Liquidity and the International Allocation of Economic Activity

Appendix — For Online Publication

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A The Model with Endogenous Acceptability

A.1 Proof of $\Xi'(\mu^*) > 0$ with Scarce Liquidity

From (47), we obtain that

$$\Xi'(\mu^*) = \gamma(1-\theta)\left\{\left[F'(y_2)-1\right]\frac{dy_2}{d\mu^*} - \left[F'(y_1)-1\right]\frac{dy_1}{d\mu^*}\right\}. \tag{A-1}$$

If liquidity is scarce in all matches, then (45) and (46) are given by

$$\theta y_2 + (1-\theta)F(y_2) = A + A^* + B + B^* \tag{A-2}$$

$$\theta y_1 + (1-\theta)F(y_1) = A + B, \tag{A-3}$$

where $y_1 < y_2 < \hat{y}$ because $y + (1-\theta)F(y)$ is increasing in y. From (A-2) and (A-3), it follows that

$$[\theta + (1-\theta)F'(y_2)]\frac{dy_2}{d\mu^*} = \frac{dA}{d\mu^*} + \frac{dA^*}{d\mu^*} \tag{A-4}$$

$$[\theta + (1-\theta)F'(y_1)]\frac{dy_1}{d\mu^*} = \frac{dA}{d\mu^*}. \tag{A-5}$$

Plugging in (A-4) and (A-5) into (A-1) to substitute for $\frac{dy_2}{d\mu^*}$ and $\frac{dy_1}{d\mu^*}$, I obtain that $\Xi'(\mu^*) > 0$ if and only if

$$\left\{(1-\theta)[F'(y_1)-1]+1\right\}\frac{dA^*}{d\mu^*} > \left[\frac{F'(y_1)-F'(y_2)}{F'(y_2)-1}\right]\frac{dA}{d\mu^*}. \tag{A-6}$$

Given that $F'(y) > 0$, $F''(y) < 0$, $F'(\hat{y}) = 1$, and $y_1 < y_2 < \hat{y}$, it is the case that $F'(y_1) > F'(y_2) > 1$, and thus, a sufficient condition for (A-6) to hold is that $\frac{dA^*}{d\mu^*} > 0$ and $\frac{dA}{d\mu^*} < 0$.

Taking the derivatives of (35), (36), (43), and (44) with respect to μ^*, and after using (A-4) and (A-5) to replace $\frac{dy_2}{d\mu^*}$ and $\frac{dy_1}{d\mu^*}$, I obtain the following system of equations that allows me to solve for $\frac{dA}{d\mu^*}$, $\frac{dA^*}{d\mu^*}$, $\frac{dr}{d\mu^*}$ and $\frac{dr^*}{d\mu^*}$:

$$\frac{dA}{d\mu^*} = \underbrace{\frac{(\sigma-1)\eta}{\sigma k H_1^2 H_2^2}}_{\mathfrak{a}} \left[\underbrace{-(\tau^{2k} H_2^2 + H_1^2)}_{\mathfrak{c}} \frac{dr}{d\mu^*} + \underbrace{\tau^k (H_1^2 + H_2^2)}_{\mathfrak{d}} \frac{dr^*}{d\mu^*} \right], \tag{A-7}$$

$$\frac{dA}{d\mu^*} + \frac{dA^*}{d\mu^*} = \underbrace{\frac{(\sigma-1)\eta}{\sigma k H_1^2 H_2^2}}_{\mathfrak{a}} \left[\underbrace{-(\tau^k - 1)(\tau^k H_2^2 - H_1^2)}_{\mathfrak{f}} \frac{dr}{d\mu^*} + \underbrace{(\tau^k - 1)(H_2^2 - \tau^k H_1^2)}_{\mathfrak{g}} \frac{dr^*}{d\mu^*} \right], \tag{A-8}$$

$$\frac{dr^*}{d\mu^*} = \underbrace{-\frac{\gamma\theta\mu^* F''(y_2)}{[\theta + (1-\theta)F'(y_2)]^3}}_{\mathfrak{h}} \left(\frac{dA}{d\mu^*} + \frac{dA^*}{d\mu^*} \right) \underbrace{-\frac{\gamma\theta[F'(y_2) - 1]}{\theta + (1-\theta)F'(y_2)}}_{\mathfrak{l}}, \tag{A-9}$$

$$\frac{dr}{d\mu^*} = \frac{dr^*}{d\mu^*} \underbrace{-\frac{\gamma\theta\mu F''(y_1)}{[\theta + (1-\theta)F'(y_1)]^3}}_{\mathfrak{k}} \frac{dA}{d\mu^*} + \underbrace{\frac{\gamma\theta[F'(y_1) - 1]}{\theta + (1-\theta)F'(y_1)}}_{\mathfrak{m}}, \tag{A-10}$$

where $H_1 \equiv \tau^k(r+\delta) - (r^*+\delta)$ and $H_2 \equiv \tau^k(r^*+\delta) - (r+\delta)$. In (A-7)–(A-10), terms $\mathfrak{a}$, $\mathfrak{d}$, $\mathfrak{h}$, $\mathfrak{k}$, and $\mathfrak{m}$ are positive, terms $\mathfrak{c}$, $\mathfrak{f}$, and $\mathfrak{l}$ are negative, and the sign of $\mathfrak{g}$ is undetermined. The solutions for $\frac{dA}{d\mu^*}$ and $\frac{dA^*}{d\mu^*}$ are

$$\frac{dA}{d\mu^*} = \frac{\mathfrak{a}\mathfrak{c}(\mathfrak{l}+\mathfrak{m}) + \mathfrak{a}\mathfrak{d}\mathfrak{l} + \mathfrak{a}^2\mathfrak{h}\mathfrak{m}(\mathfrak{d}\mathfrak{f} - \mathfrak{c}\mathfrak{g})}{1 - \mathfrak{a}\mathfrak{c}\mathfrak{k} - \mathfrak{a}\mathfrak{h}(\mathfrak{f}+\mathfrak{g}) - \mathfrak{a}^2\mathfrak{h}\mathfrak{k}(\mathfrak{d}\mathfrak{f} - \mathfrak{c}\mathfrak{g})}, \tag{A-11}$$

$$\frac{dA^*}{d\mu^*} = \frac{\mathfrak{a}(\mathfrak{l}+\mathfrak{m})(\mathfrak{f}-\mathfrak{c}) + \mathfrak{a}\mathfrak{l}(\mathfrak{g}-\mathfrak{d}) + \mathfrak{a}^2(\mathfrak{d}\mathfrak{f} - \mathfrak{c}\mathfrak{g})(\mathfrak{k}\mathfrak{l} - \mathfrak{h}\mathfrak{m})}{1 - \mathfrak{a}\mathfrak{c}\mathfrak{k} - \mathfrak{a}\mathfrak{h}(\mathfrak{f}+\mathfrak{g}) - \mathfrak{a}^2\mathfrak{h}\mathfrak{k}(\mathfrak{d}\mathfrak{f} - \mathfrak{c}\mathfrak{g})}. \tag{A-12}$$

