

# Learning and the Evolution of the Fed's Inflation Target\*

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## Abstract

This paper tries to infer and compare the evolution of the Federal Reserve's (unobserved) inflation target series by estimating a monetary model under the alternative assumptions of rational expectations or subjective expectations and learning.

In the estimated model that assumes that economic agents have rational expectations, the implied inflation target displays large shifts over time: it starts at 2% in the early 1960s, it rises to 8% in the 1970s, and it falls to 4% and 2% in the 1980s and 1990s. When the assumption of rational expectations is relaxed in favor of learning by the policymaker, the inferred target is, instead, remarkably stable over time. The target assumes values between 2 to 3% over the whole post-war sample.

The findings suggest changing beliefs and learning by the Federal Reserve as major endogenous causes of the perceived variation in the inflation target. When the model is allowed to take the central bank's evolving beliefs into account, the joint evolution of U.S. inflation, output, and monetary policy decisions can be explained without requiring large exogenous changes in the inflation target.

*Keywords:* time-varying inflation target, learning, expectations, Bayesian estimation.

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# 1 Introduction

For most of its history, the Federal Reserve has chosen to not communicate to the public the inflation target that it aimed to achieve when setting policy.<sup>1</sup> Therefore, the implicit target can only be inferred ex-post using historical data and econometric techniques.

The majority of monetary models typically assume, mostly for simplicity, that the inflation target has remained constant over time. The results by Clarida, Galí, and Gertler (2000) and Lubik and Schorfheide (2004), however, show that monetary policy failed to respond aggressively enough to the rising inflation in the 1970s and hence suggest that the Fed may have adopted a higher target in that decade. The papers by Favero and Rovelli (2003), Surico (2008), and Dennis (2004), in fact, identify one-time shifts in the inflation target after 1979. Other papers have allowed the Federal Reserve's target to vary over time. Kozicki and Tinsley (2005) estimate a backward-looking model to derive a continuously-changing inflation target. Belaygorod and Dueker (2005) and Leigh (2008) also estimate a time-varying inflation target, but they restrict their attention to the post-Volcker sample. Ireland (2007) concentrates both on estimating the changes of the Fed's inflation target over time and on analyzing their causes and consequences.<sup>2</sup>

These papers provide consistent evidence that the inflation target has fluctuated over time, rising to 6-8% in the 1970s and declining to around 2% in the 1980s-1990s. The causes of its changes, instead, remain unclear: Ireland (2007), for example, considers the hypothesis that the target is adjusted in response to supply shocks, but he cannot reject a model in which changes in the target are purely random.

One possible reading of the literature, therefore, may simply be that the Federal Reserve has intentionally adopted, for exogenous reasons, a higher target in the 1970s. But other interpretations are possible. For example, a number of studies argue that the Fed's real-time learning about the workings of the economy may have been responsible for the run-up of inflation.<sup>3</sup> This literature emphasizes how the experience of the 1950s and 1960s may have

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<sup>1</sup>The practice has changed since 2012, when after the January FOMC meeting, the Fed revealed that it was viewing a target of 2% as consistent over the long term with its mandate.

<sup>2</sup>Other papers (e.g., Smets and Wouters, 2007) estimate DSGE models that incorporate a time-varying inflation target, but their main focus is different.

<sup>3</sup>A partial list of papers that aim to study the U.S. inflation experience from the lens of different kinds of learning models include Sargent (1999), Primiceri (2006), Cogley and Sargent (2005), Milani (2008), Orphanides (2001), Bullard and Eusepi (2005), Erceg and Levin (2010), Lansing (2002), and Tambalotti (2004).

induced the Federal Reserve to perceive a low persistence in inflation and a steep inflation-output trade-off, which led policymakers to form overly optimistic forecasts for inflation in the 1970s and, as a result, caused the Fed to fail to increase interest rates by appropriate amounts.

If this interpretation is correct, the policymaker's slowness in learning that the economy was changing may be responsible for the perception of an increasing inflation target in the 1970s. That is, policymakers may have behaved *as if* they had a higher target, because they were taking policy decisions based on an outdated model of the economy.

This hypothesis has not been considered in previous research when estimating the inflation target. The cited papers, in fact, typically extract the target from an economy in which all economic agents form rational expectations. The implied target series, however, may importantly differ depending on the assumptions researchers make about the formation of expectations.

This paper, therefore, tries to estimate a potentially-changing inflation target under the assumption that both the Fed and the private sector are learning. The paper then compares the evolution of the inferred targets in different versions of a benchmark New Keynesian model under either rational expectations or learning. In this way, the paper aims to assess a potential driver of the changes in the revealed inflation target, by finding what portion of the target's time variation may be explained by policymaker's evolving beliefs and learning process.

The results indicate that in the models that impose the assumption of rational expectations, there is strong evidence of sizeable changes in the Fed's inflation target over time. The target series closely resembles the one found by Ireland (2007); the target is low at the beginning of the sample, it reaches peaks of 8% in the 1970s, and it settles down to 2-3% after Volcker's disinflation. But if the assumption of rational expectations is relaxed in favor of near-rational beliefs and learning, by letting the central bank respond to internal forecasts of inflation and output gap formed from its evolving perceived law of motion of the economy, the implied target is remarkably stable over time. When learning is taken into account, in fact, the estimated inflation target is characterized by only minimal variability, as it remains within a relatively narrow band between 1.91 and 2.87% over the sample.

This paper's findings, therefore, single out evolving beliefs and learning as a major determinant of the perceived large fluctuations in the Federal Reserve's inflation target that have been typically calculated over the post-war period in previous literature.

## 2 The Model

I estimate two versions of a benchmark New-Keynesian model (e.g., Woodford, 2003), which both allow the central bank to adopt a time-varying inflation target. The target is *exogenous* and will be treated as an unobservable in the estimation.

