Published in the Journal of Monetary Economics,
Volume 65, 80-101, July 2014

Liquidity Provision, Interest Rates, and Unemployment*

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Final version: March 2014

Abstract

The effective liquidity supply of the economy—the weighted-sum of all assets that serve as media of exchange—matters for interest rates and unemployment. We formalize this idea by adding an over-the-counter market with collateralized trades to the Mortensen-Pissarides model. An increase in public liquidity through a higher supply of real government bonds raises the real interest rate, crowding out private liquidity and increasing unemployment. If unemployment is inefficiently high, keeping liquidity scarce can be socially optimal. A liquidity crisis affecting the acceptability of private assets as collateral widens the rate-of-return difference between private and public liquidity, also increasing unemployment.

JEL Classification: D82, D83, E40, E50

Keywords: unemployment, liquidity, interest rates

*This paper was prepared for the Carnegie-Rochester-NYU Conference Series on Public Policy on "A Century of Money, Banking, and Financial Instability"(November 15-16, 2013 in Pittsburgh, PA). We thank Marvin Goodfriend (the editor) and our discussant, Nicolas Petrosky-Nadeau, for comments and suggestions. We also thank Jean-Paul Carvalho, Arvind Krishnamurthy, Edouard Schaal, Mike Woodford, Cathy Zhang, and seminar participants at Bundesbank, UC Riverside, UC Santa Barbara, the University of Melbourne, the University of Paris 2, the 2013 Summer Workshop on Money, Banking, Payments, and Finance at the Federal Reserve Bank of Chicago, and the Philadelphia Search-and-Matching Workshop at the Federal Reserve Bank of Philadelphia. All errors are our responsibility. E-mail addresses: grochete@uci.edu and jantonio@uci.edu.

1 Introduction

Since its creation in 1913 a main mission of the Federal Reserve has been to provide and manage the liquidity—broadly defined as the sum of all assets that play a role as media of exchange—required to maintain an orderly financial system while achieving maximum employment, price stability, and moderate long term interest rates.[1] Aggregate liquidity management has become increasingly important due to the reliance of economic agents on safe and liquid assets to secure their various obligations arising from their lending, hedging, and payment activities (BIS, 2001) and due to the relative scarcity of such assets in the global economy (IMF, 2012). In spite of aggregate liquidity management being a key economic policy, little theoretical work has been done to relate it to macroeconomic outcomes, such as interest rates and unemployment.

The objective of this paper is to fill this void by providing a tractable framework to analyze the joint determination of aggregate liquidity, interest rates, and labor market outcomes. Along the lines of Friedman and Schwartz (1970), throughout this paper we think of aggregate liquidity as "the weighted sum of the aggregate value of all assets, the weights varying with the degree of *moneyness*." The moneyness of an asset corresponds to its ability to serve as media of exchange, means of payment, or collateral in various transactions.

On the positive side we describe how changes in the supply and demand of liquidity affect interest rates, the supply of jobs, and unemployment. We identify a market mechanism that reduces the scarcity of liquid assets, and a liquidity channel through which monetary policy has permanent effects on the labor market. Moreover, our model provides a setting to analyze financial crises by describing how adverse shocks to the acceptability of private assets as media of exchange alter the effective liquidity supply of the economy, the structure of interest rates, and the functioning of the labor market. On the normative side, we show that the optimal provision of liquidity depends on the frictions in the labor market, and we investigate a trade-off between public provision of liquidity and unemployment.

From a methodological standpoint we develop a continuous-time model of the labor market that extends the Mortensen-Pissarides framework (MP hereafter) to include a demand and supply of liquidity and endogenous interest rates. We incorporate liquidity considerations by adding an over-the-counter (OTC) market—similar to the one in Shi (1995), Trejos and Wright (1995), and Duffie, Garleanu, and Pedersen (2005)—in which *traders* exchange services financed with collateralized

[1] The Federal Reserve was created by the Federal Reserve Act of 1913 in order to prevent financial panics such as the one in 1907. It was amended in 1977 (Section 2A) to specify the objectives of monetary policy: maximum employment, stable prices, and moderate long-term interest rates. For a description of how the mandates of the Federal Reserve have been shifting over time, see Reinhart and Rogoff (2013).

loans. This OTC market aims to capture the wholesale financial markets, including repo markets, markets for derivatives, and large-value payment systems (BIS, 2001). It can also be interpreted as a market where households finance idiosyncratic consumption opportunities or firms finance investment opportunities.

As a benchmark we first describe an economy where OTC-traders can commit to repay their debt (e.g., they can be subject to large penalties if they fail to do so). The equilibrium interest rate is the rate of time preference (as in the textbook MP model) and trades in the OTC sector are socially efficient. In the rest of the paper we relax this commitment assumption in order to make liquidity essential.

In the absence of commitment, two types of assets can serve as collateral in the OTC market: claims on firms' profits, and public assets that are backed by the ability of the policymaker to raise taxes.[2] When the supply of liquidity is abundant, the interest rate is maximum and equal to the rate of time preference (as in the economy with full commitment); in this case, the total surplus in the OTC market is maximized. When the supply of liquidity is scarce—so that OTC-traders' borrowing constraints are binding—the interest rate falls below the rate of time preference. Firms respond to the lower interest rate by opening more jobs so that total market capitalization increases, which raises the private supply of liquidity in accordance with a Tobin effect.

Our model generates the following comparative statics for the supply and demand of liquidity. Regulations that raise collateral requirements for OTC transactions (IMF, 2012, p.95) lead to a reduction in the interest rate, more job creation, and lower unemployment. Moreover, if private assets are heterogeneous in terms of their pledgeability, such regulatory changes lead to collateral expansion, i.e., assets of lower quality that are subject to lower loan-to-value ratios start being used as collateral. Along the transition, market tightness—the ratio of the number of vacancies to the number of unemployed—overshoots its new steady-state value.

When liquidity is scarce, an increase in the supply of real government bonds raises the interest rate (by reducing their convenience yield), which slows job creation and reduces the private supply of liquidity. Hence, our model predicts a crowding out of the private liquidity by the public one, in accordance with the evidence from Krishnamurthy and Vissing-Jorgensen (2013). An open-market sale of bonds in exchange for currency or reserves has a redistribution effect across trades

[2] The assumption that some assets play a special role in transactions is consistent with the evidence from Krishnamurthy and Vissing-Jorgensen (2012) according to which both government bonds and highly-rated corporate bonds exhibit convenience yields. According to BIS (2001, p.8) securities accepted as collateral in derivatives markets are limited to government securities. In contrast, in repo transactions a broad range of assets can serve as collateral, including mortgage-backed securities, corporate bonds, and equity. Recently, corporate bonds have also become acceptable for cleared interest swaps.

by shrinking narrow measures of liquidity (currency) and expanding broader measures (currency plus bonds), which leads to higher interest rates and unemployment. Conversely, an increase in the inflation rate reduces the real interest rate and unemployment.

From a normative standpoint our model identifies a trade-off between liquidity provision and unemployment. This trade-off arises because of search externalities that can make the unemployment rate inefficiently high. For instance, if the wage is too high relative to the workers' contribution to the matching process (as formally defined by the Hosios condition), then it is optimal to keep liquidity scarce to lower the cost of financing firms and to promote job creation. This finding suggests that a situation where liquidity needs are not satiated might correspond to a second-best outcome.

Lastly, we use our model to describe a liquidity "crisis" that makes private claims less acceptable as collateral in OTC transactions—for example, due to more acute informational asymmetries. Such a shock leads to a higher financing cost for firms, a higher rate-of-return differential between private and public liquidity, and higher unemployment. The policymaker can mitigate the adverse effect of this shock by committing to purchase private assets at their pre-crisis price in exchange for public liquidity.

1.1 Literature

Our model is related to the literature on unemployment and financial frictions. Wasmer and Weil (2004) extend the MP model to incorporate a credit market with search frictions.[3] In contrast to our approach, there is no OTC market and no liquidity considerations to endogenize the interest rate. There is also a literature on unemployment and money/liquidity, e.g., Shi (1998), and Berentsen, Menzio, and Wright (2011). Our description of the OTC market is similar to their search market with bilateral matches. However, the interest rate faced by firms in these models is exogenous and equal to the rate of time preference since claims on firms' profits are assumed to be illiquid.[4] Moreover, from a methodological point of view, our model is written in continuous time, which considerably simplifies the presentation and dynamics since the equilibrium is unique. The assumption of claims on capital that serve as collateral in OTC markets is also used in Ferraris and Watanabe (2008), Lagos (2010), and Rocheteau and Wright (2013). In those models, however, there is no frictional labor market and no unemployment.

A formalization of OTC markets with bilateral meetings and bargaining has been developed recently in financial economics by Duffie, Garleanu, and Pedersen (2005) and Lagos and Rocheteau

[3] This model was extended and calibrated by Petrosky-Nadeau and Wasmer (2013).

[4] There are other models of money and frictional labor markets where the goods market is frictionless, i.e., it is not described as a decentralized market with search and bargaining. See, e.g., Cooley and Quadrini (2004).

(2009), among others. We adopt the closely related description from monetary theory of Shi (1995) and Trejos and Wright (1995) as it is highly tractable and emphasizes the role of assets (money) as media of exchange, which is the purpose of our analysis.

The results according to which the interest rate falls when private liquidity is scarce and an increase in public liquidity crowds private liquidity out are analogous to those in Lagos and Rocheteau (2008) in the context of a model with fiat money and capital, and to those of Williamson (2012) in a model of costly state verification where private liquidity takes the form of loans to entrepreneurs. Goodfriend (2005) develops similar ideas in the context of a model with incomplete markets where households hold bank deposits—backed by capital and government bonds—to insure their consumption from income shocks. The model generates a liquidity structure of interest rates, where spreads depend on the liquidity services provided by each asset.[5] We also share a common focus on the provision of public and private liquidity with the corporate finance literature of Holmström and Tirole (2011). In contrast to these approaches, in our model private liquidity is composed of claims on the profits of Mortensen-Pissarides firms—which allows us to establish connections with the labor market—and the demand for liquidity comes from participants in an OTC market who are anonymous and lack commitment.

There are versions of the MP model where the interest rate is endogenous. Typically, this is achieved by assuming that households are risk-averse and accumulate assets to smooth their consumption over time. For instance, Bean and Pissarides (1993) introduce a search-labor market into an overlapping-generations economy, while Andolfatto (1996) incorporate similar frictions into a real business cycle model with perfect insurance.[6] Our model differs from these approaches in that households or workers are risk-neutral and have no need for consumption smoothing. The demand for liquid assets comes entirely from OTC-traders, and the supply of liquidity is composed of both public and heterogeneous private assets. Moreover, we characterize analytically both steady-state and non-stationary equilibria.

A related literature derives a demand for safe/liquid assets in the context of turnpike and overlapping-generations economies, e.g., Woodford (1990), Caballero and Farhi (2013), and Gorton and Ordoñez (2013). In contrast to these papers, we explicitly model the labor market and relate

[5] Goodfriend and McCallum (2007) calibrate the model to the U.S. economy and show the quantitative importance for monetary policy of a broader notion of liquidity. They also suggest that differences in liquidity services' yields across assets can help explain the equity premium puzzle.

[6] As pointed out by the editor, RBC and New Keynesian models focus entirely on fluctuations in the pure intertemporal price of consumption (the pure real interest rate), while our model focuses entirely on fluctuations in the spread between the constant pure real interest rate (the rate of time preference) and the interest rate on collateral, where the spread fluctuates over time due to fluctuations in the relative demand and supply for collateral services. These two models and explanations are not incompatible.

it to the private provision of liquidity, which varies with the entry of firms.

1.2 Some facts on the market for safe and liquid assets

This section reviews succinctly some facts on the demand and supply of liquid assets, and discusses their current trends in light of the recent financial crisis. Our model features a market for liquidity that closely resembles wholesale financial markets: agents demand liquid assets to be used as collateral to secure their loans or obligations, or as means of payment. In actual economies safe and liquid assets play essential roles in the repo and derivatives markets, and for payment and settlement activities.[7] In order to gauge the magnitude of liquidity needs, note that the repo market had an average daily trading volume of about $2.3 trillion in 2008 (see Gorton and Metrick, 2010) while the gross market value of all OTC derivatives contracts at the end of 2012 was $24.7 trillion, corresponding to some gross credit exposure of $3.6 trillion (BIS, 2013c).[8] In terms of payment and settlement activities, the value of transfers on Fedwire in 2012 was equal to $600 trillion.

In our model liquid assets are supplied by the government (in the form of bonds or fiat money) and by the private sector in the form of claims on their profits. The IMF (2012) documents that of the total world supply of safe assets in 2011 ($74.4 trillion), sovereign debt accounts for 56 percent, while securitized instruments (e.g. asset-backed and mortgage-backed securities) account for 17 percent, corporate debt for 11 percent, covered bonds for 4 percent, and gold for 11 percent. Gorton, Lewellen, and Metrick (2012) document that the percentage of all U.S. assets that are "safe" has remained stable at about 33 percent since 1952.

