Avoiding Market Dominance: Product Compatibility in Markets with Network Effects*

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Abstract

As is well-recognized, market dominance is a typical outcome in markets with network effects. A firm with a larger installed base offers a more attractive product which induces more consumers to buy its product which produces a yet bigger installed base advantage. Such a setting is investigated here but with the main difference that firms have the option of making their products compatible. When firms have similar installed bases, they make their products compatible in order to expand the market. Nevertheless, random forces could result in one firm having a bigger installed base in which case the larger firm may make its product incompatible. We find that strategic pricing tends to prevent the installed base differential from expanding to the point that incompatibility occurs. This pricing dynamic is able to neutralize increasing returns and avoid the emergence of market dominance.

1 Introduction

Markets for products with network effects face the following conundrum. The value of the good to consumers is greatest when a single product dominates, as then network effects are maximized. However, the dominance of a single product typically means the presence of a monopoly, in which case consumers suffer the usual welfare losses from an excessively high price.

One possible solution to this conundrum is to have multiple firms offer *compatible* products. If there is complete compatibility then there are no foregone network effects, while the presence of viable competitors means price competition is operative. In fact, this was the basis for one of the proposed structural remedies in the Microsoft case. Referred to as the Baby Bills solution, the proposal was to divide the Windows monopoly into several identical companies which would initially have compatible (in fact, identical) products. Key to the remedy's appeal is that by initializing the market with compatible products, these newly created competitors would have an incentive to maintain compatibility over time.[1]

For product compatibility to represent a long-run solution to the problem of network effects, two conditions must then be satisfied. First, firms must initially find it in their interests to make their products compatible. Second, there must be incentives to maintain compatibility when, in response to future developments, differences emerge in firms' installed bases.

There are a number of papers that explore the first condition including Katz and Shapiro (1986), Economides and Flyer (1998), Cremer, Rey, and Tirole (2000), Malueg and Schwartz (2006), and Tran (2006).[2] The standard model is a two stage structure; in the first stage, firms make compatibility decisions and, given products are or are not compatible, they engage in price or quantity competition (for either one or two periods). Consistent with the Microsoft setting, both firms must agree for their products to be compatible. There are two primary forces that influence whether or not compatibility occurs in equilibrium. First, compatibility enhances the value of firms' products by increasing network effects. As this draws more consumers into the market, firms have a mutual interest in making their products compatible. Second, when firms have different installed bases, the larger firm loses an advantage with compatibility. In contrast, the smaller

[1] These Microsoft clones were colloquially dubbed "Baby Bills" as a play on the term Baby Bells which is itself a colloquialism for the Regional Bell Operating Companies created with the break-up of the Bell System in 1984. For details on the Baby Bills solution, see Levinson, Romaine, and Salop (2001).

[2] Some of this work is discussed in the review of Farrell and Klemperer (2007).

firm always prefers products to be compatible since it benefits through both effects. Existing work has shown that if firms are not too different - either in terms of installed bases or other traits - then products are compatible.

Having established that there are initial market conditions that would result in firms choosing to make their products compatible, this leads us to the second issue which is the long-run viability of compatible technologies. Even if firms are initially similar and make their products compatible, randomness in demand and other shocks will surely lead to asymmetric installed bases. Could a modest difference in installed bases induce the current market leader to choose incompatibility in a march towards dominance? If so, then creating a structure with initially compatible products may only delay - but not prevent - increasing returns from kicking in and creating a monopoly. Or are there forces that would maintain incentives for compatibility even when the installed base differential is significant? More generally, are compatible products stable in the long-run or can we expect that eventually market dominance will emerge?

To explore long-run market structure issues when network effects are present, there is a growing body of work, including Mitchell and Skrzypacz (2006), Llobet and Manove (2006), Cabral (2007), Driskill (2007), and Markovich (2008). However, none of these models allow firms to make their products compatible and thus cannot address the issue of whether compatible products are stable in the long-run.

The modelling innovation of this paper is to endogenize product compatibility in a dynamic stochastic setting so as to address the long-run market structure of a product market characterized by network effects. In each period, firms first decide on compatibility and then price. Demand and customer turnover are stochastic which means that firms are very likely to end up with asymmetric installed bases even if they begin identical and choose compatible products. While consumers are myopic, firms dynamically optimize. A Markov perfect equilibrium is numerically solved for and we assess the frequency with which market dominance occurs and explore its determinants.

Our main finding is that compatible products can indeed be stable in the long-run. What underlies this finding is a dynamic that can neutralize increasing returns and prevent market dominance from emerging. As long as network effects are not too strong, firms that begin with comparably sized installed bases will choose to make their products compatible. Furthermore, if the installed base differential should grow - even to the point that the larger firm makes its product incompatible - the smaller firm prices aggressively so as to reduce the differential and thereby maintain or restore mutual incentives for product compatibility. This pricing dynamic is sufficiently powerful to sustain compatible products in the long-run and prevent market dominance from emerging. Interestingly, if a product has stronger network effects, it is possible that this strategic pricing effect is so intensified that it actually becomes *more* likely that products are compatible.

The model is described in Section 2, while the definition and computation of equilibrium are discussed in Section 3. As a benchmark, Section 4 covers the static Nash equilibrium for the compatibility-price game. Markov perfect equilibria are reviewed in Section 5, and the implications of product compatibility for market dominance are explored in Sections 6 and 7, with the latter focusing on the role of network effects. A welfare analysis of various policy regimes is examined in Section 8, and we conclude in Section 9.

2 Model

Our objective is to provide some general insight about the long-run stability of compatible technologies in the midst of network effects. Towards that end, we chose not to tailor the model to a specific product - such as operating systems - but rather to develop a more generic model that encompasses the key forces at play in many markets characterized by network effects.

2.1 State Space and Firm Decisions

The model is cast in discrete time with an infinite horizon. Though our attention in this paper is limited to when there are just two firms, the model will be described for the more general case of $N \geq 2$ firms. These firms sell to a sequence of heterogenous buyers with unit demands. At the start of a period, a firm is endowed with an installed base which represents consumers who have purchased its product in the past. Let $b_i \in \{0, 1, \ldots, M\}$ denote the installed base of firm i at the start of a period where M is the maximal size of the installed base.

Given $(b_1, \ldots, b_N)$, firms engage in a two-stage decision process in which they choose compatibility in stage 1 and then price in stage 2. In stage 1, each firm decides whether or not to "propose compatibility" with each of the other firms. Let $d_{ij} \in \{0, 1\}$ be the compatibility choice of firm i with respect to firm j where $d_{ij} = 1$ means "propose compatibility." To actually achieve compatibility requires that both firms propose it. Thus, the technologies of i and j are "compatible" if and only if $d_{ij} \cdot d_{ji} = 1$. Requiring both firms to consent is consistent with a number of markets including those involved in the Microsoft case. Furthermore, the analysis promises to be more interesting than when a firm can, by itself, make its product compatible.[3] After compatibilities are determined, firms simultaneously choose price. Let p_i denote the price of firm i.

Though firms can influence compatibility and price, we do not allow inter-firm payments which would permit a firm to induce a competitor to make its product compatible through appropriate compensation. This assumption is common in the literature on network effects. Malueg and Schwartz (2006) summarize the arguments in its favor, of which the most compelling is that such payments may not be permitted by the antitrust authority as they provide fertile grounds for firms to collude.

This is clearly a stylized modelling of compatibility but should serve our purposes well. Our primary interest is in understanding the incentives for compatibility and that means learning when firms *prefer* compatibility. We have then given them maximal flexibility by ignoring any technical constraints and assuming compatibility is costless to change. Furthermore, this modelling approach means that compatibility is not a state variable and this is important in keeping the dimensionality of the state space manageable. After presenting our main results, we argue that they are likely to be robust to having a cost to changing compatibility.

2.2 Demand

Demand in each period comes from the replacement of a randomly selected old consumer (who previously purchased) with a new consumer. There is one new consumer each period and her buying decision is based on the following discrete choice model. Let ϵ_i be the idiosyncratic preference of the buyer for firm i's product in the current period. The utility that the consumer gets from buying from firm i is

$$v_i + \theta g \left(b_i + \lambda \sum_{j \neq i} d_{ij} d_{ji} b_j \right) - p_i + \epsilon_i.$$

$b_i + \lambda \sum_{j \neq i} d_{ij} d_{ji} b_j$ is the effective installed base of firm i given the set of compatible technologies where $\lambda \in [0, 1]$ allows for the value of the installed base of other compatible technologies to be worth less to consumers of firm i's product. v_i is a measure of intrinsic product quality which is assumed to

[3] In some markets, it may be viable for consumers to purchase converters to achieve compatibility. The implications of that option are explored in Farrell and Saloner (1992) and Choi (1996, 1997).

be common across firms: $v = v_i$ and is also fixed over time.[4] Network effects are captured by the increasing function $\theta g(\cdot)$ where $\theta \geq 0$ is the parameter that controls the strength of network effects. We will refer to the sum of these two factors, $v_i + \theta g(\cdot)$, as quality. The buyer can also choose to purchase an outside good with utility $v_0 + \epsilon_0$. As the intrinsic quality parameters only affect demand through the expression $v_0 - v$, without loss of generality we set $v = 0$. The consumer's idiosyncratic preferences $(\epsilon_0, \epsilon_1, \ldots, \epsilon_N)$ are unobservable to firms.

A new consumer buys from the firm offering the highest current utility. We are then assuming consumers make myopic decisions (or, equivalently, they have static expectations about the future). By having a parsimonious representation of consumer decision-making, we are able to have a rich modelling of firm choice with respect to price and compatibility. An important though challenging extension of our work is to allow consumers to be forward-looking with rational expectations. For some recent research along those lines - though not allowing for endogenous compatibility - see Cabral (2007) and Driskill (2007).

Assuming $(\epsilon_0, \epsilon_1, \ldots, \epsilon_N)$ are independently extreme value distributed, the probability that firm i makes a sale to a new consumer is

$$\phi_i(p; d, b) \equiv \frac{\exp\left(\theta g\left(b_i + \lambda \sum_{j \neq i} d_{ij} d_{ji} b_j\right) - p_i\right)}{\exp(v_0) + \sum_{j=1}^{N} \exp\left(\theta g\left(b_j + \lambda \sum_{k \neq j} d_{jk} d_{kj} b_k\right) - p_j\right)},$$

where p is the vector of prices of all firms, d is the vector of compatibility choices, and b is the vector of installed bases. Note that if $v_0 = -\infty$ then $\phi_0(p; d, b) = 1 - \sum_{i=1}^{N} \phi_i(p; d, b) = 0$, so the outside good is hopelessly unattractive and a consumer will buy from one of the N firms with probability one. In that case, expected market demand equals one in each period and, most importantly, is independent of firms' installed bases and any decisions regarding compatibility and price. Those decisions will only influence a firm's expected market share. The case of $v_0 = -\infty$ is referred to as the case when market size (or demand) is fixed. When instead v_0 is not $-\infty$ then the expected market size is endogenous. In particular, a firm can increase its expected demand without necessarily decreasing the expected demand of its rivals.

[4] An important research and policy question is how endogenous compatibility affects innovation incentives. We intend to explore this question in the future by allowing firms to invest in quality in which case v_i will be endogenous.

2.3 Network Effects and Transition Probabilities

In modelling network effects, we will assume they are bounded in the sense that $g(b_i) = g(m)$ if $b_i \geq m$ for some $m \leq M$. Bounding the network effect is as specified in Cabral and Riordan (1994) though in their context it was learning-by-doing. Though the results reported here are based on linear network effects - $g(b_i) = \frac{b_i}{m}$ if $b_i \leq m$ - we have also allowed g to be convex, concave, and S-shaped and the main conclusions of the paper are robust.

$\Delta(b_i)$ denotes the probability that the installed base of firm i depreciates by one unit. We specify $\Delta(b_i) = 1 - (1-\delta)^{b_i}$, where $\delta \in [0,1]$ is the rate of depreciation. This specification captures the idea that the likelihood that a firm's installed base depreciates increases with the size of its installed base. δ would be expected to be higher where consumer turnover is higher or products have shorter lives so that consumers need to return to the market at a higher rate.[5]

Letting $q_i \in \{0,1\}$ indicate whether or not firm i makes the sale, its installed base changes according to the transition function

$$\Pr(b_i'|b_i, q_i) = \begin{cases} 1 - \Delta(b_i) & \text{if} \quad b_i' = b_i + q_i, \\ \Delta(b_i) & \text{if} \quad b_i' = b_i + q_i - 1, \end{cases}$$

where, at the upper and lower boundaries of the state space, we modify the transition probabilities to be $\Pr(M|M,1) = 1$ and $\Pr(0|0,0) = 1$, respectively.

3 Equilibrium

3.1 Bellman Equation and Strategies

In working backwards through the compatibility and pricing decisions, we use the following notation:

- $V_i(b)$ denotes the expected net present value of future cash flows to firm i in state b *before* the compatibility decisions have been made.
- $U_i(d, b)$ denotes the expected net present value of future cash flows to firm i in state b *after* the compatibility decisions have been made and revealed to all firms.

[5] One motivation for this specification is that if b_i old consumers were to independently "die" with probability δ, then the probability of at least one dying is $1 - (1-\delta)^{b_i}$. The number of deaths in a period is then capped at one as a simplifying approximation.

We use $d(b)$ and $p(d,b)$ to denote the compatibility and pricing strategies in equilibrium. Given compatibility choices d and installed bases b, the net present value of future cash flows to firm i is given by

$$
\begin{aligned}
U_i(d,b) &= \max_{p_i} \phi_i(p_i, p_{-i}(d,b); d,b) p_i \\
&\quad + \beta \sum_{j=0}^{N} \phi_j(p_i, p_{-i}(d_i,b); d,b) \overline{V}_{ij}(b),
\end{aligned} \tag{1}
$$

where $p_{-i}(d,b)$ are the prices charged by firm i's rivals in equilibrium (given installed bases and compatibility choices), the (constant) marginal cost of production is normalized to be zero, $\beta \in [0,1)$ is the discount factor, and $\overline{V}_{ij}(b)$ is the continuation value to firm i given that firm j wins the current consumer.

Given any feasible vector of compatibility choices d, differentiating the right-hand side of equation (1) with respect to p_i and using the properties of logit demand yields the first-order condition

$$
-\phi_i(1-\phi_i)(p_i + \beta\overline{V}_{ii}) + \phi_i + \beta \sum_{j \neq i} \phi_i \phi_j \overline{V}_{ij} = 0. \tag{2}
$$

The pricing strategies $p(d,b)$ are the solution to the system of first-order conditions.

Folding back from pricing to compatibility decisions, given installed bases b, the net present value of future cash flows to firm i is given by

$$
V_i(b) = \max_{d_i \in \{0,1\}^{N-1}} U_i(d_i, d_{-i}(b), b), \tag{3}
$$

where $d_i = (d_{i1}, \ldots, d_{ii-1}, d_{ii+1}, \ldots, d_{iN})$ and $d_{-i}(b)$ are the compatibility choices of firm i's rivals in equilibrium (given installed bases). Since firm i has 2^{N-1} feasible compatibility choices, the size of the choice set is increasing exponentially in the number of firms.

