Offshoring and Jobs: The Myriad Channels of Influence

Appendix — For Online Publication

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A The Mass of Entrants and Market Shares

To obtain N_E, note first that we can write the price for the basket of differentiated goods as

$$P = \left[N_n \bar{p}_n^{1-\sigma} + N_o \bar{p}_o^{1-\sigma}\right]^{\frac{1}{1-\sigma}}, \tag{A-1}$$

where $\bar{p}_s \equiv p(\bar{\varphi}_s)$ is the average price of firms with offshoring status s, for $s \in \{n, o\}$, with

$$\bar{\varphi}_n = \left[\int_{\hat{\varphi}}^{\hat{\varphi}_o} \varphi^{\sigma-1} g(\varphi \mid \hat{\varphi} \leq \varphi < \hat{\varphi}_o) d\varphi\right]^{\frac{1}{\sigma-1}} \quad \text{and} \quad \bar{\varphi}_o = \left[\int_{\hat{\varphi}_o}^{\infty} \varphi^{\sigma-1} g(\varphi \mid \varphi \geq \hat{\varphi}_o) d\varphi\right]^{\frac{1}{\sigma-1}}. \tag{A-2}$$

Substituting $N_n = [G(\hat{\varphi}_o) - G(\hat{\varphi})]\, N_E$ and $N_o = [1 - G(\hat{\varphi}_o)]\, N_E$ into (A-1), we solve for N_E as

$$N_E = \frac{P^{1-\sigma}}{[G(\hat{\varphi}_o) - G(\hat{\varphi})]\bar{p}_n^{1-\sigma} + [1 - G(\hat{\varphi}_o)]\bar{p}_o^{1-\sigma}}. \tag{A-3}$$

Lastly, we only have to use the zero-cutoff-profit condition in (21) to substitute for P in equation (A-3).

We can also obtain convenient expressions for the market shares of non-offshoring and offshoring firms. From the end of section 3.1 we know that the share in total differentiated-good expenditure of a producing firm with productivity φ is $\mu(\varphi) = \frac{p(\varphi)^{1-\sigma}}{P^{1-\sigma}}$. Aggregating the shares of all firms with the same offshoring status, it follows that the total market share of firms with offshoring status s is

$$\mu_s = \frac{N_s \bar{p}_s^{1-\sigma}}{P^{1-\sigma}}, \tag{A-4}$$

for $s \in \{n, o\}$. Note that $\mu_n + \mu_o = 1$ is equivalent to equation (A-1).

B Existence and Uniqueness of Equilibrium

We assume search friction parameters so that the equilibrium value for $\hat{w}_Z$—from (16)—is in the range $(\lambda k(0)w^*, \lambda k(1)w^*)$, so that an interior solution for $\hat{\alpha}$—from (17)—exists. For the existence and uniqueness of the equilibrium cutoff levels, note that after plugging in equations (22) and (23) into Π and using Leibniz's rule, we obtain

$$\frac{d\Pi}{d\hat{\varphi}} = -\frac{(\sigma-1)f[1-G(\hat{\varphi})]}{\hat{\varphi}^{\sigma}}\left[\frac{N_n}{N}\bar{\varphi}_n^{\sigma-1} + \frac{N_o}{N}\left(\frac{\bar{\varphi}_o}{c(\hat{\alpha})}\right)^{\sigma-1}\right] < 0,$$

with average productivities given by (A-2). As Π is strictly decreasing in $\hat{\varphi}$ and it converges to zero as $\hat{\varphi}$ increases, the equilibrium exists and is unique as long as Π is greater than f_E when $\hat{\varphi}$ approaches $\varphi_{\min}$ from the right, where $\varphi_{\min} \geq 0$ is the lower bound of the support of the productivity distribution. It follows from (23) that the equilibrium value for $\hat{\varphi}_o$ also exists and is unique.

C Proofs of Lemmas and Propositions

Proof of Lemma 1. If L and L^* are the amounts of domestic and foreign labor employed by a firm with productivity φ in the production of inputs, we must satisfy

$$L = \int_{\hat{\alpha}(\varphi)}^{1} \ell(\alpha)d\alpha, \tag{C-1}$$

$$L^* = \int_{0}^{\hat{\alpha}(\varphi)} \ell^*(\alpha)d\alpha, \tag{C-2}$$

where $\hat{\alpha}(\varphi)$ is this firm's fraction of offshored inputs, and $\ell(\alpha)$ and $\ell^*(\alpha)$ denote units of domestic and foreign labor employed in the production of input α.

Given equation (10) and the input production functions $y(\alpha) = \ell$ and $y^*(\alpha) = \frac{\ell^*}{\lambda k(\alpha)}$, it follows that the optimal allocation of domestic and foreign labor across different inputs for a firm with productivity φ satisfies

$$\ell(\alpha) = \ell(\alpha') \qquad \text{for } \alpha, \alpha' \in [0, \hat{\alpha}(\varphi)] \tag{C-3}$$

$$\ell^*(\alpha) = \ell^*(\alpha')\left(\frac{k(\alpha)}{k(\alpha')}\right)^{1-\rho} \qquad \text{for } \alpha, \alpha' \in (\hat{\alpha}(\varphi), 1]. \tag{C-4}$$

Thus, for domestic labor it follows from equations (C-1) and (C-3) that

$$\ell(\alpha) = \frac{L}{1-\hat{\alpha}(\varphi)} \tag{C-5}$$

for $\alpha \in (\hat{\alpha}(\varphi), 1]$. For the allocation of foreign labor, note from equation (C-4) that $\ell^*(\alpha) = \ell^*(\hat{\alpha}(\varphi))\left[\frac{k(\alpha)}{k(\hat{\alpha}(\varphi))}\right]^{1-\rho}$ for $\alpha \in [0, \hat{\alpha}(\varphi)]$ must hold. Plugging in the previous expression into equation (C-2), we solve for $\ell^*(\hat{\alpha}(\varphi))$ and substitute that expression back into $\ell^*(\alpha)$ to obtain

$$\ell^*(\alpha) = \frac{k(\alpha)^{1-\rho}L^*}{\int_0^{\hat{\alpha}(\varphi)} k(\alpha')^{1-\rho}d\alpha'}. \tag{C-6}$$

Therefore, substituting equation (C-5) into $y(\alpha)$ and equation (C-6) into $y^*(\alpha)$, and then plugging in the resulting expressions into equation (10), we get

$$Y(\varphi) = \left\{ \lambda^{\frac{1-\rho}{\rho}} \left[\int_0^{\hat{\alpha}(\varphi)} k(\alpha)^{1-\rho} d\alpha \right]^{\frac{1}{\rho}} L^{*\frac{\rho-1}{\rho}} + [1-\hat{\alpha}(\varphi)]^{\frac{1}{\rho}} L^{\frac{\rho-1}{\rho}} \right\}^{\frac{\rho}{\rho-1}}. \tag{C-7}$$

We can then rewrite $Y(\varphi)$ as

$$Y(\varphi) = \left[\kappa(\varphi) L^{*\frac{\rho-1}{\rho}} + \upsilon(\varphi) L^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}}, \tag{C-8}$$

where $\kappa(\varphi) \equiv \lambda^{\frac{1-\rho}{\rho}} K[\hat{\alpha}(\varphi)]^{\frac{1}{\rho}}$, $K[\hat{\alpha}(\varphi)] = \int_0^{\hat{\alpha}(\varphi)} k(\alpha)^{1-\rho} d\alpha$, and $\upsilon(\varphi) \equiv [1-\hat{\alpha}(\varphi)]^{\frac{1}{\rho}}$. $\square$

Proof of Lemma 2. We find the unit cost of $Y(\varphi)$ in two steps. First, we obtain the unit cost for a given $\hat{\alpha}(\varphi)$, and then obtain the optimal $\hat{\alpha}(\varphi)$. For a given $\hat{\alpha}(\varphi) > 0$, denote the domestic and foreign labor requirements to produce $Y(\varphi) = 1$ by $L_{Y(\varphi)=1}$ and $L^*_{Y(\varphi)=1}$, respectively. Next, we set the Lagrangean

$$\Upsilon = w^* L^*_{Y(\varphi)=1} + \hat{w}_Z L_{Y(\varphi)=1} + \xi \left\{ 1 - \left[\kappa(\varphi) L^{*\frac{\rho-1}{\rho}}_{Y(\varphi)=1} + \upsilon(\varphi) L^{\frac{\rho-1}{\rho}}_{Y(\varphi)=1} \right]^{\frac{\rho}{\rho-1}} \right\},$$

and obtain the following first order conditions

$$w^* - \xi \left[\kappa(\varphi) L^{*\frac{\rho-1}{\rho}}_{Y(\varphi)=1} + \upsilon(\varphi) L^{\frac{\rho-1}{\rho}}_{Y(\varphi)=1} \right]^{\frac{1}{\rho-1}} \kappa(\varphi) L^{*-\frac{1}{\rho}} = 0 \tag{C-9}$$

$$\hat{w}_Z - \xi \left[\kappa(\varphi) L^{*\frac{\rho-1}{\rho}}_{Y(\varphi)=1} + \upsilon(\varphi) L^{\frac{\rho-1}{\rho}}_{Y(\varphi)=1} \right]^{\frac{1}{\rho-1}} \upsilon(\varphi) L^{-\frac{1}{\rho}} = 0 \tag{C-10}$$

$$1 - \left[\kappa(\varphi) L^{*\frac{\rho-1}{\rho}}_{Y(\varphi)=1} + \upsilon(\varphi) L^{\frac{\rho-1}{\rho}}_{Y(\varphi)=1} \right]^{\frac{\rho}{\rho-1}} = 0. \tag{C-11}$$

Combining equations (C-9) and (C-10) we obtain the optimal relationship between $L_{Y(\varphi)=1}$ and $L^*_{Y(\varphi)=1}$:

$$L^*_{Y(\varphi)=1} = \left[\frac{\kappa(\varphi) \hat{w}_Z}{\upsilon(\varphi) w^*} \right]^{\rho} L_{Y(\varphi)=1}. \tag{C-12}$$

Plugging in equation (C-12) into (C-11), we get

$$L_{Y(\varphi)=1} = \frac{\upsilon(\varphi)^{\rho} \hat{w}_Z^{-\rho}}{\left[\kappa(\varphi)^{\rho} w^{*1-\rho} + \upsilon(\varphi)^{\rho} \hat{w}_Z^{1-\rho} \right]^{\frac{\rho}{\rho-1}}}, \tag{C-13}$$

and then substituting (C-13) into (C-12) we obtain

$$L^*_{Y(\varphi)=1} = \frac{\kappa(\varphi)^{\rho} w^{*-\rho}}{\left[\kappa(\varphi)^{\rho} w^{*1-\rho} + \upsilon(\varphi)^{\rho} \hat{w}_Z^{1-\rho} \right]^{\frac{\rho}{\rho-1}}}. \tag{C-14}$$

Thus, the unit cost of $Y(\varphi)$, denoted by $C[Y(\varphi)]$ is given by

$$C[Y(\varphi)] \equiv w^* L^*_{Y(\varphi)=1} + \hat{w}_Z L_{Y(\varphi)=1} = \left[\kappa(\varphi)^\rho w^{*1-\rho} + \upsilon(\varphi)^\rho \hat{w}_Z^{1-\rho}\right]^{\frac{1}{1-\rho}}.$$