In our model liquid assets are held by OTC-traders. In reality, banks are the largest holders of safe assets. Using 2010 data the IMF (2012) estimates that banks hold about 34 percent of worldwide government securities, while insurance companies hold 15 percent, and pension funds 7 percent (the rest were held by central banks, sovereign wealth funds, and other investors).

We will use our model to study shocks on both the supply and the demand of liquidity and their effects on the real economy and the labor market. These shocks aim to capture some of the important changes on the market for safe and liquid assets in the aftermath of the 2007-2008 financial crisis. On the supply side, a substantial amount of private assets (e.g., asset-backed securities) became illiquid due to severe informational asymmetries, reducing the effective supply of

[7] As pointed out by the IMF (2012), there are no truly safe assets—an asset is safe if it yields identical real payoffs in each state of the world. More broadly, however, an asset is considered as "safe" if it meets certain criteria such as low credit, inflation, exchange rate, and idiosyncratic risks, and high market liquidity.

[8] According to the ISDA (2012) 84 percent of all transactions in OTC derivatives are executed with the support of a collateral agreement, leading to $3.6 trillion in collateral backed trades at the end of 2011.

private liquidity. The IMF (2012) reports that 63 percent of AAA-rated mortgage-backed securities issued from 2005 to 2007 had been downgraded by 2009. The decline in the private supply of liquidity was offset by an increase in public liquidity: the amount of AAA- and AA-rated government bonds increased by \$10.8 trillion from 2007 and 2012 (BIS, 2013a).

The financial crisis raised the demand for safe and liquid assets (Fender and Lewrick, 2013). The BIS (2013c) provides estimates according to which liquidity regulation and derivatives reforms are expected to increase the demand for high-quality collateral assets by about \$4 trillion over the next several years. The BIS (2013a) highlights as sources for this shift the increase in regulation for banks and OTC derivatives markets. As an example, the Dodd-Frank Act of 2010 requires a larger fraction of derivatives transactions to be cleared in centralized exchanges with higher collateral requirements (BIS, 2013b).

2 The environment

Time is continuous and indexed by $t \in \mathbb{R}_+$. There are three categories of agents: a large measure of firms, a unit measure of workers, and a unit measure of OTC-traders. There are two types of perishable goods: a good that is consumed by all agents and that is taken as the numéraire, and a service that is produced and consumed by OTC-traders only.

Workers are endowed with one indivisible unit of labor per unit of time, they are risk-neutral, and they discount future consumption at rate $\rho > 0$, i.e., their lifetime expected utility is

$$\mathbb{E} \int_0^\infty e^{-\rho t} dC(t),$$

where $C(t)$ is their cumulative consumption of the numéraire good.[9] A firm is a technology to produce the numéraire good using a worker's indivisible labor as input.

OTC-traders exchange services in an over-the-counter market, with bilateral matching and bargaining.[10] The lifetime expected utility of an OTC-trader is

[9] The path for consumption is composed of flows (in which case $C(t)$ admits a density, $c(t)$) and lumps (in which case $C(t^+) - C(t^-) > 0$). A similar cumulative consumption process is assumed in the continuous-time models of OTC trades of Duffie, Garleanu, and Pedersen (2005).

[10] Our description of the OTC market is similar to the one used in monetary theory following Shi (1995) and Trejos and Wright (1995). According to this model the demand for liquidity originates from agents who receive random and infrequent opportunities to consume (see also Lagos and Wright, 2005 and Alvarez and Lippi, 2013). It would be straightforward to reinterpret the demand for liquidity as coming from firms with random investment opportunities (see, e.g., Holmström and Tirole, 2011, or Kiyotaki and Moore, 2005). We favor the interpretation of an OTC market for derivatives, such as the market for credit default swaps or interest rate swaps where risk-sharing services are traded for collateralized loans, or repurchase agreements. See Li, Rocheteau, and Weill (2012, Appendices G and H) for an explicit formalization. See, also, Koeppl, Monnet, and Temzelides (2008) for an application to wholesale payment and settlement systems.

$$\mathbb{E}\left\{\sum_{n=1}^{+\infty} e^{-\rho T_n}\left\{f\left[y(T_n)\right]-x(T_n)\right\}+\int_0^{\infty} e^{-\rho t} dC(t)\right\},$$

where the first term accounts for the utility from OTC trades, while the second term accounts for the utility from net consumption of the numéraire good. The process $\{T_n\}$ is Poisson with arrival rate $\alpha > 0$, and indicates the times at which the trader is matched bilaterally with another trader. Upon a bilateral match being formed, a trader is chosen at random to be either a supplier of services or a user of services. The utility from consuming y units of services is $f(y)$, where f is strictly concave, $f(0) = 0$, $f'(0) = +\infty$, and $f'(\infty) = 0$. The disutility from producing x units of services is x. For a given trader, either $y(T_n) > 0$ (he is a user of services with probability 1/2) or $x(T_n) > 0$ (he is a supplier with probability 1/2). For two traders in a match, feasibility requires that the consumption of the user, $y(T_n)$, is no greater than the production of the supplier, $x(T_n)$.

At all $t \notin \{T_n\}_{n=1}^{\infty}$ OTC-traders can consume and produce the numéraire good, $dC(t) \in \mathbb{R}$, which is not storable and can be consumed/produced in discrete quantities. The technology to consume/produce the numéraire good is not available at times $\{T_n\}$ when traders are matched. This assumption implies that the buyer of OTC services will finance its purchase with a loan to be repaid after the match is dissolved. We will consider succinctly the case where agents can commit to repay their loans. For most of the paper, however, we assume that unsecured promises to repay loans are not credible due to lack of commitment and monitoring, thereby creating a need for liquid assets from OTC market participants.[11]

Workers and firms are matched bilaterally in a labor market with search-matching frictions. The flow of hires is equal to $h(u, v)$, where u denotes the measure of unemployed workers (which is also equal to the unemployment rate) and v denotes the measure of vacancies. The matching function, h, has constant returns to scale, is strictly concave with respect to each of its arguments, and satisfies Inada-like conditions. The job finding rate of a worker is $p \equiv h(u, v)/u = h(1, \theta)$, where $\theta \equiv v/u$ is referred to as labor market tightness. The vacancy filling rate of a firm is $q \equiv h(u, v)/v = h(\theta^{-1}, 1)$. Each firm-worker match produces a constant flow of output equal to $\varphi > 0$, and the match is destroyed with Poisson arrival rate $\delta > 0$. The wage of an employed worker is $w \in (0, \varphi)$; with no loss in generality we set the income of the unemployed to 0.

In order to fill a job a firm must open a vacancy. The flow cost of advertising a vacancy in terms of the numéraire good is $\gamma > 0$. Firms' recruiting expenses are paid for by OTC-traders in exchange for the ownership in the future profits of the firm (or, equivalently, by households or firms which

[11] Following Atkeson, Eisfeldt, and Weill (2013), one can think of OTC-traders as individual traders part of large financial institutions within which assets can be reallocated. According to this interpretation, OTC-traders face trading limits determined by the amount of liquid assets that has been allocated to them by their institution.

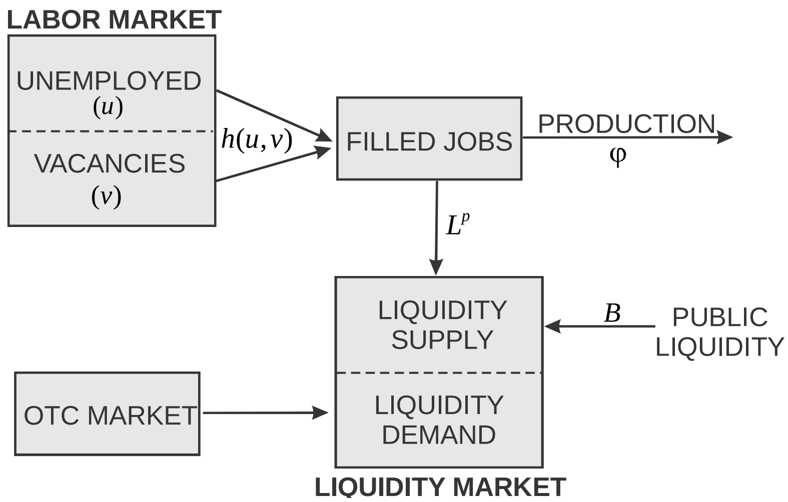

Figure 1: Sketch of the model. The top part is a labor market with search frictions captured by a matching technology $h(u, v)$. The creation of jobs contributes to the supply of liquidity. The demand for liquid assets emanate from an OTC market with bilateral meetings and bargaining.

then sell claims on filled jobs' revenue to OTC-traders in a competitive asset market). Claims on firms' revenue are liquid in the sense that they are not subject to informational asymmetries and as a result they can be used as collateral in OTC trades.[12] Later we will consider an extension where these claims are only partially acceptable as collateral.

There is a supply, B, of pure-discount government bonds that pay one unit of numéraire good according to a Poisson process with arrival rate $\varkappa > 0$, i.e., $1/\varkappa$ is a measure of the maturity of the bonds. The terminal payment of bonds is financed through lump-sum taxation.[13] Government bonds are not counterfeitable, they are perfectly divisible, and they can serve as collateral in the OTC market. The present discounted value of a bond is $\varkappa/(r+\varkappa)$, where r is the rate of return on liquid assets (both public and private). We will consider the limit when $\varkappa$ tends to infinity. The price of such a short-term bond is one and the public supply of liquidity in terms of the numéraire good is B.[14] As a summary, Figure 1 presents a sketch of the model.

[12] While we do not have an explicit intermediation sector, one interpretation is that claims on firms' profits are made liquid by a mutualization of risks engineered by financial intermediaries. See Williamson (2012) for a more detailed description of the intermediation sector.

[13] We assume here that the government can enforce the repayment of tax liabilities but it does not have the technology to monitor and enforce all private contracts. Also, by assuming lump-sum taxes we ignore a possible trade-off between the distortions induced by taxation and liquidity provision. For an analysis of this trade-off see Gorton and Ordoñez (2013).

[14] One can interpret $B < 0$ as a situation where the government withdraws liquidity from the economy, e.g., by holding private liquid assets.

3 Equilibrium under perfect commitment

We describe a benchmark economy where OTC-traders can commit to repay their unsecured debt, let say, because there is an enforcement technology that imposes large penalties for agents who default. In such an economy there is no need for liquidity.

The equilibrium of the labor market is identical to the one in the MP textbook model. A firm is a short-lived Lucas tree that generates a flow dividend, $\varphi - w$, and dies at Poisson rate δ. The price of a claim on the firm's profits is denoted by V_F and it solves $rV_F = \varphi - w - \delta V_F$, or equivalently,

$$V_F = \frac{\varphi - w}{r + \delta}. \tag{1}$$

From (1) the value of the firm is the discounted sum of its instantaneous profits, $\varphi - w$, where the effective discount rate is the real interest rate augmented with the job destruction rate. Firms are free to enter the market, in which case they open a vacancy and incur a flow cost, γ, until the job is filled. Free entry ensures that $\gamma = q(\theta)V_F$. From the definition of V_F in (1), it follows that

$$\frac{\gamma}{q(\theta)} = \frac{\varphi - w}{r + \delta}. \tag{2}$$

For a given r, (2) determines a unique $\theta > 0$. Moreover, as the real interest rate increases, the value of a filled job declines, which reduces the incentives to fund new firms, i.e., θ decreases with r.

In a steady state the number of jobs destroyed per unit of time is equal to the number of jobs created, i.e., $n\delta = p(1 - n)$, where n represents the measure of filled jobs. Solving for n we obtain

$$n = \frac{p(\theta)}{\delta + p(\theta)}. \tag{3}$$

The measure of firms increases with θ, where θ is given by (2).

Next, we turn to the OTC market. When two OTC-traders meet they must decide on a contract, (y, τ), that specifies the quantity of services produced by the seller (y) and the debt in terms of the numéraire good to be repaid by the buyer as soon as the match is dissolved (τ). By assumption, buyers can commit to repay their debt so that there is no need for collateral to secure repayment. We assume a simple bargaining protocol where the buyer of the OTC services makes a take-it-or-leave-it offer.[15] The buyer's problem is:

$$\max_{y,\tau} \{f(y) - \tau\} \quad \text{s.t.} \; -y + \tau \geq 0. \tag{4}$$

[15] It would be straightforward to generalize this trading protocol to give some bargaining power to the seller, e.g., by using the generalized Nash solution or the proportional bargaining solution. These generalizations would not affect the main insights of our model.

The buyer maximizes the utility from OTC services, $f(y)$, net of the payment made to the producer, τ, subject to the producer's participation constraint. The producer is willing to accept the buyer's offer if the payment he receives, τ, is greater or equal to the disutility of producing the OTC services, y. The solution to (4) is simply $y = y^*$, where $f'(y^*) = 1$, and $\tau = y^*$. The match surplus, $f(y) - y$, is maximum and the payment to the producer is just enough to compensate for the disutility of producing y^*.