We focus attention on Markov perfect equilibria (MPE). As firms are *ex-ante* symmetric - in the sense that they face the same demand and cost primitives - we focus on symmetric MPE. It is easiest to understand what the symmetry restriction entails if $N = 2$.[6] In this case the compatibility decision taken by firm 2 in state $(b_1, b_2) = (b', b'')$ is identical to the compatibility decision taken by firm 1 in state $(b_1, b_2) = (b'', b')$, and similarly for

[6] See e.g. Doraszelski and Satterthwaite (2007) for a formal definition of symmetry if $N > 2$.

the pricing decision and value function. This means that firms are required to behave identically if their installed bases are identical, but that they may behave differently if their installed bases are different. With a symmetric MPE, any *ex-post* asymmetries between firms arise endogenously as a consequence of firms' pricing and compatibility decisions for realized demand and the random depreciation of their installed bases. Finally, we follow the majority of the literature on numerically solving dynamic stochastic games by restricting attention to pure strategies (Pakes and McGuire, 1994, 2001).

3.2 Computation and Parameterization

As with many other dynamic models, the multiplicity of MPE is a concern. Unfortunately, it is not practical to compute all of them using the homotopy method proposed in Besanko et al (2005) because our game is more complex (with both compatibility and pricing decisions). We therefore develop an algorithm that computes a particular kind of equilibrium, namely the limit of a finite-horizon game as the horizon grows to infinity. This is a widely used selection criterion in the theoretical literature on dynamic games.

The idea is as follows: Given continuation values that encapsulate the value of future play and installed bases, in any given state it is *as if* firms are playing a two-stage game of making first compatibility and then pricing decisions. In the last period of a finite-horizon game, the continuation values are zero. Hence, we can solve for the subgame-perfect equilibrium of the two-stage game. In the previous-to-last period, the continuation values are given by the equilibrium payoffs of the last period. Continuing this line of thought, we can construct an algorithm that computes the limit of a finite-horizon game by iterating backwards in time.

It is worth pointing out two important differences to the widely-used Pakes and McGuire (1994) algorithm. First, in any given state we solve for the subgame-perfect equilibrium of the two-stage game of compatibility followed by pricing decisions whilst taking as given the continuation values of all firms. The Pakes and McGuire (1994) algorithm, in contrast, computes only a best reply for one firm taking as given both the continuation value of that firm and the strategies of its rivals. Second, while the initial guess for the value function is arbitrary in the Pakes and McGuire (1994) algorithm, using zero as the initial guess reflects the fact that the continuation values are zero in the last period of a finite-horizon game and is thus a key part of our algorithm.

Although we focus on the limit of a finite-horizon game, multiplicity of MPE remains a concern. Since products are compatible between firms i and

j if and only if both firms propose compatibility then, for any state, there is always an equilibrium outcome in which firms' products are incompatible. When it is also an equilibrium for products to be compatible, we select that equilibrium because: 1) our interest is in exploring the implications of product compatibility; and 2) the equilibrium with compatible products Pareto dominates the one with incompatible products (except when compatibility does not matter, such as when $\lambda = 0$). In the event that a firm is indifferent about whether or not to make its product compatible, we assume it proposes incompatibility.[7] At least when there are just two firms (which is the market structure of focus in this paper), this selection criteria takes care of multiplicity issues at the compatibility stage. With this selection criteria in place, our algorithm always converged and resulted in a unique equilibrium. A detailed description of our algorithm can be found in the Online Appendix.

The key parameters of the model that govern whether firms choose compatibility are the strength of the network effect θ, the degree of compatibility λ, the customer turnover rate δ, and the value of the outside option v_0. We assume $v_0 = 0$, so that market size is sensitive to firms' compatibility and price decisions, but also briefly contrast results with when $v_0 = -\infty$ so that market size is fixed.[8] The lower bound for customer turnover is zero and corresponds to the not very realistic case where consumers live or products last forever, which is achieved when $\delta = 0$. If δ is sufficiently close to one, then again the industry never takes off. We consider many values for δ between these extremes. The two extremes of $\lambda = 0$ and $\lambda = 1$ are explored along with the intermediate case of $\lambda = .5$. Finally, we investigate a range of values for the strength of network effects: $\theta \in \{0, 1, 2, 3, 4\}$. While we extensively vary the key parameters, we hold the remaining parameters constant at $N = 2$, $m = 15$, $M = 20$, and $\beta = \frac{1}{1.05}$, which corresponds to a yearly interest rate of 5%. We have no reason to think that our results are sensitive to these parameters (and we did experiment with various values for m and M).

While the model is not intended to fit any particular industry, we feel that our parameter values are reasonable when comparing the own-price elasticity for our model with empirical estimates for products with network effects. As representative examples of the equilibria for our model, the own-price elasticity is $-.77$ for one parameterization (that used in Figure 2)

[7]Experimentation with the tie-breaking rule revealed that it does not make a difference for our results.

[8]In unreported results, we find that our conclusions are robust to assuming $v_0 \in \{-3, -1, 1\}$.

and $-.92$ for another (Figure 3). As a point of comparison, Dranove and Gandal (2003) and Gandal, Kende, and Rob (2000) report own-price elasticities of -1.20 for DVD players and $-.54$ for CD players, respectively; while Clements and Ohashi (2005) report own-price elasticities ranging from -2.15 to $-.18$ for video game consoles. Other studies find evidence of much more elastic demand: Doganoglu and Grzybowski (2007) and Ohashi (2003) report own-price elasticities ranging from -5.04 to -4.20 for mobile telephony and from -18.84 to -12.51 for VCRs, respectively.

4 Static Equilibrium

Prior to characterizing equilibria for the dynamic game, it is useful to first understand the incentives for compatibility in the static model. The static equilibrium is derived by setting $\beta = 0$ in which case firms choose price to maximize current profit. Installed bases matter only because of how they affect the current value that consumers attach to firms' products; they are not an instrument to later dominance.

Firm 1's equilibrium price, $p_1(b_1, b_2, d(b_1, b_2))$, depends on its own installed base, b_1, its rival's base, b_2, and whether firms' equilibrium compatibility choices result in compatible products, which is represented by $d(b_1, b_2)$ and thus also depends on firms' installed bases. Representative of our findings is Figure 1 where we have plotted firm 1's equilibrium price against firms' installed bases. Also reported is the compatibility region - that is, $d(b_1, b_2)$ - which are the states for which both firms prefer compatibility and thus their products are compatible. When compatibility affects market demand ($\lambda = .5$ or $\lambda = 1$), products are compatible when firms' installed bases are sufficiently similar in size. The forces at work are basically the same as those in other static models that allow for compatibility choice and are most clearly identified in Cremer, Rey, and Tirole (2000). We review and elaborate upon them below.

Holding price fixed, there are two quantity effects from firms making their products compatible. Compatibility raises firm i's effective installed base from b_i to $b_i + \lambda b_j$ which then increases the value that consumers attach to its product by $\theta [g(b_i + \lambda b_j) - g(b_i)]$. Each firm's product is more attractive relative to the outside option. Firms then have a mutual interest in having compatible products because both benefit from drawing more consumers into the market. This we refer to as the *market expansion effect*.

A second quantity effect arises when firms have different installed bases. In that situation, compatibility reduces the quality differential between their products which, generally, harms the firm with a bigger installed base. In other words, the larger firm has an edge because of its installed base and that edge is partially (when $\lambda = .5$) or fully (when $\lambda = 1$) lost when products are made compatible. We call this the *business gift effect* as it means enhancing the business stealing effect of one's rival.[9]

Supplementing these quantity effects are price effects that can best be understood through the following decomposition when $\lambda = 1$. Suppose the initial state is $(b_1, b_2) = (b', b'')$ where $b' < b''$. Compatibility can then be decomposed into two parts: it causes firms' effective installed bases to shift from (b', b'') to (b'', b'') and then from (b'', b'') to $(b' + b'', b' + b'')$. As the first shift only improves the smaller firm's quality, its price rises and the larger firm's price falls.[10] The second shift causes both firms' prices to increase as the quality of their products rises relative to the outside good.[11] The first price effect is ambiguous as to how it impacts profitability though the second price effect amplifies the market expansion effect and thus further enhances the value to making products compatible.

We can now use the market expansion and business gift effects to explain why compatibility emerges when firms' installed bases are sufficiently similar in size. Suppose firms have identical installed bases and recall that firms are static profit-maximizers in this exercise. Both firms experience higher profit by having compatible products because they take demand away from the outside good (which is the market expansion effect) and neither firm loses any advantage over its competitor since relative quality is unaffected (that is, there is no business gift effect). Now suppose firms' bases are close but not identical. With compatibility, the larger firm loses only a small relative quality advantage over the smaller firm (since similar bases means similar qualities) but there is a discrete jump in absolute quality with compatibility. Hence, the market expansion effect exceeds the business gift effect when a firm's installed base is slightly larger than that of its rival. Obviously, the firm with a smaller base is better off with compatible products. This explains why there is an area around the diagonal in which firms agree to

[9] In the Online Appendix, it is shown that sufficient conditions for the business gift effect to harm the firm with the larger installed base are that g is linear or concave and/or $\lambda \simeq 1$. If g is sufficiently convex and $\lambda \ll 1$, it is possible that the business gift effect instead harms the firm with the smaller installed base.

[10] For our demand structure, Anderson et al (1992, p. 266) prove that a firm's equilibrium price is increasing in its quality (or installed base). Since prices are strategic complements then a firm's equilibrium price is decreasing in the other firm's quality.

[11] If firms have a common quality (which is composed of both intrinsic quality and network effects) then the symmetric equilibrium price can be shown to be increasing in that common quality as long as $\exp(v_0) > 0$ (that is, market size is variable).

make their products compatible, as can be seen in Figure 1. Now move the bases farther off of the diagonal. The business gift effect rises in importance - as the larger firm gives up a greater quality advantage - until it exceeds the market expansion effect; at that point the larger firm prefers that products be incompatible.[12]

This explanation is confirmed when one examines the case when there is no outside good ($v_0 = -\infty$). Since the market expansion effect vanishes, only the business gift effect is operative which would argue that the larger firm would never want to have compatible products. Indeed, when the market size is fixed, compatibility never occurs in equilibrium as long as firms have different installed bases.

5 Dynamic Equilibrium

For the primary dynamic forces of our model to be at work, the relevant part of the parameter space is when compatibility matters, network effects are not weak, and the rate of customer turnover is neither too low (so that the state stays away from the bounds) nor too high (so the "investment" incentive is not weak). In that part of the parameter space, two types of equilibria occur, which we refer to as Tipping and Compatibility. These are by far the most insightful for learning about dynamic competition and will be the focus of our attention. Throughout most of this section, we assume $v_0 = 0$, so that market demand is variable, and at its conclusion discuss what happens when market size is fixed.[13]

5.1 Tipping Equilibrium

A *Tipping* equilibrium has the following properties: i) intense price competition when firms' installed bases are of comparable size; ii) the limit distribution for installed bases is bimodal with a lot of mass at highly asymmetric states; and iii) products are generally incompatible. An example of a Tipping equilibrium is shown in Figure 2. The policy function for a Tipping equilibrium is characterized by a deep trench along and around the diagonal. In Figure 2, price is actually negative - below marginal cost - for some states near the diagonal. Sufficiently off of the diagonal, price is relatively high. This equilibrium is similar to that found in models with increasing

[12] We have indeed confirmed that where incompatibility occurs, the smaller firm prefers to have compatible products but it is vetoed by the larger firm.

[13] For a description of equilibria for a more comprehensive set of parameterizations, see the Online Appendix.

returns such as arises with advertising (Doraszelski and Markovich, 2007) and learning-by-doing (Besanko et al, 2005).[14]

When firms have sufficiently disparate installed bases, dynamic competition largely ceases as reflected in relatively high prices (these are the plateaus off of the diagonal). Due to network effects, the profitable strategy for the smaller firm is to accept having a low market share. If instead it were to try to supplant the larger firm, it would need to price at a considerable discount in light of the quality disadvantage emanating from a smaller installed base and that products are incompatible. Furthermore, low pricing would have to continue for an extended period of time in order to eliminate the installed base differential. Since such an aggressive strategy is not profitable and thus not pursued, prices are high and the larger firm reaps large profits due to its high market share by virtue of having a better product (which comes from a bigger installed base and network effects).

It is this "prize" to a firm with a significant installed base advantage that causes competition to be so intense when firms have comparable installed bases. A firm knows that if it were to gain such an advantage that the other (smaller) firm would accept its position in the market and the larger firm would reap high profits. We then have a deep trench along and around the diagonal which indicates that prices are low. Each firm focuses on fighting its rival to become the dominant firm. Note that for states in the trench, firms' products are incompatible except possibly when $b_1 = b_2$.[15]

To describe how firms' installed bases evolve, the T-period transient distribution describes the frequency with which the state (b_1, b_2) takes a particular value after T periods, starting from state $(0, 0)$ in period 0. A comparison of the transient distributions after 5, 15, and 25 periods in Figure 2 describes how the state is changing over time. Turning from the short run to the long run, the limit (or ergodic) distribution gives the frequency with which a state occurs after many periods.[16]

As shown in Figure 2, the limit distribution on installed bases is bimodal, which indicates that it is quite likely market dominance will emerge. Once

[14] This is also the case with capacity investment with price competition (Besanko and Doraszelski, 2004; Chen, 2005) though it is not an increasing returns story.

[15] Interestingly, products are not always compatible on the diagonal. Incompatibility occurs when firms price *below* marginal cost because of their eagerness to increase their installed bases. As a result, compatibility would *reduce* current profit because it increases demand and each unit sold is at a loss.

[16] More formally, let P be the $(M+1)^2 \times (M+1)^2$ transition matrix of the Markov process of industry dynamics. The transient distribution after T periods is given by $\mu_T = \mu_0 P^T$, where μ_0 is the $1 \times (M+1)^2$ initial distribution. The limit distribution μ_∞ solves the system of linear equations $\mu_\infty = \mu_\infty P$.

one of the firms gains an advantage in terms of installed bases, the strength of network effects transforms it into a long-run advantage. The movement towards skewed outcomes is apparent by following the transient distribution over time; more and more mass is dispersed away from the diagonal. The pricing behavior of firms contributes to the emergence and persistence of market dominance since the firm with the smaller installed base generally accepts its position by not pricing aggressively. The Tipping equilibrium embodies the quintessential property of network effects which is that the market "tips" to one firm dominating as soon as it has an advantage.

A Tipping equilibrium occurs when the network effect is strong - it does not occur for $\theta \in \{1, 2\}$ but does arise when $\theta \in \{3, 4\}$ - and customer turnover is modest (δ is low).[17] For a firm to price aggressively and forego current profit, the prospect of future dominance by building its installed base must be sufficiently great. This requires that the network effect is sufficiently strong and the installed base does not deteriorate too rapidly.

Result 1 (Tipping Equilibrium) *When the network effect is strong and customer turnover is modest, equilibrium is characterized by incompatible products, intense price competition when firms' installed bases are of comparable size, and tipping towards market dominance when one firm gains an advantage in terms of its installed base.*

5.2 Compatibility Equilibrium

There is another type of equilibrium which is new to the increasing returns literature and arises solely because firms have the option to make their products compatible. A *Compatibility* equilibrium has the following properties: i) high prices when firms' installed bases are of similar or highly disparate size but intense price competition when modestly different; ii) the transient and limit distributions for installed bases are unimodal with a lot of mass at reasonably symmetric states; and iii) products are compatible when firms' installed bases are comparable. A representative example is provided in Figure 3. For this type of equilibrium, let us explore compatibility and pricing in three scenarios: when the installed base differential is large, modest, and small.

