What Drives Offshoring Decisions? Selection and Escape-Competition Mechanisms

Appendix — For Online Publication

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A Proofs of Lemmas and Propositions

Proof of Proposition 1. The variable $\hat{\eta}(\varphi)$ denotes the value for η that makes a firm with productivity φ indifferent between offshoring or not. In the Bellman equation (15), this implies that

$$\frac{\pi_o(\varphi)}{\delta} - \hat{\eta}(\varphi)\left[\rho\pi_n(\varphi) + f_o\right] = \pi_n(\varphi) + (1-\delta)E\left[V(\varphi,\eta')\right].$$

Solving for $\hat{\eta}(\varphi)$ we obtain

$$\hat{\eta}(\varphi) = \frac{1}{\rho\pi_n(\varphi) + f_o}\left(\frac{\pi_o(\varphi)}{\delta} - \pi_n(\varphi)\right) - \frac{1-\delta}{\rho\pi_n(\varphi) + f_o}E\left[V(\varphi,\eta')\right]. \tag{A-1}$$

Given $\hat{\eta}(\varphi)$, we can rewrite the value function as

$$V(\varphi,\eta) = \begin{cases} \frac{\pi_o(\varphi)}{\delta} - \eta\left[\rho\pi_n(\varphi) + f_o\right] & \text{if } \eta \leq \hat{\eta}(\varphi) \\ \frac{\pi_o(\varphi)}{\delta} - \hat{\eta}(\varphi)\left[\rho\pi_n(\varphi) + f_o\right] & \text{if } \eta > \hat{\eta}(\varphi). \end{cases}$$

From this expression, we can then get that

$$E\left[V(\varphi,\eta')\right] = \frac{\pi_o(\varphi)}{\delta} - E\left[\min\left\{\eta', \hat{\eta}(\varphi)\right\}\right]\left[\rho\pi_n(\varphi) + f_o\right]. \tag{A-2}$$

Plugging in equation (A-2) into equation (A-1), we find that

$$\hat{\eta}(\varphi) = z(\varphi) + (1-\delta)E\left[\min\left\{\eta', \hat{\eta}(\varphi)\right\}\right], \tag{A-3}$$

where $z(\varphi) = \frac{\pi_o(\varphi) - \pi_n(\varphi)}{\rho\pi_n(\varphi) + f_o}$. Note that $z(\varphi) \geq 0$, as $\pi_o(\varphi) \geq \pi_n(\varphi)$ for every φ.

Let us now show that $E\left[\min\left\{\eta', \hat{\eta}(\varphi)\right\}\right] = \hat{\eta}(\varphi) - \int_0^{\hat{\eta}(\varphi)} F(\eta)d\eta$:

$$\begin{aligned}
E\left[\min\left\{\eta', \hat{\eta}(\varphi)\right\}\right] =& \Pr(\eta' \leq \hat{\eta}(\varphi))E[\eta'|\eta' \leq \hat{\eta}(\varphi)] + \Pr(\eta' > \hat{\eta}(\varphi))\hat{\eta}(\varphi) \\
=& F[\hat{\eta}(\varphi)]E[\eta'|\eta' \leq \hat{\eta}(\varphi)] + [1 - F[\hat{\eta}(\varphi)]]\hat{\eta}(\varphi) \\
=& \hat{\eta}(\varphi) - F[\hat{\eta}(\varphi)]\left[\hat{\eta}(\varphi) - E[\eta'|\eta' \leq \hat{\eta}(\varphi)]\right] \\
=& \hat{\eta}(\varphi) - F[\hat{\eta}(\varphi)]\left[\int_0^{\hat{\eta}(\varphi)} (\hat{\eta}(\varphi) - \eta)\frac{dF(\eta)}{F[\hat{\eta}(\varphi)]}\right] \\
=& \hat{\eta}(\varphi) - \int_0^{\hat{\eta}(\varphi)} (\hat{\eta}(\varphi) - \eta)dF(\eta) \\
=& \hat{\eta}(\varphi) - \int_0^{\hat{\eta}(\varphi)} F(\eta)d\eta \quad \text{(by integration by parts).}
\end{aligned}$$

Substituting the previous expression into equation (A-3), we obtain that the cutoff adjustment factor solves the equation

$$\hat{\eta}(\varphi) = \frac{z(\varphi)}{\delta} - \frac{1-\delta}{\delta}\int_0^{\hat{\eta}(\varphi)} F(\eta)d\eta. \tag{A-4}$$

To show that the solution is unique, let

$$\mathcal{G}[\hat{\eta}(\varphi)] = \hat{\eta}(\varphi) + \frac{1-\delta}{\delta}\int_0^{\hat{\eta}(\varphi)} F(\eta)d\eta - \frac{z(\varphi)}{\delta}, \tag{A-5}$$

so that $\mathcal{G}[\hat{\eta}(\varphi)] = 0$ is equivalent to equation (A-4). We know that $\hat{\eta}(\varphi) \in [0, \infty)$ and from equation (A-5) we obtain that $\mathcal{G}(0) = -\frac{z(\varphi)}{\delta} \leq 0$. Note also that $\mathcal{G}[\hat{\eta}(\varphi)] \to \infty$ as $\hat{\eta}(\varphi) \to \infty$. Therefore, given that $\mathcal{G}[\hat{\eta}(\varphi)]$ is continuous, there is at least one solution for $\mathcal{G}[\hat{\eta}(\varphi)] = 0$ in the interval $[0, \infty)$. Using Leibniz's rule, we get $\mathcal{G}'[\hat{\eta}(\varphi)] = 1 + \frac{1-\delta}{\delta}F[\hat{\eta}(\varphi)] > 0$ for every $\hat{\eta}(\varphi)$. Hence, as $\mathcal{G}[\hat{\eta}(\varphi)]$ is strictly increasing, the solution is unique. □

Proof of Proposition 2. Note that $\pi_n(\varphi) = \pi_o(\varphi) = 0$ for $\varphi \leq \varphi_o$. Then, $z(\varphi) = 0$ if $\varphi \leq \varphi_o$. From equation (A-4), note that if $z(\varphi) = 0$, the equilibrium $\hat{\eta}(\varphi)$ solves the equation

$$\hat{\eta}(\varphi) = -\frac{1-\delta}{\delta}\int_0^{\hat{\eta}(\varphi)} F(\eta)d\eta.$$

As $\hat{\eta}(\varphi) \geq 0$ and $-\frac{1-\delta}{\delta}\int_0^{\hat{\eta}(\varphi)} F(\eta)d\eta \leq 0$, it follows that the solution is $\hat{\eta}(\varphi) = 0$. As η is a continuous random variable in the interval $[0, \infty)$, it must be the case that $F(0) = 0$. Therefore, $\Lambda(\varphi) = F[\hat{\eta}(\varphi)] = 0$ if $\varphi \leq \varphi_o$.

As $\Lambda(\varphi) = 0$ when $z(\varphi) = 0$, to prove that $\Lambda(\varphi) \to 0$ as $\varphi \to \infty$, it is enough to show that $z(\varphi) \to 0$ as $\varphi \to \infty$. Note that we can rewrite $z(\varphi)$ as

$$z(\varphi) = \frac{\frac{\pi_o(\varphi)}{\pi_n(\varphi)} - 1}{\rho + \frac{f_o}{\pi_n(\varphi)}}. \tag{A-6}$$

The limit of $\pi_s(\varphi)$, for $s \in \{n, o\}$, as $\varphi \to \infty$ is given by

$$\lim_{\varphi\to\infty} \pi_s(\varphi) = \lim_{\varphi\to\infty} \left[\mathbf{\Omega}\left(\frac{\varphi}{\varphi_s} e\right) - 2 + \frac{1}{\mathbf{\Omega}\left(\frac{\varphi}{\varphi_s} e\right)} \right] \gamma\psi = \infty.$$

Hence, using L'Hôpital's rule we can write the limit of $z(\varphi)$ as

$$\lim_{\varphi\to\infty} z(\varphi) = \frac{1}{\rho} \left[\lim_{\varphi\to\infty} \frac{\pi_o'(\varphi)}{\pi_n'(\varphi)} - 1 \right]. \tag{A-7}$$

We then get

$$\lim_{\varphi\to\infty} \frac{\pi_o'(\varphi)}{\pi_n'(\varphi)} = \lim_{\varphi\to\infty} \frac{1 - \frac{1}{\mathbf{\Omega}\left(\frac{\varphi}{\varphi_o} e\right)}}{1 - \frac{1}{\mathbf{\Omega}\left(\frac{\varphi}{\varphi_n} e\right)}} = 1,$$

so that $\lim_{\varphi\to\infty} z(\varphi) = 0$.

Now, $\Lambda'(\varphi) = f[\hat{\eta}(\varphi)]\hat{\eta}'(\varphi)$, where $f(\cdot)$ is the probability density function for η. For $\hat{\eta}'(\varphi)$, we derive equation (16) with respect to φ and use Leibniz's rule to get $\hat{\eta}'(\varphi) = \frac{z'(\varphi)}{\delta + (1-\delta)\Lambda(\varphi)}$. Hence,

$$\Lambda'(\varphi) = \frac{f[\hat{\eta}(\varphi)]z'(\varphi)}{\delta + (1-\delta)\Lambda(\varphi)}. \tag{A-8}$$

Given that $f[\hat{\eta}(\varphi)]$ and the denominator are both positive, it is the case that the sign of $\Lambda'(\varphi)$ is identical to the sign of $z'(\varphi)$. I focus then on $z'(\varphi)$.

Using the envelope theorem we obtain that $\pi_s'(\varphi) = \frac{w_s y_s(\varphi)}{\varphi^2}$ for $\varphi \geq \varphi_s$. Given that $\pi_s(\varphi) = \frac{\mu_s(\varphi) w_s y_s(\varphi)}{\varphi}$, it follows that $\pi_s'(\varphi) = \frac{\pi_s(\varphi)}{\varphi \mu_s(\varphi)}$. In the interval (φ_o, φ_n), $\pi_n(\varphi) = 0$ so that $z(\varphi) = \frac{\pi_o(\varphi)}{f_o}$. Thus, we have

$$z'(\varphi) = \frac{\pi_o'(\varphi)}{f_o} = \frac{\pi_o(\varphi)}{f_o \varphi \mu_o(\varphi)} > 0 \tag{A-9}$$

for $\varphi \in (\varphi_o, \varphi_n)$. Therefore, $\Lambda(\varphi)$ is strictly increasing in the interval (φ_o, φ_n), so that a maximum for $\Lambda(\varphi)$ cannot exist in that region. Given that $\Lambda(\varphi)$ is continuous, if a maximum of $\Lambda(\varphi)$ exists, it must be in the region where $\varphi \geq \varphi_n$. I will prove that this is the case.