The interest rate is determined so that households and OTC-traders are willing to hold the claims on firms' profits. Given agents' linear preferences and the absence of credit frictions under perfect commitment, the real interest rate is equal to agents' rate of time preference,

$$r = \rho. \tag{5}$$

An equilibrium of the economy under commitment can be reduced to a list, (θ, n, y, r), that solves (2), (3), (5), and $y = y^*$. The equilibrium is unique and it exhibits a dichotomy between the labor market and the OTC sector. Market tightness is determined as in the MP model and is independent of the amount of trade in the OTC sector.

4 Essential liquidity

In the following we relax the assumption of perfect commitment. If the debt issued by an OTC-trader is not secured with some collateral, then the trader has incentives to default. As a result agents will need liquidity to make payments and secure their debt obligations. We focus on steady-state equilibria where unemployment, market tightness, and the real interest rate are constant over time. We analyze the supply of private liquidity arising from the creation of firms, the demand of liquidity by OTC-traders, and the determination of the real interest rate to clear the market for liquid assets.

4.1 Supply of liquidity

We first determine the aggregate capitalization of firms as a function of the interest rate, r. This capitalization will determine the amount of private liquidity available to OTC-traders. All claims on firms' profits are part of the liquidity of the economy (we relax this assumption in Sections 7 and 8).

The value of a firm is still determined by (1) and market tightness solves (2). In order to guarantee a positive value to firms, it must be the case that $r > -\delta$. The rate of return, r, (and hence the price of firms, V_F) will be determined in equilibrium so that agents (OTC-traders) are willing to hold the entire supply of shares. In contrast to the environment under commitment, shares provide liquidity services and—as we will show later—their rate of return can be smaller than the households' rate of time preference, ρ, which we interpret as the rate of return on illiquid assets.[16]

The private provision of liquidity, defined as $L^p = nV_F$, corresponds to the total capitalization of firms. Using (3) and $V_F = \gamma/q(\theta)$, it follows that

$$L^p(r) = \frac{\gamma\theta(r)}{\delta + p\left[\theta(r)\right]} = \frac{\gamma}{\delta/\theta(r) + q\left[\theta(r)\right]}, \tag{6}$$

where $\theta(r)$ is a decreasing function of r and hence $L^{p\prime} < 0$. As the real interest rate increases, the value of filled jobs declines (from (1)) and the number of firms declines (from (2) and (3)). As a consequence, the private supply of liquidity shrinks. Moreover, $L^p(-\delta) \to \infty$ since the discounted sum of a firm's profits becomes unbounded as r approaches $-\delta$; on the other hand, $L^p(\rho)$ is positive and finite.

The sum of L^p and B is the aggregate liquidity supply of the economy, denoted $L^s(r) \equiv L^p(r) + B$. In section 7 we refine our measure of aggregate liquidity to include fiat money and nominal bonds.[17] The curve L^s is represented graphically in Figure 2 in the case where $B = 0$.

4.2 Demand for liquidity

We now turn to the demand for liquidity by OTC-traders. Let $W(a_0)$ denote the lifetime expected discounted utility of an OTC-trader holding a_0 units of liquid assets (claims on firms' profits and government bonds). The OTC-trader's problem can be written recursively as follows:

$$W(a_0) = \max_{a(t),c(t)} \left\{ \mathbb{E} \int_0^{T_1} e^{-\rho t} c(t)dt + e^{-\rho T_1} Z\left[a(T_1)\right] \right\} \tag{7}$$

$$\text{s.t.} \quad \dot{a} = ra - c - \Upsilon \tag{8}$$

$$a(0) = a_0, \tag{9}$$

where T_1 is the random time at which the trader is matched with another trader. According to (7) the trader chooses his asset holdings, $a(t)$, and consumption path, $c(t)$, so as to maximize his discounted cumulative consumption until T_1 plus the present continuation value of a trading opportunity in the OTC market at time T_1 with $a(T_1)$ units of liquid assets, $Z\left[a(T_1)\right]$.[18] Equation

[16] For search-theoretic models of the pricing of Lucas trees in monetary economies, see Geromichalos, Licari, and Suarez-Lledo (2007), Lagos (2010), and Lester, Postlewaite, and Wright (2012).

[17] In section 7 we also introduce the concept of aggregate *effective* liquidity, which is a weighted sum of assets, with the weight of each asset determined by its acceptability as means of payment. In the current case L^p and B are equally acceptable, and hence aggregate liquidity equals aggregate effective liquidity.

[18] Recall that the consumption/production of the numéraire good can be lumpy, which allows for discrete jumps in assets holdings. For a more formal treatment of these jumps, see our working paper (Rocheteau and Rodriguez-Lopez, 2013).

(8) is a budget identity according to which the trader produces the numéraire good $(-c)$ to finance the change in asset holdings $(\dot{a})$ and taxes (Υ) net of the return on those assets (ra).

From the assumption that T_1 is exponentially distributed with arrival rate α, the maximization problem in (7) can be expressed more compactly as

$$W(a_0) = \max_{a(t),c(t)} \int_0^\infty e^{-(\alpha+\rho)t} \left\{ c(t) + \alpha Z\left[a(t)\right]\right\} dt. \tag{10}$$

From (10), the OTC-trader's problem is equivalent to one where his discount rate is $\alpha + \rho$ and his instantaneous utility is $c + \alpha Z(a)$. The current-value Hamiltonian is $H(c, a, \xi) = c + \alpha Z(a) + \xi(ra - c - \Upsilon)$, where ξ denotes the costate variable. We assume, and verify later, that Z is a concave function. A solution to $\max_c H(c, a, \xi)$ exists if $\xi = 1$ for all t and thus, the value function is linear with $W'(a) = 1$. The necessary condition for the costate variable, $(\alpha + \rho)\xi = \partial H/\partial a + \dot{\xi}$, gives the following demand for liquid assets:

$$Z'(a) = 1 + \frac{\rho - r}{\alpha}. \tag{11}$$

The left side of (11) is the benefit to a trader from holding an additional unit of assets. The right side of (11) is the cost of purchasing assets worth one unit of numéraire good augmented by the expected holding cost of the asset until the next trading opportunity in the OTC market. This holding cost is equal to the difference between the rate of time preference and the real interest rate, $\rho - r$, multiplied by the average time until the next trading opportunity in the OTC market, $\mathbb{E}[T_1] = 1/\alpha$. From (11), note that the choice for a is independent of a_0, which implies that asset holdings jump instantly to their desired value, a^*, irrespective of the initial asset holdings of the trader.[19]

The expected lifetime utility of an OTC-trader holding a units of liquid assets at time T_1—when a bilateral match occurs—is $Z(a) = \left[Z^b(a) + Z^s(a)\right]/2$, where Z^b is the value of being a buyer of OTC services and Z^s is the value of being a seller of those services. By assumption the trader has an equal chance of being a buyer or a seller.

As in section 3, the buyer unilaterally sets the terms of the OTC contract, (y, τ), where τ denotes the transfer of liquid assets to the seller, who accepts or rejects the contract.[20] Assuming

[19] The result according to which agents' choice of asset holdings when entering the competitive asset market is independent from their asset holdings when leaving the OTC market is also present in the discrete-time monetary model of Lagos and Wright (2005) and the continuous-time model of OTC trades of Lagos and Rocheteau (2009).

[20] The contract can be interpreted literally as one where the buyer is paying with assets so that the trade is final. Alternatively, the contract can be viewed as a collateralized loan where the buyer promises to repay τ units of numéraire as soon as he exits the OTC market, and the repayment of the loan is secured by the deposit of τ units of liquid assets. One can find these different interpretations in the monetary search literature. For instance, in Lagos and Rocheteau (2008) agents use capital as means of payment in bilateral matches while in Ferraris and Watanabe (2008) capital is used to collateralize loans.

that the buyer holds a^b units of liquid assets, the buyer's problem—which is analogous to (4)—is then:

$$\max_{y,\tau}\left\{f(y)-\tau\right\} \quad \text{s.t.} \; -y+\tau\geq 0 \;\; \text{and} \;\; \tau\in\left[0,a^b\right]. \tag{12}$$

According to (12) the buyer chooses his consumption of OTC services, y, and a transfer of liquid assets to the seller, τ, in order to maximize his surplus from trade, $f(y)-\tau$. The inequality $-y+\tau\geq 0$ is a participation constraint for the seller: by accepting the trade the seller must provide y units of service at a cost equal to y, but in exchange he must receive $\tau\geq y$ units of liquid assets. The novelty with respect to the full-commitment problem in (4) is the feasibility condition, $\tau\in\left[0,a^b\right]$, which states that the transfer of assets from the buyer to the seller cannot be greater than the assets held by the buyer.

The solution is $y=\tau=y^*$, where $f'(y^*)=1$, if $a^b\geq y^*$; otherwise, $y=\tau=a^b$. Hence, provided that the buyer holds enough liquid assets, he can ask for the surplus-maximizing level of services, y^*, using a fraction of his assets to collateralize the trade. If the buyer does not hold enough assets—he is liquidity constrained—then he will commit all his assets to purchase the maximum amount of services that the seller is willing to produce in exchange for those assets.

Using the solution to the bargaining problem we rewrite the value functions of the OTC-trader as follows:

$$Z^b(a) = \max_{y\leq a}\left\{f(y)-y\right\}+W(a) \tag{13}$$

$$Z^s(a) = W(a) \tag{14}$$

$$Z(a) = \frac{1}{2}\max_{y\leq a}\left\{f(y)-y\right\}+W(a). \tag{15}$$

From (13) the value of the buyer is equal to the whole surplus of the match, $f(y)-y$, augmented by the continuation value of the trader, $W(a)$. From (14) the seller receives no surplus from a match. From (15) the expected value to a trader upon being matched is half of the match surplus plus his continuation value, $W(a)$. As a result, the value of an additional unit of liquid assets when matched (before the trader's role as buyer or seller is realized) is

$$Z'(a)=\frac{\left[f'(a)-1\right]^+}{2}+1, \tag{16}$$

where $[x]^+=\max\{x,0\}$. With probability 1/2 the trader is a buyer, in which case an additional unit of assets allows him to increase his surplus by $f'(y)-1$. Using the fact that $y=a$ whenever $a<y^*$, and $y=y^*$ otherwise, (16) implies that $Z(a)$ is strictly concave for all $a<y^*$ and it is linear for all $a\geq y^*$.

Denote $\sigma = \alpha/2$ the Poisson arrival rate at which an OTC-trader gets matched as a buyer. From (11) and (16), the choice of liquid assets of the trader solves

$$f'(a) = f'(y) = 1 + \frac{\rho - r}{\sigma}. \tag{17}$$

The first equality in (17) captures the fact that $y = a$ when the trader is liquidity constrained. The second equality indicates that the trader accumulates liquid assets up to the point where the marginal surplus of an OTC trade, $f'(y) - 1$, is equal to the expected holding cost of the asset, $(\rho - r)/\sigma$. Therefore, equation (17) defines the trader's individual demand for liquid assets, $a^d = f'^{-1}[1 + (\rho - r)/\sigma]$ for all $r < \rho$. If $r = \rho$, liquidity is costless to hold so that traders hold $a^d \geq y^*$. The lifetime expected utility of the OTC trader is

$$W(a) = a + \frac{\max_y \{\sigma [f(y) - y] - (\rho - r)y\} - \Upsilon}{\rho}. \tag{18}$$

From (18), the value of an OTC-trader is equal to his initial wealth plus the discounted sum of his expected surpluses in the OTC market net of taxes and the cost of holding liquid assets to finance OTC trades.

The liquidity demand correspondence, $L^d(r)$, is obtained by aggregating the demands for liquid assets across all OTC-traders. Given that there is a unit measure of OTC-traders, it follows that

$$L^d(r) = \left\{ f'^{-1}\left(1 + \frac{\rho - r}{\sigma}\right) \right\} \text{ if } r < \rho \tag{19}$$

$$= [y^*, +\infty) \text{ if } r = \rho. \tag{20}$$

As long as liquidity is costly to hold, $r < \rho$, OTC-traders hold less than is necessary to buy y^* and the demand correspondence is a singleton. The aggregate demand for liquidity declines with the holding cost of assets, $(\rho - r)/\sigma$; it declines with $\rho - r$ and it increases with σ. It follows that when $r < \rho$, there is a positive relationship between the real interest rate and the demand for liquid assets: as r increases, the cost of holding liquid assets declines, and thus traders hold more liquidity. If $r = \rho$, then the aggregate demand for liquidity corresponds to any value above y^*. The curve L^d is represented graphically in Figure 2.

4.3 Clearing the market for liquidity

The clearing condition for the market for liquidity is

$$L^s(r) \equiv B + L^p(r) \in L^d(r). \tag{21}$$

The left side of (21) is the sum of the public and private supply of liquidity. The right side of (21) is the demand for liquidity. In Figure 2 we represent both sides in the absence of public liquidity,

$B = 0$. The demand for liquidity is upward sloping, it approaches 0 when r tends to $-\infty$, and it is indeterminate above y^* when $r = \rho$. The supply of liquidity is downward sloping, it is equal to some finite quantity when $r = \rho$, and it becomes infinite when r approaches $-\delta$. It can be seen on Figure 2 that there is a unique intersection, denoted (L^e, r^e), of the demand and supply of liquidity. The introduction of public liquidity shifts the L^s curve to the right.