Large installed base differential. When the differential is large, the outcome is basically the same as with a Tipping equilibrium. Products

[17] The exact parameter configurations for which a Tipping equilibrium occurs can be found in the Online Appendix.

are incompatible and the firm with the larger installed base dominates the market due to network effects. The smaller firm is resigned to its inferior position in the market and thus dynamic competition is minimal. Prices are relatively high as a result.

Small installed base differential. When firms have installed bases that are similar in size, prices are also relatively high though now products are compatible. Recall from our examination of the static equilibrium that compatibility reduces the quality differential emanating from firms having different installed bases, which is detrimental to the firm with a bigger installed base. At the same time, it enhances both firms' product quality and thereby expands the market. The former effect we referred to as the business gift effect and the latter as the market expansion effect. Due to these two effects, there was a region around the diagonal for which both firms choose to make their products compatible. These forces are still present in the dynamic equilibrium and are partly at work in generating the compatibility region for a Compatibility equilibrium.

Though products are compatible, this need not imply the absence of price competition. For a Tipping equilibrium, firms often make their products compatible when they have identical installed bases and, at the same time, price very low in order to acquire an advantage in its installed base. Such dynamic price competition is not observed for a Compatibility equilibrium when the installed base differential is small. To see why, suppose firms begin with identical installed bases. Regardless of which firm (if any) wins today's customer and thereby expands its installed base, firms expect their products to be compatible tomorrow; this follows from the compatibility region encompassing asymmetric as well as symmetric states. Thus, a firm which gains a small installed base advantage does not anticipate gaining a quality advantage in the near term because compatibility will be maintained; this stifles dynamic price competition.

Modest installed base differential. The most intriguing region is when firms have modestly different installed bases in which case prices are low as reflected in the dual trenches in the policy function. As explained below, pricing behavior is largely driven by dynamics associated with endogenous product compatibility. Whether or not products are compatible tomorrow depends on firms' installed bases tomorrow; only if they are sufficiently similar in size will firms mutually decide to have compatible products. Of course, tomorrow's state depends on today's pricing. With a Compatibility equilibrium, pricing is then driven not only by the prospect of dominating the market - a force that is ever present in a market with network effects - but also by the strength of firms' desire to maintain product compatibility.

To explore the incentives for compatibility, first note that the smaller firm almost always prefers compatible products - as it is benefitted both by the market expansion and business gift effects - while the larger firm prefers compatible products only when the installed base differential is sufficiently small. Thus, when products are incompatible, it is the larger firm that prevents it. Let us begin by examining how the smaller firm's desire for compatibility influences its pricing behavior.

Corresponding to Figure 3, Figure 4 reports firm 1's equilibrium price for different states. The states for which firms' products are compatible are shaded, while negative (below cost) prices are boxed. Prices are high when firms have comparable bases (that is, near the diagonal). As the state moves farther off of the diagonal - so that the difference in firms' bases increases - the smaller firm lowers its price. It does so even though firms' products are of equal quality (due to compatibility and $\lambda = 1$). In particular, the smaller firm significantly drops its price when the state approaches the (interior) border of the compatibility region. Its intent is to increase expected sales and thereby reduce the installed base differential. In Figure 4, firm 1's price drops from 1.3 to 1.0 when the state moves from $(b_1, b_2) = (4, 9)$ to $(4, 10)$ where $(4, 10)$ is just on the interior of the compatibility region. Just outside of the compatibility region, the smaller firm drops price even more; when the state moves from $(4, 10)$ to $(4, 11)$, firm 1's price drops from 1.0 to -.1. The smaller firm is trying to add to its base in order to move the state back into the region where the larger firm prefers compatibility. Compared to when the state is just inside the compatibility region, this task is made more difficult because products are no longer compatible which means the smaller firm suffers from a quality disadvantage. To compensate for that disadvantage, it needs to sell its product at an even bigger discount to the larger firm's product.

In sum, the smaller firm is pricing aggressively in order to keep the differential in installed bases sufficiently small. Its intent is to pacify, rather than fight, the larger firm so that the larger firm will "make nice" (by having a compatible product) rather than "make mean" (by pursuing monopolization through incompatible products).

Though the larger firm also drops price around the border of the compatibility region, that apparently is a response to the smaller firm's pricing behavior - as prices are strategic complements - rather than an attempt to monopolize. For example, in Figure 4, a movement in the state from

$(4, 9)$ to $(4, 10)$ results in the smaller firm dropping price from 1.3 to 1.0, while the larger firm's price only falls from 1.8 to 1.7 (and remember that consumers attach the same utility to their products as they are fully compatible). Examination of the value function shows that, along the border of the compatibility region, the larger firm only slightly prefers compatibility which is why it is willing to price much higher than the smaller firm even though it might mean the state moves out of the compatibility region. In contrast, the smaller firm strongly prefers compatibility, which explains why it is willing to price so low in order for the state to remain in that region.

It is worth emphasizing that this pricing behavior is quite distinct from what static demand effects would produce. Within the compatibility region, the relative quality of firms' products is identical since products are compatible and $\lambda = 1$. In a static model, prices would then be identical, while here the smaller firm has a lower price. Second, price falls sharply just outside of the compatibility region but eventually rises as the installed base differential becomes sufficiently large. That is also in support of our dynamic story as a firm's static equilibrium price monotonically declines as its relative quality falls.

A Compatibility equilibrium occurs when network effects are neither weak nor strong and the effect of compatibility on demand is significant. It is typical when $\theta \in \{1, 2\}$ but also occurs when $\theta \in \{3, 4\}$ as long as δ is not too low. If network effects are weak then pricing is largely uninfluenced by dynamic considerations, while if it is strong then the ability to translate a small installed base advantage into long-run dominance deters the larger firm from making its product compatible.

Result 2 (Compatibility Equilibrium) *When the network effect is modestly strong and the effect of compatibility on demand is strong, equilibrium is characterized by compatible products, mild price competition, and an absence of market dominance.*

Suppose we were now to assume that there is no viable outside option, in which case market size is fixed and each consumer buys from either firm 1 or firm 2. Running the model when $v_0 = -\infty$, firms are found never to choose to make their products compatible, except possibly when their installed bases are identical. With a fixed market size, each firm is only interested in having higher quality *relative* to its rival, in which case compatibility is always to the detriment of the larger firm. Hence, a Compatibility equilibrium does not arise, while there is a wider array of parameter values for which a Tipping equilibrium occurs. For example, when $\theta = 2$, firms do not compete aggressively for dominance when $v_0 = 0$ (that is, there are no Tipping equilibria), while they do compete for dominance for $\delta \in \{.04, \ldots, .12\}$, when $v_0 = -\infty$. In sum, fixing the market size intensifies dynamic price competition, as firms compete to gain an advantage in terms of their installed base, and there is almost no basis for compatibility.

In concluding this section, let us briefly discuss what we think might happen if there was a cost to changing compatibility that was neither small (in which case the equilibria would be almost exactly what we have now) nor large (in which case products would either always or never be compatible). The Tipping equilibrium ought to persist though there might be some mildly asymmetric states in which firms' products are compatible (assuming they began by being compatible). Still, we would expect that eventually the differential will become large enough that the larger firm will incur the cost of switching to incompatibility in order to then dominate the market. We also believe the Compatibility equilibrium would persist and, in fact, the pricing effects we have characterized could be more extreme. If firms' products are compatible and the larger firm is near the point of preferring incompatibility, the smaller firm would have an even stronger incentive to price aggressively since, once products are incompatible, it will be more difficult to return to having compatible products given that the larger firm has to be induced to incur the switching cost. Secondly, a cost to changing compatibility would mean there is a hysteresis band around the border of the compatibility region whereby firms' products remain compatible if they are currently compatible and remain incompatible if they are currently incompatible. These changes do not alter the key properties of equilibria and thus we expect the insight to be robust to having a cost to changing compatibility.

6 Product Compatibility and Market Dominance

One of the central questions of this paper is understanding to what extent endogenous product compatibility can prevent market dominance from emerging. If the transient and limit distributions with respect to installed bases are heavily skewed - putting a lot of mass on relatively asymmetric outcomes - then market dominance is likely to occur. The extent to which compatibility is feasible can be measured by the parameter λ. Firms effectively do not have the option of compatible products when $\lambda = 0$ as compatibility does not impact demand.

The pricing behavior identified in the previous section creates a *compatibility dynamic* which has the potential for maintaining some balance in the market and avoiding dominance. Suppose firms begin with identical or near-identical installed bases. They generally will find it optimal to make their products compatible in order to expand the market. With products of similar quality, firms charge similar prices. At that point, their expected market shares are comparable. Though, in expectation, future installed bases remain similar in size, random shocks to demand and customer turnover could result in one of the firms gaining a significant advantage in terms of installed bases. If that differential becomes large enough, products will no longer be compatible and firms will price in a manner to perpetuate such a skewed market structure. However, in a Compatibility equilibrium, there are forces preventing a slight advantage from growing into a large one. When firms' installed bases differ and are near the boundary of the compatibility region, the smaller firm prices aggressively in order to increase its installed base and thereby shift the state back towards symmetry. When the state is close to but outside of the compatibility region - so that the larger firm chooses to make its product incompatible - the smaller firm offers its product at an even larger discount so as to shift the state back into the compatibility region. The strategic pricing behavior of the smaller firm in the vicinity of the boundary of the compatibility region acts to keep the state within that region and thus works against market dominance.

The compatibility dynamic is revealed by reporting the resultant force, which measures the expected movement of the state as determined by the probability-weighted average of the difference between this and next period's state. Figure 5 shows the resultant forces for the parameter configurations in Figures 2 and 3. The left panel of Figure 5 has a Tipping equilibrium and, therefore, products are incompatible (except perhaps on the diagonal). Once the state is off of the diagonal, so that firms have different installed bases, the state moves away from symmetry as the larger firm builds on its advantage. Increasing returns is at work. The right panel of Figure 5 is for a Compatibility equilibrium and nicely shows how the increasing returns dynamic can be countered by the compatibility dynamic. There is a strong attraction to the diagonal for a wide range of states.

The real test of this dynamic is examining how the option of compatibility impacts the distribution on installed bases. Let us begin with a few illustrative examples and then present more systematic evidence. For two different parameter configurations, Figure 6 reports the set of state for which products are compatible and the limit distributions when $\lambda = 0$ and $\lambda = 1$. In the upper panels of Figure 6, the network effect is moderate ($\theta = 2$) and thus the limit distribution is unimodal even when compatibility is not an option. Market dominance is not likely to emerge in that case. As compatibility becomes a possibility ($\lambda = 1$), a unimodal distribution persists with more mass pushed towards symmetric outcomes. Introducing the option of compatibility makes it more likely that a roughly symmetric state occurs though does not have a significant impact.

As the strength of networks effects is increased to $\theta = 3$, endogenous product compatibility makes a striking difference; see the lower panels of Figure 6. When $\lambda = 0$, the limit distribution is significantly bimodal. Without the prospect of compatibility, it is very likely that one of the firms will dominate the market. Allowing for products to be compatible has a dramatic effect as the distribution shifts to being unimodal with a lot of mass around the diagonal. Firms are choosing to make their products compatible unless the state is reasonably asymmetric.[18] Introducing the option of compatibility makes it vastly less likely that market dominance will emerge.

Table 1 and Figure 7 provide a broader set of confirming results. Table 1 reports the mode of the limit distribution.[19] Highlighted are parameter values for which a bimodal distribution occurs when compatibility is not an option ($\lambda = 0$) and a unimodal distribution occurs when compatibility is an option ($\lambda = 1$). For example, when $(\delta, \theta) = (.07, 3)$, the lack of compatibility results in a highly skewed mode in which one firm has an installed base of 15 units and the other has only one unit. When instead firms have the option of product compatibility, the mode is symmetric with each having eight units. Figure 7 reports the expected long-run Herfindahl index (based on sales) using the limit distribution over states. To the extent that the long-run Herfindahl index exceeds .5, asymmetries arise and persist. If the customer turnover rate is not too low, the option of compatibility reduces market concentration and sometimes significantly so.

To summarize, endogenous product compatibility can neutralize the usual increasing returns mechanism associated with network effects. The trick is keeping the differential in installed bases sufficiently modest so that the larger firm chooses to make its product compatible. The burden of ensuring the differential is kept low falls on the smaller firm, whose incentive for compatibility is much greater, and is reflected in aggressive pricing when the installed base differential becomes too large. Compatible products can then be stable and, as a result, both firms can have significant market shares in the long-run.[20]

[18] For the case of a very strong network effect ($\theta = 4$), which is not shown, endogenous compatibility does not matter as, even when $\lambda = 1$, products are incompatible and a bimodal distribution arises.

[19] A bimodal distribution never occurs for $\delta > .15$.

[20] This is a useful point to contrast our model and results with another body of work

Result 3 (Avoidance of Market Dominance) *Having the option of product compatibility can result in a market achieving a relatively symmetric outcome when, in the absence of that option, there would have been market dominance.*

7 Impact of Network Effects

Consistent with previous work, Figure 7 shows that market concentration is higher when network effects are stronger. Where there is a big increase in concentration from a stronger network effect - such as when θ rises from 2 to 3 for $(\delta, \lambda) = (.08, 0)$ and from 3 to 4 for $(\delta, \lambda) = (.08, 1)$ - it is because the equilibrium is switching to a Tipping equilibrium. Not surprising, stronger network effects result in higher concentration.

To gain some insight into how network effects and product compatibility interact, let us use Figure 8 to explore how the equilibrium policy function changes with respect θ. When network effects are weak ($\theta = 1$), there is a mild Compatibility equilibrium with a large compatibility region. As θ is increased, the dual trenches deepen and the compatibility region shrinks because the larger firm increasingly prefers a monopolization strategy rather than enhancing current demand through compatibility. As a result, it is all the more important for the smaller firm to prevent the gap in bases from widening too much, which induces it to price lower along the border of the compatibility region; hence, the trench deepens as θ rises. While the limit distribution does become more dispersed as the network effect rises - indicating that it is more likely that installed bases will be highly asymmetric - the effect is relatively weak. The real impact of a stronger

dealing with endogenous market dominance. Exemplified by Budd, Harris, and Vickers (1993), this literature identifies the tendency for dynamic competition to move the industry in a joint profit-maximizing direction. As the approach uses asymptotic expansion to approximate the value and policy functions around the special cases of infinite discounting and infinite uncertainty, it has been noted that additional effects may operate when away from these special cases and could well dominate the joint-profit effect. Furthermore, and perhaps most important, firms' prices in these other models do not affect the future evolution of the state of the industry; that is, price is a static control variable while investment is a dynamic control variable. In our setting, in contrast, price and compatibility decisions affect both current profit and the future evolution of the state. Finally, the state in our model is *not* moving in the direction of maximizing joint profit. This we verified numerically but it is most easily seen by noting that joint profits are maximized by one of the firms being priced out of the market so a monopoly prevails. To the extent that endogenous compatibility helps to avoid market dominance, dynamic competition does not maximize joint profits.

network effect is to induce the smaller firm to price more aggressively in order to ensure that products are compatible. This property highlights the role of the compatibility dynamic in that market dominance is only mildly increasing when the network effect is strengthened.