If $\varphi \geq \varphi_n$, we get

$$z'(\varphi) = \left\{ \frac{\pi_o(\varphi)\left[\mu_o(\varphi) - \mu_n(\varphi)\right]}{\varphi\left[\rho\pi_n(\varphi) + f_o\right]^2 \left[\mu_o(\varphi)\right]^2 \left[1 + \mu_n(\varphi)\right]} \right\} \left[f_o - \rho\mu_o(\varphi)\mu_n(\varphi)\gamma\psi\right]. \tag{A-10}$$

Note that if $\varphi = \varphi_n$ (which yields $\mu_n(\varphi_n) = 0$), equation (A-10) collapses to equation (A-9). As $\mu_o(\varphi) > \mu_n(\varphi)$ for every $\varphi \in [\varphi_n, \infty)$, the first term is always positive. The second term gives the sign of $z'(\varphi)$ and in particular, it determines the value of φ that maximizes $z(\varphi)$—and hence $\Lambda(\varphi)$. Letting $\hat{\varphi}$ denote the argument that maximizes $z(\varphi)$, it follows that $\hat{\varphi}$ solves the equation

$$f_o - \rho\mu_o(\hat{\varphi})\mu_n(\hat{\varphi})\gamma\psi = 0. \tag{A-11}$$

To show that this is indeed a maximum and that is unique, note that $\mu_o(\varphi)\mu_n(\varphi)$ is strictly increasing in the interval $[\varphi_n, \infty)$ because $\mu_s'(\varphi) > 0$, for $s \in \{n, o\}$, with $\mu_o(\varphi)\mu_n(\varphi) = 0$ if $\varphi = \varphi_n$ (because $\mu_n(\varphi_n) = 0$) and $\mu_o(\varphi)\mu_n(\varphi) \to \infty$ as $\varphi \to \infty$. Hence, $z'(\varphi) > 0$ if $\varphi \in [\varphi_n, \hat{\varphi})$, and $z'(\varphi) < 0$ if $\varphi \in (\hat{\varphi}, \infty)$. Note also that given φ_n and φ_o, a lower f_o implies a lower $\hat{\varphi}$ (so that $\rho\mu_o(\hat{\varphi})\mu_n(\hat{\varphi})\gamma\psi$ is smaller). As f_o approaches zero, it follows that $\mu_n(\hat{\varphi})$ must get closer to zero; that is, $\hat{\varphi} \to \varphi_n$ from the right. The opposite happens with ρ: as ρ declines, the level of $\hat{\varphi}$ increases. □

Proof of Lemma 1. To obtain N_E note first from equation (6) that the log price of a producing firm with productivity φ and offshoring status s, for $s \in \{n, o\}$, is $\ln p_s(\varphi) = \ln \hat{p} - \mu_s(\varphi)$. Hence, the average log price of firms with offshoring status s is given by $\overline{\ln p}_s = \ln \hat{p} - \bar{\mu}_s$, where

$$\bar{\mu}_s = \int_{\varphi_s}^{\infty} \mu_s(\varphi) h_s(\varphi \mid \varphi \geq \varphi_s) d\varphi$$

is the average markup of this group of firms. Substituting the expressions for $\overline{\ln p}_n$ and $\overline{\ln p}_o$ into the overall average log price, $\overline{\ln p} = \frac{N_n}{N}\overline{\ln p}_n + \frac{N_o}{N}\overline{\ln p}_o$, we get

$$\ln \hat{p} - \overline{\ln p} = \frac{N_n}{N}\bar{\mu}_n + \frac{N_o}{N}\bar{\mu}_o.$$

Now, from equation (3) we know that $\ln \hat{p} - \overline{\ln p} = \frac{1}{\gamma N}$, which then implies that $\frac{1}{\gamma} = N_n\bar{\mu}_n + N_o\bar{\mu}_o$. Plugging in our expressions for N_n and N_o from (21) and (22) into the previous equation, we solve for N_E as

$$N_E = \frac{\delta}{\gamma\left[(1-\bar{\Gamma})(1-H_n(\varphi_n))\bar{\mu}_n + \bar{\Gamma}\bar{\mu}_o\right]}. \tag{A-12}$$

□

Proof of Lemma 2. We have to prove that $\zeta_\gamma > 0$, $\zeta_\psi > 0$, and that $\zeta_{f_o} < 0$. This is equivalent to proving that $\frac{d\varphi_o}{d\gamma} > 0$, $\frac{d\varphi_o}{d\psi} > 0$, and $\frac{d\varphi_o}{df_o} < 0$, respectively. Taking the total derivative of the free entry condition ($\bar{\pi}_E = f_E$) with respect to γ, we obtain

$$\frac{d\varphi_o}{d\gamma} = -\frac{\frac{\partial\bar{\pi}_E}{\partial\gamma}}{\frac{\partial\bar{\pi}_E}{\partial\varphi_o}},$$

with similar expressions for $\frac{d\varphi_o}{d\psi}$ and $\frac{d\varphi_o}{df_o}$. Under the sufficient condition that $\frac{f_o}{\rho}$ is large enough, it is the case that $\frac{\partial\bar{\pi}_E}{\partial\varphi_o} < 0$ (see section C). It is left to show that $\frac{\partial\bar{\pi}_E}{\partial\gamma} > 0$, $\frac{\partial\bar{\pi}_E}{\partial\psi} > 0$, and $\frac{\partial\bar{\pi}_E}{\partial f_o} < 0$.

I show first that $\frac{\partial\bar{\pi}_E}{\partial\gamma} > 0$. Using the expression for $\bar{\pi}_E$ on the left side of (27), I obtain

$$\begin{aligned}\frac{\partial\bar{\pi}_E}{\partial\gamma} =& \frac{\bar{\pi}_E}{\gamma} + \int_{\varphi_o}^{\infty}\Bigg\{\left[\delta + (1-\delta)\Lambda(\varphi)\right]\Lambda(\varphi)\left[\frac{E[\eta|\eta \leq \hat{\eta}(\varphi)]f_o}{\gamma} - (\rho\pi_n(\varphi) + f_o)\frac{\partial E[\eta|\eta \leq \hat{\eta}(\varphi)]}{\partial\gamma}\right] \\ &+ \left[\pi_o(\varphi) - \pi_n(\varphi) - \delta E[\eta|\eta \leq \hat{\eta}(\varphi)](\rho\pi_n(\varphi) + f_o)\right]\frac{\partial\Lambda(\varphi)}{\partial\gamma}\Bigg\}\frac{g(\varphi)}{\left[\delta + (1-\delta)\Lambda(\varphi)\right]^2}d\varphi. \end{aligned} \tag{A-13}$$

To obtain $\frac{\partial E[\eta|\eta\leq\hat{\eta}(\varphi)]}{\partial\gamma}$, note that using integration by parts, we can write $E[\eta|\eta \leq \hat{\eta}(\varphi)]$ as

$$E[\eta|\eta \leq \hat{\eta}(\varphi)] = \hat{\eta}(\varphi) - \frac{1}{\Lambda(\varphi)} \int_0^{\hat{\eta}(\varphi)} F(\eta)d\eta. \tag{A-14}$$

Hence, the partial derivative of equation (A-14) with respect to γ is given by

$$\frac{\partial E[\eta|\eta \leq \hat{\eta}(\varphi)]}{\partial\gamma} = \frac{1}{\Lambda(\varphi)^2}\left(\int_0^{\hat{\eta}(\varphi)} F(\eta)d\eta\right)\frac{\partial\Lambda(\varphi)}{\partial\gamma}.$$

Using equations (A-14) and (16), we rewrite the previous expression as

$$\frac{\partial E[\eta|\eta \leq \hat{\eta}(\varphi)]}{\partial\gamma} = \left[\frac{z(\varphi) - \delta E[\eta|\eta \leq \hat{\eta}(\varphi)]}{(\delta + (1-\delta)\Lambda(\varphi))\,\Lambda(\varphi)}\right]\frac{\partial\Lambda(\varphi)}{\partial\gamma}. \tag{A-15}$$

Finally, substituting equation (A-15) into equation (A-13), $\frac{\partial\bar{\pi}_E}{\partial\gamma}$ simplifies to

$$\frac{\partial\bar{\pi}_E}{\partial\gamma} = \frac{\bar{\pi}_E}{\gamma} + \int_{\varphi_o}^{\infty}\frac{1}{\gamma}\left[\frac{\Lambda(\varphi)E[\eta|\eta \leq \hat{\eta}(\varphi)]f_o}{\delta + (1-\delta)\Lambda(\varphi)}\right]g(\varphi)d\varphi, \tag{A-16}$$

which is unambiguously greater than zero (both components are positive).

For $\frac{\partial\bar{\pi}_E}{\partial\psi}$, we follow the same steps as with $\frac{\partial\bar{\pi}_E}{\partial\gamma}$ and obtain similar expressions: we only to replace γ with ψ in equations (A-13), (A-15), and (A-16). Hence, we get $\frac{\partial\bar{\pi}_E}{\partial\psi} > 0$.

Lastly, I show that $\frac{\partial\bar{\pi}_E}{\partial f_o} < 0$. We get

$$\begin{aligned}\frac{\partial\bar{\pi}_E}{\partial f_o} = \int_{\varphi_o}^{\infty}&\Bigg\{-\left[\delta + (1-\delta)\Lambda(\varphi)\right]\Lambda(\varphi)\left[E[\eta|\eta \leq \hat{\eta}(\varphi)] + (\rho\pi_n(\varphi) + f_o)\frac{\partial E[\eta|\eta \leq \hat{\eta}(\varphi)]}{\partial f_o}\right] \\ &+ \left[\pi_o(\varphi) - \pi_n(\varphi) - \delta E[\eta|\eta \leq \hat{\eta}(\varphi)](\rho\pi_n(\varphi) + f_o)\right]\frac{\partial\Lambda(\varphi)}{\partial f_o}\Bigg\}\frac{g(\varphi)}{(\delta + (1-\delta)\Lambda(\varphi))^2}d\varphi. \end{aligned} \tag{A-17}$$

As with $\frac{\partial E[\eta|\eta\leq\hat{\eta}(\varphi)]}{\partial\gamma}$, I get $\frac{\partial E[\eta|\eta\leq\hat{\eta}(\varphi)]}{\partial f_o} = \left[\frac{z(\varphi)-\delta E[\eta|\eta\leq\hat{\eta}(\varphi)]}{(\delta+(1-\delta)\Lambda(\varphi))\Lambda(\varphi)}\right]\frac{\partial\Lambda(\varphi)}{\partial f_o}$. Hence, after substituting the previous expression into equation (A-17), we get

$$\frac{\partial\bar{\pi}_E}{\partial f_o} = -\int_{\varphi_o}^{\infty}\left[\frac{\Lambda(\varphi)E[\eta|\eta \leq \hat{\eta}(\varphi)]}{\delta + (1-\delta)\Lambda(\varphi)}\right]g(\varphi)d\varphi, \tag{A-18}$$

which is strictly less than zero. □

Proof of Proposition 3. For an increase in γ and ψ, or for a decline in f_o, we have to prove that there exists a cutoff level, $\tilde{\varphi}$, such that the offshoring probability, $\Lambda(\varphi)$, declines if $\varphi < \tilde{\varphi}$ and increases if $\varphi > \tilde{\varphi}$.

With respect to γ and ψ shocks, it is enough to work with the response of $\Lambda(\varphi)$ to γ, as the derivatives with respect to ψ are similar (we only need to replace γ with ψ). Thus, we obtain $\frac{d\Lambda(\varphi)}{d\gamma}$ and derive the conditions that determine its sign.