### 2.1 Model Under Rational Expectations

The first model assumes that economic agents (both private agents and the monetary authority) form rational expectations. Since the simplest version of the New Keynesian model under rational expectations usually fails to match the inertia of macroeconomic variables, it becomes necessary to introduce additional features that serve to add backward-looking dynamics in the main equations. I follow this strategy here by assuming that, in the rational expectations model, households' preferences are characterized by habit formation in consumption. The model is described by the following loglinearized equations

$$(\pi_t - \pi_t^*) = \beta E_t (\pi_{t+1} - \pi_t^*) + \kappa (\omega x_t + \psi^{-1} \tilde{x}_t) + u_t \quad (1)$$

$$\tilde{x}_t = E_t \tilde{x}_{t+1} - \psi (i_t - E_t \pi_{t+1} - r_t^n) \quad (2)$$

$$i_t = \rho i_{t-1} + (1 - \rho) [r_t^n + E_t \pi_{t+1} + \chi_\pi (E_t \pi_{t+1} - \pi_t^*) + \chi_x E_t x_{t+1}] + \varepsilon_t \quad (3)$$

$$r_t^n = \rho_r r_{t-1}^n + \nu_t^r \quad (4)$$

$$u_t = \rho_u u_{t-1} + \nu_t^u \quad (5)$$

$$\pi_t^* = \rho_{\pi^*} \pi_{t-1}^* + \nu_t^{\pi^*} \quad (6)$$

where  $\tilde{x}_t \equiv x_t - \eta x_{t-1}$ , with  $x_t$  denoting the output gap,  $\pi_t$  denoting inflation, and  $i_t$  denoting the nominal interest rate; the coefficient  $0 \leq \eta \leq 1$  measures the degree of habit formation in private expenditures.<sup>4</sup> The expectation operator  $E_t$  denotes rational expectations.

Equation (1) is the New-Keynesian Phillips curve that can be derived from the optimizing behavior of monopolistically-competitive firms under Calvo price setting. The coefficient  $0 < \beta < 1$  represents the discount factor,  $\kappa$  is a decreasing function of the degree of price stickiness, and  $\omega$  denotes the sensitivity of marginal costs to income. The deviation of the current inflation rate from the target depends on its one-period-ahead expected value, on the current output

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<sup>4</sup>I have also estimated versions with inflation indexation in price setting, but this was found to be unimportant once the model includes a time-varying inflation target.

gap, and on the deviation of the current output gap from the stock of habits. Equation (2) represents the loglinearized intertemporal Euler equation that derives from households' optimal choice of consumption. The current output gap depends on the expected and past output gap, and on the deviation of the ex-ante real interest rate from its natural level. The coefficient  $\psi > 0$  represents the pseudo-intertemporal elasticity of substitution of consumption that exists in a model with habits. The terms  $u_t$  and  $r_t^n$  denote AR(1) cost-push and natural rate disturbances, with  $\nu_t^u \sim iidN(0, \sigma_u^2)$  and  $\nu_t^r \sim iidN(0, \sigma_r^2)$ . The central bank follows a Taylor rule (eq. 3) by adjusting the short-term nominal interest rate in response to deviations of expectations of inflation from its (implicit) target and to changes in the expected output gap;  $\chi_\pi$  and  $\chi_x$  denote the policy reaction coefficients, while  $\rho$  accounts for the inertia of policy decisions. The inflation target  $\pi_t^*$  is assumed to follow a very persistent AR process ( $\rho_{\pi^*} = 0.999$ ).

## 2.2 Model Under Subjective Expectations and Learning

The second version of the New Keynesian model that I will use to infer the time-varying inflation target relaxes, instead, the strong informational assumptions required by rational expectations. Economic agents are assumed to form subjective expectations and are allowed to learn about the dynamics of the economy over time. In this case, the model can capture the inertia of macroeconomic variables through the sluggishness of expectations, rather than through the so-called “mechanical” sources of persistence (e.g., Milani, 2006, 2007), such as habit formation in consumption ( $\eta$  is set to zero in this case). The model is now summarized by the following equations

$$(\pi_t - \pi_t^*) = \beta \widehat{E}_t (\pi_{t+1} - \pi_t^*) + \kappa' x_t + u_t \quad (7)$$

$$x_t = \widehat{E}_t x_{t+1} - \sigma \left( i_t - \widehat{E}_t \pi_{t+1} - r_t^n \right) \quad (8)$$

$$i_t = \rho i_{t-1} + (1 - \rho) \left[ r_t^n + \widehat{E}_t \pi_{t+1} + \chi_\pi \left( \widehat{E}_t \pi_{t+1} - \pi_t^* \right) + \chi_x \widehat{E}_t x_{t+1} \right] + \varepsilon_t \quad (9)$$

where  $\kappa' = \kappa(\omega + \sigma^{-1}) > 0$  denotes the slope of the Phillips curve, and  $\sigma > 0$  denotes the intertemporal elasticity of substitution. In this version of the model,  $\widehat{E}_t$  differs from the mathematical expectations operator  $E_t$  and it denotes, instead, subjective, or near-rational, expectations.

Equations (7) and (8) are similar to eqs. (1) and (2), under  $\eta = 0$ . The monetary authority

is assumed to follow a Taylor rule in both models (eqs. 3 and 9). But, in the first version, the central bank responds to model-consistent rational expectations of future inflation and output gap, while, in the second version, it responds to expectations derived from a perceived law of motion of the economy, whose coefficients are updated over time based on recent data. The terms  $\widehat{E}_t\pi_{t+1}$  and  $\widehat{E}_tx_{t+1}$  in the Taylor rule will be interpreted in the paper as the internal forecasts by the central bank (although for simplicity, the private sector is assumed to form forecasts from exactly the same model that the central bank uses).