In the denominator, $\mathfrak{f}+\mathfrak{g} = -(\tau^k-1)^2(H_1^2+H_2^2) < 0$ and $\mathfrak{d}\mathfrak{f} - \mathfrak{c}\mathfrak{g} = -(\tau^{2k}-1)^2 H_1^2 H_2^2 < 0$, and thus $1 - \mathfrak{a}\mathfrak{c}\mathfrak{k} - \mathfrak{a}\mathfrak{h}(\mathfrak{f}+\mathfrak{g}) - \mathfrak{a}^2\mathfrak{h}\mathfrak{k}(\mathfrak{d}\mathfrak{f} - \mathfrak{c}\mathfrak{g}) > 1$. In the numerator of (A-11), $\mathfrak{a}\mathfrak{d}\mathfrak{l} < 0$, $\mathfrak{a}^2\mathfrak{h}\mathfrak{k}(\mathfrak{d}\mathfrak{f} - \mathfrak{c}\mathfrak{g}) < 0$, and $\mathfrak{a}\mathfrak{c}(\mathfrak{l}+\mathfrak{m}) < 0$ if $\mathfrak{l}+\mathfrak{m} > 0$. The latter holds if $F'(y_1) > F'(y_2)$, which is true here because $y_1 < y_2$. It follows that $\frac{dA}{d\mu^*} < 0$. For the numerator in (A-12), $\mathfrak{f} - \mathfrak{c} = \tau^k(H_1^2+H_2^2) > 0$, $\mathfrak{g} - \mathfrak{d} = -(\tau^{2k}H_1^2 + H_2^2) < 0$, and $\mathfrak{k}\mathfrak{l} - \mathfrak{h}\mathfrak{m} < 0$. It follows that $\mathfrak{a}(\mathfrak{l}+\mathfrak{m})(\mathfrak{f}-\mathfrak{c}) > 0$, $\mathfrak{a}\mathfrak{l}(\mathfrak{g}-\mathfrak{d}) > 0$, $\mathfrak{a}^2(\mathfrak{d}\mathfrak{f} - \mathfrak{c}\mathfrak{g})(\mathfrak{k}\mathfrak{l} - \mathfrak{h}\mathfrak{m}) > 0$, and thus $\frac{dA^*}{d\mu^*} > 0$. Therefore, when liquidity is scarce in all matches, $\frac{dA}{d\mu^*} < 0$ and $\frac{dA^*}{d\mu^*} > 0$ and thus (A-6) holds so that $\Xi'(\mu^*) > 0$.

If $y_1 < y_2 = \hat{y}$, so that liquidity is scarce in matches that only accept Home assets but abundant in matches that accept both Home and Foreign assets, then by (43) and (45) it follows that $r^* = \rho$ and $\frac{dy_2}{d\mu^*} = 0$. From (47), I obtain that $\Xi'(\mu^*) = -\gamma(1-\theta)\left[F'(y_1) - 1\right]\frac{dy_1}{d\mu^*}$, which is positive if and only if $\frac{dy_1}{d\mu^*} = \left[\frac{1}{\theta+(1-\theta)F'(y_1)}\right]\frac{dA}{d\mu^*} < 0$. Thus, a sufficient condition for $\Xi'(\mu^*) > 0$ is that $\frac{dA}{d\mu^*} < 0$. With $\frac{dr}{d\mu^*} = 0$, the system of equations (A-7)–(A-10) simplifies to

$$\frac{dA}{d\mu^*} = \mathfrak{a}\mathfrak{c}\frac{dr}{d\mu^*}, \; \frac{dA}{d\mu^*} + \frac{dA^*}{d\mu^*} = \mathfrak{a}\mathfrak{f}\frac{dr}{d\mu^*}, \text{ and } \frac{dr}{d\mu^*} = \mathfrak{k}\frac{dA}{d\mu^*} + \mathfrak{m},$$

which yields $\frac{dA}{d\mu^*} = \frac{\mathfrak{acm}}{1-\mathfrak{ack}}$. Given that $\mathfrak{acm} < 0$ and $\mathfrak{ack} < 0$, it follows that $\frac{dA}{d\mu^*} < 0$ and thus $\Xi'(\mu^*) > 0$. Lastly, if liquidity is abundant in all matches so that $y_1 = y_2 = \hat{y}$, then $r = r^* = \rho$ and $\Xi'(\mu^*) = 0$.

A.2 Proof of $\frac{d\Xi(\mu^*)}{d\tau} > 0$ if $\mu^* < 1$

Given $\mu^* < 1$, I follow similar steps as in the derivation of condition (A-6) in section A.1, and obtain that if liquidity is scarce in all matches, so that $y_1 < y_2 < \hat{y}^*$, then $\frac{d\Xi(\mu^*)}{d\tau} > 0$ if and only if

$$\left\{(1-\theta)[F'(y_1)-1]+1\right\}\frac{dA^*}{d\tau} > \left[\frac{F'(y_1)-F'(y_2)}{F'(y_2)-1}\right]\frac{dA}{d\tau}. \tag{A-13}$$

Therefore, a sufficient condition for (A-13) to hold is that $\frac{dA^*}{d\tau} > 0$ and $\frac{dA}{d\tau} < 0$. As in (A-7)–(A-10), I obtain the following system of equations that allows me to solve for $\frac{dA}{d\tau}$, $\frac{dA^*}{d\tau}$, $\frac{dr}{d\tau}$ and $\frac{dr^*}{d\tau}$:

$$\frac{dA}{d\tau} = \underbrace{\frac{(\sigma-1)\eta}{\sigma k H_1^2 H_2^2}}_{\mathfrak{a}}\left[\underbrace{-k\tau^{k-1}(r^*+\delta)(H_2^2-H_1^2)}_{\mathfrak{b}}\underbrace{-(\tau^{2k}H_2^2+H_1^2)}_{\mathfrak{c}}\frac{dr}{d\tau}+\underbrace{\tau^k(H_1^2+H_2^2)}_{\mathfrak{d}}\frac{dr^*}{d\tau}\right], \tag{A-14}$$