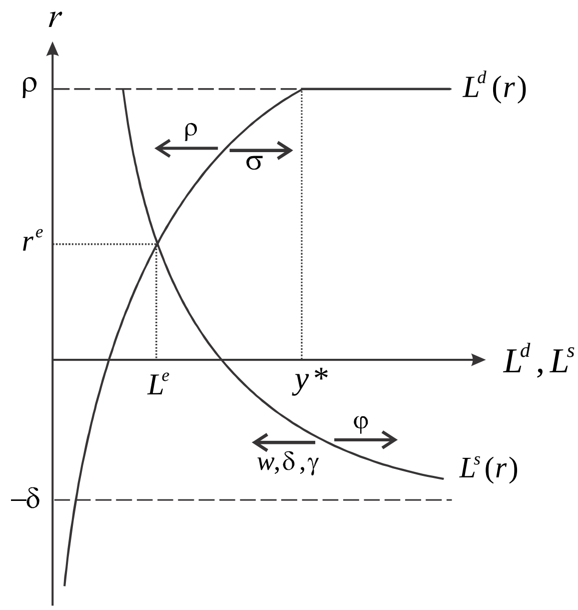

Figure 2: The market for liquidity. The supply of liquidity, $L^s(r)$, is composed of government bonds and claims on firms' profits. The demand for liquidity, $L^d(r)$, corresponds to the demand for collateral or means of payment by OTC-traders.

Definition 1 *A steady-state equilibrium is a triple,* (θ, y, r), *that solves (2), (17), and (21).*

From the discussion above and Figure 2, there is a unique r that clears the market for liquidity. In addition, θ is uniquely determined from (2) and y is uniquely determined from (17). Therefore, the steady-state equilibrium is unique. In order to characterize steady-state equilibria we distinguish two cases. In the first case liquidity is abundant in the sense that the demand for liquidity is satiated, i.e., $y = y^*$ and $r = \rho$. Graphically, the supply of liquidity intersects the demand in its horizontal part. This type of equilibrium requires $B + L^p(\rho) \geq y^*$, i.e.,

$$B + \frac{\hat{\theta}\gamma}{\delta + p(\hat{\theta})} \geq y^*, \tag{22}$$

where $\hat{\theta}$ solves

$$\frac{\gamma}{q(\hat{\theta})} = \frac{\varphi - w}{\rho + \delta}. \tag{23}$$

Condition (22) holds if firms' instantaneous profits, $\varphi - w$, are large, the cost of creating jobs, γ, is low, or the separation rate, δ, is low. From (23) note that the real interest rate is identical to the rate of time preference and hence, market tightness is determined as in the MP model. In this regime the net output in the OTC market is maximum and equal to $\sigma\left[f(y^*) - y^*\right]$ and an increase in the supply of liquidity has no effect on the real interest rate and the labor market.

Next, we consider the case in which liquidity is scarce so that the borrowing constraints of traders in the OTC market are binding, i.e., $B + L^p(\rho) < y^*$. This case corresponds to the graphical representation in Figure 2 where the equilibrium interest rate is less than the rate of time preference, $r < \rho$. From (2) and (19) the pair of endogenous variables, (θ, r), is determined jointly by the following two equations:

$$B + \frac{\theta\gamma}{\delta + p(\theta)} = f'^{-1}\left(\frac{\rho - r}{\sigma} + 1\right) \tag{24}$$

$$r = \frac{(\varphi - w)\, q(\theta)}{\gamma} - \delta. \tag{25}$$

The first condition gives a positive relationship between θ and r while the second relationship gives a negative relationship between them. The comparative statics are represented graphically in Figure 2 by arrows indicating how an increase in a parameter shifts the liquidity demand and supply curves, and they are summarized in Table 1.

exogenous $\rightarrow$ / endogeneous $\downarrow$	φ	w	δ	γ	σ	ρ	B
r	$+$	$-$	$-$	$-$	$-$	$+$	$+$
θ	$+$	$-$	$-$	$-$	$+$	$-$	$-$
u	$-$	$+$	$+$	$+$	$-$	$+$	$+$
y	$+$	$-$	$-$	$-$	$+$	$-$	$+$

Table 1: Comparative statics. Each cell indicates the sign of the partial derivative of the endogenous variable in the row with respect to the exogenous variable in the column.

Consider an increase in firm's productivity, φ. Firms become more valuable and the supply of liquidity increases, graphically L^s shifts to the right. As a consequence, both r and θ are higher, and the unemployment rate, u, is lower. OTC-traders hold more liquidity, which raises the amount of services that are produced and exchanged, y, and creates a positive spillover from the real economy to the OTC sector.

An increase in the wage, w, separation rate, δ, cost of opening a vacancy, γ, have the opposite effects on labor market outcomes and real interest rate as those stemming from an increase in productivity. As shown in Figure 2, an increase in any of these parameters shifts the private supply of liquidity to the left. For each level of $r \leq \rho$, the private supply of liquidity declines because each firm becomes less valuable when either w or δ increase, and because the steady-state number of jobs declines when γ, w, or δ increase.

An increase in σ, the frequency of meetings between OTC-traders, generates a higher demand for liquid assets. For instance, changes in regulation for OTC trades—e.g., the move of OTC derivatives contracts to central counterparties—might require a larger set of transactions to be secured with collateral.[21] Graphically, the demand for liquidity, L^d, in Figure 2 moves to the right. The price of liquid assets increases, the real interest rate declines, market tightness increases, and unemployment declines. Moreover, the increase in the private provision of liquidity due to the lower real interest rate allows traders to exchange more services, i.e., y increases. Therefore, a reform that imposes more stringent requirements on OTC trades to secure payments generates cheaper financing conditions for the real economy, thereby stimulating the private provision of liquidity.

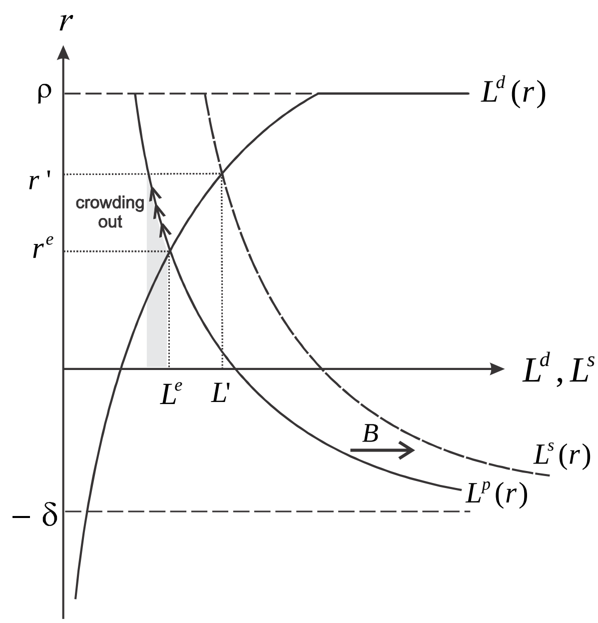

Figure 3: Public liquidity crowds out private liquidity. An increase in the supply of government bonds, B, shifts the supply of liquidity, $L^s(r)$, to the right. As a result, the equilibrium real interest rate increases from r^e to r', and the private supply of liquidity, $L^p(r)$, decreases.

Finally, let us consider an increase in the public supply of liquidity. Differentiating (24) and (25) we obtain $d\theta/dB < 0$ and $dr/dB > 0$. As shown in Figure 3, as B increases, the curve L^s moves to the right and thus, the real interest rate increases. The higher interest rate makes

[21] For instance, with probability σ^u an OTC-trader is in a match where there is some enforcement (e.g., due to reputation) and loans do not need to be secured with assets, and with probability σ^s the trader is in a match with no enforcement in which case loans need to be secured. A regulation that requires OTC trades to be collateralized would correspond to an increase in σ^s and a reduction in σ^u so that $\sigma^s + \sigma^u$ remains unchanged.

firms less valuable and drives some of them out of the market (the private supply of liquidity declines as indicated by the arrows along the curve L^p in Figure 3), market tightness decreases, and unemployment increases. Importantly, note that public liquidity crowds out private liquidity.[22] However, the crowding out is not total so that aggregate liquidity increases and the services traded in the OTC market, y, increase as well:

$$\frac{dy}{dB} = \frac{d(L^p + B)}{dB} = \left\{ \sigma f''(y)\gamma^2 \frac{\delta + p(\theta)\left[1 - \eta(\theta)\right]}{(\varphi - w)\, q'(\theta)\left[\delta + p(\theta)\right]^2} + 1 \right\}^{-1} \in (0, 1)\,.$$

These comparative statics suggest the existence of a trade-off for the policymaker between the net output of the OTC sector and the rate of unemployment. We will study the welfare implications of this trade-off in Section 6.

5 Dynamics of the labor market under scarce liquidity

We now turn to the transitional dynamics of the model. We will investigate how shocks on the supply and demand for liquidity affect the dynamics of unemployment and interest rates. Out of steady state the value of a filled job solves the following flow Bellman equation:

$$rV_F = \varphi - w - \delta V_F + \dot{V}_F. \tag{26}$$

The novelty relative to (1) is the last term on the right side that takes into account the change in the value of the firm over time. The law of motion for employment is

$$\dot{n} = p(\theta)(1 - n) - \delta n. \tag{27}$$

According to (27) the change in employment is equal to the flow of job creations—the number of unemployed, $1-n$, times the job finding rate, $p(\theta)$—net of the flow of job destructions—the number of jobs, n, times the separation rate, δ.

In order to transform (26)-(27) into a system of autonomous differential equations we use two optimality conditions. First, from the free-entry condition that must hold at any point in time, $\gamma/q(\theta) = V_F$, there is a one-to-one positive relationship between the value of a firm and market tightness, i.e., $\theta = \theta^e(V_F)$ with $\theta^{e\prime} > 0$, $\theta^e(0) = 0$, and $\theta^e(+\infty) = +\infty$. Similarly, we define the job finding rate as a function of the value of a firm, $p^e(V_F) = p\left[\theta^e(V_F)\right]$, with $p^{e\prime} > 0$, $p^e(0) = 0$, and $p^e(+\infty) = +\infty$.

[22] Similarly, in Lagos and Rocheteau (2008) a decrease in the money growth rate increases aggregate real balances (public liquidity) which reduces capital accumulation (private liquidity). See also Williamson (2012) for an environment where an increase in public liquidity crowds private liquidity out.

Second, from (17) and the market-clearing condition, $a(t) = B + n(t)V_F(t)$, the real interest rate is $r(t) = \rho - \sigma \{f'[y(t)] - 1\}$ with $y(t) = \min\{y^*, B + n(t)V_F(t)\}$. Hence, there is a one-to-one positive relationship between the real interest rate and the liquidity supply, $r = r^e(B + nV_F)$ with $r^e(0) = -\infty$ and $r^{e\prime} > 0$ if $B + nV_F < y^*$, and $r^e(B + nV_F) = \rho$ otherwise.

From these two observations we rewrite (26)-(27) as the following system of differential equations:

$$\dot{V}_F = [r^e(B + nV_F) + \delta] V_F + w - \varphi \tag{28}$$

$$\dot{n} = p^e(V_F)(1 - n) - \delta n. \tag{29}$$

The top panels of Figure 4 depict the phase diagram of the system (28)-(29). It is easy to check that the steady state is a saddle point. Hence, starting from any initial condition, n_0, there is a unique equilibrium given by the saddle path of the system.

Consider a situation of scarce liquidity, $B + n^{ss}V_F^{ss} < y^*$, as represented in the top left panel of Figure 4. If the initial level of employment is lower than its steady-state value, then the value of a filled job and market tightness are greater than their steady-state values, and they decline over time as the economy converges to its steady state along the saddle path. From (26) $\dot{V}_F < 0$ implies $V_F(t) < (\varphi - w)/[r(t) + \delta]$ for all t. Moreover, $V_F^{ss} = (\varphi - w)/(r^{ss} + \delta) < V_F(t)$. Consequently, $r(t) < r^{ss}$. The interest rate along the transition path is smaller than its steady-state value. In contrast, if liquidity is abundant, then the saddle path in the neighborhood of the steady state is such that $V_F = (\varphi - w) / (\rho + \delta)$ is constant, as shown in the top right panel of Figure 4.

Next, we illustrate the path of labor market variables under different liquidity shocks. We first describe a positive, unanticipated, liquidity demand shock that raises the frequency of trading opportunities in the OTC market, σ. The economy starts at a steady state where liquidity is scarce, $y < y^*$. In the bottom left panel of Figure 4 the economy is at the intersection of the dashed V_F-isocline and the n-isocline. An increase in σ shifts the V_F-isocline upward. The value of a firm jumps instantly upward to bring the economy to its new saddle path. Because the saddle path is downward sloping, the value of firms and market tightness overshoot their new steady-state value. Equivalently, the real interest rate falls below its new, lower steady-state value. Firms anticipate that interest rates will increase over time with the endogenous supply of private liquidity and, as a result, they open more vacancies early on following the liquidity demand shock.