Now, the optimal choice of $\hat{\alpha}(\varphi)$ is obtained by minimizing $C[Y(\varphi)]$ with respect to $\hat{\alpha}(\varphi)$:

$$\frac{dC(Y(\varphi))}{d\hat{\alpha}(\varphi)} = \frac{\rho}{1-\rho}\left[\kappa(\varphi)^\rho w^{*1-\rho} + \upsilon(\varphi)^\rho \hat{w}_Z{}^{1-\rho}\right]^{\frac{\rho}{1-\rho}}\left[w^{*1-\rho}\kappa(\varphi)^{\rho-1}\frac{d\kappa(\varphi)}{d\hat{\alpha}(\varphi)} + \hat{w}_Z{}^{1-\rho}\upsilon(\varphi)^{\rho-1}\frac{dv}{d\hat{\alpha}(\varphi)}\right] = 0$$

Using the definitions of $\kappa(\varphi)$ and $\upsilon(\varphi)$ from Lemma 1, we obtain $\frac{d\kappa(\varphi)}{d\hat{\alpha}(\varphi)}$ and $\frac{d\upsilon(\varphi)}{d\hat{\alpha}(\varphi)}$ and plug them into the previous expression. We get that $\frac{dC[Y(\varphi)]}{d\hat{\alpha}(\varphi)} = 0$ if and only if

$$\lambda w^* k[\hat{\alpha}(\varphi)] = \hat{w}_Z.$$

Solving for $\hat{\alpha}(\varphi)$ we get $\hat{\alpha}(\varphi) = k^{-1}\left(\frac{\hat{w}_Z}{\lambda w^*}\right)$. Note that the solution does not depend on φ, and thus, all offshoring firms offshore the same fraction of inputs:

$$\hat{\alpha} = k^{-1}\left(\frac{\hat{w}_Z}{\lambda w^*}\right).$$

Using this result along with the definitions of $\kappa(\varphi)$ and $\upsilon(\varphi)$ from Lemma 1, we can rewrite $C[Y(\varphi)]$ as $c(\hat{\alpha})\hat{w}_Z$, where

$$c(\hat{\alpha}) = \left[k(\hat{\alpha})^{\rho-1}K(\hat{\alpha}) + 1 - \hat{\alpha}\right]^{\frac{1}{1-\rho}}. \tag{C-15}$$

To show that $c(\hat{\alpha})\hat{w}_Z$ is decreasing in $\hat{\alpha}$, we only need to show that $c'(\hat{\alpha}) < 0$. Taking the derivative of equation (C-15) and using that $K'(\hat{\alpha}) = k(\hat{\alpha})^{1-\rho}$, we obtain

$$c'(\hat{\alpha}) = -\left[k(\hat{\alpha})^{\rho-1}K(\hat{\alpha}) + 1 - \hat{\alpha}\right]^{\frac{\rho}{1-\rho}} k(\hat{\alpha})^{\rho-2}K(\hat{\alpha})k'(\hat{\alpha}) < 0.$$

The last inequality follows from the fact that $k'(\hat{\alpha}) > 0$.

Let us now prove that $c(\hat{\alpha})\hat{w}_Z \in (w^*, \hat{w}_Z)$ for $\hat{\alpha} > 0$. First, note that $K(0) = 0$ and hence $c(0) = 1$. Given that $k(\hat{\alpha}) = \frac{\hat{w}_Z}{\lambda w^*}$ and the definition of $K(\hat{\alpha})$, we can write $c(\hat{\alpha})$ as

$$c(\hat{\alpha}) = \left\{\left(\frac{w^*}{\hat{w}_Z}\right)^{1-\rho}\int_0^{\hat{\alpha}}[\lambda k(\alpha)]^{1-\rho}\,d\alpha + 1 - \hat{\alpha}\right\}^{\frac{1}{1-\rho}}.$$

Thus, $c(1) = \frac{w^*}{\hat{w}_Z}\left\{\int_0^1[\lambda k(\alpha)]^{1-\rho}\,d\alpha\right\}^{\frac{1}{1-\rho}}$. As $\lambda k(\alpha) > 1$ for every α, it follows that

$$\left\{\int_0^1[\lambda k(\alpha)]^{1-\rho}\,d\alpha\right\}^{\frac{1}{1-\rho}} > 1$$

for every ρ, and therefore $c(1) > \frac{w^*}{\hat{w}_Z}$. Given that $c'(\hat{\alpha}) < 0$, it must be the case that $c(\hat{\alpha}) \in \left(\frac{w^*}{\hat{w}_Z}, 1\right)$ for $\hat{\alpha} > 0$. Hence, $c(\hat{\alpha})\hat{w}_Z \in (w^*, \hat{w}_Z)$ for $\hat{\alpha} > 0$. □

Proof of Lemma 3. The equilibrium gross profit function for a firm with productivity $\varphi \geq \hat{\varphi}$ can be written as

$$\pi(\varphi) = \left(p(\varphi) - \frac{c(\hat{\alpha})^{\mathbb{I}\{\varphi \geq \hat{\varphi}_o\}} \hat{w}_Z}{\varphi} \right) z(\varphi).$$

Substituting equations (19) and (22) in the previous expression, we can solve for the equilibrium output as

$$z(\varphi) = \frac{(\sigma - 1)}{\hat{w}_Z \hat{\varphi}^{\sigma-1}} \left(\frac{\varphi}{c(\hat{\alpha})^{\mathbb{I}\{\varphi \geq \hat{\varphi}_o\}}} \right)^{\sigma} f.$$

Moreover, given that $z(\varphi) = \varphi Y(\varphi)$, we get

$$Y(\varphi) = \frac{(\sigma - 1)}{\hat{w}_Z} \left(\frac{\varphi}{c(\hat{\alpha})^{\mathbb{I}\{\varphi \geq \hat{\varphi}_o\}} \hat{\varphi}} \right)^{\sigma-1} \frac{f}{c(\hat{\alpha})^{\mathbb{I}\{\varphi \geq \hat{\varphi}_o\}}}. \tag{C-16}$$

For a producing non-offshoring firm, so that $\varphi \in [\hat{\varphi}, \hat{\varphi}_o)$, we know from equation (11) that $Y(\varphi) = L$. Thus, given equation (C-16) it follows that this firm's demand for domestic labor, $L_n(\varphi)$, is given by

$$L_n(\varphi) = \frac{(\sigma - 1)}{\hat{w}_Z} \left(\frac{\varphi}{\hat{\varphi}} \right)^{\sigma-1} f \tag{C-17}$$

The subscript in $L_n(\varphi)$ identifies the firm's status: "not offshoring".

For an offshoring firm, so that $\varphi \geq \hat{\varphi}_o$, we use equation (C-13) to write its demand for labor to meet variable-input requirements as

$$L_o(\varphi) = \left[\frac{(1 - \hat{\alpha}) \hat{w}_Z{}^{-\rho}}{\left(\kappa(\varphi)^{\rho} w^{*1-\rho} + (1 - \hat{\alpha}) \hat{w}_Z{}^{1-\rho} \right)^{\frac{\rho}{\rho-1}}} \right] Y(\varphi)$$

The subscript in $L_o(\varphi)$ identifies the firm's status: "offshoring". Using the definitions of $\kappa(\varphi)$ and $\upsilon(\varphi)$ from Lemma 1, along with $k(\hat{\alpha}) = \frac{\hat{w}_Z}{\lambda w^*}$, we get $L_o(\varphi) = (1 - \hat{\alpha}) c(\hat{\alpha})^{\rho} Y(\varphi)$. Lastly, plugging in equation (C-16) into the previous expression we rewrite $L_o(\varphi)$ as

$$L_o(\varphi) = \frac{(1 - \hat{\alpha})(\sigma - 1)}{c(\hat{\alpha})^{\sigma-\rho} \hat{w}_Z} \left(\frac{\varphi}{\hat{\varphi}} \right)^{\sigma-1} f. \tag{C-18}$$

□

Proof of Lemma 4. For $\zeta_{\hat{\alpha},\lambda} = \frac{d \ln \hat{\alpha}}{d \ln \lambda}$, note first that $\frac{d \ln k(\hat{\alpha})}{d \ln \lambda} = \frac{\hat{\alpha} k'(\hat{\alpha})}{k(\hat{\alpha})} \zeta_{\hat{\alpha},\lambda}$. Now, from the optimality condition for $\hat{\alpha}$, $\lambda k(\hat{\alpha}) w^* = \hat{w}_Z$, it must be the case that $\frac{d \ln(\lambda k(\hat{\alpha}) w^*)}{d \ln \lambda} = 0$. Hence,

$$\frac{d \ln k(\hat{\alpha})}{d \ln \lambda} = -1. \tag{C-19}$$

It follows that $\frac{\hat{\alpha} k'(\hat{\alpha})}{k(\hat{\alpha})} \zeta_{\hat{\alpha},\lambda} = -1$, so we can solve for $\zeta_{\hat{\alpha},\lambda}$ as

$$\zeta_{\hat{\alpha},\lambda} = \frac{d \ln \hat{\alpha}}{d \ln \lambda} = -\frac{k(\hat{\alpha})}{\hat{\alpha} k'(\hat{\alpha})}. \tag{C-20}$$

Given that $k'(\hat{\alpha}) > 0$, it is the case that $\zeta_{\hat{\alpha},\lambda} < 0$.

Let us now obtain $\zeta_{c(\hat{\alpha}),\lambda}$. Given that $k(\hat{\alpha}) = \frac{\hat{w}_Z}{\lambda w^*}$, we can rewrite equation (C-15) as $c(\hat{\alpha}) = \left[\left(\frac{\lambda w^*}{\hat{w}_Z}\right)^{1-\rho} K(\hat{\alpha}) + 1 - \hat{\alpha}\right]^{\frac{1}{1-\rho}}$. Hence, it follows from the envelope theorem that

$$\frac{dc(\hat{\alpha})}{d\lambda} = \left[\left(\frac{\lambda w^*}{\hat{w}_Z}\right)^{1-\rho} K(\hat{\alpha}) + 1 - \hat{\alpha}\right]^{\frac{1}{1-\rho}-1} \lambda^{-\rho} \left(\frac{w^*}{\hat{w}_Z}\right)^{1-\rho} K(\hat{\alpha})$$

Therefore,

$$\zeta_{c(\hat{\alpha}),\lambda} = \frac{dc(\hat{\alpha})}{d\lambda}\frac{\lambda}{c(\hat{\alpha})} = \frac{(\lambda w^*/\hat{w}_Z)^{1-\rho} K(\hat{\alpha})}{(\lambda w^*/\hat{w}_Z)^{1-\rho} K(\hat{\alpha}) + 1 - \hat{\alpha}}.$$

Using $k(\hat{\alpha})^{\rho-1} = (\lambda w^*/\hat{w}_Z)^{1-\rho}$ and the definition of $c(\hat{\alpha})$, we rewrite the previous expression as

$$\zeta_{c(\hat{\alpha}),\lambda} = \frac{k(\hat{\alpha})^{\rho-1} K(\hat{\alpha})}{k(\hat{\alpha})^{\rho-1} K(\hat{\alpha}) + 1 - \hat{\alpha}} = 1 - (1-\hat{\alpha})c(\hat{\alpha})^{\rho-1}. \tag{C-21}$$

For $\hat{\alpha} \in (0,1)$, we have that $(1-\hat{\alpha})c(\hat{\alpha})^{\rho-1} > 0$ because $c(\hat{\alpha}) > 0$. Also, $c(\hat{\alpha})^{1-\rho} = k(\hat{\alpha})^{\rho-1}K(\hat{\alpha}) + 1 - \hat{\alpha} > 1 - \hat{\alpha}$ because $k(\hat{\alpha})^{\rho-1}K(\hat{\alpha}) > 0$, and hence $(1-\hat{\alpha})c(\hat{\alpha})^{\rho-1} < 1$. It follows that $\zeta_{c(\hat{\alpha}),\lambda} \in (0,1)$. Note that for $\rho = 1$, $\zeta_{c(\hat{\alpha}),\lambda}$ reduces to $\hat{\alpha}$.