In (28) the term in brackets is positive, and therefore, we only need to focus on $\frac{dz(\varphi)}{d\gamma}$, where $z(\varphi) = \frac{\pi_o(\varphi)-\pi_n(\varphi)}{\rho\pi_n(\varphi)+f_o}$, and $\pi_s(\varphi) = \frac{\mu_s(\varphi)^2}{1+\mu_s(\varphi)}\gamma\psi$ for $\varphi \geq \varphi_s$ (and zero otherwise). We obtain

$$\frac{dz(\varphi)}{d\gamma} = \left\{ \frac{[\mu_o(\varphi) - \mu_n(\varphi)]\psi}{[\rho\pi_n(\varphi) + f_o]^2[1 + \mu_o(\varphi)][1 + \mu_n(\varphi)]} \right\} \times \left\{ \rho\mu_o(\varphi)\mu_n(\varphi)\gamma\psi\zeta_\gamma + f_o[\mu_o(\varphi) + \mu_n(\varphi) + \mu_o(\varphi)\mu_n(\varphi)] - f_o\zeta_\gamma \right\},$$

where $\mu_s(\varphi) = 0$ if $\varphi \leq \varphi_s$, and is greater than zero otherwise, for $s \in \{n, o\}$. The first term in brackets is non-negative, and strictly positive as long as $\varphi > \varphi_o$. Then, for $\varphi > \varphi_o$, the sign of $\frac{dz(\varphi)}{d\gamma}$ is determined by the sign of

$$\Upsilon_1(\varphi) = \rho\mu_o(\varphi)\mu_n(\varphi)\gamma\psi\zeta_\gamma + f_o[\mu_o(\varphi) + \mu_n(\varphi) + \mu_o(\varphi)\mu_n(\varphi)] - f_o\zeta_\gamma.$$

By Lemma 2 ($\zeta_\gamma > 0$), we get that $\Upsilon_1(\varphi) \to -f_o\zeta_\gamma < 0$ as $\varphi \to \varphi_o$ from the right. Also $\Upsilon_1(\varphi) \to \infty$ as $\varphi \to \infty$ (because $\mu_s(\varphi) \to \infty$ as $\varphi \to \infty$ for $s \in \{n, o\}$). Therefore, given that $\Upsilon_1(\varphi)$ is continuous, there is at least one solution for $\Upsilon_1(\varphi) = 0$ in the interval (φ_o, ∞). Given that $\mu_s'(\varphi) > 0$ if $\varphi \geq \varphi_s$, for $s \in \{n, o\}$, it follows that $\Upsilon_1(\varphi)$ is strictly increasing in φ. Therefore, the solution to $\Upsilon_1(\varphi) = 0$, $\tilde{\varphi}$, is unique. Note that if $\varphi \in (\varphi_o, \tilde{\varphi})$, then $\Upsilon_1(\varphi) < 0$ and $\frac{dz(\varphi)}{d\gamma} < 0$. On the other hand, if $\varphi > \tilde{\varphi}$, then $\Upsilon_1(\varphi) > 0$ and $\frac{dz(\varphi)}{d\gamma} > 0$.

For shocks to f_o, we also get that $\text{sgn}\left(\frac{d\Lambda(\varphi)}{df_o}\right) = \text{sgn}\left(\frac{dz(\varphi)}{df_o}\right)$. We get

$$\frac{dz(\varphi)}{df_o} = \left\{ \frac{[\mu_o(\varphi) - \mu_n(\varphi)]\gamma\psi}{[\rho\pi_n(\varphi) + f_o]^2[1 + \mu_o(\varphi)][1 + \mu_n(\varphi)]f_o} \right\} \times \left\{ \rho\mu_o(\varphi)\mu_n(\varphi)\gamma\psi\zeta_{f_o} - f_o[\mu_o(\varphi) + \mu_n(\varphi) + \mu_o(\varphi)\mu_n(\varphi)] - f_o\zeta_{f_o} \right\}.$$

Similar to the previous part, the sign of $\frac{dz(\varphi)}{df_o}$ is determined by

$$\Upsilon_2(\varphi) = \rho\mu_o(\varphi)\mu_n(\varphi)\gamma\psi\zeta_{f_o} - f_o[\mu_o(\varphi) + \mu_n(\varphi) + \mu_o(\varphi)\mu_n(\varphi)] - f_o\zeta_{f_o}.$$

By Lemma 2 ($\zeta_{f_o} < 0$), $\Upsilon_2(\varphi) \to -f_o\zeta_{f_o} > 0$ as $\varphi \to \varphi_o$ from the right. Also, $\Upsilon_2(\varphi) \to -\infty$ as $\varphi \to \infty$. Given that $\Upsilon_2(\varphi)$ is continuous, there is at least one solution for $\Upsilon_2(\varphi) = 0$ in the interval (φ_o, ∞). Given that $\mu_s'(\varphi) > 0$ if $\varphi \geq \varphi_s$, for $s \in \{n, o\}$, it follows that $\Upsilon_2(\varphi)$ is strictly decreasing in φ. Therefore, the solution to $\Upsilon_2(\varphi) = 0$, $\tilde{\varphi}$, is unique. Note that if $\varphi \in (\varphi_o, \tilde{\varphi})$, then $\Upsilon_2(\varphi) > 0$ and $\frac{dz(\varphi)}{df_o} > 0$. On the other hand, if $\varphi > \tilde{\varphi}$, then $\Upsilon_2(\varphi) < 0$ and $\frac{dz(\varphi)}{df_o} < 0$. □

B The Model with Alternative Preferences

The benchmark model assumes an endogenous-markup structure that generates an inverted-U relationship between firm-level productivity and offshoring likelihood. This section shows that the same result holds if we use the quasilinear-quadratic preferences of Melitz and Ottaviano (2008), but it can only be generated in a CES setting (*i.e.*, with exogenous markups) under a very strong condition.

B.1 CES Preferences

B.1.1 Preferences, Pricing, and Production

As before, the utility function of the representative household is $U = q_h^{1-\psi} Q^{\psi}$, where q_h is the consumption of the homogeneous good, and Q is a consumption aggregator of differentiated goods. In contrast to the benchmark model, Q is now given by the constant-elasticity-of-substitution (CES) consumption aggregator

$$Q = \left(\int_{i \in \Delta} q_i^{\frac{\theta - 1}{\theta}} di \right)^{\frac{\theta}{\theta - 1}}, \tag{B-1}$$

where $\theta > 1$ denotes the elasticity of substitution between varieties, and Δ is the set of varieties available for purchase.

As in the benchmark model, the homogeneous good is the numéraire, the domestic wage is 1, and the total expenditure in differentiated goods of the representative household is $\psi < 1$. The demand of the representative household for differentiated good i is then given by

$$q_i = \frac{p_i^{-\theta}}{P^{1-\theta}} \psi, \tag{B-2}$$

where p_i is the price of good i and $P = \left[\int_{i \in \Delta} p_i^{1-\theta} di \right]^{\frac{1}{1-\theta}}$ is the price of the CES aggregator Q.

Households are located in the unit interval, and hence the market demand for differentiated good i is identical to the demand of the representative household. Assuming that the marginal cost of producer i is constant and given by c_i, CES preferences imply that this producer's profit-maximizing price is given by $p_i = (1 + \mu) c_i$, where $\mu = \frac{1}{\theta - 1}$ is the producer's proportional markup over the marginal cost. Markups are exogenous in the CES case.

As in section 3.1.2, a firm knows its productivity only after paying a sunk entry cost of f_E. The firm then can decide between using only domestic labor (L) or use also foreign labor (L^*). Recall that the foreign wage, w^*, is less than the domestic wage. In particular, the production function of a producer with productivity φ and offshoring status s is given by $y_s(\varphi) = \varphi \mathbb{L}_s$, where

$$\mathbb{L}_s = \begin{cases} L & \text{if } s = n \\ \min\left\{ \frac{L}{1-\kappa}, \frac{L^*}{\kappa \lambda} \right\} & \text{if } s = o. \end{cases}$$

As before, the price of $\mathbb{L}_s$, denoted by w_s, is either $w_n = 1$ or $w_o = 1 - \kappa + \kappa \lambda w^*$. The marginal cost of a firm with productivity φ and offshoring status s is $\frac{w_s}{\varphi}$. We assume that λ is small enough so that the marginal cost of a firm with productivity φ is always lower if the firm offshores: $\frac{w_o}{\varphi} < \frac{w_n}{\varphi}$. Therefore, the price set by a firm with productivity φ and offshoring status s is

$$p_s(\varphi) = (1+\mu)\frac{w_s}{\varphi}, \tag{B-3}$$

for $s \in \{n, o\}$.

An extra assumption of the CES model is that firms incur a fixed cost of operation, f, from selling in the market. This assumption is necessary to pin down Melitz-type cutoff productivity levels in a CES setting.[1] Using the price equation along with the demand function for each variety, we obtain that the profit function of a producing firm with productivity φ and offshoring status s is

$$\pi_s(\varphi) = \frac{1}{\theta}\left(\frac{P}{p_s(\varphi)}\right)^{\theta-1}\psi - f. \tag{B-4}$$

Hence, we define the cutoff productivity level for firms with offshoring status s as $\varphi_s = \inf\{\varphi : \pi_s(\varphi) > 0\}$ for $s \in \{n, o\}$. The zero-profit-condition for firms with offshoring status s is then given by

$$\varphi_s = \left(\frac{f\theta}{\psi}\right)^{\frac{1}{\theta-1}}\left(\frac{\theta}{\theta-1}\right)\frac{w_s}{P}. \tag{B-5}$$

A firm with productivity φ and offshoring status s does not produce if $\varphi < \varphi_s$.

Combining the expressions for φ_n and φ_o that stem from equation (B-5), we obtain

$$\varphi_o = w_o\varphi_n, \tag{B-6}$$

which is identical to equation (12) for the translog case. As before, this is one of the two expressions we need to solve for the equilibrium cutoff productivity levels. Moreover, using equation (B-5) to substitute for P, along with equation (B-3), we obtain that the equilibrium output of a firm with productivity φ and offshoring status s is

$$y_s(\varphi) = \begin{cases} 0 & \text{if } \varphi < \varphi_s \\ \left(\frac{\varphi^\theta}{\varphi_s^{\theta-1}}\right)\frac{f}{\mu w_s} & \text{if } \varphi \geq \varphi_s. \end{cases} \tag{B-7}$$

Similarly, we can rewrite the profit function for this firm as

$$\pi_s(\varphi) = \begin{cases} 0 & \text{if } \varphi < \varphi_s \\ \left[\left(\frac{\varphi}{\varphi_s}\right)^{\theta-1} - 1\right]f & \text{if } \varphi \geq \varphi_s, \end{cases} \tag{B-8}$$

for $s \in \{n, o\}$.

[1] Otherwise, firms will always find it profitable to produce a positive amount.

B.1.2 The Offshoring Decision

The offshoring decision is described as in section 3.2. Proposition 1 holds but $z(\varphi) = \frac{\pi_o(\varphi)-\pi_n(\varphi)}{\rho\pi_n(\varphi)+f_o}$ has different different properties than $z(\varphi)$ in the translog case. This creates a difference in the behavior of $\Lambda(\varphi)$, and in particular, Proposition 2 no longer holds. The following proposition describes the properties of the firm-level offshoring probability in the CES case.