Note that the compatibility region in Figure 8 shrinks as θ increases. Nevertheless, it is not always the case that products are less likely to be compatible when the network effect is stronger. Table 2 reports the long-run probability that products are compatible using the limit distribution. When the customer turnover rate is relatively low, the frequency with which products are compatible is generally lower when the network effect is stronger; for example, when $\delta = .06$, the frequency with which products are compatible falls from 96% of the time to never as θ increases from 1 to 3. However, when the customer turnover rate is modestly high, products are *more* likely to be compatible when the network effect is stronger. For example, when $\delta = .14$, the frequency with which products are compatible rises from 68% to 83% as θ increases from 1 to 4, in spite of the fact that the set of states for which products are compatible is shrinking.

The resolution of this riddle lies in the policy functions. Because the network effect is stronger, the smaller firm is more aggressive in keeping the installed base differential relatively low because it fears the larger firm may shift to a monopolization strategy. This aggressive pricing behavior makes it more likely that the state remains in the compatibility region when $\theta = 4$ than when $\theta = 1$ even though the region is smaller. Note that this surprising comparative static holds as long as the equilibrium is Compatibility. But for a Compatibility equilibrium to persist as θ is increased (and not transform into a Tipping equilibrium), it is necessary that the customer turnover rate not be too low. That is why δ must be sufficiently high for a stronger network effect to increase the frequency of compatible products.

Result 4 (Strength of Network Effect) *A stronger network effect increases market concentration. A stronger network effect decreases the frequency of compatible products when the customer turnover rate is low and increases the frequency of compatible products when the customer turnover rate is high.*

8 Welfare Effects of Compatibility

In this section we consider the welfare effects of various policies designed for a market with network effects. The first policy is one of laissez faire or, as we refer to it below, *endogenous compatibility*. The results for a policy of endogenous compatibility are derived by running the model that we have thus far been analyzing. A second policy is *mandatory compatibility* so that firms optimize with respect to price only, while we impose the condition that products are compatible. The final policy is *prohibited compatibility* which has firms optimize with respect to price only, given that products are incompatible.[21] Results are reported for when products are, in principle, fully compatible ($\lambda = 1$).[22]

In measuring the performance of these policies, we use the expected net present value of producer surplus, consumer surplus, and total welfare (the sum of the previous two measures), starting with zero installed bases. These measures are appropriate if one imagines weighing various policy options at the inception of a product market, which is typically when such discussions occur.[23] To derive producer surplus at a market's initiation, we just need to evaluate a firm's value function at $(b_1, b_2) = (0, 0)$. In the Online Appendix, we describe how we calculated the net present value of consumer surplus.

As reported in the upper panel of Table 3, firms have an unambiguous ranking of policies. Firms most prefer that products are mandated to be compatible, and least prefer a policy that prevents products from being compatible. As we have previously discussed, firms benefit from having compatible products because of the market expansion effect, and since we are evaluating these policies when firms are symmetric (both have zero installed bases), the business stealing effect is less relevant. This factor, by itself, would explain the ranking of policies. But there is a second factor which is that these policies have different implications regarding the intensity of price competition. When products are required to be compatible, there is no basis for aggressive pricing so as to dominate the market. And when products are prohibited from being compatible, there is no basis for the mild price competition that arises when products are compatible. Thus, pricing behavior is, on average, increasingly intense as we move from a policy of mandatory compatibility to one of endogenous compatibility to one of prohibited compatibility.[24] Firms then most prefer mandatory compatibility since it makes their product more attractive to consumers and it reduces

[21]Results are derived for this case by running the original model and setting $\lambda = 0$.

[22]Qualitatively similar results hold for the case of partial compatibility ($\lambda = .5$).

[23]Similar conclusions are reached when we used average profit, consumer surplus, and welfare based on the limit distribution, though there are some differences when network effects are strong ($\theta = .4$) and customer turnover is not high, $\delta \in \{.06, .08, .10\}$.

[24]For the limit distribution, we verified that average price is highest under mandatory compatibility and lowest under prohibited compatibility, when $\theta \in \{1, 2, 3\}$. When $\theta = 4$, the relative intensity of price competition with these three policy regimes depends on δ.

dynamic price competition.

In contrast to firms, the ranking of policies by consumers depends on the strength of network effects; see the middle panel of Table 3. When network effects are modest, $\theta \in \{1, 2\}$, consumers most prefer a policy of endogenous compatibility. Policies of mandatory and endogenous compatibility produce a similar frequency of compatible products; as shown in Table 2, the frequency of compatible products is relatively high under endogenous compatibility. But a policy of endogenous compatibility has an advantage in that there are episodes in which the smaller firm prices really low in order to keep the state in the compatibility region; that incentive is absent under mandatory compatibility. Thus, consumers desire a laissez faire policy which has firms determining whether their products are compatible.

When instead network effects are strong, consumers can prefer a policy of prohibited compatibility. When $\theta = 4$ and customer turnover is low, consumers are indifferent between endogenous and prohibited compatibility since, in either case, products are incompatible (because a Tipping equilibrium occurs under the endogenous compatibility regime). When instead customer turnover is modest or high, consumers strictly prefer a policy that prohibits compatible products. For those values of δ, endogenous compatibility results in a Compatibility equilibrium. When firms' products are required to be incompatible, price competition is made much more intense because network effects are strong. The gain in surplus from lower prices is sufficient to offset the loss of value from product incompatibility.

In terms of total welfare, the lower panel of Table 3 shows that a laissez faire policy is generally preferred except for some cases when network effects are strong. However, that conclusion could well depend on the particular parameter configurations which result in consumer surplus being large relative to producer surplus. More robust is the finding that firms prefer that compatibility be made mandatory, and consumers generally prefer a policy of non-intervention. Thus, we can expect industry and consumer lobbyists to be on opposing sides of a policy debate.

Result 5 (Welfare Effects of Compatibility) *When network effects are modest, firms prefer a policy of mandatory compatibility and consumers prefer a policy of endogenous compatibility. When network effects are strong, firms continue to prefer a policy of mandatory compatibility while consumers prefer a policy of prohibited compatibility (strictly so when customer turnover is not low).*

9 Concluding Remarks

The main contribution of this paper is identifying the compatibility dynamic which can prevent market dominance in markets with network effects. When firms have comparably-sized installed bases, they choose to make their products compatible in order to expand market size. This occurs at a cost to the firm with the larger installed base since its quality advantage over the other firm is diminished when products are compatible. However, as long as installed bases are sufficiently similar in size, the reduction in relative quality is small relative to the rise in absolute quality. The challenge to compatibility persisting over time is that, due to the randomness in demand and customer turnover, the differential in firms' installed bases could grow to the point that the larger firm chooses to pursue a dominance strategy and thus makes its product incompatible. However, there are strong forces preventing a slight differential from growing into a large one. When firms' installed bases near the point that the larger firm would make its product incompatible, the smaller firm prices aggressively in order to increase its installed base. Thus, strategic pricing keeps the installed base differential from expanding to the point that incompatibility occurs. Compatible products are then stable. The compatibility dynamic is able to neutralize increasing returns and result in long-run market structures that are not characterized by a single dominant firm.

This research project will continue in several directions. First is to extend the model to a triopoly and explore whether market dominance can be avoided even when market size is insensitive to firms' prices. Recall for the duopoly case that a necessary condition for compatible products is that there is an outside option whose market can be eaten into. When there is a triopoly, the third firm is an outside option from the perspective of two firms, though one whose value is endogenous. This suggests that some compatibility may arise even when market size is fixed if there are more than two firms.

A second research direction is to enrich the model by allowing firms to innovate. Prior to deciding on compatibility and price, each firm invests in R&D; the outcome of which is stochastic and affects the intrinsic quality of the good. Does the option of product compatibility reduce innovation because a firm can free ride as long as products are compatible? Does innovation offset the compatibility dynamic and allow increasing returns to flourish? Is market dominance more likely when firms can innovate? These are some of questions that will be addressed.

Finally, let us remind the reader that a strong assumption in our model is that consumers have static expectations in that they choose the product with the highest current net surplus and thus presume that installed bases will not change in the future. An alternative specification is to assume consumers have rational expectations; their beliefs being based upon the equilibrium-induced distribution over future installed bases. This modification means a far more complex model as now consumer behavior must be solved dynamically along with firms' behavior. We leave this challenging task to future research.

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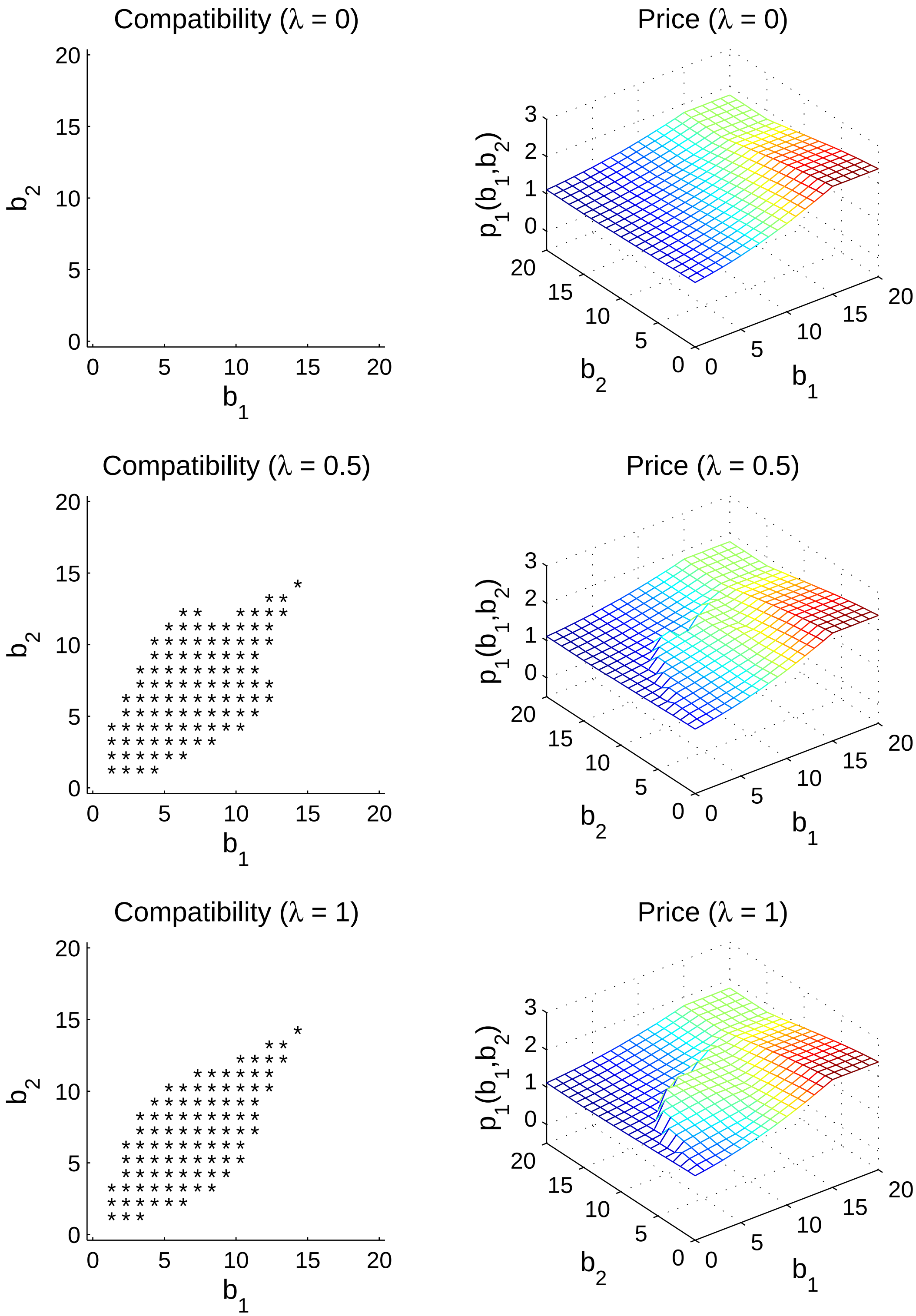

Figure 1: Static equilibria. Compatibility (a $*$ indicates compatible products) and price. $\theta = 3$, $\lambda \in \{0, .5, 1\}$.

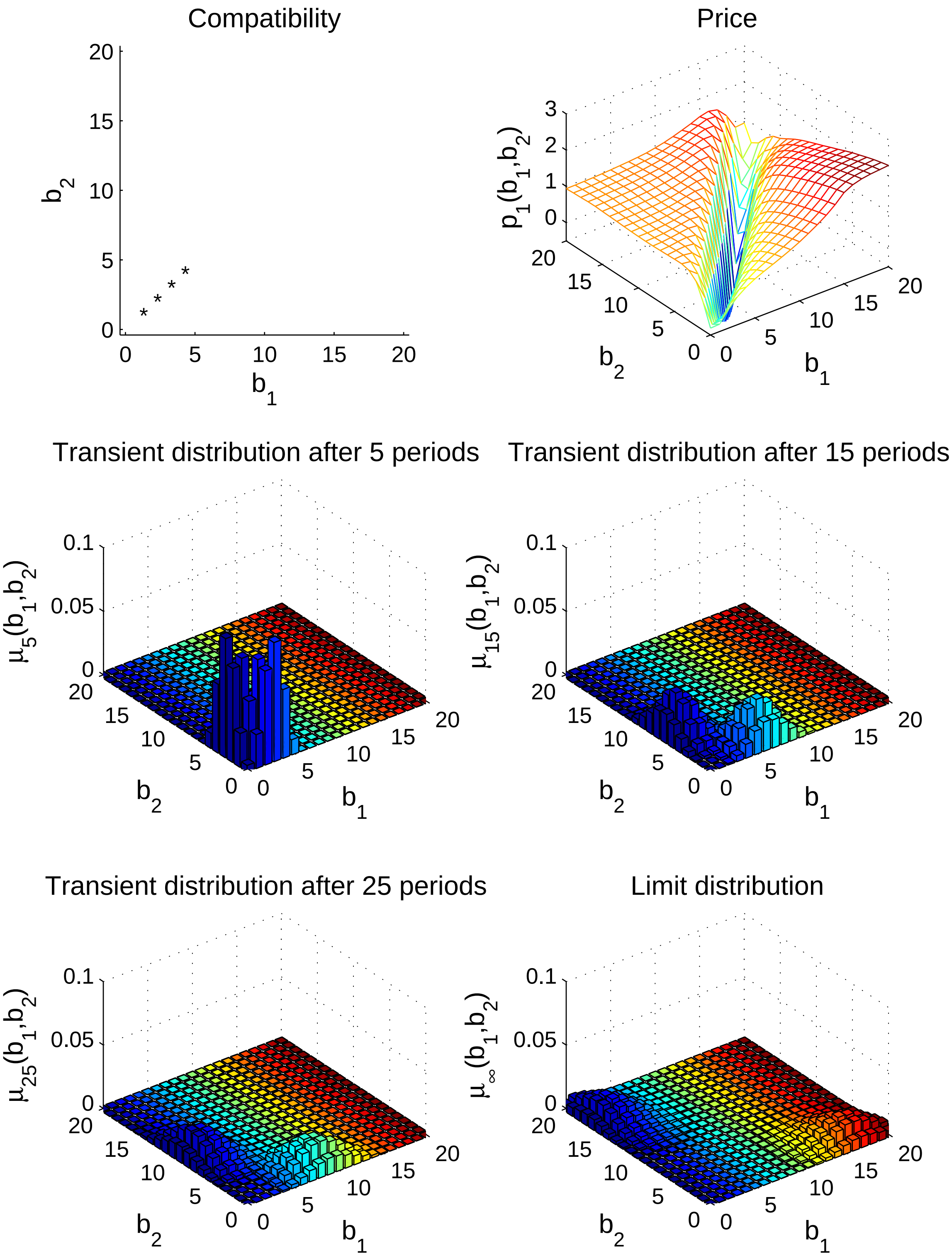

Figure 2: Tipping equilibrium. Compatibility (a $*$ indicates compatible products) and price, transient and limit distribution. $\theta = 3$, $\lambda = 1$, $\delta = .06$.