Equations (4) to (6), which describe the shock and inflation target processes, still hold in the same form in the model with learning.

### 2.3 Formation of Expectations and Learning

I will estimate the model assuming either rational expectations or learning. Under rational expectations, the model is solved as in Sims (2000), i.e. by introducing expectational errors  $\zeta_t^x = x_t - E_{t-1}x_t$  and  $\zeta_t^\pi = \pi_t - E_{t-1}\pi_t$  for the forward-looking variables  $x_t$  and  $\pi_t$ . The model is then solved by finding a mapping between the expectational errors and the structural innovations.

Under learning, I will assume that economic agents use a perceived law of motion (PLM) to form their expectations and learn about reduced-form coefficients (see Evans and Honkapohja, 2001, for a comprehensive treatment of learning models, and Evans and Honkapohja, 1999, 2011, for surveys)

$$Y_t = \phi_{0,t} + \phi_{1,t}Y_{t-1} + \epsilon_t \quad (10)$$

where  $Y_t \equiv [\pi_t, x_t]'$  and where  $\phi_{0,t}$  and  $\phi_{1,t}$  denote a vector of intercept terms and a matrix of coefficients. The PLM here doesn't correspond to the Minimum State Variable (MSV) solution under rational expectations. The agents' model is, therefore, misspecified (for example, agents are not endowed with information about the contemporaneous disturbances  $u_t$  and  $r_t^n$ , which would enter the MSV solution). They use, instead, an empirical model, which corresponds to a VAR(1) that includes lagged values of the output gap and inflation to produce forecasts for the same variables in  $t + 1$ .

As additional data become available, they update their estimates according to the constant-

gain learning formula<sup>5</sup>

$$\widehat{\phi}_t = \widehat{\phi}_{t-1} + \bar{\mathbf{g}}R_t^{-1}X_t(Y_t - \widehat{\phi}'_{t-1}X_t)' \quad (11)$$

$$R_t = R_{t-1} + \bar{\mathbf{g}}(X_tX_t' - R_{t-1}) \quad (12)$$

where  $\widehat{\phi}_t = (\phi_{0,t}, \phi_{1,t})'$  describes the updating of the learning rule coefficients, and  $R_t$  the updating of the second moments matrix of the stacked regressors  $X_t \equiv \{1, \pi_{t-1}, x_{t-1}\}'$ . The coefficient  $\bar{\mathbf{g}}$  denotes the constant gain, which governs the rate at which agents discount past information when forming their beliefs.

Economic agents form expectations about future output gap and inflation in  $t + 1$  from (10),<sup>6</sup> using the updated parameter estimates in (11) and (12). These expectations can be substituted into (7)-(9) to obtain the Actual Law of Motion of the economy (ALM), which can be represented in state-space form and estimated using likelihood-based techniques (more details are contained in Appendix A).

### 3 Estimating the Federal Reserve's Changing Inflation Target

I adopt Bayesian methods in the estimation to match quarterly U.S. data on inflation, the output gap, and the federal funds rate, spanning the period between 1960:I and 2005:II. The output gap is calculated as the log difference between real GDP and the CBO's Potential GDP estimate, inflation is calculated as the quarterly log difference of the GDP Implicit Price Deflator, and the federal funds rate is used in levels and transformed to yield quarterly rates. I jointly estimate the structural parameters of the model and the time-varying inflation target under different assumptions about the formation of expectations: rational expectations and subjective expectations with learning. The target is unobserved and derived, at each MCMC draw, through runs of the Kalman filter and the Kalman smoother. In the estimation, I fix the household's discount factor  $\beta = 0.99$ , the elasticity of marginal costs to output  $\omega = 0.5$ , and the autoregressive coefficient in the inflation target process  $\rho_{\pi^*} = 0.999$ . Table 1 reports information about the priors for the other coefficients. I select a Gamma prior distributions with mean 1 for the intertemporal elasticity of substitution coefficient  $\psi$  (or  $\sigma$  in

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<sup>5</sup>Branch and Evans (2006) show that constant-gain learning outperforms several alternatives in forecasting output and inflation.

<sup>6</sup>I assume that agents dispose only of information on economic variables up to  $t - 1$ , when forming their expectations about variables in  $t + 1$  (this assumption is common in the adaptive learning literature).

the model with learning) and with mean 0.125 for  $\kappa$  ( $\kappa'$  under learning). In the RE model, I assume a Uniform[0,1] prior distribution for the habit formation coefficient. The autoregressive coefficients  $\rho$ ,  $\rho_r$ , and  $\rho_u$  follow Beta prior distributions, while Inverse Gamma priors are chosen for the standard deviation coefficients. I assume a Normal prior with mean 1.5 for the policy reaction to inflation  $\chi_\pi$  and a Gamma prior with mean 0.25 for the reaction to the output gap  $\chi_x$ . In the model with learning, the constant gain  $\bar{g}$  is assumed equal to 0.02, which is a value often used in the adaptive learning literature (e.g., Orphanides and Williams, 2005) and which is close to the value estimated in Milani (2007). The initial values for the agents' beliefs, as well as the associated initial precision matrix, are estimated using pre-sample (1951-1959) data.<sup>7</sup>

The Metropolis-Hastings algorithm is used to generate draws from the posterior distribution. I run 500,000 draws for each model, discarding an initial burn-in of 125,000 draws.<sup>8</sup> The posterior estimates of the structural parameters for the models with RE and learning are presented in Table 2.

### 3.1 Rational Expectations

I estimate posterior means for  $\psi$  equal to 1.207 and for  $\kappa$  equal to 0.016. The Taylor rule estimates indicate a response to inflation  $\chi_\pi$  equal to 1.526 and a limited response to the output gap,  $\chi_x = 0.037$ . A large degree of habit formation in consumption is necessary to fit the data ( $\eta = 0.960$ ).