Proposition B.1. ***(The probability of offshoring with CES preferences)***

There is an inverted-U relationship between φ and $\Lambda(\varphi)$ if and only if $f_o < \rho f$, with the maximum at φ_n; otherwise, $\Lambda(\varphi)$ is non-decreasing in φ. $\Lambda(\varphi) = 0$ for $\varphi \leq \varphi_o$, and $\Lambda(\varphi) \to \underline{\Lambda}$ if $\varphi \to \infty$, where $\underline{\Lambda} = F(\underline{\hat{\eta}}) > 0$ and $\underline{\hat{\eta}}$ is the unique solution to

$$\delta\underline{\hat{\eta}} + (1-\delta)\int_0^{\underline{\hat{\eta}}} F(\eta)d\eta = \frac{1}{\rho}\left[\left(\frac{\varphi_n}{\varphi_o}\right)^{\theta-1} - 1\right]. \tag{B-9}$$

Proof. If $\varphi \leq \varphi_o$, so that $\pi_n(\varphi) = \pi_o(\varphi) = 0$, then $z(\varphi) = \hat{\eta}(\varphi) = \Lambda(\varphi) = 0$. If $\varphi \to \infty$ then $\Lambda(\varphi) \to \underline{\Lambda} = F(\underline{\hat{\eta}})$, where—from equation (16)—it must be the case that $\underline{\hat{\eta}}$ solves

$$\delta\underline{\hat{\eta}} + (1-\delta)\int_0^{\underline{\hat{\eta}}} F(\eta)d\eta = \lim_{\varphi\to\infty} z(\varphi).$$

To obtain $\lim_{\varphi\to\infty} z(\varphi)$ we use again equation (A-6), but now $\pi_s(\varphi)$ is given by (B-8). Given that $\lim_{\varphi\to\infty} \pi_s(\varphi) \to \infty$, equation (A-7) continues to hold. We obtain $\pi_s'(\varphi) = \frac{(\theta-1)f\varphi^{\theta-2}}{\varphi_s^{\theta-1}}$ for $\varphi \geq \varphi_s$ and therefore

$$\lim_{\varphi\to\infty} \frac{\pi_o'(\varphi)}{\pi_n'(\varphi)} = \frac{\pi_o'(\varphi)}{\pi_n'(\varphi)} = \left(\frac{\varphi_n}{\varphi_o}\right)^{\theta-1} > 1.$$

Plugging in the previous result into equation (A-7), we obtain

$$\lim_{\varphi\to\infty} z(\varphi) = \frac{1}{\rho}\left[\left(\frac{\varphi_n}{\varphi_o}\right)^{\theta-1} - 1\right].$$

The solution for $\underline{\hat{\eta}}$ is unique because the right-hand side of (B-9) is a positive constant, and the left-hand side is strictly increasing in $\underline{\hat{\eta}}$ (taking the value of zero if $\underline{\hat{\eta}} = 0$).

For the shape of $\Lambda(\varphi)$ in the interval $[\varphi_o, \infty)$, it is enough to focus on $z'(\varphi)$ because (A-8) continues to hold. In the interval (φ_o, φ_n), $\pi_n(\varphi) = 0$ so that $z(\varphi) = \frac{\pi_o(\varphi)}{f_o}$ and thus

$$z'(\varphi) = \frac{\pi_o'(\varphi)}{f_o} = \frac{(\theta-1)f\varphi^{\theta-2}}{f_o\varphi_o^{\theta-1}} > 0.$$

Therefore, $\Lambda(\varphi)$ is strictly increasing in the interval (φ_o, φ_n). For $\varphi \geq \varphi_n$, let us first rewrite $\pi_s'(\varphi)$ as

$$\pi_s'(\varphi) = \frac{\theta-1}{\varphi}\left[\pi_s(\varphi) + f\right].$$

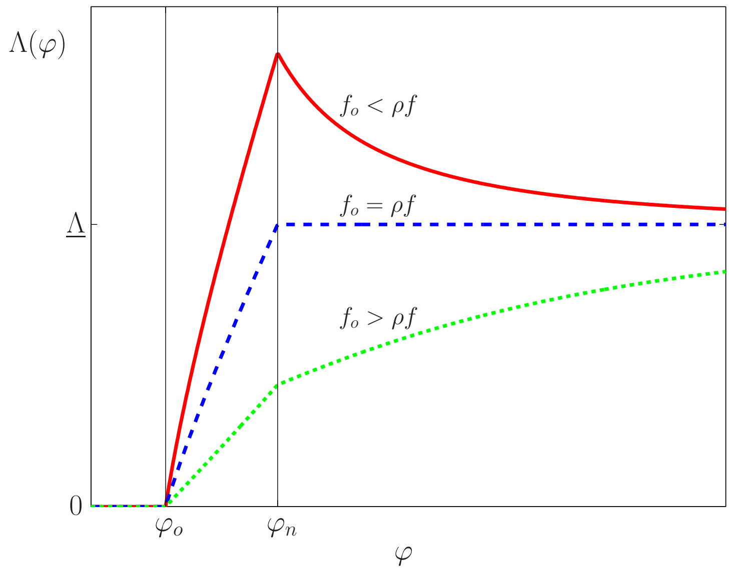

Figure B.1: Probability of offshoring in the CES case

Thus, we get

$$z'(\varphi) = \frac{(\theta - 1)[\pi_o(\varphi) - \pi_n(\varphi)]}{\varphi[\rho\pi_n(\varphi) + f_o]^2} [f_o - \rho f] . \tag{B-10}$$

The last term in brackets determines the sign of $z'(\varphi)$ and hence of $\Lambda'(\varphi)$: for $\varphi \geq \varphi_n$, $\Lambda(\varphi)$ is increasing if $f_o > \rho f$, decreasing if $f_o < \rho f$, and remains constant at $\underline{\Lambda}$ if $f_o = \rho f$ (because in that case $z(\varphi) = \frac{1}{\rho}\left[\left(\frac{\varphi_n}{\varphi_o}\right)^{\theta-1} - 1\right]$ for every $\varphi \geq \varphi_n$). Therefore, there is an inverted-U relationship between φ and $\Lambda(\varphi)$ if and only if $f_o < \rho f$, with the maximum at φ_n. □

Figure B.1 shows a graphical description of Proposition B.1. The offshoring probability increases with φ in the range $[\varphi_o, \varphi_n)$ because, as in the translog model, these firms are only subject to adjustment costs unrelated to productivity, ηf_o. The differences with respect to the translog case occur for producing non-offshoring firms (with $\varphi \geq \varphi_n$), with ρf serving as the threshold for f_o that determines whether $\Lambda(\varphi)$ is increasing or decreasing in productivity. Importantly, the model with translog preferences generates an inverted-U relationship without the need to assume any fixed costs of operation (*i.e.*, with $f = 0$).[2]

B.1.3 Distribution of Firms and Equilibrium

The expressions for $\Gamma(\varphi)$, $h_o(\varphi)$, $h_n(\varphi)$, N_n, N_o, and N follow identically as in equations (18)-(23) in section 3.3. To obtain N_E, note first that with CES preferences the aggregate price, P, can be written as

$$P = \left[N_n \bar{p}_n^{1-\theta} + N_o \bar{p}_o^{1-\theta}\right]^{\frac{1}{1-\theta}} , \tag{B-11}$$

[2]Assuming $f > 0$ in the model of section 3 will reinforce the inverted-U shape, without adding any additional insights.

where $\bar{p}_s = p_s(\bar{\varphi}_s)$ is the average price of firms with offshoring status s, with

$$\bar{\varphi}_s = \left[\int_{\varphi_s}^{\infty} \varphi^{\theta-1} h_s(\varphi \mid \varphi \geq \varphi_s) d\varphi\right]^{\frac{1}{\theta-1}} \tag{B-12}$$

denoting these firms' average productivity, for $s \in \{n, o\}$. Solving for P in equation (B-5), and plugging in the resulting expression into (B-11), along with equations (21), (22), (B-3), and (B-6), we can solve for the mass of entrants as

$$N_E = \frac{\delta\psi}{f\theta\left[(1-\bar{\Gamma})(1-H_n(\varphi_n))(\bar{\varphi}_n/\varphi_n)^{\theta-1} + \bar{\Gamma}(\bar{\varphi}_o/\varphi_o)^{\theta-1}\right]}. \tag{B-13}$$

We can also obtain market shares of non-offshoring and offshoring firms. From equation (B-2), the share of a firm with productivity φ and offshoring status s in the total expenditure on differentiated goods is given by $\sigma_s(\varphi) = \frac{p_s(\varphi)^{1-\theta}}{P^{1-\theta}}$. Aggregating over firms with the same offshoring status, we obtain that the market share of firms with status s is $\sigma_s = \frac{N_s \bar{p}_s^{1-\theta}}{P^{1-\theta}}$, with $\sigma_n + \sigma_o = 1$.

The free-entry condition in equation (27) closes the model and therefore, Definition 1 of equilibrium continues to apply. Of course, $\pi_n(\varphi)$ and $\pi_o(\varphi)$ are now given by (B-8), which then generate differences in $\hat{\eta}(\varphi)$, $\Lambda(\varphi)$, $\Gamma(\varphi)$, $h_n(\varphi)$, and $h_o(\varphi)$.

B.2 Quasilinear-Quadratic Preferences

This section shows that the inverted-U shape relationship between firm-level productivity and offshoring likelihood can also be obtained if we instead use the quasilinear-quadratic preferences of Melitz and Ottaviano (2008), which also imply a demand structure with endogenous markups. As in the translog case, we do not have to assume fixed costs of production.

The model follows almost identically the benchmark translog model, with the main differences being in the expressions for firm-level profits and markups. These are respectively given by

$$\pi_s(\varphi) = \frac{w_s^2}{4\varsigma}\left(\frac{1}{\varphi_s} - \frac{1}{\varphi}\right)^2 \tag{B-14}$$

$$\mu_s(\varphi) = \frac{1}{2}\left(\frac{\varphi}{\varphi_s} - 1\right) \tag{B-15}$$

for $\varphi \geq \varphi_s$ (and zero otherwise) and $s \in \{n, o\}$, with ς denoting a parameter that indicates the degree of substitutability between differentiated goods (a higher ς implies more substitutability). It follows that

$$\pi_s'(\varphi) = \frac{w_s^2}{2\varsigma\varphi^2}\left(\frac{1}{\varphi_s} - \frac{1}{\varphi}\right) > 0 \tag{B-16}$$

and $\mu_s'(\varphi) = \frac{1}{2\varphi_s} > 0$, so that for $\varphi \geq \varphi_s$ firm-level profits and markups are strictly increasing in productivity.

I will now show that Proposition 2 holds word-by-word for the quasilinear-quadratic preferences. The proof that $\Lambda(\varphi) = 0$ if $\varphi \leq \varphi_o$ follows exactly as in the proof of Proposition 2. To prove that $\Lambda(\varphi) \to 0$ as $\varphi \to \infty$, it is sufficient to prove that $\lim_{\varphi\to\infty} z(\varphi) = 0$. From (B-14) we can see that

$$\lim_{\varphi\to\infty} \pi_s(\varphi) = \frac{w_s^2}{4\varsigma\varphi_s^2} \tag{B-17}$$

for $s \in \{n, o\}$. Importantly, equation (12) holds (*i.e.*, $\varphi_o = w_o\varphi_n$) and thus $\lim_{\varphi\to\infty} \pi_o(\varphi) = \lim_{\varphi\to\infty} \pi_n(\varphi)$, which then implies that $\lim_{\varphi\to\infty} \frac{\pi_o(\varphi)}{\pi_n(\varphi)} = 1$. It then follows from (A-6) that $z(\varphi) \to 0$ as $\varphi \to \infty$.