For $\zeta_{\Gamma(\hat{\alpha}),\lambda}$, we use the expression for $\Gamma(\hat{\alpha})$ in equation (24) to get

$$\zeta_{\Gamma(\hat{\alpha}),\lambda} = \frac{d\ln\Gamma(\hat{\alpha})}{d\ln\lambda} = \left(1 + \frac{c(\hat{\alpha})^{\sigma-1}}{1-c(\hat{\alpha})^{\sigma-1}}\right)\zeta_{c(\hat{\alpha}),\lambda} = \left(1 + \Gamma(\hat{\alpha})^{\sigma-1}\right)\zeta_{c(\hat{\alpha}),\lambda} > \zeta_{c(\hat{\alpha}),\lambda}. \tag{C-22}$$

For $\zeta_{\hat{\varphi}_o,\lambda}$, it follows from equation (23) that

$$\zeta_{\hat{\varphi}_o,\lambda} = \zeta_{\Gamma(\hat{\alpha}),\lambda} + \zeta_{\hat{\varphi},\lambda}. \tag{C-23}$$

To obtain $\zeta_{\hat{\varphi},\lambda}$, note that given the free entry condition—equation (26)—it must be true that $\frac{d\Pi}{d\lambda} = 0$. Using equation (22) to rewrite Π as

$$\Pi = \int_{\hat{\varphi}}^{\hat{\varphi}_o}\left(\frac{\varphi}{\hat{\varphi}}\right)^{\sigma-1} fg(\varphi)d\varphi + \int_{\hat{\varphi}_o}^{\infty}\left(\frac{\varphi}{\hat{\varphi}c(\hat{\alpha})}\right)^{\sigma-1} fg(\varphi)d\varphi - f(1-G(\hat{\varphi})) - f_o(1-G(\hat{\varphi}_o)), \tag{C-24}$$

we apply Leibiniz's rule along with $B^{\sigma-1} = \frac{f_o}{f}$, and equations (23) and (C-23) to obtain that $\frac{d\Pi}{d\lambda} = 0$ is equivalent to

$$\zeta_{\hat{\varphi},\lambda}[1-G(\hat{\varphi})]\bar{\varphi}^{\sigma-1} + \zeta_{c(\hat{\alpha}),\lambda}[1-G(\hat{\varphi}_o)]\left(\frac{\bar{\varphi}_o}{c(\hat{\alpha})}\right)^{\sigma-1} = 0, \tag{C-25}$$

where

$$\bar{\varphi} = \left[\frac{N_n}{N}\bar{\varphi}_n^{\sigma-1} + \frac{N_o}{N}\left(\frac{\bar{\varphi}_o}{c(\hat{\alpha})}\right)^{\sigma-1}\right]^{\frac{1}{\sigma-1}} \tag{C-26}$$

is the average productivity of domestic active producers, with $\bar{\varphi}_n$ and $\bar{\varphi}_o$ defined as in (A-2). Given that $\frac{N_o}{N} = \frac{1-G(\hat{\varphi}_o)}{1-G(\hat{\varphi})}$, we can solve for $\zeta_{\hat{\varphi},\lambda}$ from (C-25) as

$$\zeta_{\hat{\varphi},\lambda} = -\frac{N_o}{N}\left(\frac{\bar{\varphi}_o}{c(\hat{\alpha})\bar{\varphi}}\right)^{\sigma-1}\zeta_{c(\hat{\alpha}),\lambda}. \tag{C-27}$$

Lastly, from (19), (A-1), and (A-4) it follows that $\mu_o = \frac{N_o}{N}\left(\frac{\bar{\varphi}_o}{c(\hat{\alpha})\bar{\varphi}}\right)^{\sigma-1}$ and hence, we can rewrite (C-27) as

$$\zeta_{\hat{\varphi},\lambda} = -\mu_o \zeta_{c(\hat{\alpha}),\lambda}, \tag{C-28}$$

where $\mu_o \in (0,1)$ is the market share of offshoring firms. It follows that $\zeta_{\hat{\varphi},\lambda} \in (-\zeta_{c(\hat{\alpha}),\lambda}, 0)$. This result and equations (C-22) and (C-23) imply that $\zeta_{\hat{\varphi}_o,\lambda} > 0$. □

Proof of Proposition 1. For active firms that do not change their offshoring status, we substitute the results from the proof of Lemma 4 into equation (39) and obtain

$$\zeta_{L_s(\varphi),\lambda} = \begin{cases} (\sigma-1)\mu_o\zeta_{c(\hat{\alpha}),\lambda} > 0 & \text{if } s = n \\ \underbrace{\frac{k(\hat{\alpha})}{(1-\hat{\alpha})k'(\hat{\alpha})}}_{>0} \underbrace{-(\sigma-\rho)\zeta_{c(\hat{\alpha}),\lambda}}_{<0 \text{ if } \rho<\sigma} \underbrace{+(\sigma-1)\mu_o\zeta_{c(\hat{\alpha}),\lambda}}_{>0} & \text{if } s = o. \end{cases} \tag{C-29}$$

Hence, after a decline in λ, non-offshoring firms (with productivities between $\hat{\varphi}$ and $\hat{\varphi}_o$) decrease their employment of domestic labor ($\zeta_{L_n(\varphi),\lambda} > 0$). For offshoring firms ($\varphi \geq \hat{\varphi}_o$), the job-relocation (first term) and selection (third term) effects always cause destruction of domestic labor after a decline in λ. The productivity effect (second term), however, causes job creation if $\rho < \sigma$, job destruction if $\rho > \sigma$, and has no impact if $\rho = \sigma$. Thus, $\zeta_{L_o(\varphi),\lambda} > 0$ if $\rho \geq \sigma$.

Let us now see the case when $\rho < \sigma$. Note that the sum of the second and third terms is negative—so that after a decline in λ the job creation due to the productivity effect dominates the job destruction due to the selection effect—if and only if

$$\rho < \sigma - (\sigma-1)\mu_o \equiv \bar{\rho},$$

where $\bar{\rho} \in (1, \sigma)$. Nevertheless, even in this case the final effect is ambiguous due to the job-relocation effect. Summarizing, for continuing offshoring firms $\zeta_{L_o(\varphi),\lambda} > 0$ if $\rho \geq \bar{\rho}$ and its sign is ambiguous otherwise.

After a decline in λ, $\hat{\varphi}$ increases and $\hat{\varphi}_o$ decreases. Given our assumption that $\hat{\varphi} < \hat{\varphi}_o$, we get that the firms between the old and new $\hat{\varphi}$ die (their employment drops to zero), while the firms between the new and old $\hat{\varphi}_o$ change their offshoring status from non-offshoring to offshoring. For the last group of firms, note that we can rewrite equation (30) as

$$L_s(\varphi) = \frac{(1-\hat{\alpha}\cdot\mathbb{I}\{s=o\})(\sigma-1)}{\left(c(\hat{\alpha})^{\mathbb{I}\{s=o\}}\right)^{\sigma-\rho}\hat{w}_Z}\left(\frac{\varphi}{\hat{\varphi}}\right)^{\sigma-1} f, \tag{C-30}$$

where $\mathbb{I}\{s=o\}$ is an indicator function taking the value of 1 if the firm is offshoring. Thus, we see that for firms that change their status to offshoring after a decline in λ: *i)* $\hat{\alpha}\cdot\mathbb{I}\{s=o\}$ changes from 0 to $\hat{\alpha}$, *ii)* $\hat{\varphi}$ increases, and *iii)* $c(\hat{\alpha})^{\mathbb{I}\{s=o\}}$ changes from 1 to $c(\hat{\alpha}) < 1$. *i)* and *ii)* account, respectively, for the job-relocation and selection effects and push for a decrease in domestic labor demand of new offshoring firms. On the other hand, *iii)* accounts for the productivity effect and moves in the opposite direction to the other two effects if $\rho < \sigma$, and in the same direction if $\rho > \sigma$. □

Proof of Proposition 2. For the response of the mass of entrants, N_E, to a change in λ, note first from (21) that

$$\zeta_{P,\lambda} = -\left(\frac{\sigma-1}{\sigma-\eta}\right)\zeta_{\hat{\varphi},\lambda}. \tag{C-31}$$

Using Leibniz's rule to take the derivative of the log of N_E in (A-3) with respect to the log of λ, and using (C-31) we obtain

$$\zeta_{N_E,\lambda} = \frac{\hat{\varphi}^{\sigma-1}}{(1-G(\hat{\varphi}))\bar{\varphi}^{\sigma-1}}\left(B^{\sigma-1}g(\hat{\varphi}_o)\hat{\varphi}_o\zeta_{\hat{\varphi}_o,\lambda} + g(\hat{\varphi})\hat{\varphi}\zeta_{\hat{\varphi},\lambda}\right) + \frac{(\eta-1)(\sigma-1)}{\sigma-\eta}\zeta_{\hat{\varphi},\lambda}. \tag{C-32}$$

Using the results in the proof of Lemma 4, equations (24), (C-26), the definition of $\bar{\varphi}_o$, and $\frac{N_o}{N} = \frac{1-G(\hat{\varphi}_o)}{1-G(\hat{\varphi})}$, we get

$$\frac{\zeta_{\hat{\varphi}_o,\lambda}}{\zeta_{\hat{\varphi},\lambda}} = -\frac{1}{B^{\sigma-1}}\frac{\int_{\hat{\varphi}}^{\infty}\left(\frac{\varphi}{\hat{\varphi}}\right)^{\sigma-1}g(\varphi)d\varphi}{\int_{\hat{\varphi}_o}^{\infty}\left(\frac{\varphi}{\hat{\varphi}_o}\right)^{\sigma-1}g(\varphi)d\varphi}. \tag{C-33}$$

Substituting equation (C-33) into (C-32) and rearranging terms we get

$$\zeta_{N_E,\lambda} = -\frac{g(\hat{\varphi})\hat{\varphi}^{\sigma}\zeta_{\hat{\varphi},\lambda}(J-1)}{(1-G(\hat{\varphi}))\bar{\varphi}^{\sigma-1}} + \frac{(\eta-1)(\sigma-1)\zeta_{\hat{\varphi},\lambda}}{\sigma-\eta} \tag{C-34}$$

where

$$J = \frac{\int_1^{\infty} u^{\sigma-1}\frac{g(u\hat{\varphi})}{g(\hat{\varphi})}du}{\int_1^{\infty} u^{\sigma-1}\frac{g(u\hat{\varphi}_o)}{g(\hat{\varphi}_o)}du}. \tag{C-35}$$

As $\zeta_{\hat{\varphi},\lambda} < 0$, the first term of (C-34) is positive if $J > 1$, negative if $J < 1$, and equals zero if $J = 1$, while the second term is negative (because $\eta > 1$).