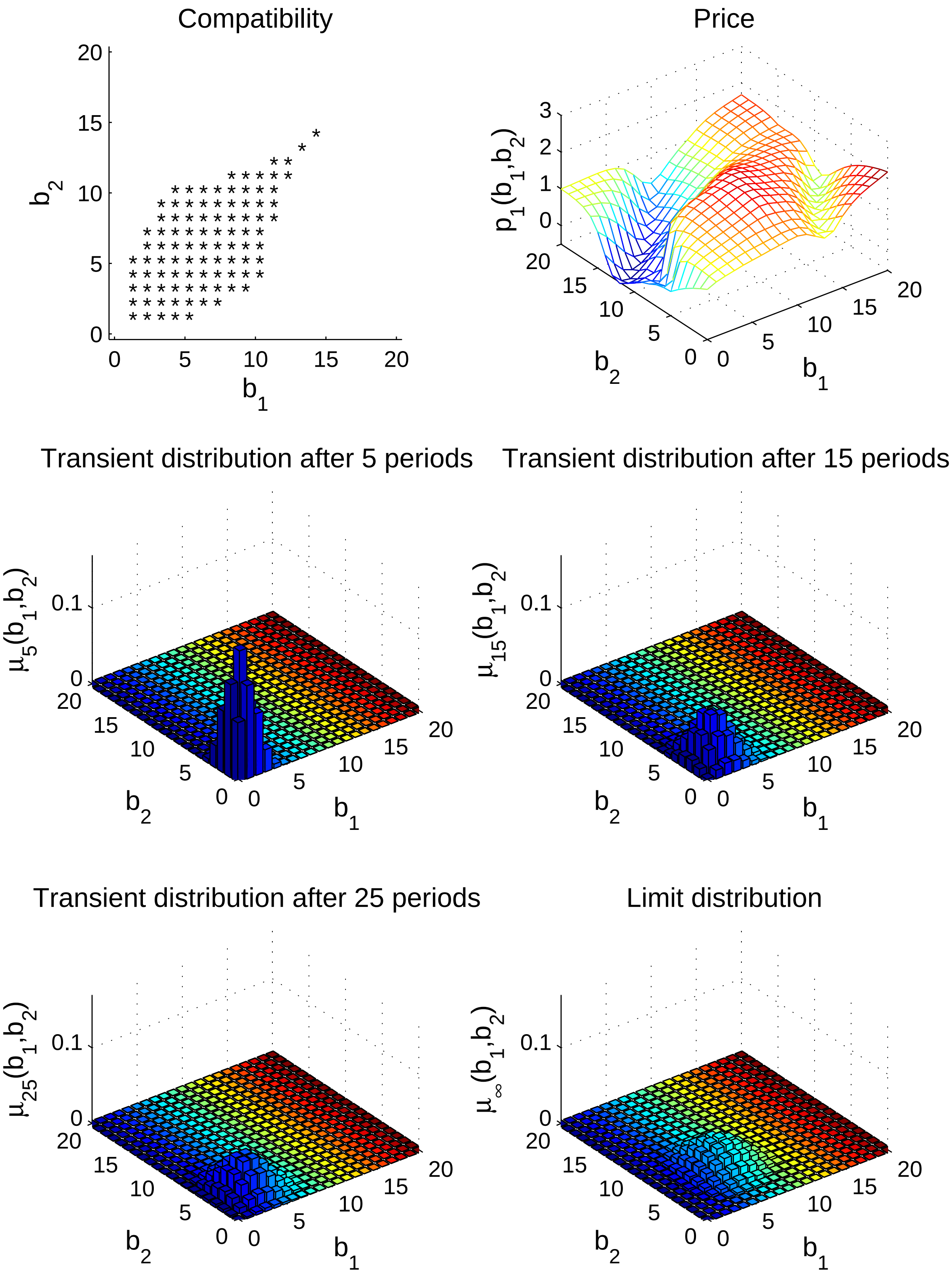

Figure 3: Compatibility equilibrium. Compatibility (a $*$ indicates compatible products) and price, transient and limit distribution. $\theta = 3$, $\lambda = 1$, $\delta = .08$.

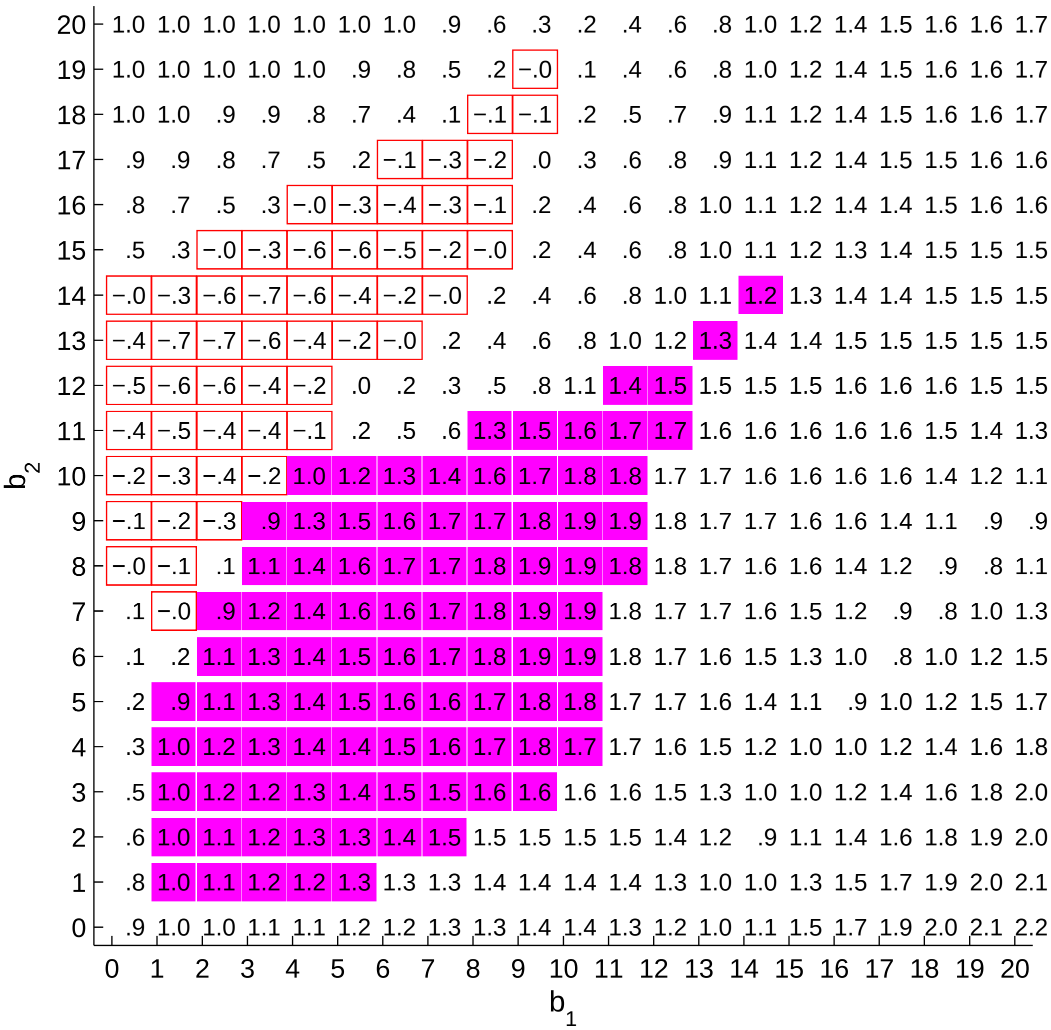

Figure 4: Compatibility equilibrium. Price. A framed box indicates a negative price. A shaded box indicates compatible products. $\theta = 3$, $\lambda = 1$, $\delta = .08$.

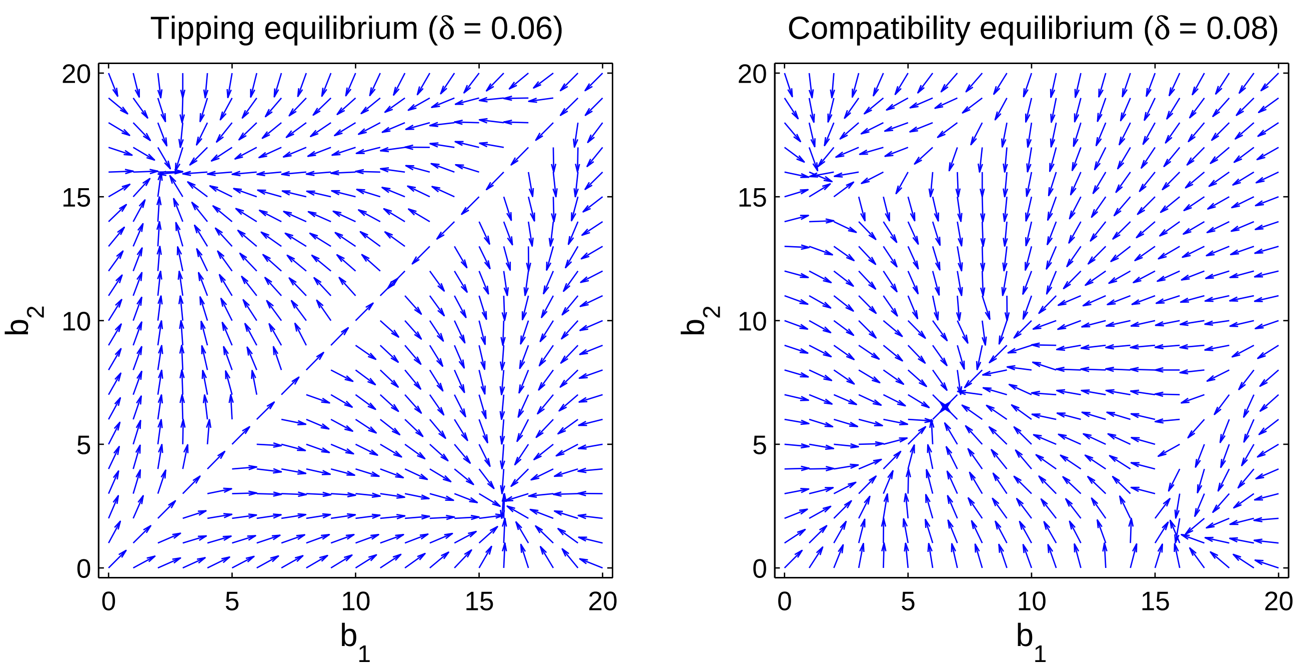

Figure 5: Resultant forces. Tipping equilibrium ($\delta = .06$, left panel) and Compatibility equilibrium ($\delta = .08$, right panel). $\theta = 3$, $\lambda = 1$.

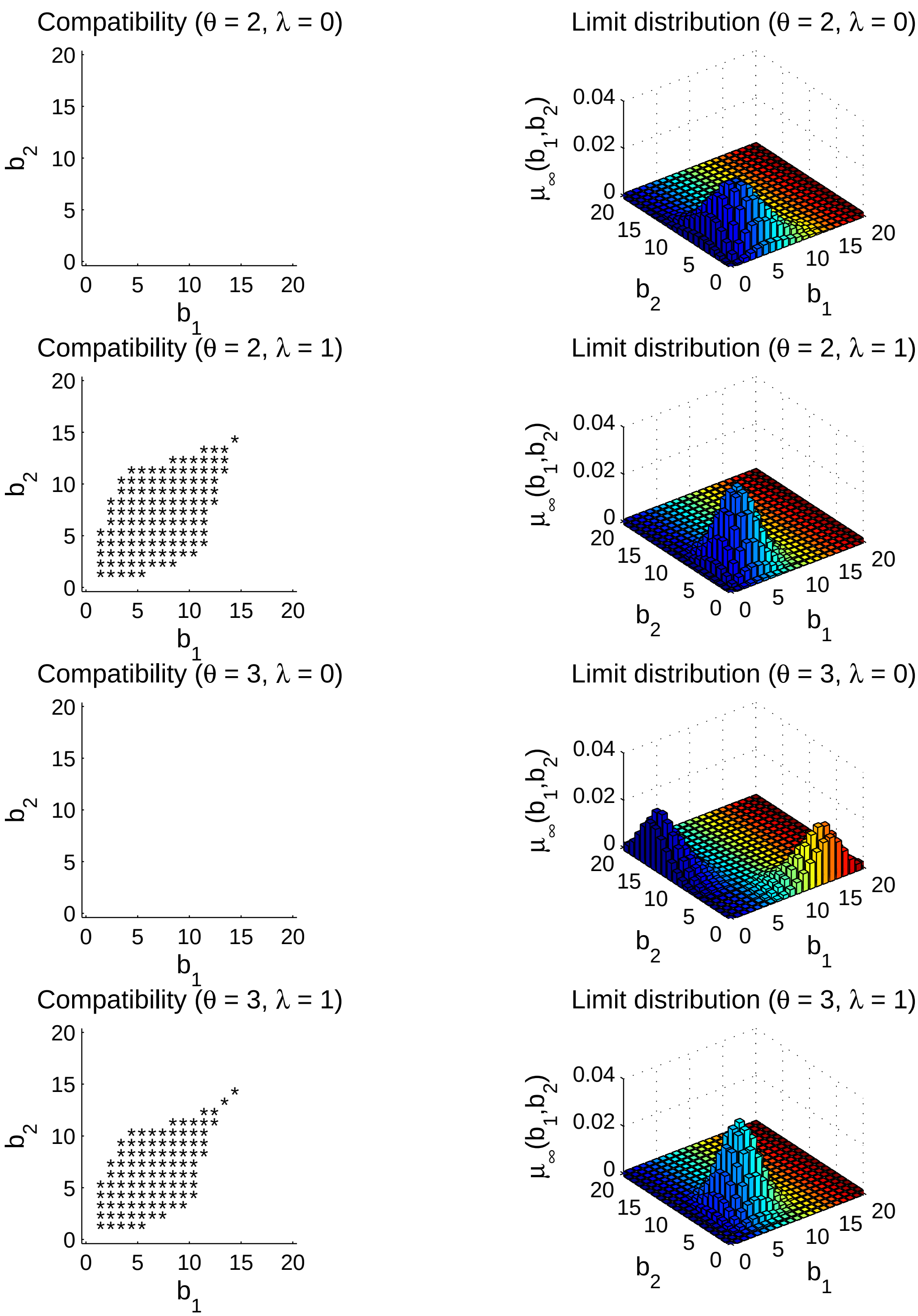

Figure 6: Compatibility (a $*$ indicates compatible products), limit distribution. $\theta \in \{2,3\}$, $\lambda \in \{0,1\}$, $\delta = .08$.