To verify that $\Lambda(\varphi)$ has a single maximum in (φ_o, ∞) we focus on $z'(\varphi)$ because (A-8) also applies for this case. From (B-14), (B-15), and (B-16), it is also the case that $\pi_s'(\varphi) = \frac{\pi_s(\varphi)}{\varphi\mu_s(\varphi)}$. In the interval (φ_o, φ_n), equation (A-9) holds and hence $\Lambda(\varphi)$ is strictly increasing in that range. Given the continuity of $\Lambda(\varphi)$, the maximum of $\Lambda(\varphi)$ (if it exists) must be in the interval $[\varphi_n, \infty)$. If $\varphi \geq \varphi_n$, we obtain

$$z'(\varphi) = \left\{\frac{\pi_o(\varphi)(\varphi_n - \varphi_o)}{2\varphi\left[\rho\pi_n(\varphi) + f_o\right]^2\left[\mu_o(\varphi)\right]^2\varphi_o^2}\right\}\underbrace{\left\{f_o\left(\varphi_o + \varphi_n - \varphi\right) - \frac{\rho(\varphi - \varphi_n)}{4\varsigma\varphi_n^2}\left(1 - \frac{\varphi_o}{\varphi}\right)\right\}}_{R(\varphi)}. \tag{B-18}$$

As in the translog case, if $\varphi = \varphi_n$ equation (B-18) collapses to equation (A-9). The first term in braces is always positive because $\varphi_n > \varphi_o$. The second term in braces, $R(\varphi)$, determines the sign of $z'(\varphi)$. Note that if $\varphi = \varphi_n$, $R(\varphi_n) = f_o\varphi_o > 0$. On the other hand, it is easy to see that $R(\varphi) \to -\infty$ if $\varphi \to \infty$. Moreover,

$$R'(\varphi) = -\left[f_o + \frac{\rho(\varphi^2 - \varphi_o\varphi_n)}{4\varsigma\varphi_n^2\varphi^2}\right] < 0$$

because $\varphi \geq \varphi_n > \varphi_o$ and hence, $R(\varphi)$ is strictly decreasing in φ, for $\varphi \geq \varphi_n$. Therefore, there exists a unique level of φ, $\hat{\varphi}$, so that

$$f_o\left(\varphi_o + \varphi_n - \hat{\varphi}\right) - \frac{\rho(\hat{\varphi} - \varphi_n)}{4\varsigma\varphi_n^2}\left(1 - \frac{\varphi_o}{\hat{\varphi}}\right) = 0. \tag{B-19}$$

That is, $\hat{\varphi}$ is the level of φ that yields the unique maximum of $z(\varphi)$ and $\Lambda(\varphi)$. Note from (B-19) that as f_o declines, $\hat{\varphi}$ declines towards φ_n. The same happens when ρ increases.

To sum up, Proposition 2 holds if we instead use the quasilinear-quadratic preferences of Melitz and Ottaviano (2008). More generally, all the theoretical results obtained with translog preferences can be replicated with quasilinear-quadratic preferences.

C Existence and Uniqueness of Equilibrium

Using (19) and (20) we can rewrite a potential entrant's expected value of entry as

$$\bar{\pi}_E = \int_{\varphi_o}^{\infty} \frac{1}{\delta + (1-\delta)\Lambda(\varphi)}\left\{[1 - \Lambda(\varphi)]\pi_n(\varphi) + \Lambda(\varphi)\left[\frac{\pi_o(\varphi)}{\delta} - E[\eta|\eta \leq \hat{\eta}(\varphi)][\rho\pi_n(\varphi) + f_o]\right]\right\} g(\varphi)d\varphi, \tag{C-1}$$

where $\Lambda(\varphi) = F[\hat{\eta}(\varphi)]$, $\hat{\eta}(\varphi)$ solves (16), and $\pi_s(\varphi)$ is given by (13), for $s \in \{n, o\}$. As is usual in Melitz-type heterogeneous-firm models, we assume that $\lim_{\varphi_o \to \varphi_{\min}} \bar{\pi}_E > f_E$, where $\varphi_{\min}$ is the lowest bound of the productivity distribution. Given that $\lim_{\varphi_o \to \infty} \bar{\pi}_E = 0$ and $\bar{\pi}_E$ is continuous, an equilibrium exists. If $\bar{\pi}_E$ is strictly decreasing in φ_o, uniqueness of equilibrium is ensured. I now describe conditions that are sufficient to obtain uniqueness.

After substituting $\varphi_o = w_o \varphi_n$ into (C-1), I obtain

$$\frac{d\bar{\pi}_E}{d\varphi_o} = -\frac{\gamma\psi}{\varphi_o} \int_{\varphi_o}^{\infty} \frac{1}{\delta + (1-\delta)\Lambda(\varphi)} \left\{ \frac{[1-\Lambda(\varphi)]\mu_n(\varphi)}{1+\mu_n(\varphi)} + \frac{\Lambda(\varphi)\mu_o(\varphi)}{\delta[1+\mu_o(\varphi)]} - \frac{\Lambda(\varphi)E[\eta|\eta \leq \hat{\eta}(\varphi)]\rho\mu_n(\varphi)}{1+\mu_n(\varphi)} \right\} g(\varphi)d\varphi \tag{C-2}$$

where $\mu_s(\varphi)$ is given by (11) if $\varphi \geq \varphi_s$, and is zero otherwise, for $s \in \{n, o\}$. The novelty in (C-2) when compared to standard heterogeneous-firm models is the last term inside the braces, which accounts for the reduction in adjustments costs when the cutoff productivity levels increase: when φ_o and φ_n increase, profits of surviving offshoring and non-offshoring firms decline, but that also implies that the expected adjustment cost declines. For uniqueness of equilibrium, the decline in expected adjustment costs should not be larger than the expected decline of offshoring and non-offshoring profits; otherwise, there would be cases in which the value of entry rises in spite of increases in the cutoff levels. In the following I discuss weak conditions that ensure that $\frac{d\bar{\pi}_E}{d\varphi_o} < 0$.

Using (16), (17), (13), (A-11), and $E[\eta|\eta < \hat{\eta}(\varphi)] = \hat{\eta}(\varphi) - \int_0^{\hat{\eta}(\varphi)} F(\eta)d\eta$, we can rewrite (C-2) as

$$\frac{d\bar{\pi}_E}{d\varphi_o} = -\frac{\gamma\psi}{\delta\varphi_o} \int_{\varphi_o}^{\infty} \frac{\mu_n(\varphi)}{1+\mu_n(\varphi)} \left\{ 1 + \rho \int_0^{\hat{\eta}(\varphi)} F(\eta)d\eta - L(\varphi) \right\} g(\varphi)d\varphi, \tag{C-3}$$

where

$$L(\varphi) = \frac{\Gamma(\varphi)[\mu_o(\varphi) - \mu_n(\varphi)][1+\mu_n(\varphi)][\mu_o(\varphi)\mu_n(\varphi) - \mu_o(\hat{\varphi})\mu_n(\hat{\varphi})]}{\mu_n(\varphi)[1+\mu_o(\varphi)][\mu_n^2(\varphi) + \mu_o(\hat{\varphi})\mu_n(\hat{\varphi})(1+\mu_n(\varphi))]}, \tag{C-4}$$

with $\hat{\varphi}$ denoting the value of φ that maximizes $z(\varphi)$ and $\Lambda(\varphi)$—see the proof of Proposition 1. A sufficient (but not necessary) condition for $\frac{d\bar{\pi}_E}{d\varphi_o} < 0$ is that

$$T(\varphi) = \frac{\mu_n(\varphi)}{1+\mu_n(\varphi)} \left\{ 1 + \rho \int_0^{\hat{\eta}(\varphi)} F(\eta)d\eta - L(\varphi) \right\} > 0$$

for every $\varphi > \varphi_o$. For $\varphi \in (\varphi_o, \varphi_n]$, so that $\mu_n(\varphi) = 0$, $T(\varphi)$ collapses to

$$T(\varphi) = \frac{\Gamma(\varphi)\mu_o(\varphi)}{1+\mu_o(\varphi)} > 0.$$

For $\varphi \in (\varphi_n, \hat{\varphi}]$ we know that $\mu_o(\varphi)\mu_n(\varphi) \leq \mu_o(\hat{\varphi})\mu_n(\hat{\varphi})$ (recall that $\mu_s'(\varphi) > 0$ if $\varphi \geq \varphi_s$) and hence $L(\varphi) \leq 0$ and $T(\varphi) > 0$. For $\varphi > \hat{\varphi}$, note first that $L(\varphi) > 0$ and thus, to satisfy $T(\varphi) > 0$ we need to describe conditions such that $L(\varphi) < 1 + \rho \int_0^{\hat{\eta}(\varphi)} F(\eta)d\eta$. We can rewrite (C-4) as

$$L(\varphi) = \Gamma(\varphi) \left[\frac{1+\mu_n(\varphi)}{1+\mu_o(\varphi)} \right] \left[1 - \frac{\mu_n(\varphi)}{\mu_o(\varphi)} \right] \left[1 - \frac{\mu_o(\hat{\varphi})\mu_n(\hat{\varphi})}{\mu_o(\varphi)\mu_n(\varphi)} \right] \left[\frac{\mu_n^2(\varphi)}{\mu_n^2(\varphi) + [1+\mu_n(\varphi)]\mu_o(\hat{\varphi})\mu_n(\hat{\varphi})} \right] \frac{\mu_o^2(\varphi)}{\mu_n^2(\varphi)}. \tag{C-5}$$

where $\Gamma(\varphi)$ and all the terms in brackets are less than 1. The last term, $\frac{\mu_o^2(\varphi)}{\mu_n^2(\varphi)} > 1$, is strictly decreasing in the interval $(\hat{\varphi}, \infty)$, and approaches 1 as $\varphi \to \infty$. Given that $\lim_{\varphi\to\infty} \frac{\mu_n(\varphi)}{\mu_o(\varphi)} = 1$, it follows that $\lim_{\varphi\to\infty} L(\varphi) = 0$; thus, it is always the case that $T(\varphi) > 0$ when φ approaches either $\hat{\varphi}$ or infinity. For other values in the $(\hat{\varphi}, \infty)$ range, note from (C-5) that $L(\varphi)$ is decreasing in $\mu_o(\hat{\varphi})\mu_n(\hat{\varphi})$. Now, from (A-11) we know that

$$\mu_o(\hat{\varphi})\mu_n(\hat{\varphi}) = \frac{f_o}{\rho\gamma\psi}, \tag{C-6}$$

and therefore, $L(\varphi)$ is decreasing in f_o. It follows that we can ensure that $L(\varphi) < 1+\rho \int_0^{\hat{\eta}(\varphi)} F(\eta)d\eta$ (so that $T(\varphi) > 0$) for every $\varphi > \hat{\varphi}$ if we assume that f_o is sufficiently large. In the paper we assume that this is the case.

To sum up, the assumption of a large enough f_o is sufficient to obtain $T(\varphi) > 0$ for every $\varphi > \varphi_o$, with the last being a sufficient but not necessary condition for $\frac{d\bar{\pi}_E}{d\varphi_o} < 0$.[3]

D The Model with Trade in Final Goods

This section contains details for the extension with trade in final goods. These details are omitted in the main text of the paper to avoid repetition with respect to the benchmark model, and to preserve space.

D.1 Prices, Markups, and Cutoff Productivity Levels

Given market segmentation, constant marginal costs, and translog preferences for differentiated goods, we obtain that the prices set by a North firm with productivity φ and offshoring status s, for $s \in \{n, o\}$, in the domestic (D) and export (X) markets are

$$p_{D,s}(\varphi) = [1 + \mu_{D,s}(\varphi)]\frac{w_s}{\varphi} \qquad \text{and} \qquad p_{X,s}(\varphi,\tau) = [1 + \mu_{X,s}(\varphi,\tau)]\frac{\tau w_s}{\varphi},$$

where

$$\mu_{D,s}(\varphi) = \mathbf{\Omega}\left(\frac{\varphi\hat{p}}{w_s}e\right) - 1 \qquad \text{and} \qquad \mu_{X,s}(\varphi,\tau) = \mathbf{\Omega}\left(\frac{\varphi\hat{p}^*}{\tau w_s}e\right) - 1. \tag{D-1}$$

Given these pricing equations and each market's demand function, we obtain that this firm's profit functions from selling in the domestic and export markets are

$$\pi_{D,s}(\varphi) = \frac{\mu_{D,s}(\varphi)^2}{1+\mu_{D,s}(\varphi)}\gamma\psi \qquad \text{and} \qquad \pi_{X,s}(\varphi,\tau) = \frac{\mu_{X,s}(\varphi,\tau)^2}{1+\mu_{X,s}(\varphi,\tau)}\gamma\psi w^*. \tag{D-2}$$

[3] Importantly, note that we do not make any assumptions regarding the distributions of productivity, $G(\varphi)$, and the cutoff adjustment factor, $F(\eta)$. Numerically, with several common distributions for $G(\varphi)$ and $F(\eta)$ (Pareto, lognormal, Weibull, Frechet, exponential, and Gamma distributions), I was not able to find a single case for which $\frac{d\bar{\pi}_E}{d\varphi_o} < 0$ was not holding even with f_o approaching zero.