$$\frac{dA}{d\tau}+\frac{dA^*}{d\tau} = \underbrace{\frac{(\sigma-1)\eta}{\sigma k H_1^2 H_2^2}}_{\mathfrak{a}}\left[\underbrace{-k\tau^{k-1}(r^*-r)(H_2^2-H_1^2)}_{\mathfrak{e}}\underbrace{-(\tau^k-1)(\tau^k H_2^2-H_1^2)}_{\mathfrak{f}}\frac{dr}{d\tau}+\underbrace{(\tau^k-1)(H_2^2-\tau^k H_1^2)}_{\mathfrak{g}}\frac{dr^*}{d\tau}\right], \tag{A-15}$$

$$\frac{dr^*}{d\tau} = \underbrace{-\frac{\gamma\theta\mu^* F''(y_2)}{[\theta+(1-\theta)F'(y_2)]^3}}_{\mathfrak{h}}\left(\frac{dA}{d\tau}+\frac{dA^*}{d\tau}\right), \tag{A-16}$$

$$\frac{dr}{d\tau} = \frac{dr^*}{d\tau}\underbrace{-\frac{\gamma\theta\mu F''(y_1)}{[\theta+(1-\theta)F'(y_1)]^3}}_{\mathfrak{k}}\frac{dA}{d\tau}, \tag{A-17}$$

where $H_1 \equiv \tau^k(r+\delta)-(r^*+\delta)$ and $H_2 \equiv \tau^k(r^*+\delta)-(r+\delta)$. In (A-14)–(A-17), terms $\mathfrak{a}$, $\mathfrak{d}$, $\mathfrak{h}$, and $\mathfrak{k}$ are positive, terms $\mathfrak{b}$, $\mathfrak{c}$, $\mathfrak{e}$, and $\mathfrak{f}$ are negative, and the sign of $\mathfrak{g}$ is undetermined. The solutions for $\frac{dA}{d\tau}$ and $\frac{dA^*}{d\tau}$ are

$$\frac{dA}{d\tau} = \frac{\mathfrak{ab}+\mathfrak{a}^2\mathfrak{h}[\mathfrak{f}(\mathfrak{e}-\mathfrak{b})-\mathfrak{bg}]}{1-\mathfrak{ack}-\mathfrak{ah}(\mathfrak{f}+\mathfrak{g})-\mathfrak{a}^2\mathfrak{hk}(\mathfrak{df}-\mathfrak{cg})}, \tag{A-18}$$

$$\frac{dA^*}{d\tau} = \frac{-\mathfrak{a}^2\mathfrak{h}[\mathfrak{f}(\mathfrak{e}-\mathfrak{b})-\mathfrak{bg}]+\mathfrak{a}(\mathfrak{e}-\mathfrak{b})+\mathfrak{a}^2\mathfrak{k}(\mathfrak{bf}-\mathfrak{ce})}{1-\mathfrak{ack}-\mathfrak{ah}(\mathfrak{f}+\mathfrak{g})-\mathfrak{a}^2\mathfrak{hk}(\mathfrak{df}-\mathfrak{cg})}. \tag{A-19}$$

The denominator is the same as in (A-11) and (A-12), and thus is positive and larger than 1. The numerator in (A-18) is negative because $\mathfrak{ab} < 0$ and $\mathfrak{f}(\mathfrak{e}-\mathfrak{b})-\mathfrak{bg} = -k\tau^{k-1}(\tau^k-1)H_1H_2(H_1+H_2)(H_2^2-H_1^2) < 0$, and thus, $\frac{dA}{d\tau} < 0$. The numerator in (A-19) is positive because $-\mathfrak{a}^2\mathfrak{h}[\mathfrak{f}(\mathfrak{e}-\mathfrak{b})-\mathfrak{bg}] > 0$, $\mathfrak{e}-\mathfrak{b} = k\tau^{k-1}(H_2^2-H_1^2)(r+\delta) > 0$ and $\mathfrak{bf}-\mathfrak{ce} = k\tau^{k-1}H_1H_2(H_2^2-H_1^2)(\tau^kH_2-H_1) > 0$, and thus $\frac{dA^*}{d\tau} > 0$. Therefore, when liquidity is scarce in all matches, $\frac{dA}{d\tau} < 0$ and $\frac{dA^*}{d\tau} > 0$ and thus (A-13) holds so that $\frac{d\Xi(\mu^*)}{d\tau} > 0$.

If $y_1 < y_2 = \hat{y}$, so that liquidity is scarce in matches that only accept Home assets but abundant in matches that accept both Home and Foreign assets, then by (43) and (45) it follows that $r^* = \rho$ and

$\frac{dy_2}{d\tau} = 0$. From (47), I obtain that $\frac{d\Xi(\mu^*)}{d\tau} = -\gamma(1-\theta)\left[F'(y_1) - 1\right]\frac{dy_1}{d\tau}$, which is positive if and only if $\frac{dy_1}{d\tau} = \left[\frac{1}{\theta+(1-\theta)F'(y_1)}\right]\frac{dA}{d\tau} < 0$. Thus, a sufficient condition for $\frac{d\Xi(\mu^*)}{d\tau} > 0$ is that $\frac{dA}{d\tau} < 0$. With $\frac{dr}{d\tau} = 0$, the system of equations (A-14)–(A-17) simplifies to

$$\frac{dA}{d\tau} = \mathfrak{a}\left(\mathfrak{b} + \mathfrak{c}\frac{dr}{d\tau}\right), \frac{dA}{d\tau} + \frac{dA^*}{d\tau} = \mathfrak{a}\left(\mathfrak{e} + \mathfrak{f}\frac{dr}{d\tau}\right), \text{ and } \frac{dr}{d\tau} = \mathfrak{k}\frac{dA}{d\tau},$$

which yields $\frac{dA}{d\tau} = \frac{\mathfrak{ab}}{1-\mathfrak{ack}}$. Given that $\mathfrak{ab} < 0$ and $\mathfrak{ack} < 0$, it follows that $\frac{dA}{d\tau} < 0$ and thus $\frac{d\Xi(\mu^*)}{d\tau} > 0$.

B The Model with Heterogeneous Liquidity Across Multiple Assets

This extension introduces liquidity differences across assets by assuming that the different categories of Home and Foreign assets have different acceptability properties in OTC matches. In particular, I assume that *(i)* Home assets are acceptable as collateral in a larger fraction of OTC matches than Foreign assets, *(ii)* for each country's assets, public liquidity is acceptable in a larger fraction of matches than private liquidity, and *(iii)* there is heterogeneity in acceptability across private assets, with firm-level productivity being positively correlated with collateral fitness. The description of preferences, production, demand, the cutoff productivity levels, and the composition of firms follow as in sections 3.1 and 3.2.

