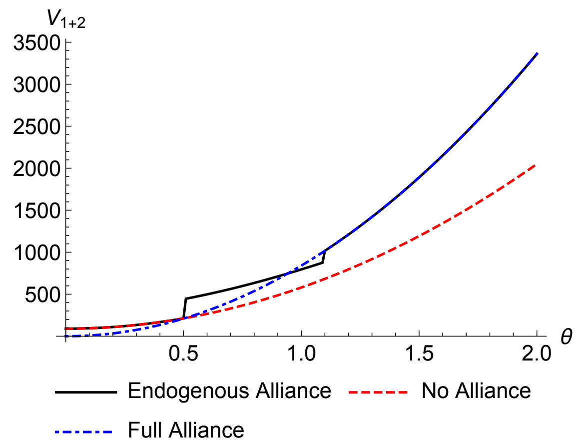
θ	No Alliance			Endogenous Alliance			Full Alliance		
	T^*	V_1^{NA}	V_{1+2}^{NA}	T^*	V_1^{EA}	V_{1+2}^{EA}	T^*	V_1^{FA}	V_{1+2}^{FA}
0.25	0	90	120.6	0	90	120.6	∞	11.4	52.5
0.5	0	90	212.5	0	90	212.5	∞	45.6	210.1
0.75	0	90	325.6	7.99	126.0	597.7	∞	102.6	472.7
1	0	90	580	10.05	188.8	795.3	∞	182.4	840.4
1.25	0	90	855.6	∞	285	1313.1	∞	285	1313.1



(a) Length of affiliation, Endogenous Alliance.



(b) Local chapter benefits.



(c) Welfare.

Figure 4: Comparative statics for θ.

ministrative bloat, or to send employees to conferences at luxury hotels. Given production inefficiencies, policies requiring affiliation are detrimental to the population as a whole, and allowing for endogenous or no affiliation can improve outcomes. The opposite holds for $\theta = 1.25$. When θ is large (there are production efficiencies), full affiliation maximizes welfare. Moreover, no policies are necessary to induce this behavior, as it is incentive compatible with the local chapters.

Results from a broader set of parameter values $\theta \in \{0, 0.01, 0.02 \ldots, 2\}$, confirming the patterns discussed above, are provided in Figure 4, with panel (a) corresponding to the affiliation period T^*, panel (b) corresponding to the local benefits, and panel (c) corresponding to welfare.

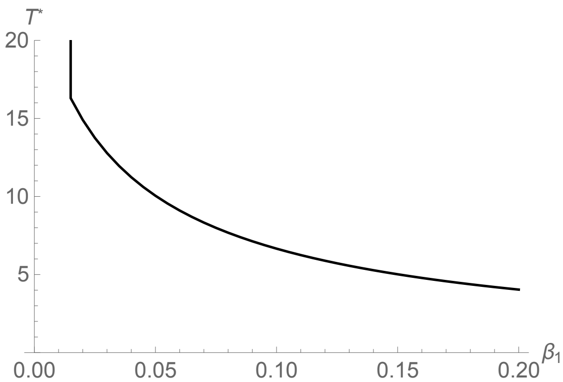
Local chapter discount rate. Table 6 reports the comparative statics results with respect to the discount rate of the local chapter, β_1. The table shows that under the Endogenous Alliance regime, if the local chapter is more patient (has a smaller discount rate), it is more likely to affiliate with the national organization. Specifically, the local chapter chooses permanent alliance when $\beta_1 = 0.01$, and chooses temporary alliance when $\beta_1 \in \{0.03, 0.05, 0.07, 0.09\}$. The length of temporary alliance decreases with β_1, from $T^* = 12.78$ at $\beta_1 = 0.03$ to $T^* = 7.13$ at $\beta_1 = 0.09$.

The intuition is as follows. As β_1 increases, two things occur: firstly fundraising efforts decrease, and secondly future payoffs are discounted at a higher rate. When the local chapter decides when to break away from the national organization, it faces the following tradeoff. If it stays in the alliance longer, it will leave with a larger installed base thanks to the higher rate of installed base accumulation when affiliated. However, lengthening the stay in the alliance also means postponing the large increase in local benefits that it gets when breaking away and focusing exclusively on local needs. Therefore, the more patient the local chapter is, the longer it chooses to stay in the alliance.

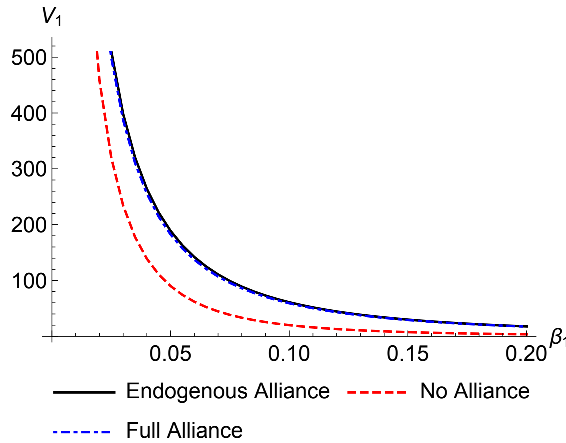
Turning to V_1, the discounted local benefits, we see that for each of the β_1 values considered in the table, the Endogenous Alliance regime dominates the Full Alliance regime (weakly so when $\beta_1 = 0.01$), which in turn dominates the No Alliance regime. As for V_{1+2}, the discounted global benefits, we see that the Full Alliance regime dominates the Endogenous Alliance regime (weakly so when $\beta_1 = 0.01$), which in turn dominates the No Alliance regime. Moreover, as β_1 goes up, the local chapter becomes less patient and discounts future benefits more heavily, therefore both V_1 and V_{1+2} decrease, for each of the three regimes.