Similarly, the prices set by a South firm with productivity φ in the domestic and export markets are, respectively, $p^*_D(\varphi) = [1 + \mu^*_D(\varphi)]\frac{w^*}{A^*\varphi}$ and $p^*_X(\varphi,\tau) = [1 + \mu^*_X(\varphi,\tau)]\frac{\tau w^*}{A^*\varphi}$, where

$$\mu^*_D(\varphi) = \mathbf{\Omega}\left(\frac{A^*\varphi\hat{p}^*}{w^*}e\right) - 1 \qquad \text{and} \qquad \mu^*_X(\varphi,\tau) = \mathbf{\Omega}\left(\frac{A^*\varphi\hat{p}}{\tau w^*}e\right) - 1. \tag{D-3}$$

This firm's profit functions from selling in each market are then given by

$$\pi^*_D(\varphi) = \frac{\mu^*_D(\varphi)^2}{1+\mu^*_D(\varphi)}\gamma\psi w^* \qquad \text{and} \qquad \pi^*_X(\varphi,\tau) = \frac{\mu^*_X(\varphi,\tau)^2}{1+\mu^*_X(\varphi,\tau)}\gamma\psi. \tag{D-4}$$

Using the markup functions from (D-1) and (D-3), we define the cutoff productivity levels for North firms with offshoring status s selling domestically as

$$\varphi_{D,s} = \inf\{\varphi : \mu_{D,s}(\varphi) > 0\} = \frac{w_s}{\hat{p}}, \tag{D-5}$$

for $s \in \{n, o\}$. In the same way, the cutoff productivity level for South firms selling domestically is given by

$$\varphi^*_D = \inf\{\varphi : \mu^*_D(\varphi) > 0\} = \frac{w^*}{A^*\hat{p}^*}. \tag{D-6}$$

For exporting, the firm's decision to export depends on both φ and τ. There are some firms, however, whose value of φ is so low that they will never export no matter their τ draw (even if τ equals 1). Let $\varphi_{X,s}$ denote the productivity level so that no firm with productivity below $\varphi_{X,s}$ and status s will ever export. If follows that $\varphi_{X,s}$ and the South equivalent φ^*_X are given by

$$\varphi_{X,s} = \inf\{\varphi : \mu_{X,s}(\varphi,1) > 0\} = \frac{w_s}{\hat{p}^*}, \tag{D-7}$$

$$\varphi^*_X = \inf\{\varphi : \mu^*_X(\varphi,1) > 0\} = \frac{w^*}{A^*\hat{p}}. \tag{D-8}$$

Thus, a North firm with productivity φ and iceberg cost τ exports if and only if $\varphi \geq \tau\varphi_{X,s}$, while a South firm exports if and only if $\varphi \geq \tau\varphi^*_X$.

Combining the six zero-cutoff-markup conditions that stem from equations (D-5)-(D-8), we obtain

$$\varphi^*_X = w^*\varphi_{D,n}/A^*, \tag{D-9}$$

$$\varphi_{X,n} = A^*\varphi^*_D/w^*, \tag{D-10}$$

$$\varphi_{D,o} = w_o\varphi_{D,n}, \tag{D-11}$$

$$\varphi_{X,o} = w_o\varphi_{X,n}. \tag{D-12}$$

Also, we can use the zero-cutoff-markup conditions to replace $\hat{p}$ and $\hat{p}^*$ in the markups equations in (D-1) and (D-3). Thus, we can conveniently rewrite the markup in the domestic and export markets of a North firm with offshoring status s as

$$\mu_{D,s}(\varphi) = \mathbf{\Omega}\left(\frac{\varphi}{\varphi_{D,s}}e\right) - 1 \qquad \text{if } \varphi \geq \varphi_{D,s}, \tag{D-13}$$

$$\mu_{X,s}(\varphi,\tau) = \mathbf{\Omega}\left(\frac{\varphi}{\tau\varphi_{X,s}}e\right) - 1 \quad \text{if } \varphi \geq \tau\varphi_{X,s}, \tag{D-14}$$

for $s \in \{n, o\}$. Similarly, we rewrite the markup in each market of a South firm as

$$\mu_D^*(\varphi) = \mathbf{\Omega}\left(\frac{\varphi}{\varphi_D^*}e\right) - 1 \qquad \text{if } \varphi \geq \varphi_D^*, \tag{D-15}$$

$$\mu_X^*(\varphi,\tau) = \mathbf{\Omega}\left(\frac{\varphi}{\tau\varphi_X^*}e\right) - 1 \quad \text{if } \varphi \geq \tau\varphi_X^*. \tag{D-16}$$

D.2 Free-Entry Conditions and Equilibrium

As in Melitz (2003), firm enter in each country up to the point that the expected value of entry equals a sunk entry cost. In terms of the homogeneous good, the sunk cost is f_E for North firms, and f_E^* for South firms. North and South firms draw their productivity from the same productivity distribution with support $[\varphi_{\min}, \infty)$, with pdf $g(\varphi)$ and cdf $G(\varphi)$. They also draw iceberg costs from the same distribution, with pdf $m(\tau)$, cdf $M(\tau)$, and support $[1, \infty)$.

Given $h_o(\varphi,\tau)$ and $h_n(\varphi,\tau)$ in (38) and following similar steps to those in section 3.4, we obtain that the free-entry condition in the North is

$$\begin{aligned}&(1-\bar{\Gamma})\int_{\varphi_{\min}}^{\infty}\int_1^{\infty}\frac{\pi_n(\varphi,\tau)}{\delta}h_n(\varphi,\tau)d\tau d\varphi + \\ &\bar{\Gamma}\int_{\varphi_{\min}}^{\infty}\int_1^{\infty}\left\{\frac{\pi_o(\varphi,\tau)}{\delta} - E[\eta \mid \eta \leq \hat{\eta}(\varphi,\tau)]\left[\rho\pi_n(\varphi,\tau) + f_o\right]\right\}h_o(\varphi,\tau)d\tau d\varphi = f_E,\end{aligned} \tag{D-17}$$

where the left-hand side is the expected value of entry for a North potential entrant.

In the South firms never offshore, but they are also subject to the exogenous death shock with rate δ. Hence, as long as it is alive, a South firm with productivity φ makes a per-period profit of $\pi^*(\varphi,\tau) = \pi_D^*(\varphi) + \pi_X^*(\varphi,\tau)$, where

$$\pi_D^*(\varphi) = \left[\frac{\mu_D^*(\varphi)^2}{1+\mu_D^*(\varphi)}\right]\gamma\psi w^*\mathbb{1}\{\varphi \geq \varphi_D^*\} \quad \text{and} \quad \pi_X^*(\varphi,\tau) = \left[\frac{\mu_X^*(\varphi,\tau)^2}{1+\mu_X^*(\varphi,\tau)}\right]\gamma\psi\mathbb{1}\{\varphi \geq \tau\varphi_X^*\}.$$

Therefore, the free-entry condition in the South is simply given by

$$\int_{\varphi_{\min}}^{\infty}\int_1^{\infty}\frac{\pi^*(\varphi,\tau)}{\delta}g(\varphi)m(\tau)d\tau d\varphi = f_E^*, \tag{D-18}$$

where the left-hand side is the expected value of entry for a South potential entrant. We can now define the equilibrium in this model.

Definition 1. *An equilibrium is a list* $(\varphi_{D,o}, \varphi_{X,o}, \varphi_{D,n}, \varphi_{X,n}, \varphi_D^*, \varphi_X^*)$ *that solves* (D-9)*,* (D-10)*,* (D-11)*,* (D-12)*,* (D-17)*, and* (D-18)*.*

In the Melitz model with a homogeneous exporting iceberg cost (*i.e.*, with τ being identical for every firm), existence and uniqueness of equilibrium require that the pre-entry expected profits from selling domestically are larger than the pre-entry expected profits from exporting, which is ensured by assuming that exporters always sell for their domestic market. This is achieved by assuming that τ is sufficiently large so that the domestic cutoff level is below the unique exporting cutoff level. In our case with random iceberg exporting costs, we also require larger pre-entry expected profits from selling domestically than from exporting, which is achieved with a sufficiently large expected value for τ. In our case, however, there may be firms that export but do not sell for the domestic market. For example, note from (D-9) and (D-10) that if $\varphi_D^* < \varphi_X^*$, it must be the case that $\varphi_{D,s} > \varphi_{X,s}$ for $s \in \{n, o\}$, so that North firms with very low τ draws only sell for the export market (recall that a North firm with the pair (φ, τ) and status s exports if $\varphi \geq \tau\varphi_{X,s}$). In addition, and similar to the model without trade in final goods, existence of equilibrium follows under standard conditions and we ensure uniqueness by assuming that f_o is sufficiently large.

D.3 Entrants and the Composition of Firms

Given $h_o(\varphi, \tau)$ and $h_n(\varphi, \tau)$ in (38), we can obtain the fraction of North firms with offshoring status s that sell in each market. Let $\varepsilon_{r,s}$ denote the fraction of North firms with offshoring status s that sell for market r, for $s \in \{n, o\}$ and $r \in \{D, X\}$. It then follows that

$$\varepsilon_{D,s} = \int_{\varphi_{D,s}}^{\infty} \int_{1}^{\infty} h_s(\varphi, \tau) d\tau d\varphi = \int_{\varphi_{D,s}}^{\infty} h_s(\varphi) d\varphi = 1 - H_s(\varphi_{D,s}), \tag{D-19}$$

$$\varepsilon_{X,s} = \int_{\varphi_{X,s}}^{\infty} \int_{1}^{\varphi/\varphi_{X,s}} h_s(\varphi, \tau) d\tau d\varphi, \tag{D-20}$$

where $h_s(\varphi)$ is defined as in (39) and $H_s(\varphi)$ is the marginal cdf of φ for North firms with offshoring status s. Similarly, let ε_r^* denote the fraction of South firms that sell for market r. Given that South firms do not offshore, their expressions for ε_D^* and ε_X^* are simpler:

$$\varepsilon_D^* = \int_{\varphi_D^*}^{\infty} \int_{1}^{\infty} g(\varphi) m(\tau) d\tau d\varphi = 1 - G(\varphi_D^*), \tag{D-21}$$

$$\varepsilon_X^* = \int_{\varphi_X^*}^{\infty} \int_{1}^{\varphi/\varphi_X^*} g(\varphi) m(\tau) d\tau d\varphi = \int_{\varphi_X^*}^{\infty} M(\varphi/\varphi_X^*) g(\varphi) d\varphi. \tag{D-22}$$

Since the masses of firms are constant in steady state, the firms that die due to the exogenous death shock must be exactly replaced by successful entrants so that

$$\delta N_{r,n} = \varepsilon_{r,n} \left(1 - \bar{\Gamma}\right) N_E, \tag{D-23}$$

$$\delta N_{r,o} = \varepsilon_{r,o} \bar{\Gamma} N_E, \tag{D-24}$$

$$\delta N_r^* = \varepsilon_r^* N_E^*, \tag{D-25}$$

for $r \in \{D, X\}$. Recall from section 5.3 that $N_{r,s}$ is the mass of North firms with offshoring status s that produce for market r, N_r^* is the mass of South firms selling for market r, N_E is the mass of North entrants every period, and N_E^* is the mass of South entrants. Hence, in each of these equations the left-hand side accounts for firms dying due to the exogenous death shock, while the right-hand side accounts for successful entrants. To obtain N_E and N_E^* in terms of the cutoff productivity levels we follow similar steps as those followed in section 3.3 for the derivation of N_E. The following lemma shows the expressions for N_E and N_E^*.