Figure B-1 presents a description of assumptions *(i)* and *(ii)*. In a fraction $\mu_{\mathfrak{g}}$ of OTC matches only Home government bonds are acceptable as collateral, in a fraction $\mu_{\mathfrak{p}}$ of matches both public and private Home assets are acceptable, in a fraction $\mu^*_{\mathfrak{g}}$ of matches Home assets and Foreign government bonds are acceptable, and in the remaining $\mu^*_{\mathfrak{p}}$ fraction of matches all categories of assets are acceptable. Analogously, Foreign private assets are acceptable in a fraction $\mu^*_{\mathfrak{p}}$ of OTC matches, Foreign bonds are acceptable in a fraction $\mu^*_{\mathfrak{p}} + \mu^*_{\mathfrak{g}}$ of matches, Home private assets in a fraction $\mu^*_{\mathfrak{p}} + \mu^*_{\mathfrak{g}} + \mu_{\mathfrak{p}}$ of matches, and Home bonds are acceptable in all matches ($\mu^*_{\mathfrak{p}} + \mu^*_{\mathfrak{g}} + \mu_{\mathfrak{p}} + \mu_{\mathfrak{g}} = 1$).

Regarding *(iii)*, to each producing Home firm (with $\varphi \geq \hat{\varphi}_D$) we associate a loan-to-value ratio, $\lambda(\varphi) \in [0, 1)$, that specifies the fraction of the asset value that can be pledged as collateral in an OTC transaction: a financier can obtain a loan of size $\lambda(\varphi)\, a(\varphi)$ if she commits $a(\varphi)$ assets of type φ as collateral. The function $\lambda(\varphi)$ satisfies $\lambda'(\varphi) > 0$ for all $\varphi \geq \hat{\varphi}_D$, $\lambda(\hat{\varphi}_D) = 0$, $\lambda(\infty) \to 1$, and $\frac{d\lambda(\varphi)}{d\hat{\varphi}_D} < 0$. Hence, firm-level productivity is positively related to collateral fitness, which captures the idea that low-productivity firms are seen by financiers as more volatile and sensitive to shocks than more productive firms and thus they get lower loan-to-value ratios. Note that a firm at the cutoff $\hat{\varphi}_D$ is illiquid and hence must yield a return of ρ—financiers know that this firm will die for any minimal shock causing an increase in $\hat{\varphi}_D$, so they are unwilling to accept assets of type $\hat{\varphi}_D$ in OTC transactions. Analogous properties hold for loan-to-value ratios of Foreign private assets, which are described by the function $\lambda^*(\varphi)$.

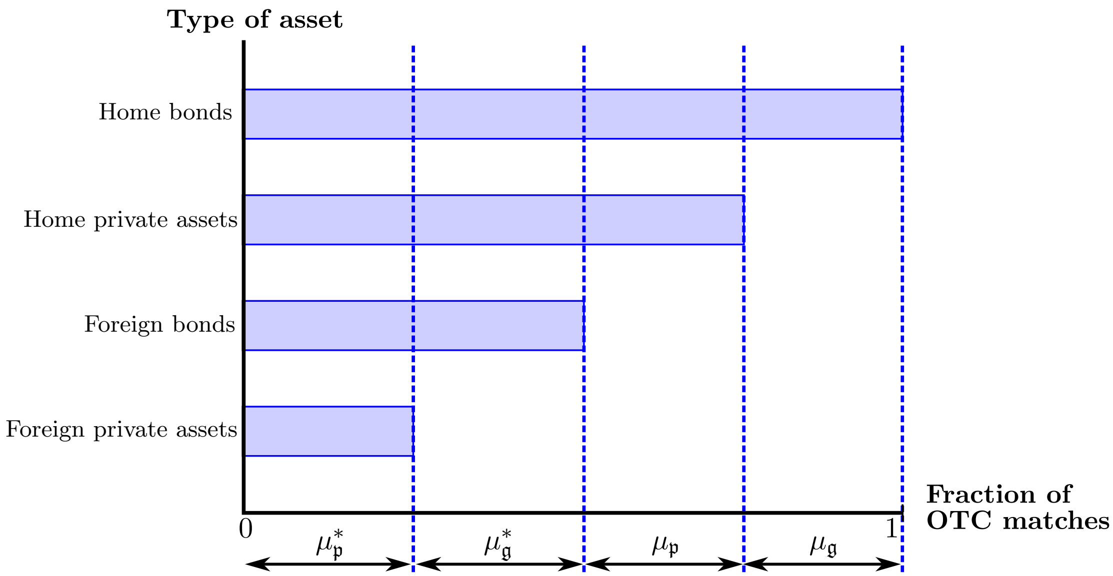


Figure B-1: Acceptability of Home and Foreign assets in OTC matches

Although the analysis below only requires $\lambda(\varphi)$ and $\lambda^*(\varphi)$ to meet the properties described above, I assume a useful functional form that depends on a single parameter:

$$\lambda(\varphi) = 1 - \left(\frac{\hat{\varphi}_D}{\varphi}\right)^{\kappa} \quad \text{and} \quad \lambda^*(\varphi) = 1 - \left(\frac{\hat{\varphi}_D^*}{\varphi}\right)^{\kappa^*}$$

where $\varphi \geq \hat{\varphi}_D$ for Home firms, $\varphi \geq \hat{\varphi}_D^*$ for Foreign firms, $\kappa > 0$, and $\kappa^* > 0$. If $\kappa \to \infty$, then $\lambda(\varphi) \to 1$ for all $\varphi > \hat{\varphi}_D$, which approximates the case in which all claims on producing firms are equally liquid. Note also that $\frac{d\lambda(\varphi)}{d\kappa} > 0$ for all $\varphi > \hat{\varphi}_D$, so that a decline in κ is useful to analyze the effects of a liquidity reduction stemming from lower loan-to-value ratios of Home private assets.

Furthermore, I assume that for a Home or Foreign private asset to be part of the available liquidity to financiers, the asset must be certified by a rating agency that makes public the asset's underlying productivity. Each private asset's certification process involves a sunk cost of f_A (in terms of the homogeneous good), which implies the existence of two more cutoff productivity levels, $\hat{\varphi}_A$ and $\hat{\varphi}_A^*$, that separate assets into "non-certified" and "certified" categories. Non-certified assets have underlying productivities in the range $[\hat{\varphi}_D, \hat{\varphi}_A)$, they are illiquid, and hence pay the illiquid interest rate, ρ. Certified assets have underlying productivities in the range $[\hat{\varphi}_A, \infty)$, they are liquid, and hence pay an interest rate below ρ.