Table 6: Forward-looking behavior of the local chapter

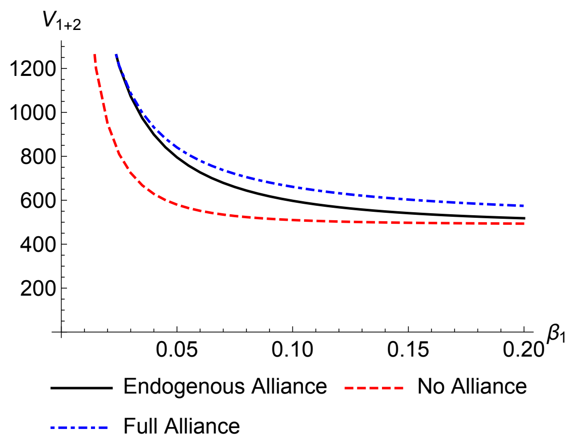
β_1	No Alliance			Endogenous Alliance			Full Alliance		
	T^*	V_1^{NA}	V_{1+2}^{NA}	T^*	V_1^{EA}	V_{1+2}^{EA}	T^*	V_1^{FA}	V_{1+2}^{FA}
0.01	0	1250	1740	∞	1600	2370	∞	1600	2370
0.03	0	234.4	724.4	12.78	398.5	1073	∞	387.5	1087.5
0.05	0	90	580	10.05	188.8	795.3	∞	182.4	840.4
0.07	0	44.6	534.6	8.33	110.8	679.1	∞	107.1	737.1
0.09	0	25.5	515.5	7.13	73	617.8	∞	70.4	680.7



(a) Length of affiliation, Endogenous Alliance.



(b) Local chapter benefits.

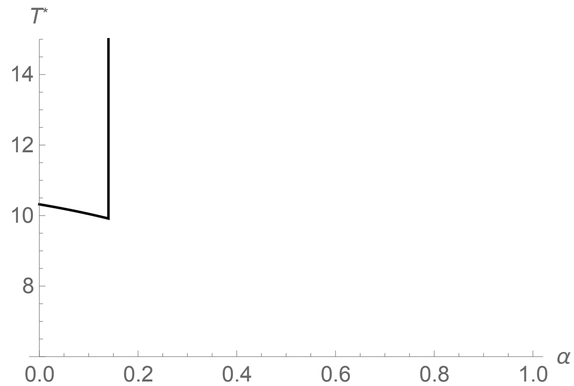


(c) Welfare.

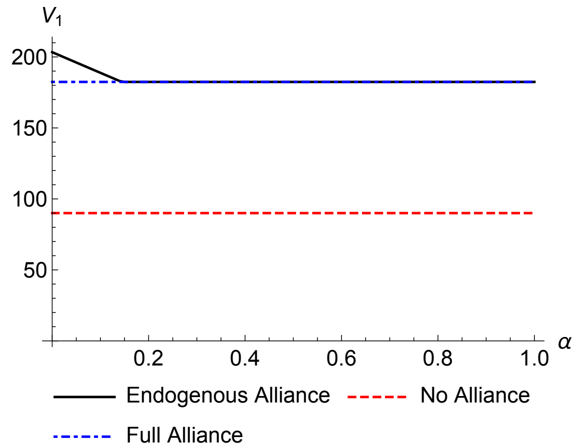
Figure 5: Comparative statics for β_1.

Figure 5 plots the results from a broader set of parameter values, $\beta_1 \in \{0.01, 0.015, ..., 0.2\}$. Figure 5(a) shows that when $\beta_1 < 0.014$, the local chapter chooses to affiliate with the national organization forever. When $\beta_1 \geq 0.014$, the local chapter chooses temporary alliance, and the length of alliance decreases with β_1. Figure 5(b) shows that local benefits are highest under the Endogenous Alliance regime and lowest under the No Alliance regime. Figure 5(c) shows that global benefits are highest under the Full Alliance regime and lowest under the No Alliance regime.