Consider next an increase in the public supply of liquidity, B. In the bottom right panel of Figure 4 the V_F-isocline moves downward. The value of filled jobs and market tightness fall below their new steady-state value because agents anticipate that the increase in public liquidity will

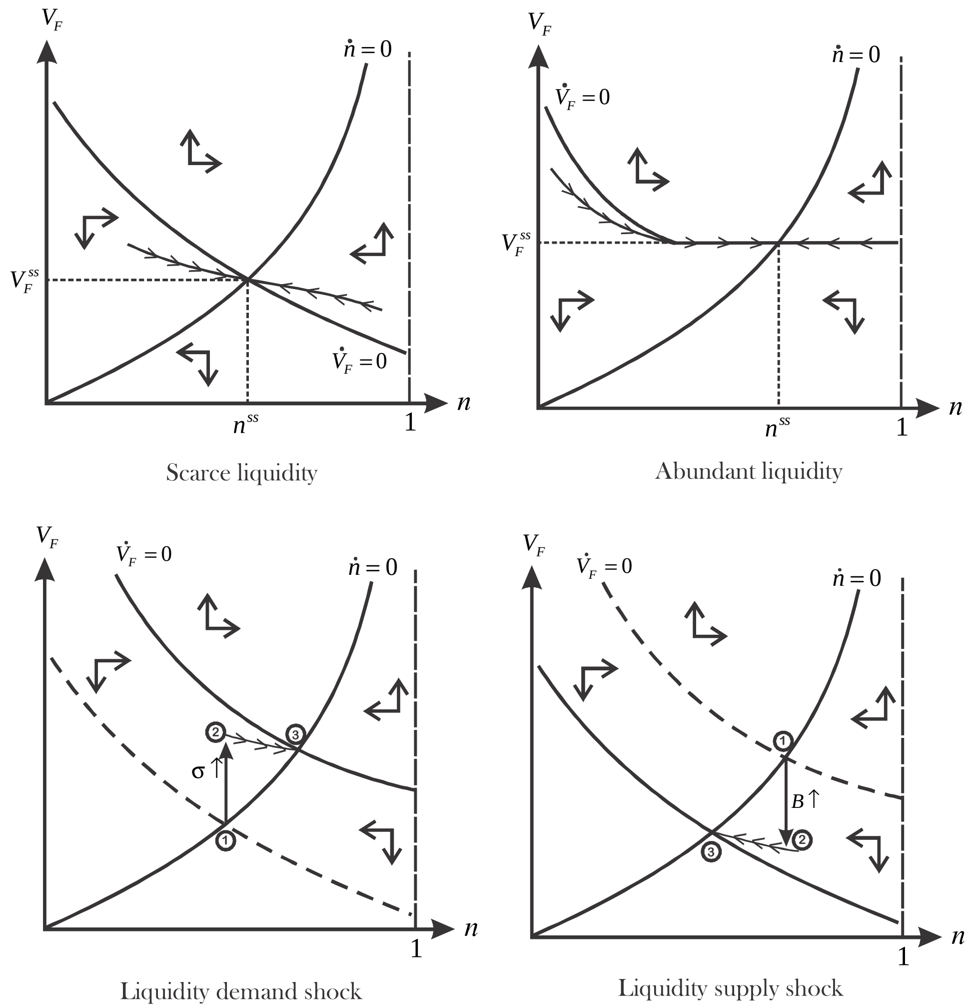

Figure 4: Phase diagrams. An equilibrium can be reduced to a pair of trajectories for employment, $n(t)$, and the value of a firm, $V_F(t)$. When liquidity is scarce (top left panel) there is a negative relationship between n and V_F along the equilibrium path. When liquidity is abundant (top right panel) V_F is constant at its steady-state value. An increase in the demand for liquid assets (bottom left panel) shifts the V_F-locus upward while an increase in the supply of government bonds (bottom right panel) has the opposite effect.

crowd out private liquidity gradually over time. As the private liquidity declines, the real interest rate decreases making it optimal for firms to postpone the opening of some vacancies.

So far we have kept the public supply of liquidity constant over time. Alternatively, the path for the public supply of liquidity could be chosen so as to keep the real interest rate constant. In that case the steady state associated with (28)-(29) is a saddle point and the saddle path is a horizontal line, as in the top right diagram in Figure 4. This means that along the transitional path the value of jobs and market tightness are constant, and only the level of employment changes over time. From (29) the path for employment is

$$n(t) = \frac{p(\theta)}{\delta + p(\theta)} + \left[n(0) - \frac{p(\theta)}{\delta + p(\theta)} \right] e^{-[\delta + p(\theta)]t}, \tag{30}$$

where $p(\theta)/[\delta + p(\theta)]$ is the steady-state employment rate.

6 Optimal liquidity provision

We have shown in Sections 4 and 5 that an increase in public liquidity raises the quantities traded in OTC matches, but it reduces job creation by raising the interest rate. This trade-off between the total surplus of the OTC market and aggregate employment is illustrated in Figure 5. In the right panel we plot the equilibrium market tightness, θ^e, as a decreasing function of the real interest rate, r, while in the left panel we plot the equilibrium output in OTC matches, y^e, as an increasing function of r. In the following we explore the normative implications of this trade-off.

We measure social welfare by the discounted sum of the utility flows of all agents (OTC-traders and workers) in the economy, i.e.,

$$\mathcal{W} = \int_0^{+\infty} e^{-\rho t} \left\{ \sigma \left\{ f\left[y(t)\right] - y(t) \right\} + n(t)\varphi - \theta(t)\left[1 - n(t)\right]\gamma \right\} dt. \tag{31}$$

According to (31) a measure $\sigma = \alpha/2$ of matches are formed in the OTC market, and in each match the net output is $f(y) - y$. In the labor market there is a measure n of filled jobs, where each job produces φ units of output. Finally, each of the $\theta(1-n) = v$ vacancies incurs a flow cost γ.

The constrained-efficient allocation is the triple $\{y(t), v(t), n(t)\}$ that maximizes $\mathcal{W}$ subject to the law of motion for employment, $\dot{n} = h(1-n, v) - \delta n$. The solution to the planner's problem is such that $y(t) = y^*$ (where $f'(y^*) = 1$) and $\theta(t) = \theta^*$ for all t, where θ^* solves

$$(\rho + \delta)\frac{\gamma}{q(\theta^*)} = \eta(\theta^*)\varphi - \left[1 - \eta(\theta^*)\right]\gamma\theta^*, \tag{32}$$

where $\eta(\theta) \equiv \theta p'(\theta)/p(\theta)$ is the elasticity of the matching function.

Next we determine the conditions under which the equilibrium allocation coincides with the constrained-efficient one. From (17) $y(t) = y^*$ in equilibrium if and only if $r(t) = \rho$ for all t, i.e., there is no cost of holding liquidity.[23] This condition, which requires that liquid assets have the same rate of return than illiquid ones, is a version of the Friedman rule. It holds if and only if liquidity is abundant, $B + L^p(\rho) \geq y^*$. In Figure 5 we denote r^{Friedman} the value of the interest rate that achieves efficiency in the OTC sector.

The comparison from (2) and (32) shows that $\theta(t) = \theta^*$ if and only if $w = w^*$, where

$$w^* = [1 - \eta(\theta^*)] (\varphi + \theta^* \gamma). \tag{33}$$

The requirement $w = w^*$ corresponds to the Hosios condition for efficiency in markets with search externalities. In Figure 5, the constrained-efficient allocation is achieved in equilibrium if the dashed line representing θ^* intersects the curve θ^e at $r = \rho$.

If $w > w^*$, then market tightness when $r = \rho$ is too low and unemployment too high relative to the constrained-efficient benchmark. This inefficiency arises because of a congestion externality according to which firms do not internalize the effect of their entry decisions on other firms' vacancy filling rate.[24] In Figure 5 the dashed line, θ^*, intersects $\theta^e(r)$ for a value of r that is below the rate of time preference, i.e., $\theta^e(\rho) < \theta^*$. We denote r^{Hosios} the value of $r < \rho$ such that $\theta^e(r) = \theta^*$.

Next, we show that a policy that keeps liquidity scarce when the unemployment rate is inefficiently high can lower the interest rate and raise welfare. In order to establish this result we focus on equilibria where the supply of liquidity, $L^p + B$, is constant over time.[25] As a result the real interest rate, r, the services traded in OTC matches, y, and market tightness, θ, are also constant. Substituting $n(t)$ by its expression in (30) into the expression for social welfare in (31), we obtain:

$$\mathcal{W} = \overbrace{\frac{\sigma [f(y) - y]}{\rho}}^{\text{OTC-sector}} + \overbrace{\frac{\varphi}{\rho} - \frac{\rho [1 - n(0)] + \delta}{[\rho + \delta + p(\theta)] \rho} (\varphi + \theta \gamma)}^{\text{Labor market}}. \tag{34}$$

The first term on the right side of (34) is the discounted sum of OTC-traders' surpluses. The second and third terms correspond to the net output in the labor market. From (34) a change in

[23] Whether or not the constrained-efficient allocation is implementable depends crucially on the choice of the trading mechanism in the OTC market. If the terms of trade in the OTC matches are determined according to the Nash solution, then the constrained-efficient allocation is not achievable since at $r = \rho$ the output is inefficiently low, $y < y^*$. See, e.g., Lagos and Wright (2005) and Rocheteau and Wright (2005) in a model with free entry of sellers.

[24] This finding is related to the result according to which in monetary economies with search frictions, social efficiency requires both the Friedman rule and the Hosios condition to hold. See Cooley and Quadrini (2004) and Berentsen, Rocheteau, and Shi (2007). In our context, the Friedman rule corresponds to $r = \rho$, i.e., liquidity is not costly to hold.

[25] This means that the policymaker adjusts the public liquidity in order to compensate for any change in private liquidity, $\dot{B} = -\dot{L}^p$.

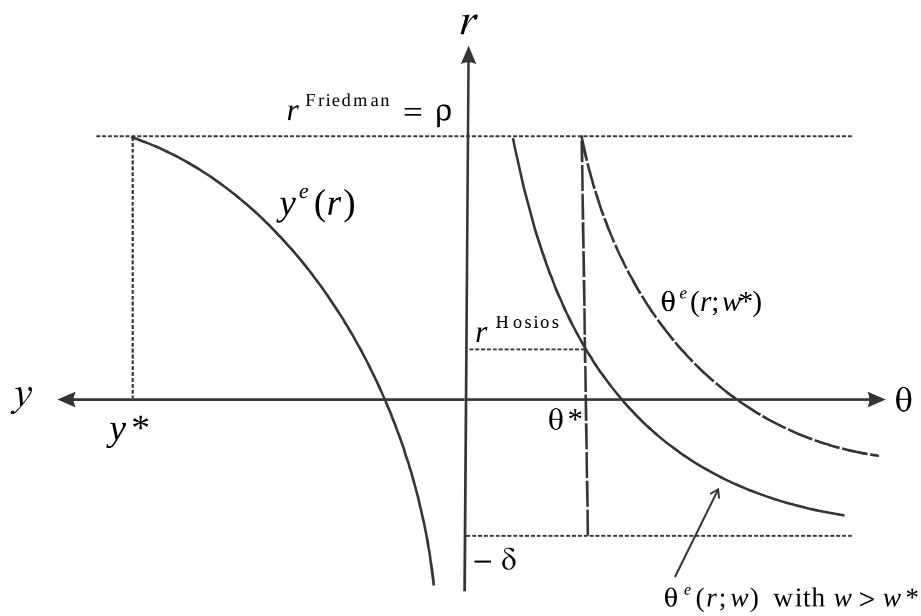

Figure 5: The trade-off between liquidity provision and labor market tightness. Equilibrium output in the OTC sector, $y^e(r)$, is equal to its socially-efficient value, y^*, when $r = \rho$ (Friedman rule). If $r = \rho$, equilibrium market tightness, $\theta^e(r, w)$, is equal to its socially-efficient value, θ^*, when $w = w^*$ (Hosios condition). If $w > w^*$ efficiency in the labor market requires $r < \rho$.

the interest rate has the following effect on welfare:

$$\frac{d\mathcal{W}}{dr} = \overbrace{\frac{\sigma\left[f'(y) - 1\right]}{\rho}\frac{\partial y}{\partial r}}^{\text{OTC-sector}} + \overbrace{\frac{\partial \mathcal{W}}{\partial \theta}\frac{\partial \theta}{\partial r}}^{\text{Labor market}}, \tag{35}$$

where

$$\frac{\partial \mathcal{W}}{\partial \theta} = -q(\theta)\left\{\frac{[1 - n(0)] + \delta/\rho}{[\rho + \delta + p(\theta)]^2}\right\}\left\{\frac{\gamma}{q(\theta)}(\rho + \delta) - \varphi\eta(\theta) + \theta\gamma\left[1 - \eta(\theta)\right]\right\}. \tag{36}$$

The first term on the right side of (35) is the effect of a change in the interest rate on the total surplus of the OTC sector. An increase in the interest reduces the cost of holding liquidity, which from (17) induces OTC-traders to hold more liquid assets and to trade larger quantities. In the left panel of Figure 5, an increase in r moves the economy upward and to the left along the y^e curve, which corresponds to a higher level of output in the OTC sector.