Let $r(\varphi)$ denote the rate of return of Home private assets with underlying productivity φ, so that $r(\varphi) = \rho$ if $\varphi \in [\hat{\varphi}_D, \hat{\varphi}_A)$ and $r(\varphi) < \rho$ if $\varphi \in [\hat{\varphi}_A, \infty)$. Similarly, let $r^*(\varphi)$ denote the rate of return of Foreign assets with underlying productivity φ. To pin down $\hat{\varphi}_A$ and $\hat{\varphi}_A^*$, note that an asset with underlying productivity φ will be certified if and only if the value of the firm when certified minus the sunk certification cost, is no less than the value of the firm when not certified; this condition holds with equality for a firm at the cutoff. Thus, $\hat{\varphi}_A$ and $\hat{\varphi}^*_A$ solve

$$[[\pi_D(\hat{\varphi}_A) - f]\,\mathbb{1}\{\hat{\varphi}_A \geq \hat{\varphi}_D\} + [\pi_X(\hat{\varphi}_A) - f]\,\mathbb{1}\{\hat{\varphi}_A \geq \hat{\varphi}_X\}]\left[\frac{1}{r(\hat{\varphi}_A)+\delta} - \frac{1}{\rho+\delta}\right] = f_A \tag{B-1}$$

$$\left[\left[\pi^*_D(\hat{\varphi}^*_A) - f\right]\mathbb{1}\{\hat{\varphi}^*_A \geq \hat{\varphi}^*_D\} + \left[\pi^*_X(\hat{\varphi}^*_A) - f\right]\mathbb{1}\{\hat{\varphi}^*_A \geq \hat{\varphi}^*_X\}\right]\left[\frac{1}{r(\hat{\varphi}^*_A)+\delta} - \frac{1}{\rho+\delta}\right] = f_A, \tag{B-2}$$

where $\mathbb{1}\{\cdot\}$ is an indicator function taking the value of 1 if the condition inside the brackets is satisfied (and is zero otherwise). The left-hand side in (B-1) and (B-2) shows the difference between the discounted sum of instantaneous profits when certified (with an effective discount rate of $r(\hat{\varphi}_A) + \delta$) and the discounted sum of instantaneous profits when not certified (with an effective discount rate of $\rho + \delta$). The right-hand side in (B-1) and (B-2) shows the sunk certification cost.

B.1 Supply of Private Liquid Assets

Financiers fund the entry of differentiated-good firms in both countries in exchange for claims on firms' profits. As mentioned before, financiers may use these claims as private liquidity (*i.e.*, as collateral in their financial transactions). In contrast to the benchmark case, however, the total market capitalization of firms is no longer equivalent to the amount of private liquidity available. In particular, in the presence of loan-to-value ratios below 1 and certification costs that give rise to the cutoffs $\hat{\varphi}_A$ and $\hat{\varphi}^*_A$, the total supply of Home private liquidity, A, and the total supply of Foreign private liquidity, A^*, are now a fraction of the total market capitalization of firms in each country.

At Home, the value of a firm with productivity φ is defined as

$$V(\varphi) = \frac{[\pi_D(\varphi) - f]\,\mathbb{1}\{\varphi \geq \hat{\varphi}_D\} + [\pi_X(\varphi) - f]\,\mathbb{1}\{\varphi \geq \hat{\varphi}_X\}}{r(\varphi) + \delta} \tag{B-3}$$

where $r(\varphi) = \rho$ if $\varphi \in [\hat{\varphi}_D, \hat{\varphi}_A)$ and $r(\varphi) < \rho$ if $\varphi \in [\hat{\varphi}_A, \infty)$, and $\mathbb{1}\{\cdot\}$ is the indicator function. As a firm knows its productivity only after entry, the pre-entry expected value of a firm for Home potential entrants is $V_E = \int_{\hat{\varphi}_D}^{\infty} V(\varphi) g(\varphi) d\varphi$. With similar expressions for Foreign firms, and assuming identical entry costs for Home entrants and Foreign entrants, f_E, the free-entry conditions for differentiated-good firms at Home and Foreign are

$$\int_{\hat{\varphi}_D}^{\infty}\left[\frac{\pi_D(\varphi) - f}{r(\varphi)+\delta}\right] g(\varphi)d\varphi + \int_{\hat{\varphi}_X}^{\infty}\left[\frac{\pi_X(\varphi) - f}{r(\varphi)+\delta}\right] g(\varphi)d\varphi = f_E + [1 - G(\hat{\varphi}_A)]f_A, \tag{B-4}$$

$$\int_{\hat{\varphi}^*_D}^{\infty}\left[\frac{\pi^*_D(\varphi) - f}{r^*(\varphi)+\delta}\right] g(\varphi)d\varphi + \int_{\hat{\varphi}^*_X}^{\infty}\left[\frac{\pi^*_X(\varphi) - f}{r^*(\varphi)+\delta}\right] g(\varphi)d\varphi = f_E + [1 - G(\hat{\varphi}^*_A)]f_A. \tag{B-5}$$

In (B-4), the left-hand side is V_E, with the first term showing the expected discounted profits from selling domestically, and the second term showing the expected discounted profits from exporting; the right-hand side shows the sunk entry cost plus the expected certification cost (which is only paid if the entrant's productivity draw is $\hat{\varphi}_A$ or higher). Equation (B-5) has an analogous interpretation for Foreign potential entrants.

Let $A(\varphi)$ denote the supply of liquidity stemming from Home firms with productivity φ, and let $A^*(\varphi)$ denote the supply of liquidity stemming from Foreign firms with productivity φ. The aggregate value of Home firms with productivity φ is given by $N_A V(\varphi)g(\varphi|\varphi \geq \hat{\varphi}_A)$, where $N_A = [1-G(\hat{\varphi}_A)]N_E/\delta$ denotes the measure of certified Home firms. Analogously, $N_A^* V^*(\varphi)g(\varphi|\varphi \geq \hat{\varphi}_A^*)$ is the aggregate value of Foreign firms with productivity φ for all $\varphi \geq \hat{\varphi}_A^*$, where $N_A^* = [1-G(\hat{\varphi}_A^*)]N_E^*/\delta$ is the measure of certified Foreign firms. Given that only fractions $\lambda(\varphi)$ and $\lambda^*(\varphi)$ of the value of these firms can serve as collateral in the OTC market, it follows that

$$\begin{aligned} A(\varphi) =& \lambda(\varphi)N_A V(\varphi)g(\varphi|\varphi \geq \hat{\varphi}_A) \\ A^*(\varphi) =& \lambda^*(\varphi)N_A^* V^*(\varphi)g(\varphi|\varphi \geq \hat{\varphi}_A^*) \end{aligned}$$