Lemma D.1. ***(North and South entrants)***

The measures of N_E and N_E^* are given by

$$N_E = \frac{\delta}{\gamma}\left[\frac{\tilde{\mu}_D^* - \tilde{\mu}_X^*}{(\tilde{\mu}_{D,n} + \tilde{\mu}_{D,o})\,\tilde{\mu}_D^* - (\tilde{\mu}_{X,n} + \tilde{\mu}_{X,o})\,\tilde{\mu}_X^*}\right], \tag{D-26}$$

$$N_E^* = \frac{\delta}{\gamma}\left[\frac{(\tilde{\mu}_{D,n} - \tilde{\mu}_{X,n}) + (\tilde{\mu}_{D,o} - \tilde{\mu}_{X,o})}{(\tilde{\mu}_{D,n} + \tilde{\mu}_{D,o})\,\tilde{\mu}_D^* - (\tilde{\mu}_{X,n} + \tilde{\mu}_{X,o})\,\tilde{\mu}_X^*}\right], \tag{D-27}$$

where

$$\begin{aligned}
\tilde{\mu}_{D,n} &= \int_{\varphi_{D,n}}^{\infty}\int_{1}^{\infty} \mu_{D,n}(\varphi)[1-\Gamma(\varphi,\tau)]g(\varphi)m(\tau)d\tau d\varphi = (1-\bar{\Gamma})\int_{\varphi_{D,n}}^{\infty} \mu_{D,n}(\varphi)h_n(\varphi)d\varphi \\
\tilde{\mu}_{X,n} &= \int_{\varphi_{X,n}}^{\infty}\int_{1}^{\varphi/\varphi_{X,n}} \mu_{X,n}(\varphi,\tau)[1-\Gamma(\varphi,\tau)]g(\varphi)m(\tau)d\tau d\varphi \\
&= (1-\bar{\Gamma})\int_{\varphi_{X,n}}^{\infty}\int_{1}^{\varphi/\varphi_{X,n}} \mu_{X,n}(\varphi,\tau)h_n(\varphi,\tau)d\tau d\varphi \\
\tilde{\mu}_{D,o} &= \int_{\varphi_{D,o}}^{\infty}\int_{1}^{\infty} \mu_{D,o}(\varphi)\Gamma(\varphi,\tau)g(\varphi)m(\tau)d\tau d\varphi = \bar{\Gamma}\int_{\varphi_{D,o}}^{\infty} \mu_{D,o}(\varphi)h_o(\varphi)d\varphi \\
\tilde{\mu}_{X,o} &= \int_{\varphi_{X,o}}^{\infty}\int_{1}^{\varphi/\varphi_{X,o}} \mu_{X,o}(\varphi,\tau)\Gamma(\varphi,\tau)g(\varphi)m(\tau)d\tau d\varphi = \bar{\Gamma}\int_{\varphi_{X,o}}^{\infty}\int_{1}^{\varphi/\varphi_{X,o}} \mu_{X,o}(\varphi,\tau)h_o(\varphi,\tau)d\tau d\varphi.
\end{aligned}$$

denote the unconditional expected markups for a potential North entrant from selling in market r under offshoring status s, for $r \in \{D, X\}$ and $s \in \{n, o\}$, and

$$\tilde{\mu}_D^* = \int_{\varphi_D^*}^{\infty} \mu_D^*(\varphi)g(\varphi)d\varphi \quad \text{and} \quad \tilde{\mu}_X^* = \int_{\varphi_X^*}^{\infty}\int_{1}^{\varphi/\varphi_X^*} \mu_X^*(\varphi,\tau)g(\varphi)m(\tau)d\tau d\varphi$$

are the unconditional expected markups for a potential South entrant from selling in market r, for $r \in \{D, X\}$.

Proof. Note first from equation (6) that for a North firm with productivity φ and offshoring status $s \in \{n, o\}$: *(i)* $\ln p_{D,s}(\varphi) = \ln \hat{p} - \mu_{D,s}(\varphi)$ if $\varphi \geq \varphi_{D,s}$, and *(ii)* $\ln p_{X,s}(\varphi) = \ln \hat{p}^* - \mu_{X,s}(\varphi,\tau)$ if $\varphi \geq \tau\varphi_{X,s}$. Therefore, the average log price of North firms with offshoring status s is $\overline{\ln p}_{D,s} =$

$\ln \hat{p} - \bar{\mu}_{D,s}$ in the domestic market, and $\overline{\ln p}_{X,s} = \ln \hat{p}^* - \bar{\mu}_{X,s}$ in the export market, where

$$\bar{\mu}_{D,s} = \int_{\varphi_{D,s}}^{\infty} \mu_{D,s}(\varphi) h_s(\varphi \mid \varphi \geq \varphi_{D,s}) d\varphi = (1/\varepsilon_{D,s}) \int_{\varphi_{D,s}}^{\infty} \mu_{D,s}(\varphi) h_s(\varphi) d\varphi \tag{D-28}$$

$$\bar{\mu}_{X,s} = (1/\varepsilon_{X,s}) \int_{\varphi_{X,s}}^{\infty} \int_{1}^{\varphi/\varphi_{X,s}} \mu_{X,s}(\varphi,\tau) h_s(\varphi,\tau) d\tau d\varphi \tag{D-29}$$

are the average markups of North firms with offshoring status s in the domestic (D) and export (X) markets, for $s \in \{n, o\}$. On the other hand, the average log prices of South firms are $\overline{\ln p}_D^* = \ln \hat{p}^* - \bar{\mu}_D^*$ and $\overline{\ln p}_X^* = \ln \hat{p} - \bar{\mu}_X^*$ where

$$\bar{\mu}_D^* = \int_{\varphi_D^*}^{\infty} \mu_D^*(\varphi) g(\varphi \mid \varphi \geq \varphi_D^*) d\varphi = (1/\varepsilon_D^*) \int_{\varphi_D^*}^{\infty} \mu_D^*(\varphi) g(\varphi) d\varphi \tag{D-30}$$

$$\bar{\mu}_X^* = (1/\varepsilon_X^*) \int_{\varphi_X^*}^{\infty} \int_{1}^{\varphi/\varphi_X^*} \mu_X^*(\varphi,\tau) g(\varphi) m(\tau) d\tau d\varphi \tag{D-31}$$

are the average markups of South firms selling in each market.

Substituting the equations for $\overline{\ln p}_{D,n}$, $\overline{\ln p}_{D,o}$, and $\overline{\ln p}_X^*$ into the overall average log price in the North market, $\overline{\ln p} = \frac{N_{D,n}}{N} \overline{\ln p}_{D,n} + \frac{N_{D,o}}{N} \overline{\ln p}_{D,o} + \frac{N_X^*}{N} \overline{\ln p}_X^*$, we get

$$\ln \hat{p} - \overline{\ln p} = \frac{N_{D,n}}{N} \bar{\mu}_{D,n} + \frac{N_{D,o}}{N} \bar{\mu}_{D,o} + \frac{N_X^*}{N} \bar{\mu}_X^*. \tag{D-32}$$

Also, substituting the equations for $\overline{\ln p}_D^*$, $\overline{\ln p}_{X,n}$, and $\overline{\ln p}_{X,o}$ into the overall average log price in the South market, $\overline{\ln p}^* = \frac{N_D^*}{N^*} \overline{\ln p}_D^* + \frac{N_{X,n}}{N^*} \overline{\ln p}_{X,n} + \frac{N_{X,o}}{N^*} \overline{\ln p}_{X,o}$, we obtain

$$\ln \hat{p}^* - \overline{\ln p}^* = \frac{N_D^*}{N^*} \bar{\mu}_D^* + \frac{N_{X,n}}{N^*} \bar{\mu}_{X,n} + \frac{N_{X,o}}{N^*} \bar{\mu}_{X,o}. \tag{D-33}$$

From equation (3) it follows that $\ln \hat{p} - \overline{\ln p} = \frac{1}{\gamma N}$, with an analogous expression holding in the South market, $\ln \hat{p}^* - \overline{\ln p}^* = \frac{1}{\gamma N^*}$. Therefore, we can rewrite (D-32) and (D-33) as

$$\frac{1}{\gamma} = N_{D,n} \bar{\mu}_{D,n} + N_{D,o} \bar{\mu}_{D,o} + N_X^* \bar{\mu}_X^*, \tag{D-34}$$

$$\frac{1}{\gamma} = N_D^* \bar{\mu}_D^* + N_{X,n} \bar{\mu}_{X,n} + N_{X,o} \bar{\mu}_{X,o}. \tag{D-35}$$

From the definitions of the unconditional expected markups above and equations (D-28)-(D-31), note that $\tilde{\mu}_{r,n} = (1 - \bar{\Gamma}) \varepsilon_{r,n} \bar{\mu}_{r,n}$, $\tilde{\mu}_{r,o} = \bar{\Gamma} \varepsilon_{r,o} \bar{\mu}_{r,o}$, and $\tilde{\mu}_r^* = \varepsilon_r^* \bar{\mu}_r^*$, for $r \in \{D, X\}$. Lastly, using the previous equations and substituting the expressions for $N_{D,n}$, $N_{D,o}$, $N_{X,n}$, $N_{X,o}$, N_D^* and N_X^* from (D-23)-(D-25) into (D-34) and (D-35) we obtain the system of equations that allows us to obtain (D-26) and (D-27). □

Once we obtain the equilibrium cutoff productivity levels, we obtain N_E and N_E^* using (D-26) and (D-27), and then plug them into (D-23)-(D-25) to obtain $N_{D,o}$, $N_{D,n}$, $N_{X,o}$, $N_{X,o}$, N_D^* and N_X^*, which are then plugged into $N = N_{D,n} + N_{D,o} + N_X^*$ and $N^* = N_D^* + N_{X,n} + N_{X,o}$.

Let N_P denote the mass of North producing firms, and N_P^* the mass of South producing firms. In contrast to the case without export opportunities, N_P is now different from the number of varieties sold in the North, N. If $\varphi_{D,s} \leq \varphi_{X,s}$, so that exporting firms are a subset of firms selling to the domestic market, it follows that $N_P = N_{D,o} + N_{D,n}$. However, if $\varphi_{D,s} > \varphi_{X,s}$ there will be some low-τ firms that export but do not sell domestically. Let ϖ_s denote the fraction of North exporters with status s, $N_{X,s}$, whose productivities are between $\varphi_{X,s}$ and $\varphi_{D,s}$. It follows that

$$\varpi_s = \mathbb{1}\{\varphi_{D,s} > \varphi_{X,s}\}(1/\varepsilon_{X,s}) \int_{\varphi_{X,s}}^{\varphi_{D,s}} \int_{1}^{\varphi/\varphi_{X,s}} h_s(\varphi,\tau) d\tau d\varphi.$$

Therefore,

$$N_P = N_{D,o} + N_{D,n} + \varpi_o N_{X,o} + \varpi_n N_{X,n}. \tag{D-36}$$

Analogously, for South firms we have that

$$\varpi^* = \mathbb{1}\{\varphi_D^* > \varphi_X^*\}(1/\varepsilon_X^*) \int_{\varphi_X^*}^{\varphi_D^*} M(\varphi/\varphi_X^*) g(\varphi) d\varphi.$$

Hence,

$$N_P^* = N_D^* + \varpi^* N_X^*. \tag{D-37}$$

We can obtain further useful expressions for different masses of firms. The mass of North offshoring firms, N_o, the mass or North non-offshoring firms, and the mass of North exporters, N_X, are given by

$$N_o = N_{D,o} + \varpi_o N_{X,o}, \tag{D-38}$$

$$N_n = N_P - N_o, \tag{D-39}$$

$$N_X = N_{X,o} + N_{X,n}. \tag{D-40}$$

D.4 Average Prices, Average Productivities, and Market Shares

Average productivities, $\bar{\varphi}_{r,s}$ and $\bar{\varphi}_r^*$, and average prices, $\bar{p}_{r,s}$ and $\bar{p}_r^*$, for $r \in \{D, X\}$ and $s \in \{n, o\}$, follow similar definitions to those of average markups in (D-28)-(D-31). The overall average prices can then be written as $\bar{p} = \frac{N_{D,n}}{N}\bar{p}_{D,n} + \frac{N_{D,o}}{N}\bar{p}_{D,o} + \frac{N_X^*}{N}\bar{p}_X^*$ in the North, and as $\bar{p}^* = \frac{N_D^*}{N^*}\bar{p}_D^* + \frac{N_{X,n}}{N^*}\bar{p}_{X,n} + \frac{N_{X,o}}{N^*}\bar{p}_{X,o}$ in the South.