The second term on the right side of (35) is the effect of a change in the interest rate on the labor market. In the right panel of Figure 5, an increase in r moves the economy upward and to the left along the θ^e curve, which corresponds to a lower level of market tightness. From the discussion above, the overall effect on welfare depends on whether the wage is larger than w^* (in which case $\theta^e(\rho) < \theta^*$) or smaller than w^* (in which case $\theta^e(\rho) > \theta^*$).

Suppose that r is close to ρ, i.e., liquidity is close to being abundant. From (17) y is close to y^* so that the first term on the right side of (35) is close to 0, i.e.,

$$\left.\frac{d\mathcal{W}}{dr}\right|_{r\approx\rho} = \frac{\partial\mathcal{W}}{\partial\theta}\frac{\partial\theta}{\partial r}.$$

It follows from (36) that if $w > w^*$ then $d\mathcal{W}/dr|_{r\approx\rho} < 0$, i.e., it is optimal to keep liquidity scarce so as to reduce the interest rate below the rate of time preference. By reducing the interest rate the policymaker raises the inefficiently low market tightness and reduces the inefficiently high unemployment. It also reduces OTC-traders' surpluses by making liquidity more costly. Provided that the decrease in the interest rate is not too large, the welfare gain for the labor market outweighs the welfare loss for the OTC sector. In Figure 5, assuming $w > w^*$, a decrease in the interest rate below ρ reduces y^e below y^* in the left panel but brings θ^e closer to θ^* in the right panel. The optimal policy is such that $r \in \left(r^{\text{Hosios}}, \rho\right)$.

Lastly, if $w < w^*$ then unemployment is inefficiently low when $r = \rho$. Graphically, the dashed line, θ^*, is located to the left of $\theta^e(r)$. Reducing r below ρ would reduce y below its efficient level and it would make θ^e even higher, thereby widening the gap between θ^e and θ^*. As a result, the Friedman rule ($r = \rho$) is optimal even though it fails to implement the constrained-efficient allocation.

7 Monetary policy

In this section we investigate how liquidity considerations matter for the conduct of monetary policy and its effects on the labor market. We extend our model to allow for two types of public liquidity: fiat money and nominal bonds. Fiat money is an intrinsically useless asset that pays no dividend, and nominal bonds are pure discount bonds that yield one unit of fiat money at a Poisson rate equal to one. The supply of fiat money, $\mathcal{M}(t)$, and the supply of nominal bonds, $\mathcal{B}(t)$, grow at a constant rate, π. Consequently, the ratio $\mathcal{B}(t)/\mathcal{M}(t) \in \mathbb{R}$ is constant over time. The government's budget constraint is

$$\upsilon^m\mathcal{B} = \Upsilon + g_{\mathcal{M}}\mathcal{M}\upsilon^m + g_{\mathcal{B}}\mathcal{B}\upsilon^b, \tag{37}$$

where $g_{\mathcal{B}}$ and $g_{\mathcal{M}}$ denote the rates at which new bonds and money, respectively, are issued, υ^m denotes the real value of a unit of money in terms of the numéraire good, υ^b the real value of a nominal bond, and Υ is a lump-sum tax on OTC traders. According to (37) the government redeems bonds that mature, $\upsilon^m\mathcal{B}$, by raising lump-sum taxes, Υ, issuing money, $g_{\mathcal{M}}\mathcal{M}\upsilon^m$, and

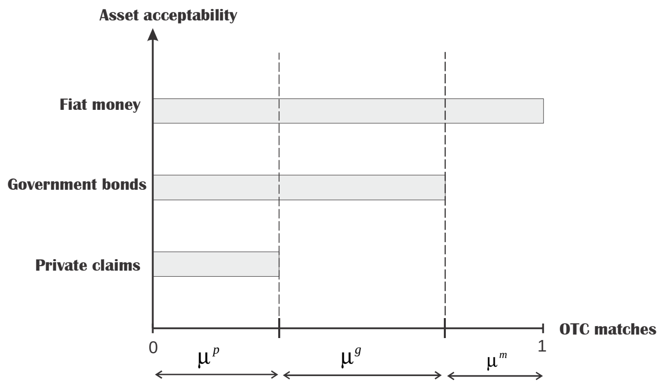

Figure 6: Assets' acceptability in OTC matches. In a fraction μ_m of all matches only fiat money is accepted. In a fraction μ^g of all matches both money and government bonds are accepted. In remaining matches all assets are accepted.

new bonds, $g_{\mathcal{B}}\mathcal{B}v^b$.[26]

We introduce liquidity differences across assets by assuming that fiat money is acceptable as means of payment in all matches while nominal bonds and private assets are eligible as collateral in a fraction of all OTC matches.[27] Formally, there is a fraction μ^m of matches where only fiat money is acceptable, and a fraction μ^g of matches where only public liquidity—i.e., fiat money and government bonds—can be used as media of exchange. In the remaining fraction of matches, $\mu^p = 1 - \mu^m - \mu^g$, all assets are acceptable.[28] See Figure 6 for a graphical representation of assets' acceptability in OTC matches.

We denote by m the real money holdings of an OTC-trader, by g his holdings of government bonds, and by a his holdings of private assets (in terms of the numéraire). The rate of return of fiat money is r^m, the rate of return of bonds is r^g, and the rate of return of private assets is r. The budget constraint of the OTC-trader becomes

$$\dot{a} + \dot{m} + \dot{g} = r^m m + r^g g + ra - c - \Upsilon. \tag{38}$$

[26] The laws of motion for $\mathcal{B}$ and $\mathcal{M}$ are $\dot{\mathcal{B}} = g_{\mathcal{B}}\mathcal{B} - \mathcal{B}$ and $\dot{\mathcal{M}} = g_{\mathcal{M}}\mathcal{M} + \mathcal{B}$. Using the assumption that $\dot{\mathcal{M}}/\mathcal{M} = \dot{\mathcal{B}}/\mathcal{B} = \pi$, these two equations can be rearranged as $g_{\mathcal{B}} = 1 + \pi$ and $g_{\mathcal{M}} = \pi - \mathcal{B}/\mathcal{M}$. Thus, given π and $\mathcal{B}/\mathcal{M}$, the rates of bonds and money creation are uniquely determined.

[27] See the ISDA (1996, Chapter 2, Section 3) for criteria for collateral eligibility in derivatives transactions. Considerations for eligibility include liquidity, volatility, collateral quality (credit rating), and time remaining to maturity, among many other factors. Also, the Federal Reserve accepts a narrow range of securities as collateral while other central banks (e.g., Bank of Japan) accept a wider set of securities—see Table 3 in BIS (2001). Private fixed income securities are less liquid than public ones because private issues tend to be smaller and more heterogeneous than those of the government, and they are more difficult to value and to hedge than government securities (BIS, 2001).

[28] One can endogenize the μ's by introducing a costly technology to authenticate assets (see, e.g., Kim, 1996 and Lester, Postlewaite, and Wright, 2012), an informational asymmetry regarding the terminal value of the asset through an adverse selection problem (e.g., Guerrieri, Shimer, and Wright, 2010 and Rocheteau, 2011) or a moral hazard problem (e.g., Li, Rocheteau, and Weill, 2012). The liquidity differences across assets can also be generated by the trading mechanism in pairwise meetings as in Zhu and Wallace (2007) and Nosal and Rocheteau (2013).

The change in the trader's wealth, $\dot{a}+\dot{m}+\dot{g}$, is equal to the interest payments on his portfolio, $r^m m + r^g g + ra$, net of consumption, c, and taxes, Υ. The main difference with respect to our benchmark model is the fact that the trader's portfolio is now composed of assets with different liquidity properties and rates of return. The continuation value of a trader upon being matched, $Z(m,g,a)$, solves

$$\begin{aligned} Z(m,g,a) = & \frac{\mu^p}{2} \max_{y^p \leq m+g+a} \{f(y^p) - y^p\} + \frac{\mu^g}{2} \max_{y^g \leq m+g} \{f(y^g) - y^g\} \\ & + \frac{\mu^m}{2} \max_{y^m \leq m} \{f(y^m) - y^m\} + W(m,g,a). \end{aligned} \tag{39}$$

With probability 1/2 the trader is the buyer in the match, in which case he can make a take-it-or-leave-it offer to the seller in order to maximize his surplus, $f(y)-y$. With probability μ^p all assets are acceptable and the trader can transfer up to $m+g+a$ in exchange for y^p. With probability μ^g fiat money and government bonds are acceptable so that the trader can transfer up to $m+g$ to purchase y^g. Lastly, with probability μ^m only fiat money is acceptable and the trader can only transfer up to m in exchange for y^m.

The OTC-trader's optimal portfolio solves

$$\frac{\rho - r}{\sigma} = \mu^p \left[f'(y^p) - 1\right] \tag{40}$$

$$\frac{\rho - r^g}{\sigma} = \mu^p \left[f'(y^p) - 1\right] + \mu^g \left[f'(y^g) - 1\right] \tag{41}$$

$$\frac{\rho - r^m}{\sigma} = \mu^p \left[f'(y^p) - 1\right] + \mu^g \left[f'(y^g) - 1\right] + \mu^m \left[f'(y^m) - 1\right]. \tag{42}$$

Equation (40) defines the optimal choice of private assets. The left side is the holding cost of private assets. The right side indicates the expected marginal surplus from holding an additional unit of private assets. Those assets can only be used in a fraction μ^p of all matches, in which case the marginal surplus of the trader is $f'(y^p)-1$. Equations (41) and (42) have a similar interpretation. Substracting (40) from (41) and (41) from (42), the rate-of-return differences across assets are

$$r - r^g = \mu^g \sigma \left[f'(y^g) - 1\right] \geq 0 \tag{43}$$

$$r^g - r^m = \mu^m \sigma \left[f'(y^m) - 1\right] \geq 0. \tag{44}$$

Private assets dominate government bonds in their rate of return provided that $\mu^g > 0$ and $y^g < y^*$. Similarly, government bonds dominate fiat money in their rate of return if $\mu^m > 0$ and $y^m < y^*$.

We focus on steady-state equilibria where the real supply of money, $M \equiv \upsilon^m \mathcal{M}$, and the real supply of bonds, $B \equiv \upsilon^b \mathcal{B}$, are constant over time. It follows that $\dot{\upsilon}^m/\upsilon^m = \dot{\upsilon}^b/\upsilon^b = -\pi$. Since fiat money yields no dividend its rate of return is

$$r^m = \dot{\upsilon}^m/\upsilon^m = -\pi. \tag{45}$$

The price of bonds solves the following asset pricing condition:

$$r^g \upsilon^b = \upsilon^m - \upsilon^b + \dot{\upsilon}^b. \tag{46}$$

According to (46) a nominal bond matures at Poisson rate equal to one, in which case the bond holder enjoys a capital gain equal to $\upsilon^m - \upsilon^b$. The last term on the right side of (46) is the change in the value of bonds over time. Using that υ^b/υ^m is the nominal price of a newly-issued bond, the nominal interest rate on government bonds is

$$i^g = \frac{\upsilon^m}{\upsilon^b} - 1 = r^g + \pi. \tag{47}$$

From the buyer-takes-all bargaining procedure, the quantity traded in an OTC match is the minimum between the real value of the buyer's acceptable assets in that match and the socially efficient quantity. By market clearing this gives:

$$y^m = \min\{M, y^*\} \tag{48}$$

$$y^g = \min\left\{\upsilon^m \mathcal{M} + \upsilon^b \mathcal{B}, y^*\right\} = \min\left\{M\left(1 + \frac{\mathcal{B}}{(1+i^g)\mathcal{M}}\right), y^*\right\} \tag{49}$$

$$y^p = \min\left\{\upsilon^m \mathcal{M} + \upsilon^b \mathcal{B} + L^p, y^*\right\} = \min\left\{M\left(1 + \frac{\mathcal{B}}{(1+i^g)\mathcal{M}}\right) + L^p, y^*\right\}. \tag{50}$$

Definition 2 *A steady-state equilibrium is a list, $(y^m, y^g, y^p, r^m, r^g, r, i^g, M, \theta)$, that solves (2), (40)-(42), (45), (47), and (48)-(50). An equilibrium is monetary if $M>0$.*

In this setting, we define aggregate liquidity as a collection of nested aggregates,

$$\mathcal{L}_1 \equiv M \subseteq \mathcal{L}_2 \equiv M + B \subseteq \mathcal{L}_3 \equiv M + B + L^p. \tag{51}$$

The narrowest measure of liquidity, $\mathcal{L}_1$, corresponds to the real supply of fiat money, which is acceptable as media of exchange in all trades. The intermediate aggregate, $\mathcal{L}_2$, is composed of $\mathcal{L}_1$ plus government bonds, with the latter being acceptable in a fraction $\mu^p + \mu^g$ of trades. The larger aggregate, $\mathcal{L}_3$, includes $\mathcal{L}_2$ plus private assets, the latter of which can serve as media of exchange in a fraction μ^p of trades.