Figure 1 shows the evolution of the estimated Federal Reserve's inflation target obtained in the model that assumes that both the private sector and the central bank have rational expectations. The changing target closely resembles the one estimated by Ireland (2007) in a related, yet somewhat different, setting. The target starts low in the 1960s (with values below 2%), but it quickly drifts upward in the late 1960s and early 1970s (to values slightly above 4.5%). The inferred target then reaches peaks of 8% in the 1970s, before falling to 3-4%

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<sup>7</sup>Some of these initial beliefs may start far from their values under the invariant equilibrium under learning. Indeed, while the initial perceptions regarding the output gap equation are not far from the equilibrium values, the initial beliefs regarding the autoregressive coefficient for inflation and the intercept in the inflation equation fall far from those. The idea is that these initial values are obtained from a previous regime, characterized by a lower mean for inflation, and a modest persistence (this is consistent with what agents would have learned from the history of inflation before 1960, and as documented by Sargent, 1999, and Primiceri, 2006). Afterwards, they revise their beliefs based on new data. While this is the preferred approach in the paper, I will also check the robustness of the results to different initializations in the Robustness section.

<sup>8</sup>See Milani (2007, 2008) and Appendix A for more details on the estimation.



around the Volcker’s disinflation period. After 1990, the target stabilizes around 2% (a large decline in the target is observed in concomitance with the 1990-1991 recession, a pattern that is consistent with the “opportunistic approach to disinflation”’s interpretation, and which has also been obtained in Leigh, 2008).

### 3.2 Subjective Expectations and Learning

In the model with learning, the posterior estimates indicate a mean equal to 0.643 for  $\sigma$  and to 0.023 for the slope of the Phillips curve  $\kappa'$ . The estimates of the monetary policy rule coefficients reveal a reaction coefficient to inflation equal to 1.459 and a reaction to the output gap equal to 0.201.<sup>9</sup>

Figure 2 illustrates the updating of private agents and policymakers’ beliefs over the sample. As in Milani (2004), by looking at the historical evidence, economic agents perceive a negative intercept and a low autoregressive term in the inflation equation until the early 1970s (the perceived autocorrelation coefficient  $\phi_{11,t}^{\pi}$  starts close to zero in the 1960s and it is revised upward to 0.8 only in the late 1970s), together with a relatively large sensitivity of inflation to changes in the output gap (which is perceived by agents to be equal to 0.06 in the early 1970s, before falling to values around 0.02 after 1980). These perceptions lead economic agents and the central bank to underestimate inflation in the late 1960s and for most of the 1970s (Figure 3). The underestimation of inflation in the second half of the 1960s and in the 1970s, along with its overestimation in the early 1960s and in the 1980s and 1990s, is consistent with the available evidence from survey forecasts (as discussed, for example, in Croushore, 1998).

Figure 4 displays the inferred inflation target when the Fed is allowed to update its beliefs and learn about the economy over time. The target is much more stable over time. While under rational expectations the estimated target jumps from 2% to 8% in the 1970s, under learning, the target moves in a narrower band between 1.91% and 2.87% over the whole post-war sample.

Therefore, the evolution of the Federal Reserve’s inflation target can be seen through two different lenses. The first indicates that the Federal Reserve has set policy under rational expectations and full knowledge of the dynamics of the economy over the post-war period, but

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<sup>9</sup>At each draw, I check whether the corresponding equilibrium under rational expectations would be determinate. Given the strong estimated reaction to inflation, the equilibrium is always determinate. The system would also be E-stable under learning if agents used a minimum state variable solution including the exogenous disturbances as their PLM.

its target for inflation has been subject to large changes for some exogenous reasons. Or, in the second case, the central bank may be assumed to have imperfect beliefs about the structure of the economy and to have attempted to learn it over time. Incorrect beliefs in real time – such as a perception of an extremely low persistence in inflation – have led the central bank to underestimate future inflation in the 1970s and, hence, to policy decisions that look ex-post overly accommodative, even though its desired target for inflation has remained quite stable over the entire sample.<sup>10</sup> This learning story, therefore, provides a way to endogenize and explain the sizeable shifts in the inflation target that are obtained under rational expectations; when policymakers’ learning is taken into account, the estimated inflation target remains characterized by only minor movements over time.

Moreover, it is possible to evaluate which of the two accounts provides a better match of the data by computing the models’ marginal likelihoods (using Geweke’s modified harmonic mean estimator). The model with learning attains a much better fit of the data. The marginal likelihood is -259.09, compared with -278.766 for the model with rational expectations, which was further enriched to include habit formation.

## 4 Robustness

This section evaluates the sensitivity of the results to a range of alternative assumptions.

First, it might be assumed, as in Ireland (2007), that the central bank adjusts its inflation target in response to positive or negative supply shocks (this permits to account for the ‘opportunistic approach to disinflation’ theory). The process for the inflation target is now given by

$$\pi_t^* = \rho_{\pi^*} \pi_{t-1}^* + \delta_u u_t + \nu_t^{\pi^*} \quad (13)$$

where  $\delta_u$  is the coefficient that denotes the extent to which the central bank adjusts its target in response to supply shocks. In the estimation,  $\delta_u$  is assumed to follow a  $\Gamma(2, 30)$  prior, implying a prior mean equal to 0.067. The posterior mean estimates for  $\delta_u$  are found to be equal to 0.046 under rational expectations and 0.012 under learning.

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<sup>10</sup>The main interpretation offered in the paper looks as the expectation terms in the Taylor rule as the internal central bank’s forecasts. Under the alternative interpretation that considers those expectations as the observed private sector forecasts, the findings would highlight some potential dangers of responding to unaligned expectations by private agents.