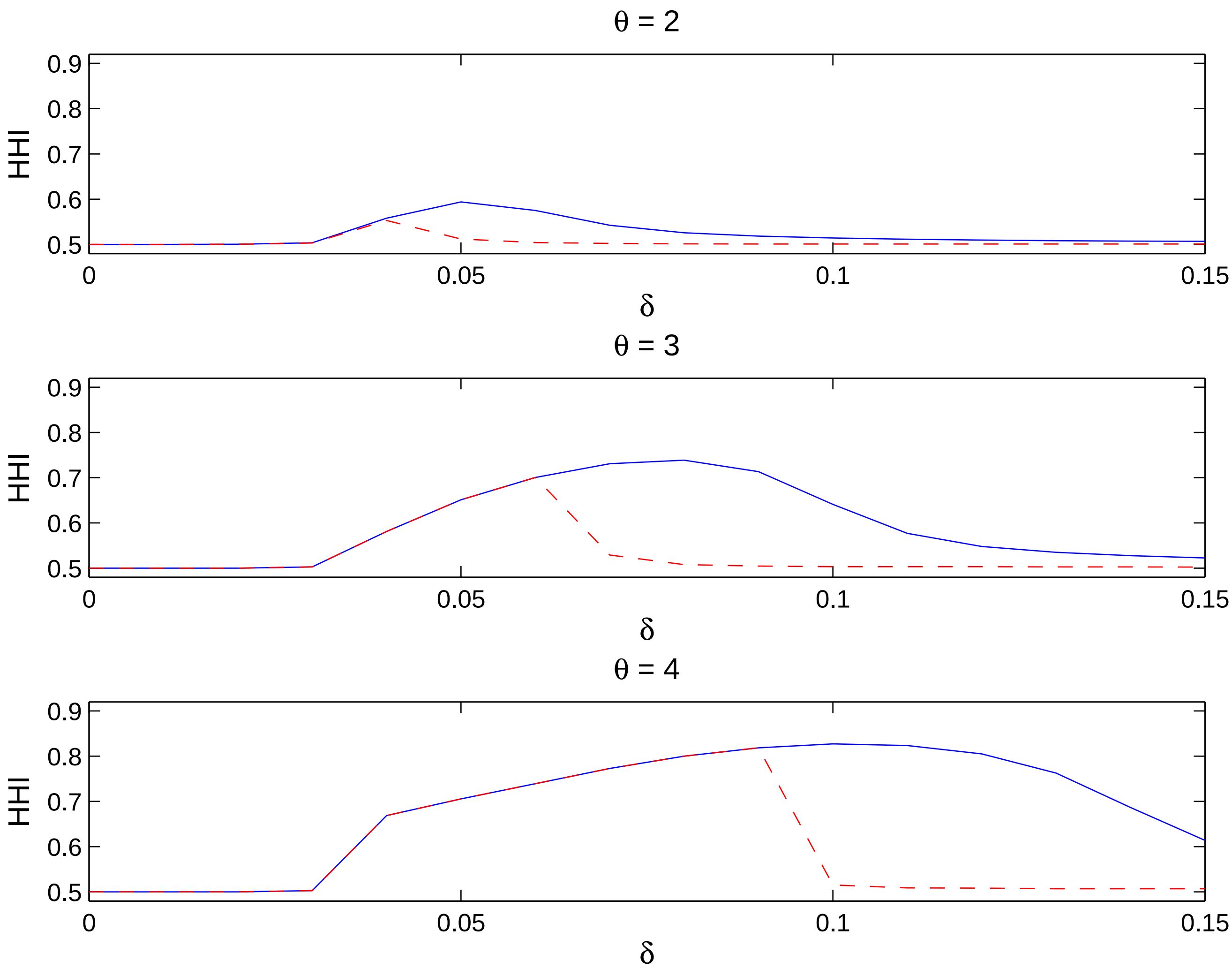

Figure 7: Herfindahl index. $\theta \in \{2, 3, 4\}$, $\lambda = 0$ (solid line) and $\lambda = 1$ (dashed line), $\delta \in [0, .15]$.

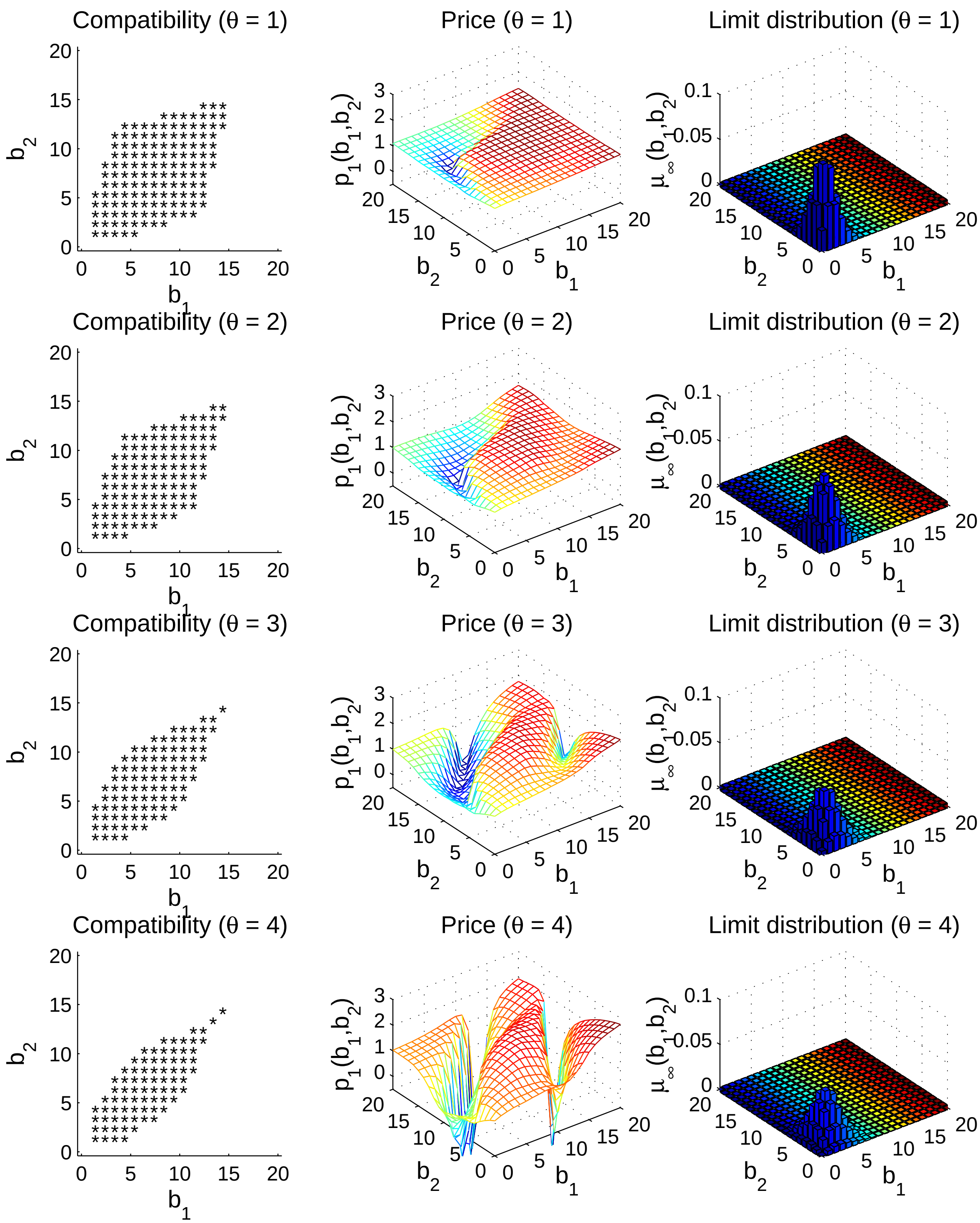

Figure 8: Compatibility (a $*$ indicates compatible products) and price, limit distribution. $\theta \in \{1, 2, 3, 4\}$, $\lambda = 1$, $\delta = .14$.

$\delta\backslash\theta$	2		3		4	
	$\lambda = 0$	$\lambda = 1$	$\lambda = 0$	$\lambda = 1$	$\lambda = 0$	$\lambda = 1$
0	(20,20)	(20,20)	(20,20)	(20,20)	(20,20)	(20,20)
.01	(20,20)	(20,20)	(20,20)	(20,20)	(20,20)	(20,20)
.02	(19,20)	(19,20)	(20,20)	(20,20)	(20,20)	(20,20)
.03	(16,16)	(16,16)	(18,18)	(18,18)	(18,20)	(18,20)
.04	(6,16)	(6,16)	(5,20)	(5,20)	(4,20)	(4,20)
.05	$\boxed{(3,15)}$	$\boxed{(9,9)}$	(3,18)	(3,18)	(2,20)	(2,20)
.06	$\boxed{(3,11)}$	$\boxed{(8,8)}$	(2,16)	(2,16)	(1,20)	(1,20)
.07	(4,5)	(6,6)	$\boxed{(1,15)}$	$\boxed{(8,8)}$	(1,17)	(1,17)
.08	(3,4)	(5,5)	$\boxed{(1,14)}$	$\boxed{(7,7)}$	(1,16)	(1,16)
.09	(3,3)	(4,4)	$\boxed{(1,12)}$	$\boxed{(5,5)}$	(0,15)	(0,15)
.10	(2,3)	(3,3)	$\boxed{(1,9)}$	$\boxed{(4,4)}$	$\boxed{(0,15)}$	$\boxed{(6,6)}$
.11	(2,2)	(3,3)	(3,3)	(4,4)	$\boxed{(0,14)}$	$\boxed{(5,5)}$
.12	(2,2)	(2,2)	(2,3)	(3,3)	$\boxed{(0,13)}$	$\boxed{(4,4)}$
.13	(2,2)	(2,2)	(2,2)	(3,3)	$\boxed{(0,11)}$	$\boxed{(4,4)}$
.14	(1,2)	(2,2)	(2,2)	(2,2)	$\boxed{(0,9)}$	$\boxed{(3,3)}$
.15	(1,1)	(2,2)	(1,2)	(2,2)	(2,2)	(3,3)

Table 1: Mode of limit distribution. If the distribution is bimodal, then just one of the modes is reported. A framed box indicates that there is a bimodal distribution under $\lambda = 0$ and a unimodal distribution under $\lambda = 1$. $\theta \in \{2, 3, 4\}$, $\lambda \in \{0, 1\}$, $\delta \in \{0, .01, \ldots, .20\}$.

$\delta \backslash \theta$	1	2	3	4
.04	.87	.03	.00	.00
.06	.96	.90	.00	.00
.08	.91	.95	.93	.00
.10	.84	.90	.94	.93
.12	.76	.81	.86	.90
.14	.68	.73	.80	.83
.16	.59	.66	.73	.72
.18	.52	.59	.66	.66

Table 2: Probability of compatible products. $\theta \in \{1, 2, 3, 4\}$, $\lambda = 1$, $\delta \in \{.04, .06, \ldots, .18\}$.

Producer surplus												
$\delta\backslash\theta$	1			2			3			4		
	EC	MC	PC	EC	MC	PC	EC	MC	PC	EC	MC	PC
.06	5.51	5.60	5.03	6.76	6.96	5.16	3.98	8.78	3.98	3.03	1.65	3.03
.08	5.35	5.45	4.99	6.29	6.51	5.17	7.66	8.02	4.47	3.12	9.76	3.12
.10	5.24	5.34	4.96	5.97	6.20	5.15	7.03	7.41	5.02	8.41	8.92	3.47
.12	5.16	5.26	4.93	5.72	5.97	5.11	6.49	6.96	5.16	7.51	8.23	4.11
.14	5.10	5.20	4.92	5.56	5.81	5.08	6.19	6.63	5.19	6.93	7.69	4.84

Consumer surplus												
$\delta\backslash\theta$	1			2			3			4		
	EC	MC	PC	EC	MC	PC	EC	MC	PC	EC	MC	PC
.06	12.60	12.45	11.79	17.48	17.20	16.27	28.79	23.77	28.58	39.94	31.63	39.94
.08	12.09	11.94	11.41	16.12	15.73	14.51	21.89	21.43	24.15	36.91	28.53	36.91
.10	11.73	11.59	11.15	14.98	14.61	13.53	19.84	19.21	18.95	25.82	25.27	33.14
.12	11.46	11.33	1.96	14.17	13.81	12.89	18.24	17.52	16.44	23.59	22.55	27.86
.14	11.25	11.13	1.81	13.54	13.23	12.43	16.94	16.27	15.12	21.55	2.45	21.68

Total welfare												
$\delta\backslash\theta$	1			2			3			4		
	EC	MC	PC	EC	MC	PC	EC	MC	PC	EC	MC	PC
.06	18.11	18.05	16.82	24.24	24.16	21.42	32.78	32.55	32.55	42.97	42.28	42.97
.08	17.44	17.39	16.40	22.41	22.24	19.68	29.55	29.44	28.62	4.03	38.29	4.03
.10	16.97	16.93	16.11	2.95	2.80	18.67	26.88	26.62	23.97	34.23	34.19	36.61
.12	16.62	16.59	15.89	19.89	19.78	18.00	24.73	24.48	21.61	31.10	3.78	31.97
.14	16.34	16.34	15.72	19.10	19.04	17.51	23.13	22.90	2.31	28.48	28.14	26.51

Table 3: Net present value of producer surplus, consumer surplus, and total welfare. Endogenous compatibility (EC, $\lambda = 1$), mandatory compatibility (MC, $\lambda = 1$), and prohibited compatibility (PC, $\lambda = 0$). $\theta \in \{1, 2, 3, 4\}$, $\delta \in \{.06, .08, \dots, .14\}$.

Avoiding Market Dominance: Product Compatibility in Markets with Network Effects
– Online Appendix –

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1 Computation and Parameterization: Technical Details

We describe our algorithm here for $N = 2$. While the state space is

$$B = \left\{ (b_1, b_2) \in \{0, 1, \dots, M\}^2 \right\},$$

with the symmetry restriction in place, it suffices to consider the reduced states space

$$B^{\diamondsuit} = \{(b_1, b_2) \in B | b_1 \geq b_2\}.$$

Moreover, it suffices to consider two compatibility outcomes in any given state $(b_1, b_2) \in B^{\diamondsuit}$, namely $d_{12}(b_1, b_2) = d_{21}(b_1, b_2) = 0$ (incompatible products) and $d_{12}(b_1, b_2) = d_{21}(b_1, b_2) = 1$ (compatible products). Hence, our goal is to determine the following value and policy functions:

$$\begin{gathered} V_1(b_1, b_2), V_2(b_1, b_2), \\ U_1(0, 0, b_1, b_2), U_2(0, 0, b_1, b_2), U_1(1, 1, b_1, b_2), U_2(1, 1, b_1, b_2), \\ d_1(b_1, b_2), d_2(b_1, b_2), \\ p_1(0, 0, b_1, b_2), p_2(0, 0, b_1, b_2), p_1(1, 1, b_1, b_2), p_2(1, 1, b_1, b_2), \end{gathered}$$

where $(b_1, b_2) \in B^{\diamondsuit}$. The value and policy functions on the full state space can be recovered as needed from the value and policy functions on the reduced state space by exploiting the symmetry restriction. For example, the value function of firm 2 in state $(b_1, b_2) \notin B^{\diamondsuit}$, $V_2(b_1, b_2)$, is identical to the value function of firm 1 in state $(b_2, b_1) \in B^{\diamondsuit}$, $V_1(b_2, b_1)$.