Following Friedman and Schwartz (1970) we can also define a single measure of aggregate effective liquidity as "the weighted sum of the aggregate value of all assets, the weights varying with the degree of moneyness." In our context, the moneyness of an asset is measured by the acceptability of that asset in a match. Hence, one can measure aggregate effective liquidity by

$$\mathcal{L}^e = M + (\mu^g + \mu^p)B + \mu^p L^p. \tag{52}$$

Note that $\mathcal{L}^e$ can also be written as $\mathcal{L}^e = \mu^m \mathcal{L}_1 + \mu^g \mathcal{L}_2 + \mu^p \mathcal{L}_3$: in a fraction μ^m of matches only $\mathcal{L}_1$ is acceptable, in a fraction μ^g any asset in $\mathcal{L}_2$ is acceptable, and in a fraction μ^p any asset in $\mathcal{L}_3$ is acceptable. We can rewrite (48)-(50) as $y^m = \min\{\mathcal{L}_1, y^*\}$, $y^g = \min\{\mathcal{L}_2, y^*\}$, and $y^p = \min\{\mathcal{L}_3, y^*\}$.

In the following we study two types of policies: *(i)* an open-market operation that changes the ratio $\mathcal{B}/\mathcal{M}$ without affecting the inflation rate; *(ii)* a change in the rate of growth of money supply, π, keeping the ratio $\mathcal{B}/\mathcal{M}$ constant. To simplify the presentation, we assume that public bonds and private assets are perfect substitutes by setting $\mu^g = 0$.[29] (We relax this assumption in the following section.) Hence, in a fraction μ^p of matches all assets are eligible as collateral while in the remaining fraction of matches, $\mu^m = 1 - \mu^p$, only fiat money can serve as medium of exchange. From (43), $\mu^g = 0$ implies $r = r^g$, so that bonds and private claims have the same rate of return.

Under this simplification an equilibrium can be reduced to a pair (i^g, M). From (44), (47), and (48), the nominal interest rate is

$$i^g = \sigma \mu^m \left[f'(M) - 1 \right]^+ \tag{53}$$

where $[x]^+ = \max\{x, 0\}$. This relationship is a standard aggregate money demand according to which the convenience yield of fiat money, which is equalized to i^g, decreases with aggregate real balances. It is represented by the curve LPM (Liquidity Premium Money) in Figure 7. From (40), (47), and (50) we obtain a second condition,

$$i^g = \rho + \pi - \sigma \mu^p \left\{ f' \left[M \left(1 + \frac{\mathcal{B}}{(1+i^g)\mathcal{M}} \right) + L^p \left(i^g - \pi \right) \right] - 1 \right\}^+ . \tag{54}$$

As M increases, the rate-of-return difference between illiquid assets and bonds, $\rho + \pi - i^g$, declines; i.e., i^g increases. This positive relationship between i^g and M is represented by the curve LPB (Liquidity Premium Bonds) in Figure 7. Graphically, an equilibrium is obtained at the intersection of LPM and LPB.

Consider first equilibria where overall liquidity is abundant ($\mathcal{L}_3 \geq y^*$) so that $y^p = y^*$. As shown in the left panel of Figure 7, the LPM curve intersects the LPB curve in its horizontal part. From (54) $i^g = \rho + \pi$ and $r = \rho$. Therefore, the real interest rate, r, is independent of monetary factors. It follows that neither open-market operations nor changes in the rate of growth of money supply can affect market tightness and the unemployment rate. From (53), aggregate real balances are uniquely determined by

$$\rho + \pi = \sigma \mu^m \left[f'(M) - 1 \right].$$

[29] This case is similar to the one studied in Williamson (2012, section 6) where the fraction μ^p of matches are interpreted as monitored trades and the remaining $1 - \mu^p$ matches are interpreted as unmonitored trades. In addition, Williamson (2012) allows agents to reallocate their portfolios through a deposit contract arrangement after their type of meeting (monitored vs unmonitored) has been realized.

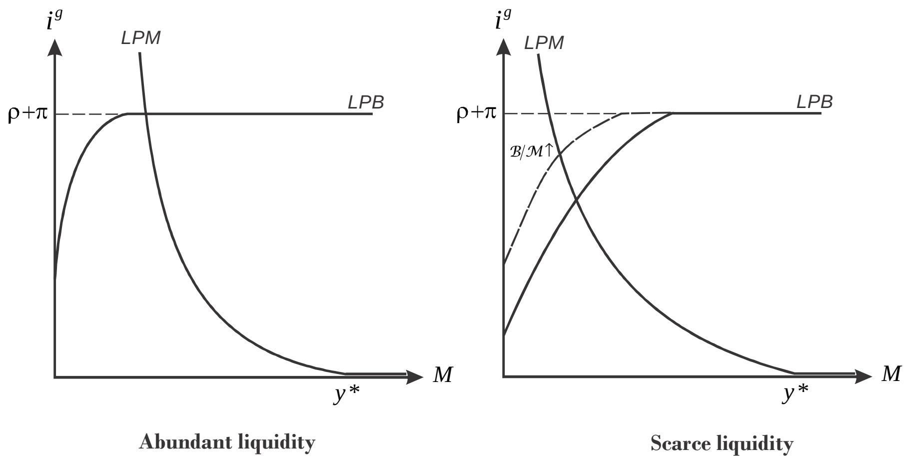

Figure 7: Real money balances and the nominal interest rate. The curve LPM is a money demand function that gives a negative relationship between real balances, M, and the nominal interest rate, i^g. The curve LPB indicates that the liquidity premium on bonds, $(\rho+\pi)-i^g$, decreases with M. When liquidity is abundant (left panel) this premium is driven to 0. When liquidity is scarce (right panel) this premium is positive and it decreases following an open-market sale of bonds.

An increase in the inflation rate reduces aggregate real balances, which reduces output in the OTC sector. From (50), an equilibrium with abundant liquidity exists if

$$M\left[1+\frac{\mathcal{B}}{(1+\rho+\pi)\mathcal{M}}\right]+L^p(\rho)\geq y^*. \tag{55}$$

Condition (55) is satisfied for low inflation rates.

Suppose next that (55) does not hold, i.e., liquidity is scarce in all matches ($\mathcal{L}_3<y^*$). As shown in the right panel of Figure 7, the LPM curve intersects the LPB curve in its upward-sloping part. We describe first the effects of an open-market sale of bonds according to which the ratio $\mathcal{B}/\mathcal{M}$ increases, but the rate of growth of $\mathcal{B}$ and $\mathcal{M}$ is unchanged. Graphically, the LPB curve moves to the left (as represented by the dashed line in the right panel of Figure 7). Hence, i^g increases and $\mathcal{L}_1=M$ declines. By changing the composition of money and bonds, the open-market operation redistributes liquidity across matches by shrinking the narrow measure of liquidity, $\mathcal{L}_1$, and expanding the broader measures of liquidity, $\mathcal{L}_2$ and $\mathcal{L}_3$.[30] As a result, in matches where money is the only means of payment, output (y^m) is lower, while in the μ^p matches where all assets are acceptable, output (y^p) is higher. The real interest rate, $r=i^g-\pi$, is higher, which leads to a lower market tightness, and a higher unemployment rate. Within the liquidity aggregate $\mathcal{L}_3$, public liquidity ($\mathcal{L}_2$) crowds out private liquidity (L^p).

[30] The net effect on aggregate effective liquidity is, however, ambiguous. Indeed, from (42), (45), and $\mu^g=0$, note that output in the OTC sector must satisfy $(\rho+\pi)/\sigma=\mu^p f'(y^p)+\mu^m f'(y^m)-1$, and therefore, the effect of the open-market sale on $\mathcal{L}^e=\mu^m y^m+\mu^p y^p$ depends on f''. In particular, if f'' is constant, then $\mathcal{L}^e$ is constant; if f'' is increasing, then $\mathcal{L}^e$ shrinks; and if f'' is decreasing, then $\mathcal{L}^e$ expands.

We now investigate the effects of an increase in the rate of growth of money supply, π, keeping the ratio $\mathcal{B}/\mathcal{M}$ constant. For given i^g the real rate of return of bonds falls. Graphically, the LPB curve moves upward and therefore, the nominal interest rate increases and aggregate real balances, M, decline, in accordance with a negative real balance effect. In order to determine the effects of inflation on the real interest rate, rewrite (54) as

$$r = \rho - \sigma\mu^p \left\{ f' \left[M \left(1 + \frac{\mathcal{B}}{(1 + r + \pi)\mathcal{M}} \right) + L^p(r) \right] - 1 \right\}^+ . \tag{56}$$

As π increases and M declines, the right side of (56) decreases and hence, r declines. In words, inflation reduces the rate of return of fiat money and the demand for real balances. As a result the demand for bonds increases, which reduces their real rate of return. In terms of the liquidity aggregates, $\mathcal{L}_1$, $\mathcal{L}_3$, and $\mathcal{L}^e$ shrink; that is, inflation reduces all measures of liquidity. The consequences for the labor market are a higher market tightness and a lower unemployment rate. Consequently, our model predicts a long-run Phillips curve according to which higher inflation is associated with lower unemployment.

8 Liquidity crises

A key role of the Federal Reserve since its creation in 1913 is to maintain the stability of the financial system by providing liquidity in situations of financial crises. We investigate this role in the context of our model by formalizing a shock resembling the one that triggered the 2007-08 financial crisis. According to Robert Lucas,

> *"the shock came because complex mortgage-related securities minted by Wall Street and certified as safe by rating agencies had become part of the effective liquidity supply of the system. All of a sudden, a whole bunch of this stuff turns out to be crap."* Wall Street Journal (09/24/2011)

In accordance with this description we consider an unanticipated shock that reduces the acceptability of private assets as collateral, i.e., μ^p falls. For instance, OTC-traders realize that there are severe informational asymmetries regarding the value of asset-backed securities. To simplify the analysis we assume that there is a single form of public liquidity, real bonds, in fixed supply B. Moreover, we endogenize the *effective liquidity supply of the system* by introducing heterogeneity across private assets and a costly process to certify assets as safe or genuine.

Suppose that when production starts the type of the firm, $\omega \in [0, 1]$, is drawn from a uniform distribution, where the type is a measure of the asset's fitness as collateral (e.g., redeployability of capital, credit rating, volatility, sensitivity to private information).[31] To each type we associate a loan-to-value ratio, $\lambda(\omega) \in [0,1]$, that specifies the fraction of the asset value that can be pledged—the buyer can obtain a loan of size $\lambda(\omega)a(\omega)$ if he commits $a(\omega)$ assets of type ω as collateral. The function $\lambda(\omega)$ is continuous and increasing, with $\lambda(0)=0$ and $\lambda(1)=1$.[32]

A private asset must be certified by a third party (e.g., a credit rating agency) in order to be acceptable as collateral. Certification makes the type of an asset common-knowledge and it guarantees that the asset is not fraudulent. The certification cost in terms of the numéraire good is $\zeta > 0$.[33] Government bonds do not need to be certified—they are perfectly recognizable. As before, in a fraction μ^p of matches public liquidity and all certified private assets are acceptable as collateral, whereas in the remaining $1-\mu^p$ matches only public liquidity is acceptable.

Let $\Omega \subset [0,1]$ denote the set of assets that are certified, and let $\Omega^c = [0,1] \setminus \Omega$. The budget constraint of the OTC-trader can be rewritten as:

$$\dot{g} = \int_{\Omega} r(\omega)a(d\omega) + \int_{\Omega^c} \rho a(d\omega) + r^g g - \int \varepsilon(d\omega) - c - \Upsilon, \tag{57}$$

where $a(d\omega)$ is the measure of assets of type ω, and $\varepsilon(d\omega)$ is the investment in private assets of type ω, with $\dot{a} = \varepsilon$. According to the first term on the right side of (57), each unit of asset of type $\omega \in \Omega$, yields an interest payment equal to $r(\omega)$. According to the second term on the right side of (57) assets that are not certified, $\omega \in \Omega^c$, are illiquid and pay an interest rate equal to ρ. The third term is the return on government bonds, and the fourth term is the total investment in private assets.

Following the same reasoning as in the previous section, the rate of return of asset ω conditional on being certified is

$$r(\omega) = \rho - \mu^p \lambda(\omega)\sigma \left[f'(y^p) - 1\right], \tag{58}$$

where the output traded in a bilateral match is

$$y^p = \min\left\{\mathrm{A} + B, y^*\right\}, \tag{59}$$

[31] The relationship between the redeployability of capital (the value in its next best use) and asset liquidity is discussed in Shleifer and Vishny (1992).

[32] We follow Kiyotaki and Moore (2005) and keep the loan-to-value ratios, $\lambda(\omega)$, as exogenous. The monetary literature has provided several ways to endogenize such constraints. Rocheteau (2011) explains resalability constraints in a model of an OTC market where the asset holder has private information about the terminal value of the asset and uses asset retention as a signaling mechanism. Li, Rocheteau, and Weill (2012) obtain a distribution of resalability constraints and haircuts in an OTC market where assets can be subject to fraudulent practices. Finally, Nosal and Rocheteau (2013) show that liquidity differences similar to the ones described in this section can emerge as the result of a pairwise Pareto-efficient bargaining protocol.