We can also obtain the average productivity of all producing North firms with the same offshoring status, $\bar{\varphi}_o$ and $\bar{\varphi}_n$. All offshoring firms produce, and hence, $\bar{\varphi}_o$ is simply given by

$$\bar{\varphi}_o = \int_{\min(\varphi_{D,o},\varphi_{X,o})}^{\infty} \varphi h_o(\varphi) d\varphi. \tag{D-41}$$

If $\min(\varphi_{D,o}, \varphi_{X,o}) = \varphi_{D,o}$, then $\bar{\varphi}_o = \bar{\varphi}_{D,o}$ (in that case $\varepsilon_{D,o} = 1$ because $H_o(\varphi_{D,o}) = 0$). For $\bar{\varphi}_n$, we need to describe first the distribution of non-offshoring firms conditional on producing—recall that $h_n(\varphi)$ denotes the marginal pdf of productivity for all non-offshoring firms, producing or not. If $\varphi_{D,n} \leq \varphi_{X,n}$, so that exporting non-offshoring firms are a subset of firms that produce for the domestic market, the conditional distribution is simply $h_n(\varphi|\varphi \geq \varphi_{D,n}) = (1/\varepsilon_{D,n})h_n(\varphi)$ and $\bar{\varphi}_n = \bar{\varphi}_{D,n}$. If $\varphi_{D,n} > \varphi_{X,n}$, however, the conditional distribution is not that simple because only a subset of non-offshoring firms in the range $[\varphi_{X,n}, \varphi_{D,n})$ produce (those that export due to their low τ draw). In this case the conditional distribution of non-offshoring firms is

$$h_n(\varphi|active) = \begin{cases} 0 & \text{if } \varphi < \varphi_{X,n} \\ [1/(\varepsilon_{X,n}\varpi_n + \varepsilon_{D,n})] \int_1^{\varphi/\varphi_{X,n}} h_n(\varphi,\tau)d\tau & \text{if } \varphi \in [\varphi_{X,n}, \varphi_{D,n}) \\ [1/(\varepsilon_{X,n}\varpi_n + \varepsilon_{D,n})]h_n(\varphi) & \text{if } \varphi \geq \varphi_{D,n}. \end{cases} \quad \text{(D-42)}$$

It follows that

$$\bar{\varphi}_n = \frac{\varpi_n N_{X,n}}{N_n}\left[\frac{1}{\varpi_n \varepsilon_{X,n}}\int_{\varphi_{X,n}}^{\varphi_{D,n}}\int_1^{\varphi/\varphi_{X,n}} \varphi h_n(\varphi,\tau)d\tau d\varphi\right] + \frac{N_{D,n}}{N_n}\bar{\varphi}_{D,n}. \quad \text{(D-43)}$$

Using $\bar{\varphi}_o$ and $\bar{\varphi}_n$, we obtain the average productivity of all producing North firms, $\bar{\varphi}$, as

$$\bar{\varphi} = \frac{N_o}{N_P}\bar{\varphi}_o + \frac{N_n}{N_P}\bar{\varphi}_n. \quad \text{(D-44)}$$

As before, the *effective* productivity of offshoring firms considers the decline in marginal costs due to offshoring. Hence, the average effective productivity of offshoring firms selling in market r is $\bar{\varphi}^E_{r,o} = \bar{\varphi}_{r,o}/w_o$, for $r \in \{D, X\}$, the overall average effective productivity of offshoring firms is $\bar{\varphi}^E_o = \bar{\varphi}_o/w_o$, and the average effective productivity of all North firms is

$$\bar{\varphi}^E = \frac{N_o}{N_P}\bar{\varphi}^E_o + \frac{N_n}{N_P}\bar{\varphi}_n. \quad \text{(D-45)}$$

For South firms, their average productivity, $\bar{\varphi}^*$, equals $\bar{\varphi}^*_D$ if $\varphi^*_D \leq \varphi^*_X$. Otherwise, we have that

$$\bar{\varphi}^* = \frac{\varpi^* N^*_X}{N^*_P}\left[\frac{1}{\varpi^*\varepsilon^*_X}\int_{\varphi^*_X}^{\varphi^*_D} \varphi M(\varphi/\varphi^*_X)g(\varphi)d\varphi\right] + \frac{N^*_D}{N^*_P}\bar{\varphi}^*_D, \quad \text{(D-46)}$$

where the term inside the brackets is the average productivity of South producing firms with productivities in the range $[\varphi^*_X, \varphi^*_D)$.

Lastly, from equation (7) we know that $\sigma_{D,s}(\varphi) = \gamma\mu_{D,s}(\varphi)$ and $\sigma_{X,s}(\varphi,\tau) = \gamma\mu_{X,s}(\varphi,\tau)$ are the market share densities in each destination of a North firm with productivity φ and offshoring status s, and $\sigma^*_D(\varphi) = \gamma\mu^*_D(\varphi)$ and $\sigma^*_X(\varphi,\tau) = \gamma\mu^*_X(\varphi,\tau)$ are the market share densities in destination r of a South firm with productivity φ. Aggregating, it follows that the market share in destination r of North firms with offshoring status s, $\sigma_{r,s}$, and the market share in destination r of South firms, σ^*_r, are given by

$$\sigma_{r,s} = \gamma N_{r,s}\bar{\mu}_{r,s} \quad \text{and} \quad \sigma^*_r = \gamma N^*_r\bar{\mu}^*_r,$$

for $r \in \{D, X\}$ and $s \in \{n, o\}$. Of course, it is the case that $\sigma_{D,o} + \sigma_{D,n} + \sigma^*_X = 1$ and $\sigma^*_D + \sigma_{X,o} + \sigma_{X,n} = 1$.

D.5 The Impact of Trade Liberalization in the South

This section describes the effects of trade liberalization in final goods and of reductions in the variable offshoring cost, λ, on outcomes for South firms and the South market. Recall that South firms never offshore, but the South is the source of offshored labor for North firms. Table D.1, which is simply a continuation of Table 4, shows the results.

Table D.1: The effects of trade liberalization in the South

	Autarky	**Offshoring**	**Final-good trade and offshoring**			
	$(\tau^\infty, \lambda^\infty)$	(τ^∞, λ^H)	(τ^H, λ^H)	(τ^H, λ^L)	(τ^L, λ^H)	(τ^L, λ^L)
Productivity:						
φ_D^*	0.427	0.427	0.464	0.457	0.538	0.503
φ_X^*	—	—	0.585	0.624	0.621	0.692
$\bar{\varphi}_D^*$	1.327	1.327	1.330	1.330	1.341	1.335
$\bar{\varphi}_X^*$	—	—	1.722	1.745	1.580	1.645
$\bar{\varphi}^*$	1.327	1.327	1.330	1.330	1.341	1.335
Prices:						
$\hat{p}^*$	2.211	2.211	2.035	2.065	1.752	1.877
$\bar{p}^*$	1.250	1.250	1.341	1.363	1.236	1.289
$\bar{p}_D^*$	1.250	1.250	1.206	1.214	1.126	1.163
$\bar{p}_X^*$	—	—	1.310	1.238	1.214	1.114
Markups and shares:						
$\bar{\mu}_D^*$	0.599	0.599	0.550	0.559	0.468	0.505
$\bar{\mu}_X^*$	—	—	0.224	0.214	0.239	0.215
σ_D^*	1.000	1.000	0.771	0.715	0.539	0.285
σ_X^*	—	—	0.080	0.062	0.174	0.065
Composition of firms:						
N^*	0.835	0.835	1.110	1.111	1.337	1.243
N_P^*	0.835	0.835	0.701	0.640	0.576	0.282
N_X^*/N_P^*	—	—	0.255	0.226	0.630	0.536

The $(\tau^\infty, \lambda^\infty)$ and (τ^∞, λ^H) outcomes are identical for South firms: with no trade in final goods, whether North firms offshore or not is irrelevant for South firms in the differentiated good sector (when moving from $(\tau^\infty, \lambda^\infty)$ to (τ^∞, λ^H), offshoring North firms hire South labor previously employed in the South homogeneous-good sector).

Similar to North firms, South firms face a tougher competitive environment in both markets after trade liberalization in final goods (going from $\tau^\infty \to \tau^H \to \tau^L$, keeping λ constant) as both φ_D^* and φ_X^* increase, and hence, all producing firms in the South are forced to reduce their markups. On the other hand, reductions in the variable cost of offshoring ($\lambda^\infty \to \lambda^H \to \lambda^L$, keeping the same distribution of τ) are irrelevant for South firms if $\tau \equiv \tau^\infty$, but otherwise create an easier competitive environment in the domestic market (φ_D^* declines), but a tougher competitive environment in the export market (φ_X^* rises). Importantly, with trade in final goods the environment becomes easier in the South market due to the effect of a reduction in λ on entry of South firms, which declines as potential South entrants realize it is hard to compete with now more efficient offshoring North firms; in the end, a large fraction of South firms that die due to the exogenous death shock are never replaced by new South firms and thus N_P^* falls.

In all cases with trade in final goods in Table D.1 we get $\varphi_D^* < \varphi_X^*$, so that South exporting firms are a subset of South firms producing for their domestic market. It follows that $\bar{\varphi}_D^* = \bar{\varphi}^*$ and $N_P^* = N_D^*$. Note that trade liberalization (of any type) hardly affects the average productivity of South firms selling domestically, $\bar{\varphi}_D^*$. On the other hand, the average productivity of South exporters, $\bar{\varphi}_X^*$, declines with final-good trade liberalization, but increases with a reduction in λ.

Regarding prices, an important message is that trade liberalization does not necessarily reduce average prices in the South. Note that $\bar{p}^*$ rises when the South opens to trade in final goods (from τ^∞ to τ^H) due to the high average price of imports from the North, which are subject to high iceberg costs (on average). As liberalization in final goods deepens ($\tau^H \to \tau^L$), the average price declines. A reduction in λ, however, rises $\bar{p}^*$ with both low and high levels of final-good trade liberalization. This happens as South firms that die due to the exogenous death shock are replaced in the South market by North firms that are on average less productive than before.

Trade liberalization in final goods increases the number of varieties that are consumed in the South, N^*, but a reduction in λ causes a negligible increase in N^* if $\tau \equiv \tau^H$ and a decline if $\tau \equiv \tau^L$. Independently of this, note that both types of trade liberalization shrink the size of the heterogeneous-good sector in the South, with N_P^* and σ_D^* decreasing after any type of liberalization. In other words, as trade liberalization deepens, the South transforms from a producer of final differentiated goods to an offshoring hub for North firms. This process would be attenuated for higher levels of A^* or lower levels of f_E^*.

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