The algorithm is iterative. The initial guess for the value functions $\widetilde{V}_1(b_1, b_2)$ and $\widetilde{V}_2(b_1, b_2)$, where $(b_1, b_2) \in B^{\diamondsuit}$, is zero to capture the fact that the continuation values are zero in the last period of a finite-horizon game. The algorithm takes value functions $\widetilde{V}_1(b_1, b_2)$ and $\widetilde{V}_2(b_1, b_2)$ as the starting point for an iteration and generates updated value functions $V_1(b_1, b_2)$ and $V_2(b_1, b_2)$. Along the way it also computes the remaining value and policy functions.

Each iteration cycles through the reduced state space in some predetermined (but arbitrary) order. In any given state $(b_1, b_2) \in B^{\diamondsuit}$ it solves for the subgame-perfect Nash equilibrium of the two-stage game of compatibility followed by pricing decisions whilst taking as given the continuation values of both firms. Specifically, the algorithm proceeds as follows:

1. Compute the continuation values in state $(b_1, b_2) \in B^{\diamondsuit}$:

$$\overline{V}_{10}(b_1, b_2) = \sum_{(b_1', b_2') \in B} \widetilde{V}_1(b_1', b_2') \Pr(b_1'|b_1, 0) \Pr(b_2'|b_2, 0),$$
$$\overline{V}_{11}(b_1, b_2) = \sum_{(b_1', b_2') \in B} \widetilde{V}_1(b_1', b_2') \Pr(b_1'|b_1, 1) \Pr(b_2'|b_2, 0),$$
$$\overline{V}_{12}(b_1, b_2) = \sum_{(b_1', b_2') \in B} \widetilde{V}_1(b_1', b_2') \Pr(b_1'|b_1, 0) \Pr(b_2'|b_2, 1),$$
$$\overline{V}_{20}(b_1, b_2) = \sum_{(b_1', b_2') \in B} \widetilde{V}_2(b_1', b_2') \Pr(b_1'|b_1, 0) \Pr(b_2'|b_2, 0),$$
$$\overline{V}_{21}(b_1, b_2) = \sum_{(b_1', b_2') \in B} \widetilde{V}_2(b_1', b_2') \Pr(b_1'|b_1, 1) \Pr(b_2'|b_2, 0),$$
$$\overline{V}_{22}(b_1, b_2) = \sum_{(b_1', b_2') \in B} \widetilde{V}_2(b_1', b_2') \Pr(b_1'|b_1, 0) \Pr(b_2'|b_2, 1).$$

2. Assume first $d_{12}(b_1, b_2) = d_{21}(b_1, b_2) = 0$ and obtain pricing decisions given incompatible products, $p_1(0, 0, b_1, b_2)$ and $p_2(0, 0, b_1, b_2)$, in state $(b_1, b_2) \in B^{\diamondsuit}$ by solving the following system of first-order conditions for p_1 and p_2:

$$\begin{aligned}
&-\phi_1(\cdot)(1 - \phi_1(\cdot)) \left(p_1 + \beta \overline{V}_{11}(b_1, b_2)\right) + \phi_1(\cdot) \\
&+\beta\phi_1(\cdot) \left(\phi_0(\cdot)\overline{V}_{10}(b_1, b_2) + \phi_2(\cdot)\overline{V}_{12}(b_1, b_2)\right) = 0, \\
&-\phi_2(\cdot)(1 - \phi_2(\cdot)) \left(p_2 + \beta \overline{V}_{22}(b_1, b_2)\right) + \phi_2(\cdot) \\
&+\beta\phi_2(\cdot) \left(\phi_0(\cdot)\overline{V}_{20}(b_1, b_2) + \phi_1(\cdot)\overline{V}_{21}(b_1, b_2)\right) = 0.
\end{aligned}$$

where $\phi_0(\cdot)$, $\phi_1(\cdot)$, and $\phi_2(\cdot)$ is shorthand for $\phi_0(p_1, p_2; 0, 0, b_1, b_2)$, $\phi_1(p_1, p_2; 0, 0, b_1, b_2)$, and $\phi_2(p_1, p_2; 0, 0, b_1, b_2)$, respectively. Next assume $d_{12}(b_1, b_2) = d_{21}(b_1, b_2) = 1$ and obtain pricing decisions given compatible products, $p_1(1, 1, b_1, b_2)$ and $p_2(1, 1, b_1, b_2)$, in state $(b_1, b_2) \in B^{\diamondsuit}$ by solving the analogous system of first-order conditions for p_1 and p_2.

3. Compute the value functions given incompatible products, $U_1(0, 0, b_1, b_2)$ and $U_2(0, 0, b_1, b_2)$,

in state $(b_1, b_2) \in B^{\diamondsuit}$:

$$\begin{aligned}
U_1(0,0,b_1,b_2) &= \phi_1(p_1(\cdot),p_2(\cdot);0,0,b_1,b_2)p_1(\cdot) \\
&\quad +\beta\Big(\phi_0(p_1(\cdot),p_2(\cdot);0,0,b_1,b_2)\overline{V}_{10}(b_1,b_2) \\
&\quad +\phi_1(p_1(\cdot),p_2(\cdot);0,0,b_1,b_2)\overline{V}_{11}(b_1,b_2) \\
&\quad +\phi_2(p_1(\cdot),p_2(\cdot);0,0,b_1,b_2)\overline{V}_{12}(b_1,b_2)\Big), \\
U_2(0,0,b_1,b_2) &= \phi_2(p_1(\cdot),p_2(\cdot);0,0,b_1,b_2)p_2(\cdot) \\
&\quad +\beta\Big(\phi_0(p_1(\cdot),p_2(\cdot);0,0,b_1,b_2)\overline{V}_{20}(b_1,b_2) \\
&\quad +\phi_1(p_1(\cdot),p_2(\cdot);0,0,b_1,b_2)\overline{V}_{21}(b_1,b_2) \\
&\quad +\phi_2(p_1(\cdot),p_2(\cdot);0,0,b_1,b_2)\overline{V}_{22}(b_1,b_2)\Big),
\end{aligned}$$

where $p_1(\cdot)$ and $p_2(\cdot)$ is shorthand for $p_1(0,0,b_1,b_2)$ and $p_2(0,0,b_1,b_2)$, respectively. Compute analogously the value functions given compatible products, $U_1(1,1,b_1,b_2)$ and $U_2(1,1,b_1,b_2)$, in state $(b_1,b_2) \in B^{\diamondsuit}$.

4. Determine compatibility decisions in state $(b_1,b_2) \in B^{\diamondsuit}$:

$$d_{12}(b_1,b_2) = d_{21}(b_1,b_2)$$
$$= \begin{cases} 1 & \text{if } U_1(1,1,b_1,b_2) > U_1(0,0,b_1,b_2), U_2(1,1,b_1,b_2) > U_2(0,0,b_1,b_2), \\ 0 & \text{otherwise.} \end{cases}$$

   Note that $d_{12}(b_1,b_2) = d_{21}(b_1,b_2) = 1$ if and only if both firms strictly prefer compatible over incompatible products, meaning that neither firm is willing to deviate from compatibility and that ties are broken in favor of incompatibility.

5. Compute the value functions in state $(b_1,b_2) \in B^{\diamondsuit}$:

$$\begin{aligned}
V_1(b_1,b_2) &= U_1(d_{12}(b_1,b_2), d_{21}(b_1,b_2), b_1, b_2), \\
V_2(b_1,b_2) &= U_2(d_{12}(b_1,b_2), d_{21}(b_1,b_2), b_1, b_2).
\end{aligned}$$

Once the computations for a state are completed, the algorithm moves on to another state. After all states have been visited, the algorithm updates the current guess for the value functions by assigning $\widetilde{V}_1(b_1,b_2) \leftarrow V_1(b_1,b_2)$ and $\widetilde{V}_2(b_1,b_2) \leftarrow V_2(b_1,b_2)$, where $(b_1,b_2) \in B^{\diamondsuit}$. This completes the iteration. Our procedure is thus a Gauss-Jacobi scheme. See Judd (1998) for a comparison of Gauss-Jacobi and Gauss-Seidel schemes.

The algorithm continues to iterate until the relative change in the value and the policy functions from one iteration to the next is below a pre-specified tolerance. See Doraszelski

& Judd (2004) for a detailed discussion of stopping criteria.

2 Static Equilibrium: Business Gift Effect

In what follows we provide sufficient conditions for the business gift effect to harm the firm with the larger installed base. Let $N = 2$ and consider the effect on, say, firm 1's demand from having compatible instead of incompatible products:

$$
\begin{aligned}
& \phi_1 \left(p_1, p_2; (1,1), (b_1, b_2)\right) - \phi_1 \left(p_1, p_2; (0,0), (b_1, b_2)\right) \\
= \; & \frac{\exp\left(\theta g\left(b_1 + \lambda b_2\right) - p_1\right)}{\exp\left(v_0\right) + \exp\left(\theta g\left(b_1 + \lambda b_2\right) - p_1\right) + \exp\left(\theta g\left(b_2 + \lambda b_1\right) - p_2\right)} \\
& - \frac{\exp\left(\theta g\left(b_1\right) - p_1\right)}{\exp\left(v_0\right) + \exp\left(\theta g\left(b_1\right) - p_1\right) + \exp\left(\theta g\left(b_2\right) - p_2\right)}.
\end{aligned}
$$

Rearranging yields:

$$
\begin{aligned}
& \phi_1 \left(p_1, p_2; (1,1), (b_1, b_2)\right) - \phi_1 \left(p_1, p_2; (0,0), (b_1, b_2)\right) \\
= \; & \left[\frac{\exp\left(\theta g\left(b_1 + \lambda b_2\right) - p_1\right) + \exp\left(\theta g\left(b_2 + \lambda b_1\right) - p_2\right)}{\exp\left(v_0\right) + \exp\left(\theta g\left(b_1 + \lambda b_2\right) - p_1\right) + \exp\left(\theta g\left(b_2 + \lambda b_1\right) - p_2\right)}\right] \times \\
& \left[\frac{\exp\left(\theta g\left(b_1 + \lambda b_2\right) - p_1\right)}{\exp\left(\theta g\left(b_1 + \lambda b_2\right) - p_1\right) + \exp\left(\theta g\left(b_2 + \lambda b_1\right) - p_2\right)}\right] \\
& - \left[\frac{\exp\left(\theta g\left(b_1\right) - p_1\right) + \exp\left(\theta g\left(b_2\right) - p_2\right)}{\exp\left(v_0\right) + \exp\left(\theta g\left(b_1\right) - p_1\right) + \exp\left(\theta g\left(b_2\right) - p_2\right)}\right] \times \\
& \left[\frac{\exp\left(\theta g\left(b_1\right) - p_1\right)}{\exp\left(\theta g\left(b_1\right) - p_1\right) + \exp\left(\theta g\left(b_2\right) - p_2\right)}\right] \\
= \; & \Delta^c \left(b_1, b_2, \lambda\right) \Omega^c \left(b_1, b_2, \lambda\right) - \Delta^{in} \left(b_1, b_2\right) \Omega^{in} \left(b_1, b_2\right).
\end{aligned}
$$

$\Delta^c (b_1, b_2, \lambda)$ and $\Delta^{in} (b_1, b_2)$ measure the market size, for the case of compatible and incompatible products respectively, in terms of total demand for the two firms as a proportion of total demand including the outside option. The market expansion effect from making products compatible is then necessarily positive if

$$
\begin{aligned}
& \Delta^c \left(b_1, b_2, \lambda\right) - \Delta^{in} \left(b_1, b_2\right) \\
= \; & \frac{\exp\left(\theta g\left(b_1 + \lambda b_2\right) - p_1\right) + \exp\left(\theta g\left(b_2 + \lambda b_1\right) - p_2\right)}{\exp\left(v_0\right) + \exp\left(\theta g\left(b_1 + \lambda b_2\right) - p_1\right) + \exp\left(\theta g\left(b_2 + \lambda b_1\right) - p_2\right)} \\
& - \frac{\exp\left(\theta g\left(b_1\right) - p_1\right) + \exp\left(\theta g\left(b_2\right) - p_2\right)}{\exp\left(v_0\right) + \exp\left(\theta g\left(b_1\right) - p_1\right) + \exp\left(\theta g\left(b_2\right) - p_2\right)} > 0.
\end{aligned}
$$

This is true as long as $\lambda > 0$ and $g' > 0$.

The business gift effect concerns the impact of compatibility on each firm's share of market demand (excluding the outside option). It is measured by:

$$\begin{aligned}
&\Omega^{c}\left(b_{1},b_{2},\lambda\right)-\Omega^{in}\left(b_{1},b_{2}\right)\\
=\;&\frac{\exp\left(\theta g\left(b_{1}+\lambda b_{2}\right)-p_{1}\right)}{\exp\left(\theta g\left(b_{1}+\lambda b_{2}\right)-p_{1}\right)+\exp\left(\theta g\left(b_{2}+\lambda b_{1}\right)-p_{2}\right)}\\
&-\frac{\exp\left(\theta g\left(b_{1}\right)-p_{1}\right)}{\exp\left(\theta g\left(b_{1}\right)-p_{1}\right)+\exp\left(\theta g\left(b_{2}\right)-p_{2}\right)}.
\end{aligned}$$

Since $\Omega^{c}\left(b_{1},b_{2},0\right)=\Omega^{in}\left(b_{1},b_{2}\right)$, we have

$$\Omega^{c}\left(b_{1},b_{2},\lambda\right)=\Omega^{in}\left(b_{1},b_{2}\right)+\int_{0}^{\lambda}\left(\frac{\partial\Omega^{c}\left(b_{1},b_{2},\lambda'\right)}{\partial\lambda'}\right)d\lambda'.$$

Moreover, since

$$\begin{aligned}
\frac{\partial\Omega^{c}\left(b_{1},b_{2},\lambda\right)}{\partial\lambda}\;=\;&\frac{\exp\left(\theta g\left(b_{1}+\lambda b_{2}\right)-p_{1}\right)\exp\left(\theta g\left(b_{2}+\lambda b_{1}\right)-p_{2}\right)}{\left[\exp\left(\theta g\left(b_{1}+\lambda b_{2}\right)-p_{1}\right)+\exp\left(\theta g\left(b_{2}+\lambda b_{1}\right)-p_{2}\right)\right]^{2}}\times\\
&\theta\left[b_{2}g'\left(b_{1}+\lambda b_{2}\right)-b_{1}g'\left(b_{2}+\lambda b_{1}\right)\right],
\end{aligned}$$

we have

$$\operatorname{sign}\left\{\frac{\partial\Omega^{c}\left(b_{1},b_{2},\lambda\right)}{\partial\lambda}\right\}=\operatorname{sign}\left\{b_{2}g'\left(b_{1}+\lambda b_{2}\right)-b_{1}g'\left(b_{2}+\lambda b_{1}\right)\right\}. \tag{A1}$$

It follows that if $b_{2}g'\left(b_{1}+\lambda' b_{2}\right)-b_{1}g'\left(b_{2}+\lambda' b_{1}\right)\gtreqless 0$ for all $\lambda'\in(0,\lambda)$,then $\Omega^{c}\left(b_{1},b_{2},\lambda\right)-\Omega^{in}\left(b_{1},b_{2}\right)\gtreqless 0$.