As a second robustness check, I re-estimate the learning model under higher gain coefficients (setting  $\bar{g} = 0.03$  and  $0.05$ ).

The implied inflation target series are shown in Figure 5. The targets under rational expectations and learning, when allowed to react to supply shocks, remain similar to those obtained in the previous cases. The inflation targets obtained under learning have levels that vary somewhat depending on the speed of the learning process (the faster the learning process, the sooner the central bank recognizes that inflation follows a persistent process, the higher the implied estimated target over the first half of the sample), but they are still substantially stable over time. Therefore, the main conclusions of the paper remain unaffected.

The results are also robust to the use of alternative initializations for the learning process. I assume three additional cases besides the initialization based on pre-sample beliefs. In one case, the initial beliefs are set at zero, but with the perceived autoregressive coefficients starting from 0.5. In the second case, the beliefs about autoregressive coefficients are initialized at 0.9. In the final case, the initial beliefs about autoregressive coefficient are left unspecified and jointly estimated along with the structural parameters. The inferred inflation target series are shown in Figure 6. The figure shows some differences in the early part of the sample, and a slightly higher target under the 0.5 initialization, but, overall, the target series remain very close to each other and stable over the full sample period.

Finally, in the baseline estimation, the RE model included habit formation in consumption, while the learning model abstracted from such assumption. I re-estimate a model now that includes both learning and habit formation. Figure 7 shows the estimated inflation target obtained in this case. The target remains similar to the one estimated in the model with no habits. The posterior estimates for the model with habit formation and learning are presented in Table 3. Under learning, the degree of habit formation is modest, with a posterior mean for  $\eta$  estimated at 0.11, while it was 0.96 under rational expectations. The assumption of adaptive learning in the formation of expectations provides, in fact, an alternative to habit formation for introducing endogenous persistence in the model. Both add a degree of backward-lookingness into the model, either by making output a function of its lagged value in the Euler equation (with habits) or through the updating of beliefs based on past data (under learning). In the case of learning, however, the implied persistence may also vary over time depending on the state of beliefs. Previous papers by Milani (2007) and Milani (2017) similarly show that

habit formation may become attenuated, or even redundant, when learning replaces rational expectations.

## 4.1 Survey Expectations

Figure 8 compares the inflation expectations formed by agents through their PLM over the sample with the corresponding inflation expectation series from the Survey of Professional Forecasters (I use the mean forecast across individual forecasters for the Price Deflator measure of Inflation, and referring to  $t + 1$ , the quarter after the survey date). The forecasts from the learning model appear in line with the forecasts from the survey. In particular, the forecasts obtained from the case with initial beliefs for the autoregressive coefficients equal to 0.5 track the survey series most closely. The similarity in model-implied and survey forecasts is reassuring that the results are not driven by highly unrealistic beliefs.<sup>11</sup>

## 4.2 Estimation for an Inflation Targeter: United Kingdom

An indirect way to corroborate the evidence of a roughly constant inflation target in a model with subjective expectations is to repeat the analysis using United Kingdom data. With the Bank of England Act announced in May 1997, the Bank of England was granted (instrument) independence and moved to an inflation targeting regime. The inflation target was clearly set and communicated to the private sector. It is, therefore, possible to repeat the estimation on U.K. data and check whether we are able to infer an evolution of the inflation target roughly around 2% over the sample.

The estimation results are reported in Table 4. Figure 9 shows the estimated inflation target from the model, along with actual inflation. The target falls slightly below 2% as a result of the initial observations, but it remains roughly around 2% over the post-1998 sample. The 95% bands contain the 2% value at all times.

## 5 Conclusions

This paper has attempted to infer the evolution of the Fed's (unobserved) inflation target over the post-war period from an estimated monetary model under the alternative assumptions of

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<sup>11</sup>Some recent papers (Molnar and Ormeno, 2015, Del Negro and Eusepi, 2011, and Milani, 2011, 2017) take this exercise further by estimating the model adding survey expectations to the list of observables.

rational expectations and learning.

Under rational expectations, the estimated inflation target is characterized by large fluctuations over time. The target implicitly adopted by the Federal Reserve reaches peaks of 8% in the 1970s, while dropping to 2% in the post-Volcker sample. When the assumption of rational expectations is relaxed, by allowing the central bank to adjust its beliefs and learn about the structure of the economy, the inferred Fed's inflation target is almost constant over the sample. The model with learning and a more stable target provides a substantially better fit of the data than does the model with rational expectations with a widely fluctuating target. The results hence indicate that when learning is taken into account, there is no need to appeal to large swings in the exogenous inflation target to explain the historical U.S. monetary policy experience. The Federal Reserve's evolving beliefs and learning, therefore, appear as major endogenous causes of the perception of a rising and falling inflation target over the sample, as they can successfully account for almost all its time variation.

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# A Appendix

## A.1 Estimation of Model with Learning

The model equations in expressions (7) to (9), along with the processes for the unobserved disturbances (4) to (6), can be written in state-space form as:

$$A_0(\Theta)\Upsilon_t = A_1(\Theta)\Upsilon_{t-1} + A_2(\Theta)\widehat{E}_{t-1}\Upsilon_{t+1} + A_3(\Theta)\nu_t, \quad (14)$$

where  $\Upsilon_t = [\pi_t, x_t, i_t, u_t, r_t^n, \pi_t^*]'$ ,  $\nu_t = [\nu_t^u, \nu_t^r, \nu_t^{\pi^*}]'$ , and where  $\widehat{E}_{t-1}$  denotes subjective expectations, which are in this case formed on the basis of a  $(t-1)$  information set;  $A_0$ ,  $A_1$ ,  $A_2$ , and  $A_3$ , are matrices of coefficients, which are functions of the model's structural parameters collected in the vector  $\Theta = \{\beta, \kappa', \sigma, \rho, \chi_\pi, \chi_x, \rho_u, \rho_r, \rho_{\pi^*}, \sigma_u, \sigma_r, \sigma_{\pi^*}, \sigma_\varepsilon, \bar{\mathbf{g}}\}$ .