for all $\varphi \geq \hat{\varphi}_A$ at Home and $\varphi \geq \hat{\varphi}_A^*$ at Foreign. The supplies of Home private liquidity, $A = \int_{\hat{\varphi}_A}^{\infty} A(\varphi)d\varphi$, and Foreign private liquidity, $A^* = \int_{\hat{\varphi}_A^*}^{\infty} A^*(\varphi)d\varphi$, can then be written as

$$A =N_A \int_{\hat{\varphi}_A}^{\infty} \lambda(\varphi) \left[\frac{[\pi_D(\varphi) - f] + [\pi_X(\varphi) - f]\,\mathbb{1}\{\varphi \geq \hat{\varphi}_X\}}{r(\varphi)+\delta}\right] g(\varphi|\varphi \geq \hat{\varphi}_A)d\varphi, \tag{B-6}$$

$$A^* =N_A^* \int_{\hat{\varphi}_A^*}^{\infty} \lambda^*(\varphi) \left[\frac{[\pi_D^*(\varphi) - f] + [\pi_X^*(\varphi) - f]\,\mathbb{1}\{\varphi \geq \hat{\varphi}_X^*\}}{r^*(\varphi)+\delta}\right] g(\varphi|\varphi \geq \hat{\varphi}_A^*)d\varphi. \tag{B-7}$$

B.2 Demand for Multiple Liquid Assets

Let $a(\varphi)$ and $a^*(\varphi)$ denote the financier's holdings of Home and Foreign assets of type φ, and let b and b^* denote the financier's holdings of Home and Foreign government bonds. In addition to Home and Foreign private assets' interest rates, $r(\varphi)$ and $r^*(\varphi)$, the interest rates of Home and Foreign government bonds are r_b and r_b^*. Hence, the budget constraint of a Home financier is

$$\int_{\hat{\varphi}_D}^{\infty} \dot{a}(\varphi)d\varphi + \int_{\hat{\varphi}_D^*}^{\infty} \dot{a}^*(\varphi)d\varphi + \dot{b} + \dot{b}^* = \int_{\hat{\varphi}_D}^{\infty} r(\varphi)a(\varphi)d\varphi + \int_{\hat{\varphi}_D^*}^{\infty} r^*(\varphi)a^*(\varphi)d\varphi + r_b b + r_b^* b^* - h - \Upsilon.$$

The left-hand side presents the change in the financier's wealth, which is given by the financier's total investment in private assets and government bonds from both Home and Foreign. The right-hand side shows the interest payments on the financier's portfolio net of homogeneous-good consumption (h) and taxes (Υ). In contrast to the closed-economy model, the financier's portfolio is now composed of assets with different liquidity properties and interest rates. The budget constraint of a Foreign financier is identical to Home financier's budget constraint, with the exception of the last term, which changes to Υ^* (taxes in Foreign).

The total amounts of Home private liquidity, a, and Foreign private liquidity, a^*, held by a financier are given by

$$a = \int_{\hat{\varphi}_A}^{\infty} \lambda(\varphi)a(\varphi)d\varphi \quad \text{and} \quad a^* = \int_{\hat{\varphi}_A^*}^{\infty} \lambda^*(\varphi)a^*(\varphi)d\varphi,$$

which weight the holdings of each asset by its loan-to-value ratio, and take into account that liquid assets have underlying productivities no less than $\hat{\varphi}_A$ and $\hat{\varphi}^*_A$.

Following similar steps to those in section 2.2.2 to obtain (16), we find that the continuation value of a financier upon being matched (but before realizing its buyer or seller role), $Z(a, a^*, b, b^*)$, is given by

$$\begin{aligned} Z(a, a^*, b, b^*) =& \frac{\mu^*_{\mathfrak{p}}}{2} \max_{y^*_{\mathfrak{p}} \leq a+a^*+b+b^*} \left\{F(y^*_{\mathfrak{p}}) - y^*_{\mathfrak{p}}\right\} + \frac{\mu^*_{\mathfrak{g}}}{2} \max_{y^*_{\mathfrak{g}} \leq a+b+b^*} \left\{F(y^*_{\mathfrak{g}}) - y^*_{\mathfrak{g}}\right\} \\ &+ \frac{\mu_{\mathfrak{p}}}{2} \max_{y_{\mathfrak{p}} \leq a+b} \left\{F(y_{\mathfrak{p}}) - y_{\mathfrak{p}}\right\} + \frac{\mu_{\mathfrak{g}}}{2} \max_{y_{\mathfrak{g}} \leq b} \left\{F(y_{\mathfrak{g}}) - y_{\mathfrak{g}}\right\} + W(a, a^*, b, b^*) \,. \end{aligned}$$

This expression shows that with probability $1/2$ the financier is the buyer in the OTC match, in which case she can make a take-it-or-leave-it offer to the seller in order to maximize her surplus, $F(y) - y$. With probability $\mu^*_{\mathfrak{p}}$, the financier is in a match in which private and public assets from both Home and Foreign are acceptable as collateral and thus, she can transfer up to $a + a^* + b + b^*$ in exchange for $y^*_{\mathfrak{p}}$. With probability $\mu^*_{\mathfrak{g}}$ Home assets and Foreign government bonds are acceptable, so that the financier can transfer up to $a + b + b^*$ to purchase $y^*_{\mathfrak{g}}$. With probability $\mu_{\mathfrak{p}}$ only Home assets are acceptable, so that the financier can transfer up to $a + b$ to purchase $y_{\mathfrak{p}}$. Lastly, with probability $\mu_{\mathfrak{g}}$ only Home government bonds are acceptable, so that the financier can transfer up to b to purchase $y_{\mathfrak{g}}$.

Similar to the derivation of equation (19) in the closed-economy model, the financier's optimal portfolio solves

$$\frac{\rho - r^*(\varphi)}{\gamma} = \mu^*_{\mathfrak{p}} \lambda^*(\varphi) \left[F'(y^*_{\mathfrak{p}}) - 1\right] \tag{B-8}$$

$$\frac{\rho - r^*_b}{\gamma} = \mu^*_{\mathfrak{p}} \left[F'(y^*_{\mathfrak{p}}) - 1\right] + \mu^*_{\mathfrak{g}} \left[F'(y^*_{\mathfrak{g}}) - 1\right] \tag{B-9}$$

$$\frac{\rho - r(\varphi)}{\gamma} = \mu^*_{\mathfrak{p}} \lambda(\varphi) \left[F'(y^*_{\mathfrak{p}}) - 1\right] + \mu^*_{\mathfrak{g}} \lambda(\varphi) \left[F'(y^*_{\mathfrak{g}}) - 1\right] + \mu_{\mathfrak{p}} \lambda(\varphi) \left[F'(y_{\mathfrak{p}}) - 1\right] \tag{B-10}$$

$$\frac{\rho - r_b}{\gamma} = \mu^*_{\mathfrak{p}} \left[F'(y^*_{\mathfrak{p}}) - 1\right] + \mu^*_{\mathfrak{g}} \left[F'(y^*_{\mathfrak{g}}) - 1\right] + \mu_{\mathfrak{p}} \left[F'(y_{\mathfrak{p}}) - 1\right] + \mu_{\mathfrak{g}} \left[F'(y_{\mathfrak{g}}) - 1\right] . \tag{B-11}$$