Given that $u \geq 1$, a sufficient condition for $J \geq 1$ is

$$\frac{d\left[\frac{d\ln g(\varphi)}{d\ln\varphi}\right]}{d\varphi} \leq 0 \tag{C-36}$$

for every φ. This condition is by no means restrictive, as it is satisfied by several commonly used distributions for non-negative random variables including the Pareto, lognormal, Weibull, F, Chi-squared, Exponential, Fréchet, and Gamma distributions.[1] We assume that condition (C-36) holds,

[1] To our knowledge, this list covers all the distributions that have been used in heterogeneous-firm models. The condition is satisfied with equality for the Pareto distribution.

and hence

$$\zeta_{N_E,\lambda} = \underbrace{-\frac{g(\hat{\varphi})\hat{\varphi}^{\sigma}\zeta_{\hat{\varphi},\lambda}(J-1)}{(1-G(\hat{\varphi}))\bar{\varphi}^{\sigma-1}}}_{\geq 0} + \underbrace{\frac{(\eta-1)(\sigma-1)\zeta_{\hat{\varphi},\lambda}}{\sigma-\eta}}_{<0} \tag{C-37}$$

The sign of $\zeta_{N_E,\lambda}$ is ambiguous, but it is more likely to become negative as $\eta \to \sigma$.

For the mass of firms, N, note from $N = [1-G(\hat{\varphi})]N_E$ that $\zeta_{N,\lambda} = \zeta_{N_E,\lambda} - \frac{g(\hat{\varphi})\hat{\varphi}\zeta_{\hat{\varphi},\lambda}}{1-G(\hat{\varphi})}$. Substituting (C-32) into the previous expression we get

$$\zeta_{N,\lambda} = \underbrace{\frac{\hat{\varphi}^{\sigma-1}}{(1-G(\hat{\varphi}))\bar{\varphi}^{\sigma-1}}\left(B^{\sigma-1}g(\hat{\varphi}_o)\hat{\varphi}_o\zeta_{\hat{\varphi}_o,\lambda} - g(\hat{\varphi})\hat{\varphi}\zeta_{\hat{\varphi},\lambda}\left[\left(\frac{\bar{\varphi}}{\hat{\varphi}}\right)^{\sigma-1} - 1\right]\right)}_{>0} + \underbrace{\frac{(\eta-1)(\sigma-1)}{\sigma-\eta}\zeta_{\hat{\varphi},\lambda}}_{<0}. \tag{C-38}$$

The first term is positive because $\zeta_{\hat{\varphi}_o,\lambda} > 0$, $\zeta_{\hat{\varphi},\lambda} < 0$, and $\bar{\varphi} > \hat{\varphi}$ (recall that $\bar{\varphi}$ is the average productivity conditioning on $\varphi \geq \hat{\varphi}$). The second term moves in the opposite direction and therefore, the final effect on N is ambiguous.

For the mass of offshoring firms, N_o, we know that $N_o = [1-G(\hat{\varphi}_o)]N_E$ and therefore, $\zeta_{N_o,\lambda} = \zeta_{N_E,\lambda} - \frac{g(\hat{\varphi}_o)\hat{\varphi}_o\zeta_{\hat{\varphi}_o,\lambda}}{1-G(\hat{\varphi}_o)}$. Plugging in (C-32) into the previous equation, we obtain

$$\zeta_{N_o,\lambda} = \underbrace{\frac{\hat{\varphi}^{\sigma-1}}{(1-G(\hat{\varphi}))\bar{\varphi}^{\sigma-1}}\left(-\frac{g(\hat{\varphi}_o)\hat{\varphi}_o\zeta_{\hat{\varphi}_o,\lambda}}{\hat{\varphi}^{\sigma-1}}\left[\frac{N}{N_o}\bar{\varphi}^{\sigma-1} - B^{\sigma-1}\hat{\varphi}^{\sigma-1}\right] + g(\hat{\varphi})\hat{\varphi}\zeta_{\hat{\varphi},\lambda}\right)}_{<0} + \underbrace{\frac{(\eta-1)(\sigma-1)}{\sigma-\eta}\zeta_{\hat{\varphi},\lambda}}_{<0}.$$

The first term is negative because $\zeta_{\hat{\varphi}_o,\lambda} > 0$, $\zeta_{\hat{\varphi},\lambda} < 0$, and $\frac{N}{N_o}\bar{\varphi}^{\sigma-1} > B^{\sigma-1}\hat{\varphi}^{\sigma-1}$—the last result follows from (23), (24), and (C-26). Hence, $\zeta_{N_o,\lambda} < 0$.

Lastly, for the mass of non-offshoring firms, N_n, we know that $N_n = [G(\hat{\varphi}_o) - G(\hat{\varphi})]N_E$ and therefore

$$\zeta_{N_n,\lambda} = \underbrace{\left[\frac{g(\hat{\varphi}_o)\hat{\varphi}_o\zeta_{\hat{\varphi}_o,\lambda} - g(\hat{\varphi})\hat{\varphi}\zeta_{\hat{\varphi},\lambda}}{G(\hat{\varphi}_o) - G(\hat{\varphi})}\right]}_{>0} + \zeta_{N_E,\lambda}. \tag{C-39}$$

Although the first term is positive, the ambiguity in the sign of $\zeta_{N_E,\lambda}$ carries over to $\zeta_{N_n,\lambda}$. As with N_E, N_n is more likely to increase after a decline in λ when η approaches σ.

The case with $\eta \to 1$: Note that the last term is (C-37) and (C-38) approaches zero as $\eta \to 1$, and therefore, $\zeta_{N_E,\lambda} \geq 0$ and $\zeta_{N,\lambda} > 0$ as $\eta \to 1$. From (C-39), it follows that $\zeta_{N,\lambda} > 0$. Hence, when $\eta \to 1$, a reduction in λ causes a decline in both N and N_n, and N_E either declines or remains constant. □

Proof of Proposition 3. Let us work with each of the components of equation (40).

Extensive margin. In equation (40) we have that the net extensive margin (NEM_λ) has two components: the change in domestic employment due to non-offshoring firms that stop (or start) producing due to the effect of λ on $\hat{\varphi}$, $\mathrm{EM}_{1,\lambda} = \left[-L_n(\hat{\varphi})g(\hat{\varphi})\frac{d\hat{\varphi}}{d\lambda}\right] N_E$; and the change in domestic employment due to the effect of λ on the mass of entrants, $\mathrm{EM}_{2,\lambda} = \frac{L_Z}{N_E}\frac{dN_E}{d\lambda}$.

For $\mathrm{EM}_{1,\lambda}$, we use equation (30) to rewrite it as

$$\mathrm{EM}_{1,\lambda} = \left[-\frac{(\sigma-1)fg(\hat{\varphi})\hat{\varphi}\zeta_{\hat{\varphi},\lambda}}{\lambda\hat{w}_Z}\right] N_E > 0. \tag{C-40}$$

$\mathrm{EM}_{1,\lambda}$ is positive because $\zeta_{\hat{\varphi},\lambda} < 0$.

For $\mathrm{EM}_{2,\lambda}$, note that we can rewrite it as $\mathrm{EM}_{2,\lambda} = \frac{L_Z}{\lambda}\zeta_{N_E,\lambda}$. We know from Proposition 2 that the sign of $\zeta_{N_E,\lambda}$ is ambiguous. It follows that $\mathrm{NEM}_\lambda = \mathrm{EM}_{1,\lambda} + \mathrm{EM}_{2,\lambda}$ is also ambiguous.

We will now rewrite $\mathrm{EM}_{2,\lambda}$ in a way that will allow us to make comparisons with the other margins. For $L_Z \equiv N_n\bar{L}_n + N_o\bar{L}_o$, note that given the definitions of $\bar{L}_n$ and $\bar{L}_o$ in section 4 and given Lemma 3, it follows that

$$\bar{L}_n = L_n(\bar{\varphi}_n) = \frac{(\sigma-1)}{\hat{w}_Z}\left(\frac{\bar{\varphi}_n}{\hat{\varphi}}\right)^{\sigma-1} f \quad \text{and} \quad \bar{L}_o = L_o(\bar{\varphi}_o) = \frac{(1-\hat{\alpha})(\sigma-1)}{c(\hat{\alpha})^{\sigma-\rho}\hat{w}_Z}\left(\frac{\bar{\varphi}_o}{\hat{\varphi}}\right)^{\sigma-1} f.$$

Using the expressions for N_n and N_o, along with the equations for $\zeta_{c(\hat{\alpha}),\lambda}$ and $\zeta_{\hat{\varphi},\lambda}$ from Lemma 4, we obtain

$$L_Z = \left[\frac{(1-G(\hat{\varphi}))(\sigma-1)}{\hat{w}_Z}\left(\frac{\bar{\varphi}}{\hat{\varphi}}\right)^{\sigma-1} f(1+\zeta_{\hat{\varphi},\lambda})\right] N_E. \tag{C-41}$$

Lastly, from equations (C-40), (C-34), (C-41) we can write $\mathrm{EM}_{2,\lambda} = \frac{L_Z}{\lambda}\zeta_{N_E,\lambda}$ as

$$\mathrm{EM}_{2,\lambda} = \mathrm{EM}_{1,\lambda}(1+\zeta_{\hat{\varphi},\lambda})\left[J - 1 - \frac{(\sigma-1)(\eta-1)[1-G(\hat{\varphi})]\bar{\varphi}^{\sigma-1}}{(\sigma-\eta)\hat{\varphi}^{\sigma}g(\hat{\varphi})}\right]. \tag{C-42}$$

Intensive margin. From equation (40) we know that the net intensive margin (NIM_λ) has three components: the change in domestic employment of firms that change their offshoring status, $\mathrm{IM}_{1,\lambda} = \left[(L_n(\hat{\varphi}_o) - L_o(\hat{\varphi}_o))g(\hat{\varphi}_o)\frac{d\hat{\varphi}_o}{d\lambda}\right] N_E$; the change in domestic employment of continuing non-offshoring firms, $\mathrm{IM}_{2,\lambda} = \left[\int_{\hat{\varphi}}^{\hat{\varphi}_o}\frac{dL_n(\varphi)}{d\lambda}g(\varphi)d\varphi\right] N_E$; and the change in domestic employment of continuing offshoring firms, $\mathrm{IM}_{3,\lambda} = \left[\int_{\hat{\varphi}_o}^{\infty}\frac{dL_o(\varphi)}{d\lambda}g(\varphi)d\varphi\right] N_E$.

For $\mathrm{IM}_{1,\lambda}$, we use the expressions for $L_n(\varphi)$ and $L_o(\varphi)$ in equation (30), along with equations (23), (24), and Lemma 4 to obtain

$$\mathrm{IM}_{1,\lambda} = \mathrm{EM}_{1,\lambda}\left[-\frac{B^{\sigma-1}\Gamma(\hat{\alpha})^{\sigma-1}g(\hat{\varphi}_o)\hat{\varphi}_o\zeta_{\hat{\varphi}_o,\lambda}}{g(\hat{\varphi})\hat{\varphi}\zeta_{\hat{\varphi},\lambda}}\right]\left[1 - \frac{1-\hat{\alpha}}{c(\hat{\alpha})^{\sigma-\rho}}\right].$$

Given the definition of J in (C-35), we rewrite equation (C-33) as $\frac{\zeta_{\hat{\varphi}_o,\lambda}}{\zeta_{\hat{\varphi},\lambda}} = -\frac{g(\hat{\varphi})\hat{\varphi}J}{B^{\sigma-1}g(\hat{\varphi}_o)\hat{\varphi}_o}$. Thus, the expression for $\mathrm{IM}_{1,\lambda}$ simplifies to

$$\mathrm{IM}_{1,\lambda} = \mathrm{EM}_{1,\lambda}J\Gamma(\hat{\alpha})^{\sigma-1}\left[1 - \frac{1-\hat{\alpha}}{c(\hat{\alpha})^{\sigma-\rho}}\right].$$

The sign of $\mathrm{IM}_{1,\lambda}$ is ambiguous if $\rho < \sigma$—the term in brackets determines the sign. Note that $\mathrm{IM}_{1,\lambda} < 0$, implying job creation by expansion—for new offshoring firms—after a decline in λ, if

$c(\hat{\alpha})^{\sigma-\rho} < 1-\hat{\alpha}$ (so that the productivity effect outweighs the job-relocation effect). If $\rho \geq \sigma$, it follows that $c(\hat{\alpha})^{\sigma-\rho} > 1-\hat{\alpha}$ and thus, $\mathrm{IM}_{1,\lambda} > 0$—job destruction by contraction for existing firms that begin to offshore after a decline in λ.