Expectations in the model are formed from the Perceived Law of Motion (PLM) described in expression (10) as

$$\widehat{E}_{t-1}\Upsilon_{t+1} = (I + \widehat{\Phi}_{1,t-1})\widehat{\Phi}_{0,t-1} + \widehat{\Phi}_{1,t-1}^2\Upsilon_{t-1}, \quad (15)$$

where  $\widehat{\Phi}_{0,t} = [\widehat{\phi}_{0,t}', 0_{1 \times 4}]'$ ,  $\widehat{\Phi}_{1,t} = \begin{bmatrix} \widehat{\phi}_{1,t} & \mathbf{0}_{2 \times 4} \\ \mathbf{0}_{4 \times 2} & \mathbf{0}_{4 \times 4} \end{bmatrix}$ ,  $I$  denotes a  $6 \times 6$  identity matrix, and  $\widehat{\phi}_{0,t}$ ,  $\widehat{\phi}_{1,t}$  are obtained from the constant-gain recursion in (11).

Expectations from (15) can be substituted into the original system (14) to obtain the Actual Law of Motion, or ALM, of the economy

$$\Upsilon_t = C(\Theta, \widehat{\Phi}_{t-1}) + F(\Theta, \widehat{\Phi}_{t-1})\Upsilon_{t-1} + D(\Theta)\nu_t \quad (16)$$

where  $C(\Theta, \widehat{\Phi}_{t-1}) = A_0^{-1}A_2(I + \widehat{\Phi}_{1,t-1})\widehat{\Phi}_{0,t-1}$ ,  $F(\Theta, \widehat{\Phi}_{t-1}) = A_0^{-1}(A_1 + A_2\widehat{\Phi}_{1,t-1}^2)$ ,  $D = A_0^{-1}A_3$ . The coefficients in  $C_t$ ,  $F_t$ , are, therefore, time-varying as a result of the updating of agents' beliefs, which are updated according to the learning formulae (11)-(12) reported earlier in the text.

To the ALM (16), which governs the dynamics of the structural model, we add the following observation equation:

$$\text{Observables}_t = H_0 + H\Upsilon_t, \quad (17)$$

where  $H_0$  collects average, or steady-state, values, and  $H$  is a matrix of ones and zeroes, which selects observables from the state vector  $\Upsilon_t$ . As discussed in the text, the vector of observable variables includes the inflation rate, the output gap, and the Federal Funds rate, for  $\pi_t$ ,  $x_t$ ,  $i_t$ .

The vector of structural parameters  $\Theta$  is estimated using Bayesian methods. We generate draws using a Random-Walk Metropolis-Hastings (MH) algorithm, which will be described below. The likelihood of the system (16)-(17) is obtained at each MH draw using the Kalman filter. At each iteration  $t$  of the Kalman filter, with  $t = 1, \dots, T$ , the beliefs of the agents are updated according to (11)-(12). In terms of timing, the endogenous variables at date  $t$  depend on expectations that are formed conditioning on information sets including the values of endogenous variables in  $t-1$ . When forming those expectations, agents update their beliefs by regressing endogenous variables up to  $t-1$  on variables dated  $t-2$  (in this case, simply the endogenous variables' lagged values): the resulting beliefs are denoted  $\widehat{\phi}_{0,t-1}$ , and  $\widehat{\phi}_{1,t-1}$ , and enter (15), and thus (16).

More specifically, the Metropolis-Hastings procedure works as follows:

1. Start from an initial guess for the parameter vector  $\Theta_0$ . Set  $j = 1$ .



2. Evaluate the likelihood function  $p(\Upsilon^T|\Theta_0)$  for the system's ALM using the Kalman filter. The vector  $C_{t-1}$  and the matrix  $F_{t-1}$  are functions of agents' beliefs  $\Phi_{t-1}$  and hence time-varying. We run the Kalman filter from  $t = 1$  to  $T$  in the sample. At each iteration  $t$ ,  $C_{t-1}$  and  $F_{t-1}$  are updated using the newly computed beliefs  $\Phi_{t-1}$ , obtained from the constant-gain learning formulas (11)-(12).
3. Evaluate the prior probability  $p(\Theta_0)$ . The priors for the coefficients are assumed independent.
4. Obtain the posterior density as  $p(\Upsilon^T|\Theta_0)p(\Theta_0)$ .
5. For  $j = 1$ , and for all subsequent draws  $j$ , generate  $\Theta_j^* = \Theta_{j-1} + \varepsilon$ , where  $\Theta_j^*$  is the proposal draw and  $\varepsilon \sim N(0, c\Sigma_\varepsilon)$ .  $c$  is a scale factor that is usually adjusted to keep the acceptance ratio of the MH algorithm at a satisfactory rate (e.g., within a 25%-50% range).
6. Generate  $u$  from a *Uniform*[0, 1]
7. Set 
$$\begin{cases} \Theta_j = \Theta_j^* & \text{if } u \leq \alpha(\Theta_{j-1}, \Theta_j^*) = \min \left\{ \frac{p(\Upsilon^T|\Theta_j^*)p(\Theta_j^*)}{p(\Upsilon^T|\Theta_{j-1})p(\Theta_{j-1})}, 1 \right\} \\ \Theta_j = \Theta_{j-1} & \text{if } u > \alpha(\Theta_{j-1}, \Theta_j^*) \end{cases}$$
8. Repeat for  $j + 1$  from step 2. until  $j = D$ , where  $D$  denotes the total number of draws. In our estimations we set  $D = 500,000$ .