Equations (B-8)-(B-11) define the optimal choice of each type of asset.[1] On one extreme, the left-hand side of equation (B-8) is the holding cost of Foreign private asset of type φ, while the right-hand side indicates the expected marginal surplus from holding an additional unit of that asset. That Foreign asset can only be used in a fraction $\mu^*_{\mathfrak{p}}$ of all matches with a pledgeability ratio of $\lambda^*(\varphi)$, in which case the marginal surplus of the financier is $F'(y^*_{\mathfrak{p}}) - 1$. On the other extreme, the left-hand side of (B-11) shows the holding cost of a Home government bond, while the right-hand side is its marginal surplus from its use in all matches in the financial market.

Similar to the closed-economy case, the quantity of financial services traded in an OTC match is the minimum between the value of the buyer's liquidity in that match and the surplus-maximizing

[1] Note that the closed-economy equation (19) can be obtained from (B-10) and (B-11) by assuming that $\mu^*_{\mathfrak{p}} = \mu^*_{\mathfrak{g}} = \mu_{\mathfrak{g}} = 0$, $\mu_{\mathfrak{p}} = 1$, and $\kappa \to \infty$ so that $\lambda(\varphi) \to 1$ for all $\varphi \geq \hat{\varphi}_D$.

quantity, $\hat{y}$. The difference is that in the closed-economy case only domestic private and public assets are used, and they are all fully acceptable in all matches. Given that financiers hold identical portfolios and that there is a unit measure of financiers in the world, market clearing implies that $a = A$, $a^* = A^*$, $b = B$, and $b = B^*$. Thus, we get

$$y_{\mathfrak{p}}^* = \min\{A + A^* + B + B^*, \hat{y}\}, \tag{B-12}$$

$$y_{\mathfrak{g}}^* = \min\{A + B + B^*, \hat{y}\}, \tag{B-13}$$

$$y_{\mathfrak{p}} = \min\{A + B, \hat{y}\}, \tag{B-14}$$

$$y_{\mathfrak{g}} = \min\{B, \hat{y}\}. \tag{B-15}$$

The definition of a steady-state equilibrium in the two-country model follows.

Definition. *A steady-state equilibrium in the two-country model is a list*

$$\left\langle \hat{\varphi}_D, \hat{\varphi}_X, \hat{\varphi}_D^*, \hat{\varphi}_X^*, \hat{\varphi}_A, \hat{\varphi}_A^*, A, A^*, y_{\mathfrak{p}}^*, y_{\mathfrak{g}}^*, y_{\mathfrak{p}}, y_{\mathfrak{g}}, r^*(\varphi), r_b^*, r(\varphi), r_b \right\rangle$$

that solves (22)*,* (23)*,* (B-1)*,* (B-2)*,* (B-4)*-*(B-7)*, and* (B-8)*-*(B-15)*.*

Similar to (41) and (42), note from (B-8)-(B-15) that we can write the full structure of interest rates as

$$r^*(\varphi) = \rho - \mu_{\mathfrak{p}}^* \gamma \lambda^*(\varphi) \left[F'(A + A^* + B + B^*) - 1\right]^+, \tag{B-16}$$

$$r_b^* = \rho - \mu_{\mathfrak{p}}^* \gamma \left[F'(A + A^* + B + B^*) - 1\right]^+ - \mu_{\mathfrak{g}}^* \gamma \left[F'(A + B + B^*) - 1\right]^+, \tag{B-17}$$

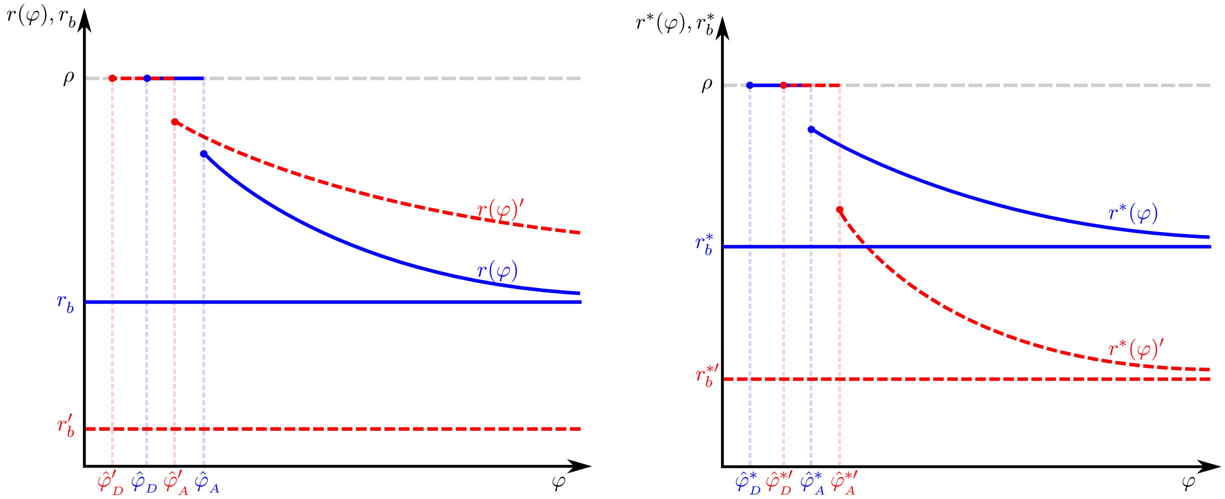
$$\begin{aligned} r(\varphi) = \rho &- \mu_{\mathfrak{p}}^* \gamma \lambda(\varphi) \left[F'(A + A^* + B + B^*) - 1\right]^+ - \mu_{\mathfrak{g}}^* \gamma \lambda(\varphi) \left[F'(A + B + B^*) - 1\right]^+ \\ &- \mu_{\mathfrak{p}} \gamma \lambda(\varphi) \left[F'(A + B) - 1\right]^+, \end{aligned} \tag{B-18}$$