The business gift effect is said to harm the firm with the larger installed base when its market share declines with compatibility. That is, if $b_1>b_2$, then $\Omega^{c}\left(b_{1},b_{2},\lambda\right)-\Omega^{in}\left(b_{1},b_{2}\right)<0$. Using (A1), a sufficient condition is:

$$b_{2}g'\left(b_{1}+\lambda b_{2}\right)<b_{1}g'\left(b_{2}+\lambda b_{1}\right),\quad \lambda\in(0,1). \tag{A2}$$

Since $b_1>b_2$, a sufficient condition for (A2) is:

$$g'\left(b_{2}+\lambda b_{1}\right)\geq g'\left(b_{1}+\lambda b_{2}\right),\quad \lambda\in(0,1). \tag{A3}$$

Since $b_1>b_2$ implies $b_1+\lambda b_2\geq b_2+\lambda b_1$, (A3) holds when g is linear or concave. Thus, if there is a constant or diminishing marginal effect of the installed base on a product's value, then the business gift effect harms the firm with the larger installed base and benefits the firm with the smaller installed base.

The business effect also harms the larger firm when spillovers are complete ($\lambda = 1$):

$$\begin{aligned}
& \Omega^{c}\left(b_{1}, b_{2}, 1\right)-\Omega^{in}\left(b_{1}, b_{2}\right)<0 \\
\Leftrightarrow \quad & \frac{\exp \left(\theta g\left(b_{1}+b_{2}\right)-p_{1}\right)}{\exp \left(\theta g\left(b_{1}+b_{2}\right)-p_{1}\right)+\exp \left(\theta g\left(b_{2}+b_{1}\right)-p_{2}\right)}<\frac{\exp \left(\theta g\left(b_{1}\right)-p_{1}\right)}{\exp \left(\theta g\left(b_{1}\right)-p_{1}\right)+\exp \left(\theta g\left(b_{2}\right)-p_{2}\right)} \\
\Leftrightarrow \quad & \exp \left(\theta g\left(b_{1}+b_{2}\right)-p_{1}\right) \exp \left(\theta g\left(b_{2}\right)-p_{2}\right)<\exp \left(\theta g\left(b_{1}\right)-p_{1}\right) \exp \left(\theta g\left(b_{2}+b_{1}\right)-p_{2}\right) \\
\Leftrightarrow \quad & \exp \left(\theta g\left(b_{1}+b_{2}\right)-p_{1}+\theta g\left(b_{2}\right)-p_{2}\right)<\exp \left(\theta g\left(b_{1}\right)-p_{1}+\theta g\left(b_{2}+b_{1}\right)-p_{2}\right) \\
\Leftrightarrow \quad & g\left(b_{2}\right)<g\left(b_{1}\right) \Leftrightarrow b_{2}<b_{1}
\end{aligned}$$

because $g' > 0$.

In sum, product compatibility reduces the market share of the firm with the larger installed base when either g is linear or concave and/or λ is sufficiently close to one. For the business gift effect to instead imply that the smaller firm's market share is reduced with compatible products, necessary conditions are that $\lambda \ll 1$ and g is sufficiently convex. However, we do not yet have a numerical that shows that the business gift effect can harm the smaller firm.

3 Dynamic Equilibrium: Flat and Rising Equilibria

In the main paper we focus on Tipping and Compatibility equilibria in order to showcase the primary dynamic forces of our model. Table A1 reports the type of equilibrium for an array of values for $\lambda \in \{0, .5, 1\}$, $\theta \in \{1, 2, 3, 4\}$, and $\delta \in \{0, .01, \ldots, .2\}$. As can be seen, Tipping and Compatibility equilibria arise in a part of the parameter space where compatibility matters, network effects are not weak, and the rate of customer turnover is neither too low (so that the state stays away from the bounds) nor too high (so the "investment" incentive is not weak). Outside this region, other types of equilibria arise. Below we discuss these Flat and Rising equilibria in more detail.

It is important to keep in mind that the types of equilibria, helpful as they are in understanding the range of behaviors that can occur, lie on a continuum and thus morph into each other as we change the parameter values.

A *Flat* equilibrium is a modest perturbation of a static equilibrium. Figure A1 presents an illustrative example. Not surprisingly, a Flat equilibrium arises when dynamic effects are minimal because the network effect is weak (θ is low), spillovers are absent ($\lambda = 0$), or customer turnover is high (δ is high). Note that when δ is high, there is little point for firms to compete aggressively for customers in order to build an installed base since the gains are likely to fritter away due to customer turnover; in other words, the return to investment is low when the depreciation rate is high. Given that in a Flat equilibrium a firm largely

$\lambda = 0$																					
$\theta\backslash\delta$	0	.01	.02	.03	.04	.05	.06	.07	.08	.09	.10	.11	.12	.13	.14	.15	.16	.17	.18	.19	.20
1	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
2	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R
3	R	R	R	RT	RT	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T
4	R	R	R	RT	RT	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T

$\lambda = .5$																					
$\theta\backslash\delta$	0	.01	.02	.03	.04	.05	.06	.07	.08	.09	.10	.11	.12	.13	.14	.15	.16	.17	.18	.19	.20
1	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C
2	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C
3	C	C	R	RT	RT	T	T	T	T	T	TC	TC	C	C	C	C	C	C	C	C	C
4	R	R	R	RT	RT	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T

$\lambda = 1$																					
$\theta\backslash\delta$	0	.01	.02	.03	.04	.05	.06	.07	.08	.09	.10	.11	.12	.13	.14	.15	.16	.17	.18	.19	.20
1	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C
2	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C
3	C	C	C	RT	RT	T	T	C	C	C	C	C	C	C	C	C	C	C	C	C	C
4	R	R	R	RT	RT	T	T	T	T	T	C	C	C	C	C	C	C	C	C	C	C

Table A1: Types of equilibria: Flat (F), Rising (R), Tipping (T), and Compatibility (C). Multiple entries indicate that the equilibrium morphs from one type to another. $\theta \in \{1, 2, 3, 4\}$, $\lambda \in \{0, .5, 1\}$, $\delta \in \{0, .01, \ldots, .20\}$, $v_0 = 0$.

ignores its rival's installed base when setting its own price, it is not surprising that the industry evolves towards a symmetric structure. Figure A1 illustrates.

A *Rising* equilibrium is characterized by a fairly monotonic policy function for which price is increasing in a firm's own base but relatively insensitive to its rival's base; an example is shown in Figure A2. Products are typically incompatible. This equilibrium arises when compatibility does not impact demand ($\lambda = 0$) or when the rate of customer turnover rate is very low. Again the industry evolves towards a symmetric structure.

4 Welfare: Consumer Surplus

Given our demand specification, the expected consumer surplus in state b is (Anderson, de Palma & Thisse 1992, p. 45):

$$w(b) = \ln \left(\exp(v_0) + \sum_{i=1}^{N} \exp \left(v_i + \theta g \left(b_i + \lambda \sum_{j \neq i} d_{ij} d_{ji} b_j \right) - p_i \right) \right).$$

Let $W(b)$ denote the expected net present value of consumer surplus in state b defined as

$$W(b) = E \left(\sum_{t=0}^{\infty} \beta^t w(b^t) \mid b^0 = b \right),$$

where b^t is the state in period t. Theorem 3.22 in Kulkarni (1995) shows that $W(b)$ satisfies the recursive equation

$$W(b) = w(b) + \beta \sum_{b' \in B} q_{b,b'} W(b'),$$

where $B = \left\{ (b_1, b_2, \ldots, b_N) \in \{0, 1, \ldots, M\}^N \right\}$ is the state space and $q_{b,b'}$ is the probability that next period's state is b' given that this period's state is b. This equation can be written in matrix form as

$$(I - \beta Q)W = w,$$

where W and w are column vectors and Q is a square matrix. Since Q is a stochastic matrix and $\beta \in [0, 1)$, $I - \beta Q$ is invertible and we can compute the expected net present value of consumer surplus as

$$W = (I - \beta Q)^{-1} w.$$

References

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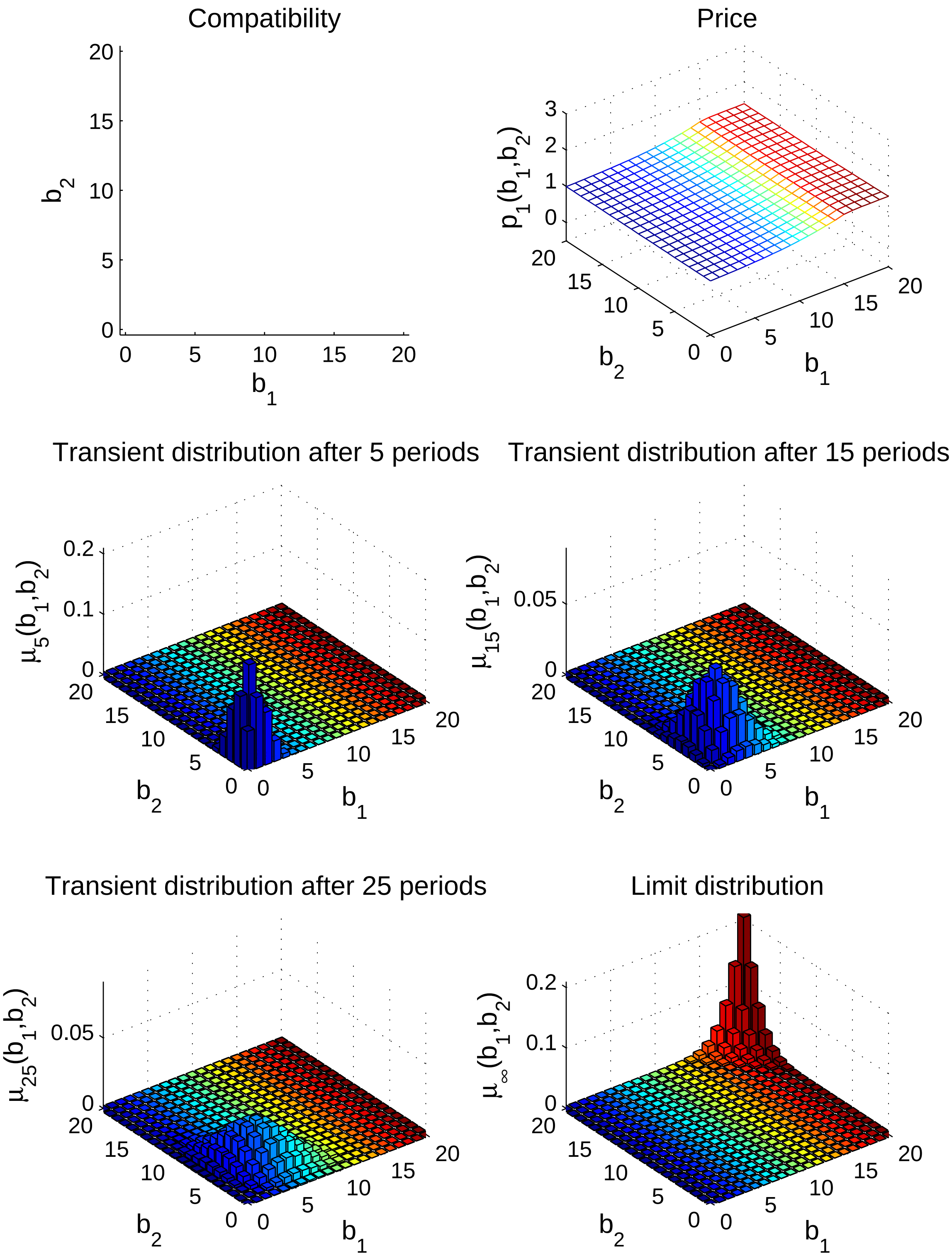

Figure A1: Flat equilibrium. Compatibility (a $*$ indicates compatible products) and price, transient and limit distribution. $\theta = 1$, $\lambda = 0$, $\delta = .01$, $v_0 = 0$.

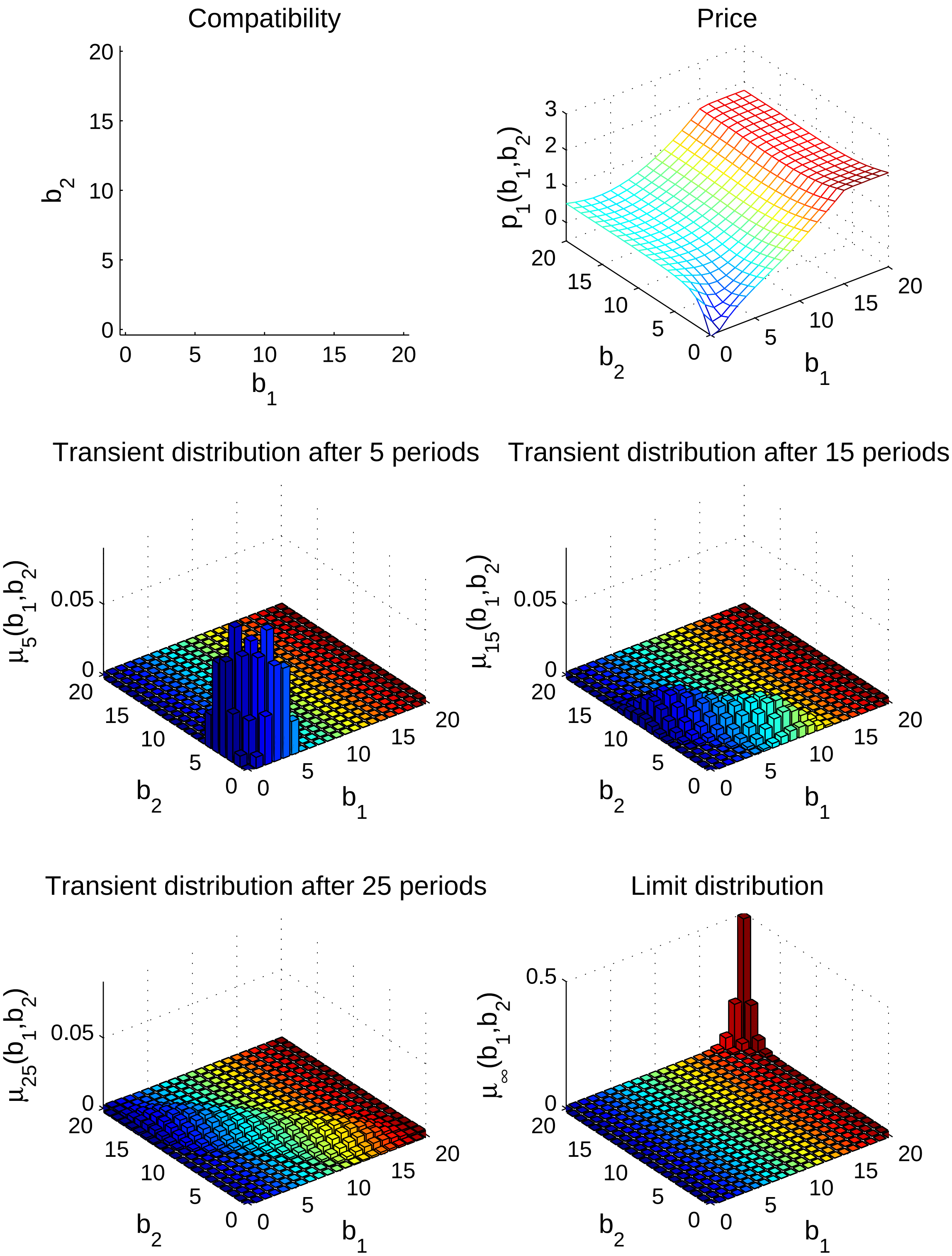

Figure A2: Rising equilibrium. Compatibility (a $*$ indicates compatible products) and price, transient and limit distribution. $\theta = 3$, $\lambda = 0$, $\delta = .01$, $v_0 = 0$.

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