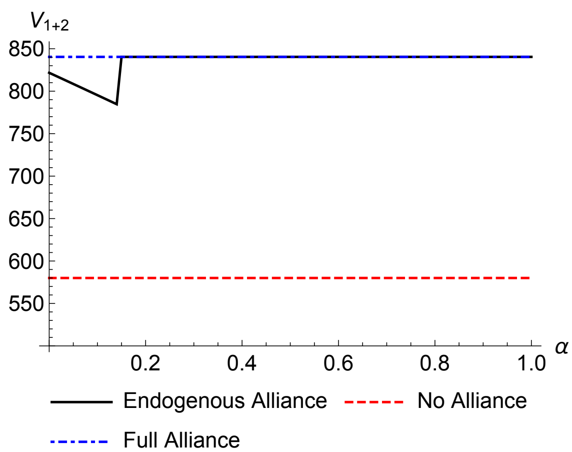
Degree of excludability of installed base. Table 7 reports the comparative statics results with respect to the degree of excludability of installed base, α. The table shows that as the excludability goes up, the length of alliance under the Endogenous Alliance regime



(a) Length of affiliation, Endogenous Alliance.



(b) Local chapter benefits.



(c) Welfare.

Figure 6: Comparative statics for α.

first decreases slightly, from $T^* = 10.32$ at $\alpha = 0$ to $T^* = 10.05$ at $\alpha = 0.1$, then goes up to infinity (i.e. permanent affiliation) at $\alpha = 0.15$.

Here's the intuition. As the proportion of installed base lost at breakup increases, staying

Table 7: Excludability of treatment

α	No Alliance			Endogenous Alliance			Full Alliance		
	T^*	V_1^{NA}	V_{1+2}^{NA}	T^*	V_1^{EA}	V_{1+2}^{EA}	T^*	V_1^{FA}	V_{1+2}^{FA}
0	0	90	580	10.32	618.4	821.7	∞	182.4	840.4
0.05	0	90	580	10.19	196.1	808.6	∞	182.4	840.4
0.1	0	90	580	10.05	188.8	795.3	∞	182.4	840.4
0.15	0	90	580	∞	182.4	840.4	∞	182.4	840.4

in a temporary alliance becomes less valuable. That's because the size of the jump in the local installed base at the time of exit is given by $\frac{1-\alpha}{\mu(M_1,M_p)}$. As α increases, this value becomes smaller, and temporary affiliation becomes less desirable. Consequently the length of time the local chapter is willing to affiliate with the national organization decreases. However, as permanent affiliation is more valuable than non-affiliation, once α grows to be too large so that temporary alliance is no longer worthwhile, the local chapter chooses permanent affiliation (over non-affiliation).

The above result shows that the local chapter chooses temporary affiliation when the good is non-excludable, but chooses permanent affiliation when there is a high degree of excludability. Since non-excludability is one of the defining features of public goods as compared to private goods, this finding suggests that the phenomenon of temporary affiliation is closely associated with the good being public. Therefore our model predicts that strategic affiliation followed by breaking away from the parent organization is more common in the nonprofit sector (which mostly deals with public goods) than in the for-profit sector (which mostly deals with private goods). This is a testable prediction of the model that can be brought to data for empirical examination. To the best of our knowledge, currently there does not exist either detailed data or large-scale descriptive statistics on strategic and temporary affiliation in for-profits and non-profits. Therefore the findings in this paper call for new research efforts on compiling data and empirically investigating this topic.

The rest of the information presented in Table 7 shows that the discounted local benefits (V_1) are highest under Endogenous Alliance and lowest under No Alliance, whereas the discounted global benefits (V_{1+2}) are highest under Full Alliance and lowest under No Alliance.

The above patterns are further confirmed in Figure 6, which plots the results from a broader set of parameter values, $\alpha \in \{0, 0.01, ..., 1\}$. In particular, Figure 6(a) shows that when $\alpha < 0.14$, the local chapter chooses temporary alliance and the length of alliance decreases with α, whereas when $\alpha \geq 0.14$, the local chapter chooses to affiliate with the national organization forever. Figures 6(b) and 6(c) similarly confirm the intuitions discussed above.

5 Concluding Remarks

In this paper we investigate strategic alliance in the nonprofit sector using a dynamic model of clubs with endogenous affiliation and split. We show that local organizations may choose to affiliate with the national organization for faster capital accumulation. Temporary alliance occurs when a local organization strategically affiliates with the national organization only to break away after accumulating enough capital.

Our results show that alliance is more likely to arise and persist when the local chapter is smaller, when the local chapter's mission is closer to the national organization's, when the national organization is more efficient in production, and when the local chapter is more patient. Moreover, a regulation that requires the local chapter to be affiliated with the national organization would be welfare reducing from the social point of view when the local chapter is large, when the local and national missions differ substantially, and when production at the national organization is inefficient.

A particularly interesting avenue for future research on collaborative nonprofits emerge from this line of work. In many instances, local chapters only send a fraction of the funds raised to the national organization. Both the local and parent organizations are thus developing complementary treatment independently. Affiliation slows the growth rate of the local treatment, but increases both the growth rate and sustainable size of the global treatment. Coupling this feature with the strategic nature of alliances creates the possibility for a wide range of behaviors to be studied, both theoretically and empirically, such as affiliation cycles, optimal affiliation contracts (e.g., requiring $x\%$ of all funds raised by the local chapter to be sent to the parent organization), and optimal excludability restrictions (e.g., non-compete clauses).

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