$$\begin{aligned} r_b = \rho &- \mu_{\mathfrak{p}}^* \gamma \left[F'(A + A^* + B + B^*) - 1\right]^+ - \mu_{\mathfrak{g}}^* \gamma \left[F'(A + B + B^*) - 1\right]^+ \\ &- \mu_{\mathfrak{p}} \gamma \left[F'(A + B) - 1\right]^+ - \mu_{\mathfrak{g}} \gamma \left[F'(B) - 1\right]^+, \end{aligned} \tag{B-19}$$

where $[x]^+ = \max\{x, 0\}$. From (B-16) and given that $\frac{d\lambda^*(\varphi)}{d\varphi} > 0$, Foreign private asset of type $\varphi \geq \hat{\varphi}_A^*$ dominates private asset of type $\tilde{\varphi} > \varphi$ in interest rate, $r^*(\varphi) > r^*(\tilde{\varphi})$, provided that $\mu_{\mathfrak{p}}^* > 0$ and $A + A^* + B + B^* < \hat{y}$. Similarly, Foreign private asset of type $\tilde{\varphi}$ dominates Foreign government bonds in interest rate, $r^*(\tilde{\varphi}) > r_b^*$, if either $\mu_{\mathfrak{p}}^* > 0$ and $A + A^* + B + B^* < \hat{y}$, or $\mu_{\mathfrak{g}}^* > 0$ and $A + A^* + B < \hat{y}$, or both. Similar interest-rate comparisons across multiple assets can be obtained from (B-16)-(B-19).

B.3 The Effects of a Permanent Change in Asset Liquidity

Using the extended model, this section describes how a permanent reduction in the acceptability of Home private assets affects the full structure of interest rates. In this framework, a permanent acceptability reduction of Home private assets can arise from either *(i)* a decline in $\mu_{\mathfrak{p}}$ while keeping $\mu_{\mathfrak{p}}^*$



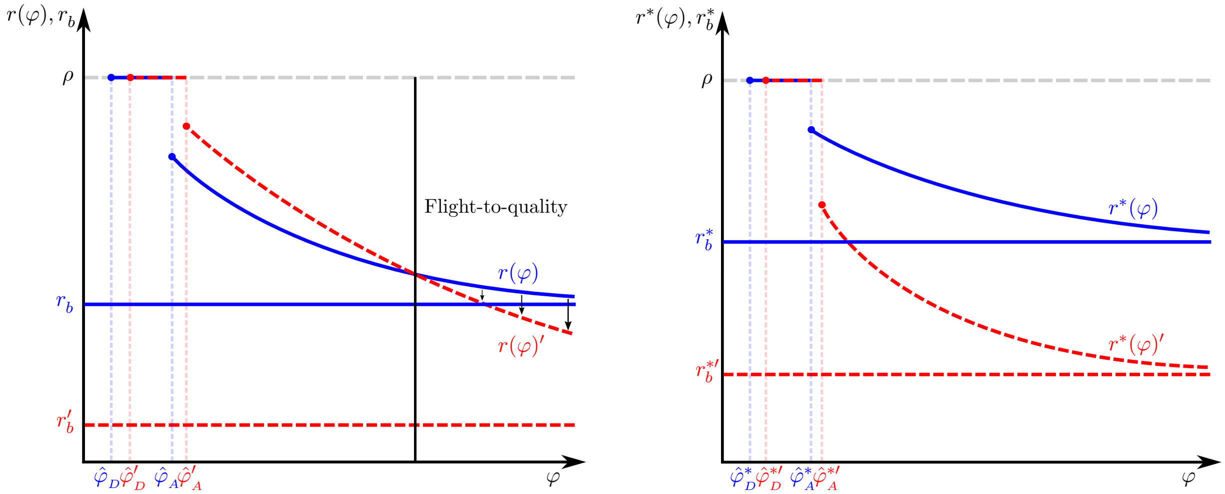
(a) Home interest rates (b) Foreign interest rates

Figure B-2: Lower acceptability of Home private assets: A decline in $\mu_{\mathfrak{p}}$ (new—dashed lines)

and $\mu_{\mathfrak{g}}^*$ constant, or *(ii)* a decline in the parameter κ of the loan-to-value function, $\lambda(\varphi) = 1 - (\hat{\varphi}_D/\varphi)^\kappa$. In the first case, the fraction of financial matches in which Home private assets are acceptable, $\mu_{\mathfrak{p}}^* + \mu_{\mathfrak{g}}^* + \mu_{\mathfrak{p}}$, declines, and in the second case, the fraction of a liquid asset's value that can be pledged as collateral is lower. The first case resembles a world financial system's general rejection of Home private assets (whether the Home asset comes, for example, from Apple or Dell), while the second case resembles downgrades of ratings for Home private assets.

Figure B-2 shows the effects of a decline in $\mu_{\mathfrak{p}}$ on the full structure of Home and Foreign interest rates. The interest rate on Home government bonds decreases despite Home being the origin of the acceptability reduction. On the other hand, interest rates increase for most Home private assets, with the exception of some low-productivity assets that become liquid after the decline in $\hat{\varphi}_A$. Intuitively, the reduction in acceptability has such a strong negative effect on the aggregate amount of Home private liquidity, that financiers start using some lower-quality assets to compensate. This type of event has negative real effects in Home that are very similar to those observed in section 4 for Foreign after an increase in μ. In contrast, most Foreign assets experience higher liquidity premiums (*i.e.*, lower interest rates). The exceptions are some low-productivity firms' assets that become illiquid after the increase in $\hat{\varphi}_A^*$.

Lastly, Figure B-3 presents the structure of interest rates before and after a decline in κ. For Foreign the effects are similar to those described in Figure B-2. For Home, however, there are important differences. In contrast to Figure B-2a, $\hat{\varphi}_D$ and $\hat{\varphi}_A$ increase in Figure B-3a, which imply increases in Home aggregate productivity and in the average quality of Home collateral used in financial transactions. Moreover, Figure B-3a shows a flight-to-quality phenomenon that not only involves Home government bonds (whose interest rate declines), but also assets issued by high-productivity Home



(a) Home interest rates (b) Foreign interest rates

Figure B-3: Lower acceptability of Home private assets: A decline in κ (new—dashed lines)

firms—note that the interest rate increases for assets from low-productivity firms but declines for high-productivity firms' assets.[2] In the end, these flight-to-quality effects cushion the negative impact of the reduction in κ on the real economy: although the amount of Home private liquidity, A, declines, the total capitalization of Home firms may increase.

References

Dick-Nielsen, J., P. Feldhütter, and D. Lando (2012): "Corporate Bond Liquidity Before and After the Onset of the Subprime Crisis," *Journal of Financial Economics*, 103(3), 471–492.

[2]These findings are consistent with the results of Dick-Nielsen, Feldhütter, and Lando (2012), who find evidence of flight-to-quality towards AAA-rate corporate bonds during the subprime crisis.