For $\mathrm{IM}_{2,\lambda}$, note that we can rewrite it as

$$\mathrm{IM}_{2,\lambda} = \frac{\zeta_{L_n(\varphi),\lambda}}{\lambda}\left[\int_{\hat{\varphi}}^{\hat{\varphi}_o} L_n(\varphi)g(\varphi)d\varphi\right] N_E = \frac{\zeta_{L_n(\varphi),\lambda}}{\lambda} N_n \bar{L}_n.$$

From equation (C-29) in the proof of Proposition 1 we know that $\zeta_{L_n(\varphi),\lambda} > 0$ and thus, $\mathrm{IM}_{2,\lambda} > 0$. Then, after a decline in λ, existing non-offshoring firms generate job destruction by contraction.

Following the same steps, for existing offshoring firms we obtain $\mathrm{IM}_{3,\lambda} = \frac{\zeta_{L_o(\varphi),\lambda}}{\lambda} N_o \bar{L}_o$. The sign for $\mathrm{IM}_{3,\lambda}$ is then determined by the sign of $\zeta_{L_o(\varphi),\lambda}$. Again, from equation (C-29) in the proof of Proposition 1, we get that $\mathrm{IM}_{3,\lambda} > 0$ if $\rho \geq \sigma-(\sigma-1)\mu_o$ and its sign is ambiguous if $\rho < \sigma-(\sigma-1)\mu_o$. Therefore, after a decline in λ, existing offshoring firms generate job destruction by contraction if $\rho \geq \sigma-(\sigma-1)\mu_o$ and the effect is ambiguous otherwise.

The sum of $\mathrm{IM}_{2,\lambda}$ and $\mathrm{IM}_{3,\lambda}$ is given by $\mathrm{IM}_{2,\lambda}+\mathrm{IM}_{3,\lambda} = \frac{\zeta_{L_n(\varphi),\lambda}}{\lambda} N_n \bar{L}_n + \frac{\zeta_{L_o(\varphi),\lambda}}{\lambda} N_o \bar{L}_o$. Using the expressions for N_n, N_o, $\bar{L}_n$, and $\bar{L}_o$ obtained above, along with equations (C-29) and (C-40) and Lemma 4, we rewrite the sum as

$$\mathrm{IM}_{2,\lambda}+\mathrm{IM}_{3,\lambda} = \underbrace{\frac{\mathrm{EM}_{1,\lambda}(1-G(\hat{\varphi}))\tilde{\varphi}^{\sigma-1}}{\hat{\varphi}^{\sigma} g(\hat{\varphi})}}_{>0}\left[\underbrace{-\frac{\hat{\alpha}\zeta_{\hat{\alpha},\lambda}}{c(\hat{\alpha})^{1-\rho}\zeta_{c(\hat{\alpha}),\lambda}}}_{>0}\underbrace{+(\sigma-1)(1-\mu_o)\zeta_{c(\hat{\alpha}),\lambda}}_{>0}\underbrace{-(1-\rho)(1-\zeta_{c(\hat{\alpha}),\lambda})}_{<0 \text{ if } \rho<1}\right]$$

Note that the last term inside the brackets is negative only when $\rho < 1$. More specifically, note that the sum of the second and third terms inside the brackets is only negative when

$$\rho < 1-\frac{(\sigma-1)(1-\mu_o)\zeta_{c(\hat{\alpha}),\lambda}}{1-\zeta_{c(\hat{\alpha}),\lambda}}.$$

Thus, we have that $\mathrm{IM}_{2,\lambda}+\mathrm{IM}_{3,\lambda} > 0$ if $\rho \geq 1-\frac{(\sigma-1)(1-\mu_o)\zeta_{c(\hat{\alpha}),\lambda}}{1-\zeta_{c(\hat{\alpha}),\lambda}}$. Otherwise, the sign of $\mathrm{IM}_{2,\lambda}+\mathrm{IM}_{3,\lambda}$ is ambiguous.

The net intensive margin effect (NIM_λ) is given by the sum of $\mathrm{IM}_{1,\lambda}$, $\mathrm{IM}_{2,\lambda}$, and $\mathrm{IM}_{3,\lambda}$. Even though $\mathrm{IM}_{2,\lambda}+\mathrm{IM}_{3,\lambda}$ is ambiguous only when $\rho < 1-\frac{(\sigma-1)(1-\mu_o)\zeta_{c(\hat{\alpha}),\lambda}}{1-\zeta_{c(\hat{\alpha}),\lambda}}$, the ambiguity of $\mathrm{IM}_{1,\lambda}$ when $\rho < \sigma$ is carried out to the sign of NIM_λ. Hence, $\mathrm{NIM}_\lambda > 0$ if $\rho \geq \sigma$ and its sign is ambiguous otherwise.

Overall effect. The net effect on employment, $\frac{dL_Z}{d\lambda} = \mathrm{NEM}_\lambda + \mathrm{NIM}_\lambda$, is ambiguous even if $\rho \geq \sigma$. This overall ambiguity is driven by the ambiguous effect of λ on the mass of entrants, $\mathrm{EM}_{2,\lambda}$, due to the role of η. Note that as $\eta \to \sigma$, the negative term in (C-42) tends to $-\infty$. As η declines, the negative term approaches 0 and overall job destruction after a decline in λ is more likely.

The case with $\eta \to 1$: Note that $\text{EM}_{2,\lambda} \to \text{EM}_{1,\lambda}(1+\zeta_{\hat{\varphi},\lambda})\,(J-1) \geq 0$ and

$$\text{NEM}_\lambda \equiv \text{EM}_{1,\lambda} + \text{EM}_{2,\lambda} \to \text{EM}_{1,\lambda}\left[-\zeta_{\hat{\varphi},\lambda} + (1+\zeta_{\hat{\varphi},\lambda})J\right] > 0$$

because $\zeta_{\hat{\varphi},\lambda} \in (-\zeta_{c(\hat{\alpha}),\lambda}, 0)$, $\zeta_{c(\hat{\alpha}),\lambda} \in (0,1)$, and $J \geq 1$. In addition,

$$\text{NEM}_\lambda + \text{IM}_{1,\lambda} \to \text{EM}_{1,\lambda}\left[-\zeta_{\hat{\varphi},\lambda} + J\left(1+\zeta_{\hat{\varphi},\lambda} + \Gamma(\hat{\alpha})^{\sigma-1} - \frac{\Gamma(\hat{\alpha})^{\sigma-1}(1-\hat{\alpha})}{c(\hat{\alpha})^{\sigma-\rho}}\right)\right].$$

Using $\zeta_{\hat{\varphi},\lambda} = -\mu_o\zeta_{c(\hat{\alpha}),\lambda}$, $(1-\hat{\alpha})c(\hat{\alpha})^{\rho-1} = 1-\zeta_{c(\hat{\alpha}),\lambda}$, and $\Gamma(\hat{\alpha})^{\sigma-1} = \frac{c(\hat{\alpha})^{\sigma-1}}{1-c(\hat{\alpha})^{\sigma-1}}$ into the previous expression, it simplifies to

$$\text{NEM}_\lambda + \text{IM}_{1,\lambda} \to \text{EM}_{1,\lambda}\zeta_{c(\hat{\alpha}),\lambda}\left[\mu_o + J(1-\mu_o+\Gamma(\hat{\alpha})^{\sigma-1})\right] > 0.$$

The previous expression is positive because $\mu_o \in (0,1)$, $J \geq 1$, $\Gamma(\hat{\alpha}) > 0$, and $\zeta_{c(\hat{\alpha}),\lambda} > 0$. Therefore, if $\eta \to 1$, any possible job creation due to $\text{IM}_{1,\lambda}$ is dominated by the job destruction generated at the extensive margin.

It follows that with $\eta \to 1$, $\frac{dL_Z}{d\lambda} > 0$ if $\rho \geq 1 - \frac{(\sigma-1)(1-\mu_o)\zeta_{c(\hat{\alpha}),\lambda}}{1-\zeta_{c(\hat{\alpha}),\lambda}}$, where $\frac{(\sigma-1)(1-\mu_o)\zeta_{c(\hat{\alpha}),\lambda}}{1-\zeta_{c(\hat{\alpha}),\lambda}} > 0$, and the sign is ambiguous otherwise. □

Proof of Lemma 5. Note from the condition for the optimal choice of $\hat{\alpha}$ that

$$\lambda k(\hat{\alpha})w^* = \hat{w}_Z$$

Therefore, $\frac{d\ln k(\hat{\alpha})}{d\ln \hat{w}_Z} = 1$ or $\frac{\hat{\alpha}k'(\hat{\alpha})}{k(\hat{\alpha})}\frac{d\ln\hat{\alpha}}{d\ln\hat{w}_Z} = 1$ or $\zeta_{\hat{\alpha},\hat{w}_Z} = \frac{k(\hat{\alpha})}{\hat{\alpha}k'(\hat{\alpha})} > 0$.