[33] This formalization is related to the costly-state-verification assumption introduced by Williamson (1987). It is also related to the assumption in Lester, Postlewaite, and Wright (2012) according to which buyers of assets must incur a cost in order to be able to authenticate and accept an asset.

and the private liquidity of a trader is

$$\mathrm{A} = \int_{\Omega} \lambda(\omega) a(d\omega). \tag{60}$$

From (58) there are two sources of asset liquidity. On the extensive margin, private assets are eligible as collateral in a fraction μ^p of all matches. On the intensive margin, the buyer can pledge a fraction $\lambda(\omega)$ of asset ω. By a similar reasoning, the rate of return on public liquidity is

$$r^g = \rho - \sigma \left\{ \mu^p \left[f'(y^p) - 1 \right] + (1 - \mu^p) \left[f'(y^g) - 1 \right] \right\}, \tag{61}$$

with

$$y^g = \min \{ B, y^* \}. \tag{62}$$

The certification decision of a type-ω firm is determined by $\max\left\{V_F(\omega) - \zeta, \hat{V}_F\right\}$, where $V_F(\omega) = (\varphi - w)/[r(\omega) + \delta]$ is the value of the certified firm and $\hat{V}_F = (\varphi - w)/(\rho + \delta)$ is the value of a non-certified firm. Therefore, the threshold for ω below which it is not optimal to certify the firm is $\hat{\omega}$, which is the solution to $V_F(\hat{\omega}) - \zeta \leq \hat{V}_F$ (with an equality if $\hat{\omega} < 1$). That is,

$$\frac{\varphi - w}{r(\hat{\omega}) + \delta} - \zeta \leq \frac{\varphi - w}{\rho + \delta}, \quad \text{“} = \text{”} \text{ if } \hat{\omega} < 1. \tag{63}$$

Notice that $\hat{\omega} > 0$ since from (58) it follows that $r(0) = \rho$ and hence $V_F(0) = \hat{V}_F$. Substituting $r(\hat{\omega})$ by its expression given in (58) into (63), the loan-to-value ratio associated with the critical type, $\hat{\omega}$, is

$$\lambda(\hat{\omega}) = \min \left\{ \frac{\zeta \left(\rho + \delta \right)^2}{\mu^p \sigma \left[f'(y^p) - 1 \right] \left[\zeta \left(\rho + \delta \right) + \varphi - w \right]}, 1 \right\}. \tag{64}$$

The set of assets that are accepted as collateral, $\Omega = [\hat{\omega}, 1]$, expands as μ^p or σ increases, but shrinks as y^p increases.

Finally, we clear asset markets by requiring that $a(\omega) = nV_F(\omega)$ for all ω, which from (1) and (60) implies

$$\mathrm{A} = \int_{\hat{\omega}}^{1} \frac{\lambda(\omega) n(\theta)(\varphi - w)}{r(\omega) + \delta} d\omega, \tag{65}$$

where θ is determined by the free-entry condition,

$$\frac{\gamma}{q(\theta)} = \int_0^1 \max \left\{ V_F(\omega) - \zeta, \hat{V}_F \right\} d\omega = \hat{\omega} \left(\frac{\varphi - w}{\rho + \delta} \right) + \int_{\hat{\omega}}^1 \left[\frac{\varphi - w}{r(\omega) + \delta} - \zeta \right] d\omega. \tag{66}$$

From (66) the average cost of opening a vacancy is equal to the expected value of a filled job, where the expectation is taken over the distribution of firm types (as the type of the firm determines the rate of return on claims on the firm's profits).

Definition 3 *A steady-state equilibrium is a list, $\langle \theta, \hat{\omega}, \mathrm{A}, y^p, y^g, r(\omega), r^g \rangle$, that solves (59), (58), (61), (62), (64), (65), and (66).*

We reduce an equilibrium to a single equation in A as follows. Substituting $r(\omega)$ by its expression given in (58) into (66), θ is the solution to

$$\frac{\gamma}{q(\theta)} = \max_{\hat{\omega}} \left\{ \frac{\hat{\omega}\left(\varphi - w\right)}{\rho + \delta} + \int_{\hat{\omega}}^{1} \left[\frac{\varphi - w}{\rho + \delta - \mu^p \lambda(\omega) \sigma \left[f'(\mathrm{A} + B) - 1\right]^{+}} - \zeta \right] d\omega \right\}, \tag{67}$$

where $[x]^+ = \max\{x, 0\}$. The market tightness defined by (67) is a continuous, nonincreasing function of private liquidity, $\theta = \theta(\mathrm{A})$, represented graphically in the bottom panel of Figure 8. Next, we substitute $\theta(\mathrm{A})$ coming from (67) into (65) to determine A as the solution to

$$\mathrm{A} = \int_{\hat{\omega}(\mathrm{A})}^{1} \frac{\lambda(\omega) n \left[\theta(\mathrm{A})\right] \left(\varphi - w\right)}{\rho + \delta - \mu^p \lambda(\omega) \sigma \left[f'(\mathrm{A} + B) - 1\right]^{+}} d\omega. \tag{68}$$

The right side of (68) is positive and decreasing in A, and is equal to 0 when A is above a threshold, A_1. Hence, there is a unique solution for A in (65). Graphically, the right and left sides of (65) are represented by two curves labeled RHS and LHS, respectively, in Figure 8. Given A, θ is uniquely determined by (67), y is uniquely determined by (59) with $g = B$, and $\hat{\omega}$ is uniquely determined by (64). Thus, there is a unique steady-state equilibrium.

Suppose that the following condition holds:

$$f'(B) - 1 > \frac{\zeta \left(\rho + \delta\right)^2}{\mu^p \sigma \left[\zeta \left(\rho + \delta\right) + \varphi - w\right]}. \tag{69}$$

Public liquidity is sufficiently scarce so that some private assets are part of the effective liquidity supply of the economy. In this context we investigate a negative shock on μ^p. The right side of (68) decreases, which leads to a decline in private liquidity, A. In terms of the distribution of interest rates, the predictions of the model are in accordance with a flight to liquidity/quality. From (67) and (68) the liquidity premium of private assets, $\mu^p \lambda(\omega) \sigma \left[f'(\mathrm{A} + B) - 1\right]^{+}$, decreases and hence real interest rates, $r(\omega)$, increase. In Figure 9 the curve representing the distribution of interest rates, $r(\omega)$, moves upward and the set of eligible assets gets smaller, i.e., $\hat{\omega}$ increases. From (61) the real interest rate on government bonds declines. In terms of labor market outcomes, our model predicts that the measure of firms declines and unemployment increases as a result of higher interest rates. Hence, there is endogenous destruction of private liquidity that amplifies the initial shock both because fewer firms enter the market and because there is a flight to quality ($\hat{\omega}$ increases).

Suppose that the policymaker responds to the shock by raising the supply of public liquidity, B. The right side of (68) decreases so that A is lower and y^p is higher. From (67), the fact

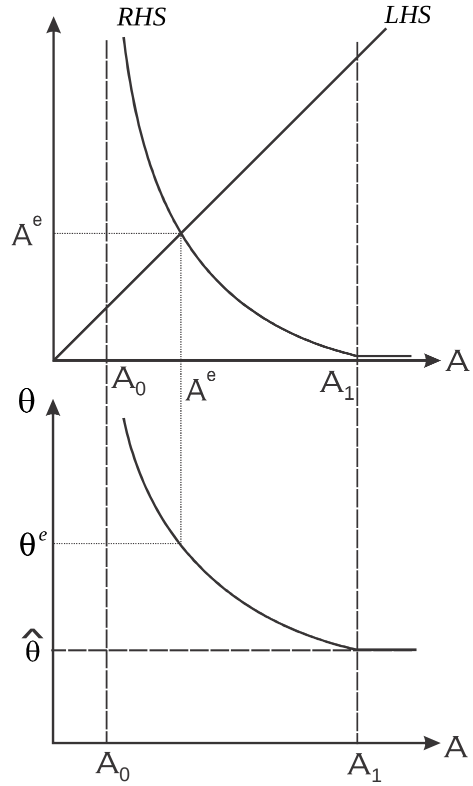

Figure 8: Determination of the equilibrium values of A and θ. The top panel plots the right and left sides of (68). The private supply of liquidity, A, is uniquely determined at the intersection of these two curves. Given A, the bottom panel determines market tightness, θ.

that $\sigma\left[f'(y^p)-1\right]^+$ declines implies that market tightness decreases further and unemployment increases. These results provide another example of public liquidity crowding out private liquidity.[34]

An alternative policy consists in committing to purchase the private assets at the price implied by the pre-crisis interest rate, $r(\omega)$, i.e., $V_F(\omega)=(\varphi-w)/\left[r(\omega)+\delta\right]$, and to replace the private liquidity by public assets so as to keep $y^p = \mathrm{A}+B$ unchanged. Because private assets have become less liquid, $\mu^{p\prime} < \mu^p$, their market price, $(\varphi-w)/(r'+\delta)$, is less than the one offered by the policymaker. Consequently, all private assets that were previously liquid are sold to the policymaker, i.e., the new supply of public liquidity is $B' = y^p$. From (61), with $y^p = y^g$ and $\mu^p+\mu^g=1$ the interest rate on public liquidity is $\rho-r^g=\sigma\left[f'(y^p)-1\right]$, which implies $r^g<r(\omega)$ for all ω. Thus, the policymaker can finance the interest payment on public liquidity with the interest payment it collects on private assets. The state of the labor market is unchanged, and the

[34] Public liquidity also raises the average quality of assets (as measured by $\hat{\omega}$) used as collateral and saves on certification costs. This result is in accordance with an old idea in monetary theory according to which money is a substitute for investment in information because it is recognizable. See Brunner and Meltzer (1971) for one of the very first statements of this idea.

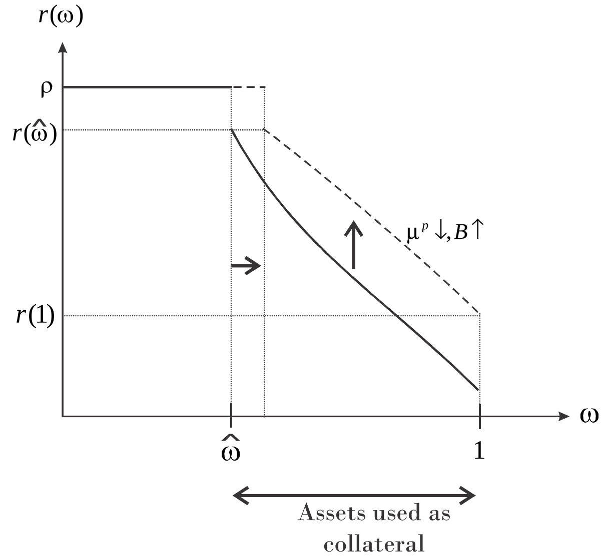

Figure 9: Liquidity structure of asset returns. The rate of return of an asset, r, is a non-increasing function of its fitness as collateral, ω. Given a fixed cost to certify assets, only a subset of assets, $[\hat{\omega}, 1]$, are used as collateral. A shock that reduces the acceptability of private assets, μ^p, leads to an increase in interest rates and a contraction of the set of assets used as collateral.

output of the OTC market is increased.

9 Conclusion

We have developed a tractable model of liquidity provision and the labor market by introducing an explicit market for liquidity into the canonical model of equilibrium unemployment of Mortensen and Pissarides (1994). Our model allowed us to study the interactions between the key missions of a monetary authority: providing liquidity to the financial sector, households, and firms, and keeping interest rates moderate to achieve full employment.

In terms of policies, we showed that an increase in public liquidity through an increase in the supply of real government bonds, raises interest rates by reducing the convenience yield on liquid assets, reduces entry of firms, and increases unemployment. An open-market sale of nominal government bonds by a central bank which withdraws currency or bank reserves from the economy, redistributes liquidity across trades as narrow liquidity (currency) shrinks while broad liquidity (including bonds) expands. These changes in the composition of liquidity lead to higher interest rates and unemployment. In contrast, an increase in the inflation rate reduces the real interest rate and leads to more job creation and a lower unemployment rate. More generally, we identified a trade-off between liquidity provision and the objective of keeping moderate interest rates for the private sector. Under some conditions this trade-off makes it optimal to keep liquidity scarce in order to reduce an inefficiently high unemployment rate.

In terms of financial stability we studied a financial crisis triggered by a lower acceptability of some assets as collateral due, for instance, to severe informational asymmetries regarding the quality of assets. The interest rate on private assets increases along with the rate-of-return differential between private and public liquidity, and unemployment rises. The government can mitigate this shock by offering to purchase private assets at their pre-crisis prices. Regulations that raise collateral requirements in OTC transactions reduce interest rates and unemployment. If assets are heterogeneous in terms of their ability to serve as collateral, then an increase in collateral requirements leads to lower unemployment and collateral expansion.

In our working paper (Rocheteau and Rodriguez-Lopez, 2013) we extended our model to add a channel through which the provision of liquidity to the OTC sector could mitigate search-like frictions in credit markets. In that version of the model an increase in public liquidity raises the expected surplus of participants in the OTC market leading to more entry. As the number of OTC-traders increases, firms can access funds more rapidly, which may promote job creation and cause a decline in unemployment. We are planing to calibrate this version of the model to study quantitatively the non-monotonic effects of liquidity provision on the labor market and unemployment.

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