Posterior means for the model coefficients, as well as the evolution of beliefs, and of unobserved variables, such as the central bank's inflation target, are obtained as averages across MH draws, after discarding an initial burn-in period, corresponding to the first 125,000 draws.

Parameters	Prior Distr.	Prior Mean	95% Prior Prob. Interval
$\beta$		0.99	
$\psi, \sigma$	$\Gamma$	1	[0.12, 2.79]
$\kappa, \kappa'$	$\Gamma$	0.125	[0.015, 0.35]
$\rho$	$B$	0.7	[0.32, 0.96]
$\chi_\pi$	$N$	1.5	[1.25, 1.75]
$\chi_x$	$\Gamma$	0.25	[0.03, 0.70]
$\rho_r$	$B$	0.5	[0.09, 0.91]
$\rho_u$	$B$	0.5	[0.09, 0.91]
$\sigma_r$	$\Gamma^{-1}$	0.5	[0.14, 1.62]
$\sigma_u$	$\Gamma^{-1}$	0.5	[0.14, 1.62]
$\sigma_\varepsilon$	$\Gamma^{-1}$	0.5	[0.14, 1.62]
$\rho_{\pi^*}$		0.999	
$\sigma_{\pi^*}$	$\Gamma^{-1}$	0.01	[0.001, 0.04]
$\eta$	$U$	0.5	[0.025, 0.975]
$\bar{g}$		0.02	

Table 1 - Prior Distributions: model parameters under rational expectations and learning.

Notes: The symbols in the table denote the following prior distribution:  $B$  = Beta,  $N$  = Normal,  $\Gamma$  = Gamma,  $\Gamma^{-1}$  = Inverse Gamma,  $U$  = Uniform.

Parameters	Rational Expectations		Learning	
	Post. Mean	95% HPD Int.	Post. Mean	95% HPD Int.
$\beta$	.99	–	0.99	–
$\psi$	1.207	[.73, 1.85]		
$\sigma$			0.643	[.44, .86]
$\kappa$	0.016	[.005, .035]		
$\kappa'$			0.023	[.004, .05]
$\rho$	0.708	[.60, .81]	0.905	[.86, .94]
$\chi_\pi$	1.526	[1.29, 1.78]	1.459	[1.22, 1.71]
$\chi_x$	0.037	[.003, .12]	0.201	[.07, .37]
$\rho_r$	0.772	[.65, .88]	0.371	[.24, .51]
$\rho_u$	0.44	[.12, .67]	0.631	[.51, .75]
$\sigma_r$	0.631	[.44, .90]	1.178	[.87, 1.66]
$\sigma_u$	0.146	[.10, .20]	0.322	[.29, .36]
$\sigma_\varepsilon$	0.174	[.15, .20]	0.208	[.19, .23]
$\rho_{\pi^*}$	0.999	–	0.999	–
$\sigma_{\pi^*}$	0.111	[.08, .15]	0.008	[.002, .03]
$\eta$	0.960	[.87, .99]		
$\bar{g}$			0.02	–
MargL		-278.766		-259.09

Table 2 - Posterior Parameter Estimates: Monetary Model with Rational Expectations and Learning.

Notes: the table reports the posterior mean estimates for each parameter, along with the corresponding 95% Highest Posterior Density (HPD) intervals. The last row displays the models' marginal likelihoods, computed using Geweke's modified harmonic mean approximation.

<b>Learning</b>		
Parameters	Post. Mean	95% HPD Int.
$\beta$	0.99	—
$\psi$		
$\sigma$	0.90	[.57, 1.30]
$\kappa$		
$\kappa'$	0.02	[.002, .04]
$\rho$	0.87	[.79, .92]
$\chi_\pi$	1.52	[1.30, 1.75]
$\chi_x$	0.15	[.05, .30]
$\rho_r$	0.38	[.22, .54]
$\rho_u$	0.62	[.51, .73]
$\sigma_r$	0.97	[.68, 1.37]
$\sigma_u$	0.32	[.29, .35]
$\sigma_\varepsilon$	0.20	[.18, .23]
$\rho_{\pi^*}$	0.999	—
$\sigma_{\pi^*}$	0.007	[.002, .02]
$\eta$	0.11	[.003, .34]
$\bar{g}$	0.02	—
MargL	-260.7632	

Table 3 - Robustness: Posterior Parameter Estimates for model with both Learning and Habit Formation.

Notes: the table reports the posterior mean estimates for each parameter, along with the corresponding 95% Highest Posterior Density (HPD) intervals. The last row displays the models' marginal likelihoods, computed using Geweke's modified harmonic mean approximation.

<b>Learning</b>		
Parameters	Post. Mean	95% HPD Int.
$\beta$	0.99	—
$\psi$		
$\sigma$	0.73	[.42, 1.19]
$\kappa'$	0.09	[.1, .20]
$\rho$	0.96	[.94, .97]
$\chi_\pi$	1.46	[1.22, 1.69]
$\chi_x$	0.17	[.02, .45]
$\rho_r$	0.43	[.21, .71]
$\rho_u$	0.71	[.47, .89]
$\sigma_r$	0.71	[.41, 1.17]
$\sigma_u$	0.60	[.49, .75]
$\sigma_\varepsilon$	0.09	[.08, .12]
$\rho_{\pi^*}$	0.999	—
$\sigma_{\pi^*}$	0.007	[.001, .015]
$\bar{\mathbf{g}}$	0.02	—

Table 4 - Posterior Parameter Estimates for United Kingdom.

Notes: the table reports the posterior mean estimates for each parameter, along with the corresponding 95% Highest Posterior Density (HPD) intervals. The last row displays the models' marginal likelihoods, computed using Geweke's modified harmonic mean approximation.