Let us now obtain $\zeta_{c(\hat{\alpha}),\hat{w}_Z}$. Given that $k(\hat{\alpha}) = \frac{\hat{w}_Z}{\lambda w^*}$, we can rewrite equation (C-15) as $c(\hat{\alpha}) = \left[\left(\frac{\lambda w^*}{\hat{w}_Z}\right)^{1-\rho}K(\hat{\alpha}) + 1 - \hat{\alpha}\right]^{\frac{1}{1-\rho}}$. Hence, it follows from the envelope theorem that

$$\frac{dc(\hat{\alpha})}{d\hat{w}_Z} = -\left[\left(\frac{\lambda w^*}{\hat{w}_Z}\right)^{1-\rho}K(\hat{\alpha}) + 1 - \hat{\alpha}\right]^{\frac{1}{1-\rho}-1}\frac{1}{\hat{w}_Z}\left(\frac{\lambda w^*}{\hat{w}_Z}\right)^{1-\rho}K(\hat{\alpha})$$

Therefore,

$$\zeta_{c(\hat{\alpha}),\hat{w}_Z} = \frac{dc(\hat{\alpha})}{d\hat{w}_Z}\frac{\hat{w}_Z}{c(\hat{\alpha})} = -\frac{(\lambda w^*/\hat{w}_Z)^{1-\rho}K(\hat{\alpha})}{(\lambda w^*/\hat{w}_Z)^{1-\rho}K(\hat{\alpha}) + 1 - \hat{\alpha}}.$$

Using $k(\hat{\alpha})^{\rho-1} = (\lambda w^*/\hat{w}_Z)^{1-\rho}$ and the definition of $c(\hat{\alpha})$, we rewrite the previous expression as

$$\zeta_{c(\hat{\alpha}),\hat{w}_Z} = -\frac{k(\hat{\alpha})^{\rho-1}K(\hat{\alpha})}{k(\hat{\alpha})^{\rho-1}K(\hat{\alpha}) + 1 - \hat{\alpha}} = (1-\hat{\alpha})c(\hat{\alpha})^{\rho-1} - 1 = -\zeta_{c(\hat{\alpha}),\lambda} \in (-1,0).$$

For $\zeta_{\Gamma(\hat{\alpha}),\hat{w}_Z}$, we use the expression for $\Gamma(\hat{\alpha})$ in equation (24) to get

$$\zeta_{\Gamma(\hat{\alpha}),\hat{w}_Z} = \left(1 + \frac{c(\hat{\alpha})^{\sigma-1}}{1-c(\hat{\alpha})^{\sigma-1}}\right)\zeta_{c(\hat{\alpha}),\hat{w}_Z} = \left(1+\Gamma(\hat{\alpha})^{\sigma-1}\right)\zeta_{c(\hat{\alpha}),\hat{w}_Z} < \zeta_{c(\hat{\alpha}),\hat{w}_Z} < 0.$$

To obtain $\zeta_{\hat{\varphi},\hat{w}_Z}$ we follow the same steps as in the proof for $\zeta_{\hat{\varphi},\lambda}$ in Lemma 4 to obtain that

$$\zeta_{\hat{\varphi},\hat{w}_Z} = -\mu_o \zeta_{c(\hat{\alpha}),\hat{w}_Z} \in (0, -\zeta_{c(\hat{\alpha}),\hat{w}_Z}) > 0.$$

Given that $\zeta_{\hat{\varphi}_o,\hat{w}_Z} = \zeta_{\Gamma(\hat{\alpha}),\hat{w}_Z} + \zeta_{\hat{\varphi},\hat{w}_Z}$, it follows that

$$\zeta_{\hat{\varphi}_o,\hat{w}_Z} = \left(1 - \mu_o + \Gamma(\hat{\alpha})^{\sigma-1}\right) \zeta_{c(\hat{\alpha}),\hat{w}_Z} < 0.$$

□

Proof of Proposition 5. For active firms that do not change their offshoring status we know from Lemma 5 and its proof that

$$\zeta_{L_s(\varphi),\hat{w}_Z} = \begin{cases} \underbrace{(\sigma-1)\mu_o\zeta_{c(\hat{\alpha}),\hat{w}_Z}}_{<0} - 1 < 0 & \text{if } s = n \\ \underbrace{-\frac{\hat{\alpha}}{1-\hat{\alpha}}\zeta_{\hat{\alpha},\hat{w}_Z}}_{<0} \underbrace{-(\sigma-\rho)\zeta_{c(\hat{\alpha}),\hat{w}_Z}}_{>0 \text{ if } \rho<\sigma} \underbrace{+(\sigma-1)\mu_o\zeta_{c(\hat{\alpha}),\hat{w}_Z}}_{<0} - 1 & \text{if } s = o. \end{cases}$$

Hence, after a decline in $\hat{w}_Z$, non-offshoring firms increase their employment of domestic labor ($\zeta_{L_n(\varphi),\hat{w}_Z} < 0$). For offshoring firms, the job-relocation (first term), selection (third term), and direct marginal cost (fourth term, -1) effects always cause creation of domestic labor after a decline in $\hat{w}_Z$. The productivity effect (second term), however, causes job destruction if $\rho < \sigma$, job creation if $\rho > \sigma$, and has no impact if $\rho = \sigma$. Thus, $\zeta_{L_o(\varphi),\hat{w}_Z} < 0$ if $\rho \geq \sigma$.

For the $\rho < \sigma$ case, the sum of the second, third, and fourth terms is positive if and only if

$$\rho < \underbrace{\left[\frac{1 + \zeta_{c(\hat{\alpha}),\hat{w}_Z}}{\zeta_{c(\hat{\alpha}),\hat{w}_Z}}\right]}_{<0} \underbrace{+(\sigma-1)(1-\mu_o)}_{(0,\sigma-1)} \equiv \tilde{\rho},$$

where $\tilde{\rho} < \sigma - 1$. Then, for continuing offshoring firms $\zeta_{L_o(\varphi),\hat{w}_Z} < 0$ if $\rho \geq \tilde{\rho}$ and its sign is ambiguous otherwise.

After a decline in $\hat{w}_Z$, $\hat{\varphi}$ declines and $\hat{\varphi}_o$ increases. Then, firms between the new and old $\hat{\varphi}$ are being born, while the firms between the old and new $\hat{\varphi}_o$ change their offshoring status from offshoring to non-offshoring. Note from (C-30) that for firms that stop offshoring after a decline in $\hat{w}_Z$, their demand for domestic labor increases due to: *i)* $\hat{\alpha} \cdot \mathbb{I}\{s = o\}$ changes from $\hat{\alpha}$ to 0 (job relocation), *ii)* $\hat{\varphi}$ declines (selection effect), *iii)* the direct effect of the decline in $\hat{w}_Z$. Also, $c(\hat{\alpha})^{\mathbb{I}\{s=o\}}$ changes from $c(\hat{\alpha}) < 1$ to 1 (productivity effect) and causes a decline in the demand for domestic labor if $\rho < \sigma$, and an increase if $\rho > \sigma$. Hence, the effect is ambiguous for these firms if $\rho < \sigma$, but they expand labor if $\rho \geq \sigma$.

For the response of the mass of entrants, N_E, to a change in $\hat{w}_Z$, note first from (21) that

$$\zeta_{P,\hat{w}_Z} = \left(\frac{\sigma-1}{\sigma-\eta}\right)(1-\zeta_{\hat{\varphi},\hat{w}_Z}). \tag{C-43}$$

Using Leibniz's rule to take the derivative of the log of equation (A-3) with respect to the log of $\hat{w}_Z$, and using (C-43) we obtain

$$\zeta_{N_E,\hat{w}_Z} = \frac{\hat{\varphi}^{\sigma-1}}{(1-G(\hat{\varphi}))\bar{\varphi}^{\sigma-1}}\left(B^{\sigma-1}g(\hat{\varphi}_o)\hat{\varphi}_o\zeta_{\hat{\varphi}_o,\hat{w}_Z} + g(\hat{\varphi})\hat{\varphi}\zeta_{\hat{\varphi},\hat{w}_Z}\right) - \frac{(\eta-1)(\sigma-1)}{\sigma-\eta}(1-\zeta_{\hat{\varphi},\hat{w}_Z}). \tag{C-44}$$

A useful result that spans from the proofs of Lemmas 4 and 5 is that $\zeta_{\hat{\alpha},\hat{w}_Z} = -\zeta_{\hat{\alpha},\lambda}$, $\zeta_{c(\hat{\alpha}),\hat{w}_Z} = -\zeta_{c(\hat{\alpha}),\lambda}$, $\zeta_{\hat{\varphi},\hat{w}_Z} = -\zeta_{\hat{\varphi},\lambda}$, $\zeta_{\hat{\varphi}_o,\hat{w}_Z} = -\zeta_{\hat{\varphi}_o,\lambda}$. Therefore, from the proof of Proposition 2 we can rewrite (C-44) as

$$\zeta_{N_E,\hat{w}_Z} = \frac{g(\hat{\varphi})\hat{\varphi}^{\sigma}\zeta_{\hat{\varphi},\lambda}(J-1)}{(1-G(\hat{\varphi}))\bar{\varphi}^{\sigma-1}} - \frac{(\eta-1)(\sigma-1)}{\sigma-\eta}(1+\zeta_{\hat{\varphi},\lambda}) < 0, \tag{C-45}$$

because $J \geq 1$ (condition (C-36) holds) and $\zeta_{\hat{\varphi},\lambda} \in (-1,0)$.

For the mass of firms, N, note from $N = [1-G(\hat{\varphi})]N_E$ that $\zeta_{N,\hat{w}_Z} = \zeta_{N_E,\hat{w}_Z} - \frac{g(\hat{\varphi})\hat{\varphi}\zeta_{\hat{\varphi},\hat{w}_Z}}{1-G(\hat{\varphi})}$. Given that $\zeta_{N_E,\hat{w}_Z} < 0$ and $\zeta_{\hat{\varphi},\hat{w}_Z} > 0$, it follows that $\zeta_{N,\hat{w}_Z} < 0$. For the mass of non-offshoring firms, N_n, we know that $N_n = [G(\hat{\varphi}_o) - G(\hat{\varphi})]N_E$ and therefore

$$\zeta_{N_n,\hat{w}_Z} = \left[\frac{g(\hat{\varphi}_o)\hat{\varphi}_o\zeta_{\hat{\varphi}_o,\hat{w}_Z} - g(\hat{\varphi})\hat{\varphi}\zeta_{\hat{\varphi},\hat{w}_Z}}{G(\hat{\varphi}_o)-G(\hat{\varphi})}\right] + \zeta_{N_E,\hat{w}_Z} < 0$$

because first term is negative ($\zeta_{\hat{\varphi}_o,\hat{w}_Z} < 0$, $\zeta_{\hat{\varphi},\hat{w}_Z} > 0$, and $G(\hat{\varphi}_o) > G(\hat{\varphi})$) and $\zeta_{N_E,\hat{w}_Z} < 0$. For the mass of offshoring firms, N_o, we know that $N_o = [1-G(\hat{\varphi}_o)]N_E$ and therefore, $\zeta_{N_o,\hat{w}_Z} = \zeta_{N_E,\hat{w}_Z} - \frac{g(\hat{\varphi}_o)\hat{\varphi}_o\zeta_{\hat{\varphi}_o,\hat{w}_Z}}{1-G(\hat{\varphi}_o)}$. The second term is positive, and thus, the sign of $\zeta_{N_o,\hat{w}_Z}$ is ambiguous.

For the last part of the proposition, note that

$$\frac{dL_Z}{d\hat{w}_Z} = \underbrace{\left[-L_n(\hat{\varphi})g(\hat{\varphi})\frac{d\hat{\varphi}}{d\hat{w}_Z}\right]N_E + \frac{L_Z}{N_E}\frac{dN_E}{d\hat{w}_Z}}_{\textit{Net extensive margin}} +$$
$$\underbrace{\left[\left(L_n(\hat{\varphi}_o) - L_o(\hat{\varphi}_o)\right)g(\hat{\varphi}_o)\frac{d\hat{\varphi}_o}{d\hat{w}_Z} + \int_{\hat{\varphi}}^{\hat{\varphi}_o}\frac{dL_n(\varphi)}{d\hat{w}_Z}g(\varphi)d\varphi + \int_{\hat{\varphi}_o}^{\infty}\frac{dL_o(\varphi)}{d\hat{w}_Z}g(\varphi)d\varphi\right]N_E}_{\textit{Net intensive margin}}. \tag{C-46}$$

Let us work with each of the components of equation (C-46).

Extensive margin. In equation (C-46) we have that the net extensive margin ($\text{NEM}_{\hat{w}_Z}$) has two components: the change in domestic employment due to non-offshoring firms that stop (or start) producing due to the effect of $\hat{w}_Z$ on $\hat{\varphi}$, $\text{EM}_{1,\hat{w}_Z} = \left[-L_n(\hat{\varphi})g(\hat{\varphi})\frac{d\hat{\varphi}}{d\hat{w}_Z}\right]N_E$; and the change in domestic employment due to the effect of $\hat{w}_Z$ on the mass of entrants, $\text{EM}_{2,\hat{w}_Z} = \frac{L_Z}{N_E}\frac{dN_E}{d\hat{w}_Z}$.

For $\text{EM}_{1,\hat{w}_Z}$, we use equation (30) to rewrite it as

$$\text{EM}_{1,\hat{w}_Z} = \left[-\frac{(\sigma-1)fg(\hat{\varphi})\hat{\varphi}\zeta_{\hat{\varphi},\hat{w}_Z}}{\hat{w}_Z^2}\right]N_E < 0. \tag{C-47}$$

$\mathrm{EM}_{1,\hat{w}_Z}$ is negative because $\zeta_{\hat{\varphi},\hat{w}_Z} > 0$.

For $\mathrm{EM}_{2,\hat{w}_Z}$, note that we can rewrite it as $\mathrm{EM}_{2,\hat{w}_Z} = \frac{L_Z}{\hat{w}_Z}\zeta_{N_E,\hat{w}_Z}$. We know from (C-45) that $\zeta_{N_E,\hat{w}_Z} < 0$ and hence $\mathrm{EM}_{2,\hat{w}_Z} < 0$. It follows that $\mathrm{NEM}_{\hat{w}_Z} = \mathrm{EM}_{1,\hat{w}_Z} + \mathrm{EM}_{2,\hat{w}_Z} < 0$. Thus, a decline in $\hat{w}_Z$ causes net job creation at the extensive margin.

Intensive margin. From equation (C-46) we know that the net intensive margin ($\mathrm{NIM}_{\hat{w}_Z}$) has three components: the change in domestic employment of firms that change their offshoring status, $\mathrm{IM}_{1,\hat{w}_Z} = \left[(L_n(\hat{\varphi}_o) - L_o(\hat{\varphi}_o))g(\hat{\varphi}_o)\frac{d\hat{\varphi}_o}{d\hat{w}_Z}\right] N_E$; the change in domestic employment of continuing non-offshoring firms, $\mathrm{IM}_{2,\hat{w}_Z} = \left[\int_{\hat{\varphi}}^{\hat{\varphi}_o} \frac{dL_n(\varphi)}{d\hat{w}_Z} g(\varphi)d\varphi\right] N_E$; and the change in domestic employment of continuing offshoring firms, $\mathrm{IM}_{3,\hat{w}_Z} = \left[\int_{\hat{\varphi}_o}^{\infty} \frac{dL_o(\varphi)}{d\hat{w}_Z} g(\varphi)d\varphi\right] N_E$.

For $\mathrm{IM}_{1,\hat{w}_Z}$, we use the expressions for $L_n(\varphi)$ and $L_o(\varphi)$ in equation (30), along with equations (23), (24), (C-33), and the results in the proof of Lemma 5 to obtain

$$\mathrm{IM}_{1,\hat{w}_Z} = \mathrm{EM}_{1,\hat{w}_Z} J\Gamma(\hat{\alpha})^{\sigma-1}\left[1 - \frac{1-\hat{\alpha}}{c(\hat{\alpha})^{\sigma-\rho}}\right].$$

The sign of $\mathrm{IM}_{1,\hat{w}_Z}$ is ambiguous if $\rho < \sigma$—the term in brackets determines the sign. If $\rho \geq \sigma$, it follows that $c(\hat{\alpha})^{\sigma-\rho} > 1 - \hat{\alpha}$ and thus, $\mathrm{IM}_{1,\hat{w}_Z} < 0$—job creation by expansion for existing firms that begin to offshore after a decline in $\hat{w}_Z$.

For $\mathrm{IM}_{2,\hat{w}_Z}$, note that we can rewrite it as

$$\mathrm{IM}_{2,\hat{w}_Z} = \frac{\zeta_{L_n(\varphi),\hat{w}_Z}}{\hat{w}_Z}\left[\int_{\hat{\varphi}}^{\hat{\varphi}_o} L_n(\varphi)g(\varphi)d\varphi\right] N_E = \frac{\zeta_{L_n(\varphi),\hat{w}_Z}}{\hat{w}_Z} N_n \bar{L}_n.$$

From above we know that $\zeta_{L_n(\varphi),\hat{w}_Z} < 0$ and thus, $\mathrm{IM}_{2,\hat{w}_Z} < 0$. Then, after a decline in $\hat{w}_Z$, existing non-offshoring firms create jobs by expansion.

Following the same steps, for existing offshoring firms we obtain $\mathrm{IM}_{3,\hat{w}_Z} = \frac{\zeta_{L_o(\varphi),\hat{w}_Z}}{\hat{w}_Z} N_o \bar{L}_o$. The sign for $\mathrm{IM}_{3,\hat{w}_Z}$ is then determined by the sign of $\zeta_{L_o(\varphi),\hat{w}_Z}$. Above we got that $\zeta_{L_o(\varphi),\hat{w}_Z} < 0$ if $\rho \geq \tilde{\rho}$, and then, $\mathrm{IM}_{3,\hat{w}_Z} < 0$ if $\rho \geq \tilde{\rho}$ and its sign is ambiguous if $\rho < \tilde{\rho}$. Therefore, after a decline in $\hat{w}_Z$, existing offshoring firms create jobs by expansion if if $\rho \geq \tilde{\rho}$ and the effect is ambiguous otherwise.

The net intensive margin effect ($\mathrm{NIM}_{\hat{w}_Z}$) is given by the sum of $\mathrm{IM}_{1,\hat{w}_Z}$, $\mathrm{IM}_{2,\hat{w}_Z}$, and $\mathrm{IM}_{3,\hat{w}_Z}$. The ambiguity of $\mathrm{IM}_{1,\hat{w}_Z}$ when $\rho < \sigma$ is carried out to the sign of $\mathrm{NIM}_{\hat{w}_Z}$. Hence, $\mathrm{NIM}_{\hat{w}_Z} < 0$ if $\rho \geq \sigma$ and its sign is ambiguous otherwise.

Overall effect. The net effect on employment, $\frac{dL_Z}{d\hat{w}_Z} = \mathrm{NEM}_{\hat{w}_Z} + \mathrm{NIM}_{\hat{w}_Z}$, is ambiguous if $\rho < \sigma$. Otherwise, $\frac{dL_Z}{d\hat{w}_Z} < 0$. Note that as $\eta \to \sigma$, $\mathrm{NEM}_{\hat{w}_Z}$ tends to $-\infty$. Note also that $\mathrm{NIM}_{\hat{w}_Z}$ is more likely be negative for higher ρ. Thus, overall net job creation is more likely with higher η and higher ρ. $\square$

D Fixed Costs of Offshoring and Employment

The channels through which a decline in the fixed cost of offshoring, f_o, affect employment are the same as those described for a decline in λ. The following proposition summarizes the effects of a decline in f_o on employment.

Proposition D.1. ***(The fixed cost of offshoring and employment)***

A decline in f_o causes: (i) the death of the least productive non-offshoring firms, who then destroy all their jobs; (ii) job destruction (by contraction) at surviving firms that do not change their offshoring status; (iii) an ambiguous domestic labor response for existing firms that begin to offshore if $\rho < \sigma$ (and job destruction otherwise); (iv) and increase in N_o, but ambiguous responses of N_E, N, and N_n; (v) an ambiguous domestic labor response at the extensive margin; (vi) an ambiguous response at the intensive margin if $\rho < \sigma$ (and job destruction otherwise); (vii) an overall ambiguous effect, but net job creation is more likely for higher η and lower ρ.

Proof. Parts *(i)*, *(ii)*, and *(iii)* of Proposition D.1 are the equivalent to Proposition 1 for a change in f_o. In this case, $\hat{\alpha}$ and $c(\hat{\alpha})$ are not affected by a change in f_o; that is, the elasticities $\zeta_{\hat{\alpha},f_o}$ and $\zeta_{c(\hat{\alpha}),f_o}$ are zero. Hence, from equation (30) we obtain that for active firms that do not change their offshoring status

$$\zeta_{L_s(\varphi),f_o} = -(\sigma-1)\zeta_{\hat{\varphi},f_o} \tag{D-1}$$

for $s \in \{n, o\}$. Following similar steps as those followed in the proof of Lemma 4 to obtain $\zeta_{\hat{\varphi},\lambda}$, we get that

$$\zeta_{\hat{\varphi},f_o} = -\frac{\mu_o(1-c(\hat{\alpha})^{\sigma-1})}{\sigma-1}\left(\frac{\hat{\varphi}_o}{\bar{\varphi}_o}\right)^{\sigma-1} < 0. \tag{D-2}$$

Given that $\mu_o \in (0,1)$, $c(\hat{\alpha}) \in (w^*/\hat{w}_Z, 1)$, $\hat{\varphi}_o < \bar{\varphi}_o$, and $\sigma > 1$, it follows that $\zeta_{\hat{\varphi},f_o} \in (-\frac{1}{\sigma-1}, 0)$. Therefore, plugging in equation (D-2) into (D-1), we get

$$\zeta_{L_n(\varphi),f_o} = \zeta_{L_o(\varphi),f_o} = \mu_o(1-c(\hat{\alpha})^{\sigma-1})\left(\frac{\hat{\varphi}_o}{\bar{\varphi}_o}\right)^{\sigma-1} \in (0,1) \tag{D-3}$$

for active firms (with $\varphi \geq \hat{\varphi}$) that do not change their offshoring status. For these firms, only the selection effect is present when f_o changes. Thus, after a decline in f_o, continuing firms that do not change their offshoring status destroy labor by contraction.

As with a change in λ, there will be firms that change their offshoring status after a change in f_o. From equation (23), note that $\zeta_{\hat{\varphi}_o,f_o} = \zeta_{B,f_o} + \zeta_{\hat{\varphi},f_o}$ ($\Gamma(\hat{\alpha})$ does not depend on f_o). Given that $\zeta_{B,f_o} = \frac{1}{\sigma-1}$ and equation (D-2), we obtain

$$\zeta_{\hat{\varphi}_o,f_o} = \frac{1}{\sigma-1}\left[1-\mu_o(1-c(\hat{\alpha})^{\sigma-1})\left(\frac{\hat{\varphi}_o}{\bar{\varphi}_o}\right)^{\sigma-1}\right] \in \left(0, \frac{1}{\sigma-1}\right). \tag{D-4}$$

This implies that, as with a decline in λ, $\hat{\varphi}$ increases and $\hat{\varphi}_o$ decreases with a decline in f_o. Hence, for new offshoring firms, the last paragraph of the proof of Proposition 1 applies identically.

Part *(iv)* of Proposition D.1 is the equivalent to Proposition 2 for a change in f_o. For the response of the mass of entrants, N_E, to a change in f_o, note first from (21) that

$$\zeta_{P,f_o} = -\left(\frac{\sigma-1}{\sigma-\eta}\right)\zeta_{\hat{\varphi},f_o}. \tag{D-5}$$

Using Leibniz's rule to take the derivative of the log of equation (A-3) with respect to the log of f_o, and using (D-5) we obtain

$$\zeta_{N_E,f_o} = \underbrace{\zeta_{\hat{\varphi},f_o}}_{<0}\left[\underbrace{\frac{(\sigma-1)^2}{\sigma-\eta} + \frac{g(\hat{\varphi})\hat{\varphi}^{\sigma}}{[1-G(\hat{\varphi})]\bar{\varphi}^{\sigma-1}}}_{>0}\underbrace{-\frac{(\sigma-1)g(\hat{\varphi}_o)\hat{\varphi}_o\zeta_{\hat{\varphi}_o,f_o}}{1-G(\hat{\varphi}_o)}}_{<0}\right], \tag{D-6}$$

which has an ambiguous sign.

For the mass of firms, N, note from $N = [1-G(\hat{\varphi})]N_E$ that $\zeta_{N,f_o} = \zeta_{N_E,f_o} - \frac{g(\hat{\varphi})\hat{\varphi}\zeta_{\hat{\varphi},f_o}}{1-G(\hat{\varphi})}$. Substituting (D-6) into the previous expression we get

$$\zeta_{N,f_o} = \underbrace{\zeta_{\hat{\varphi},f_o}}_{<0}\left\{\underbrace{\frac{(\sigma-1)^2}{\sigma-\eta}}_{>0}\underbrace{-\frac{g(\hat{\varphi})\hat{\varphi}}{1-G(\hat{\varphi})}\left[1-\left(\frac{\hat{\varphi}}{\bar{\varphi}}\right)^{\sigma-1}\right] - \frac{(\sigma-1)g(\hat{\varphi}_o)\hat{\varphi}_o\zeta_{\hat{\varphi}_o,f_o}}{1-G(\hat{\varphi}_o)}}_{<0}\right\}, \tag{D-7}$$

Without further assumptions on the distribution of φ, the sign of ζ_{N,f_o} is ambiguous. For the mass of offshoring firms, N_o, we know that $N_o = [1-G(\hat{\varphi}_o)]N_E$ and therefore $\zeta_{N_o,f_o} = \zeta_{N_E,f_o} - \frac{g(\hat{\varphi}_o)\hat{\varphi}_o\zeta_{\hat{\varphi}_o,f_o}}{1-G(\hat{\varphi}_o)}$. Plugging in (D-6) into the previous equation, we obtain

$$\zeta_{N_o,f_o} = -\frac{(\sigma-1)g(\hat{\varphi}_o)\hat{\varphi}_o\zeta^2_{\hat{\varphi}_o,f_o}}{1-G(\hat{\varphi}_o)} + \zeta_{\hat{\varphi},f_o}\left[\frac{(\sigma-1)^2}{\sigma-\eta} + \frac{g(\hat{\varphi})\hat{\varphi}^{\sigma}}{[1-G(\hat{\varphi})]\bar{\varphi}^{\sigma-1}}\right] < 0.$$

Then, a decline in f_o causes an increase in the mass of offshoring firms. For the mass of non-offshoring firms, N_n, we know that $N_n = [G(\hat{\varphi}_o) - G(\hat{\varphi})]N_E$ and therefore

$$\zeta_{N_n,f_o} = \left[\frac{g(\hat{\varphi}_o)\hat{\varphi}_o\zeta_{\hat{\varphi}_o,f_o} - g(\hat{\varphi})\hat{\varphi}\zeta_{\hat{\varphi},f_o}}{G(\hat{\varphi}_o)-G(\hat{\varphi})}\right] + \zeta_{N_E,f_o}.$$

As in the proof of Proposition 2 for a change in λ, the first term is positive but the ambiguity in the sign of $\zeta_{N_E,\lambda}$ carries over to $\zeta_{N_n,\lambda}$.

Parts *(v)*, *(vi)*, and *(vii)* of Proposition D.1 are the equivalent to Proposition 3 for a change in f_o. Using Leibniz's rule to take the derivative of equation (32) with respect to f_o, we decompose the effect of f_o on L_Z into its extensive- and intensive-margin components as

$$\frac{dL_Z}{df_o} = \underbrace{\left[-L_n(\hat{\varphi})g(\hat{\varphi})\frac{d\hat{\varphi}}{df_o}\right]N_E + \frac{L_Z}{N_E}\frac{dN_E}{df_o}}_{\text{Net extensive margin}} +$$
$$\underbrace{\left[\left(L_n(\hat{\varphi}_o) - L_o(\hat{\varphi}_o)\right)g(\hat{\varphi}_o)\frac{d\hat{\varphi}_o}{df_o} + \int_{\hat{\varphi}}^{\hat{\varphi}_o}\frac{dL_n(\varphi)}{df_o}g(\varphi)d\varphi + \int_{\hat{\varphi}_o}^{\infty}\frac{dL_o(\varphi)}{df_o}g(\varphi)d\varphi\right]N_E}_{\text{Net intensive margin}}. \tag{D-8}$$

Extensive margin. The net extensive margin (NEM_{f_o}) in equation (D-8) has two components: the change in domestic employment due to non-offshoring firms that stop (or start) producing due to the effect of f_o on $\hat{\varphi}$, $\text{EM}_{1,f_o} = \left[-L_n(\hat{\varphi})g(\hat{\varphi})\frac{d\hat{\varphi}}{df_o}\right]N_E$; and the change in domestic employment due to the effect of f_o on the mass of entrants, $\text{EM}_{2,f_o} = \frac{L_Z}{N_E}\frac{dN_E}{df_o}$.

For EM_{1,f_o}, we use equation (30) to rewrite it as

$$\text{EM}_{1,f_o} = \left[-\frac{(\sigma-1)fg(\hat{\varphi})\hat{\varphi}\zeta_{\hat{\varphi},f_o}}{f_o\hat{w}_Z}\right]N_E > 0. \tag{D-9}$$

EM_{1,f_o} is positive because $\zeta_{\hat{\varphi},f_o} < 0$.

For EM_{2,f_o}, note that we can rewrite it as $\text{EM}_{2,f_o} = \frac{L_Z}{f_o}\zeta_{N_E,f_o}$. Substituting equations (C-41) and (D-6) into EM_{2,f_o}—and using equations (23), (24), and (D-2)—we obtain

$$\text{EM}_{2,f_o} = \underbrace{\frac{(\sigma-1)(1+\zeta_{\hat{\varphi},\lambda})}{\hat{w}_Z}}_{>0}\left\{\underbrace{-\frac{[1-G(\hat{\varphi}_o)](\sigma-1)}{\sigma-\eta} + \frac{fg(\hat{\varphi})\hat{\varphi}\zeta_{\hat{\varphi},f_o}}{f_o}}_{<0}\underbrace{+g(\hat{\varphi}_o)\hat{\varphi}_o\zeta_{\hat{\varphi}_o,f_o}}_{>0}\right\}N_E. \tag{D-10}$$

Given (D-6), the sign of EM_{2,f_o} is also ambiguous. This ambiguity carries over to $\text{NEM}_{f_o} = \text{EM}_{1,f_o} + \text{EM}_{2,f_o}$. Note that EM_{2,f_o}—and hence NEM_{f_o}—is more likely to cause job creation after a decline in f_o if η approaches σ.

Intensive margin. From equation (D-8) we know that the net intensive margin (NIM_{f_o}) has three components: the change in domestic employment of firms that change their offshoring status, $\text{IM}_{1,f_o} = \left[(L_n(\hat{\varphi}_o) - L_o(\hat{\varphi}_o))g(\hat{\varphi}_o)\frac{d\hat{\varphi}_o}{df_o}\right]N_E$; the change in domestic employment of continuing non-offshoring firms, $\text{IM}_{2,f_o} = \left[\int_{\hat{\varphi}}^{\hat{\varphi}_o}\frac{dL_n(\varphi)}{df_o}g(\varphi)d\varphi\right]N_E$; and the change in domestic employment of continuing offshoring firms, $\text{IM}_{3,f_o} = \left[\int_{\hat{\varphi}_o}^{\infty}\frac{dL_o(\varphi)}{df_o}g(\varphi)d\varphi\right]N_E$.

For IM_{1,f_o}, we use the expressions for $L_n(\varphi)$ and $L_o(\varphi)$ in equation (30), along with equation (23) and $B^{\sigma-1} = \frac{f_o}{f}$, to obtain

$$\text{IM}_{1,f_o} = g(\hat{\varphi}_o)\hat{\varphi}_o\zeta_{\hat{\varphi}_o,f_o}\left[\frac{(\sigma-1)\Gamma(\hat{\alpha})^{\sigma-1}}{\hat{w}_Z}\left(1-\frac{1-\hat{\alpha}}{c(\hat{\alpha})^{\sigma-\rho}}\right)\right]N_E.$$

As with $\text{IM}_{1,\lambda}$, the sign of IM_{1,f_o} is ambiguous if $\rho < \sigma$. Note that if $c(\hat{\alpha})^{\sigma-\rho} \leq (1-\hat{\alpha})$—which can only happen if $\rho < \sigma$—new offshoring firms create jobs by expansion after a decline in f_o ($\text{IM}_{1,f_o} < 0$). The sign of IM_{1,f_o} is positive if $\rho \geq \sigma$.

For IM_{2,f_o}, we rewrite it as

$$\mathrm{IM}_{2,f_o} = \frac{\zeta_{L_n(\varphi),f_o}}{f_o}\left[\int_{\hat{\varphi}}^{\hat{\varphi}_o} L_n(\varphi)g(\varphi)d\varphi\right] N_E = \frac{\zeta_{L_n(\varphi),f_o}}{f_o} N_n \bar{L}_n,$$

where we use that $N_n = (G(\hat{\varphi}_o) - G(\hat{\varphi}))N_E$. From equation (D-1) we know that $\zeta_{L_n(\varphi),f_o} > 0$ and thus, $\mathrm{IM}_{2,f_o} > 0$. Similarly, for existing offshoring firms we obtain

$$\mathrm{IM}_{3,f_o} = \frac{\zeta_{L_o(\varphi),f_o}}{f_o} N_o \bar{L}_o,$$

where we use that $N_o = (1 - G(\hat{\varphi}_o))N_E$. Again, from equation (D-1) we know that $\zeta_{L_o(\varphi),f_o} > 0$ and therefore, $\mathrm{IM}_{3,f_o} > 0$. Then, after a decline in f_o, existing firms that do not change their offshoring status destroy jobs by contraction.

The net intensive margin effect (NIM_{f_o}) is given by the sum of IM_{1,f_o}, IM_{2,f_o}, and IM_{3,f_o}. Even though $\mathrm{IM}_{2,f_o} + \mathrm{IM}_{3,f_o} > 0$, the ambiguity of IM_{1,f_o} is carried out to the sign of NIM_{f_o}.

Overall effect. The net effect on employment, $\frac{dL_Z}{df_o} = \mathrm{NEM}_{f_o} + \mathrm{NIM}_{f_o}$, is ambiguous. Even though $\mathrm{EM}_{1,f_o} + \mathrm{IM}_{2,f_o} + \mathrm{IM}_{3,f_o} > 0$, the ambiguity of EM_{2,f_o} carries over to the overall effect. Note that a decline in f_o is more likely to cause net job creation for higher η (through its impact on EM_{2,f_o}) and lower ρ. □

Compared to the effects of a decline in λ, the key difference in Proposition D.1 comes from the fact that there are no productivity and job-relocation effects for continuing offshoring firms—note that $\hat{\alpha}$ and $c(\hat{\alpha})$ do not depend on f_o. As with the employment of continuing non-offshoring firms, the only effect for continuing offshoring firms is the selection effect and hence, their employment decreases after a decline in f_o. The productivity and job-relocation effects are, however, present for firms that switch from no offshoring to offshoring, and the impact of a decline in f_o on their domestic employment is ambiguous.