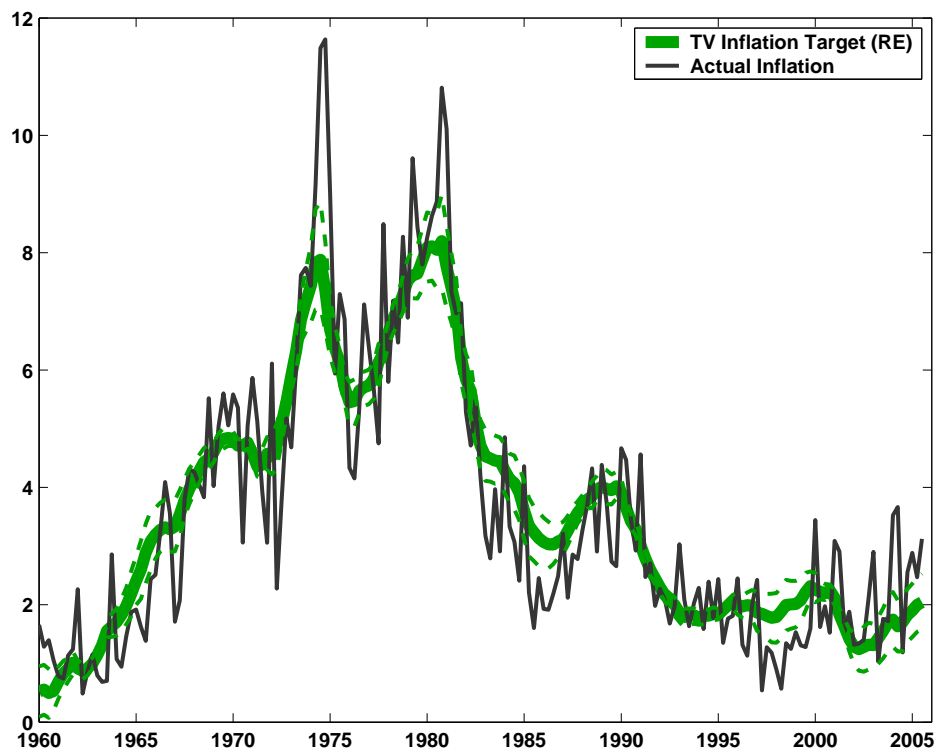


Figure 1: Actual Inflation and Inferred Time-Varying Inflation Target in a Model with Rational Expectations.

*Notes:* the inflation target series shown in the figure is the mean across MCMC draws (solid line); the dashed lines represent the corresponding 5 and 95% percentiles.

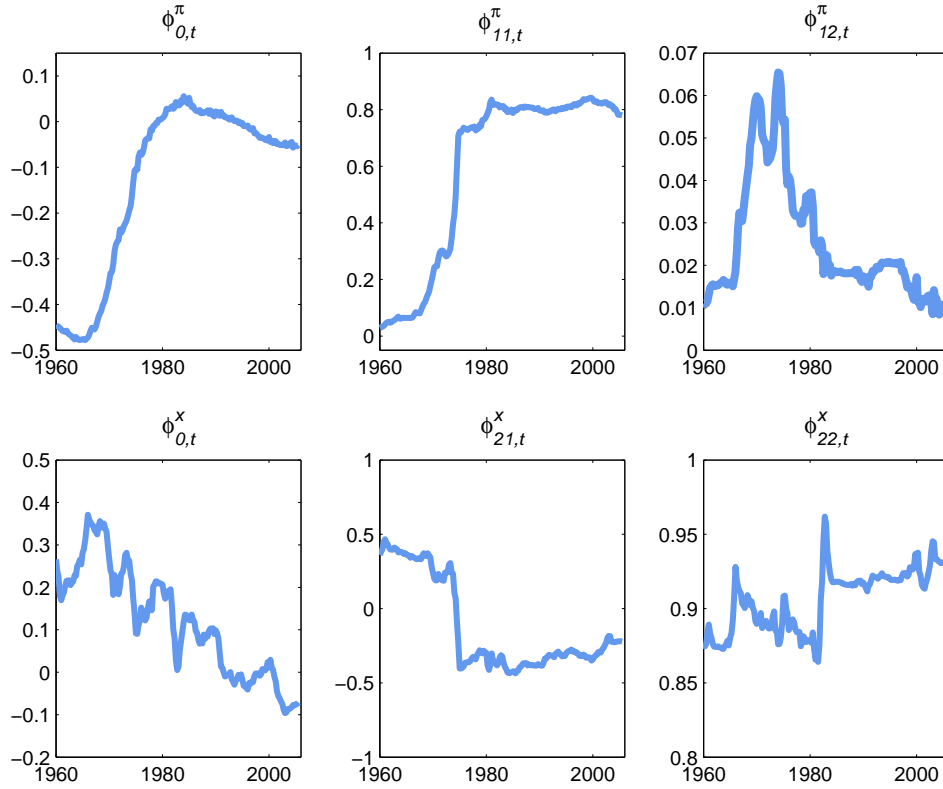


Figure 2: Evolution of agents' beliefs, 1960:I-2005:II.

Note: The agents' learning rule is

$$\begin{pmatrix} \pi_t \\ x_t \end{pmatrix} = \begin{pmatrix} \phi_{0,t}^\pi \\ \phi_{0,t}^x \end{pmatrix} + \begin{pmatrix} \phi_{11,t}^\pi & \phi_{12,t}^\pi \\ \phi_{21,t}^x & \phi_{22,t}^x \end{pmatrix} \begin{pmatrix} \pi_{t-1} \\ x_{t-1} \end{pmatrix} + \begin{pmatrix} \epsilon_t^\pi \\ \epsilon_t^x \end{pmatrix}.$$

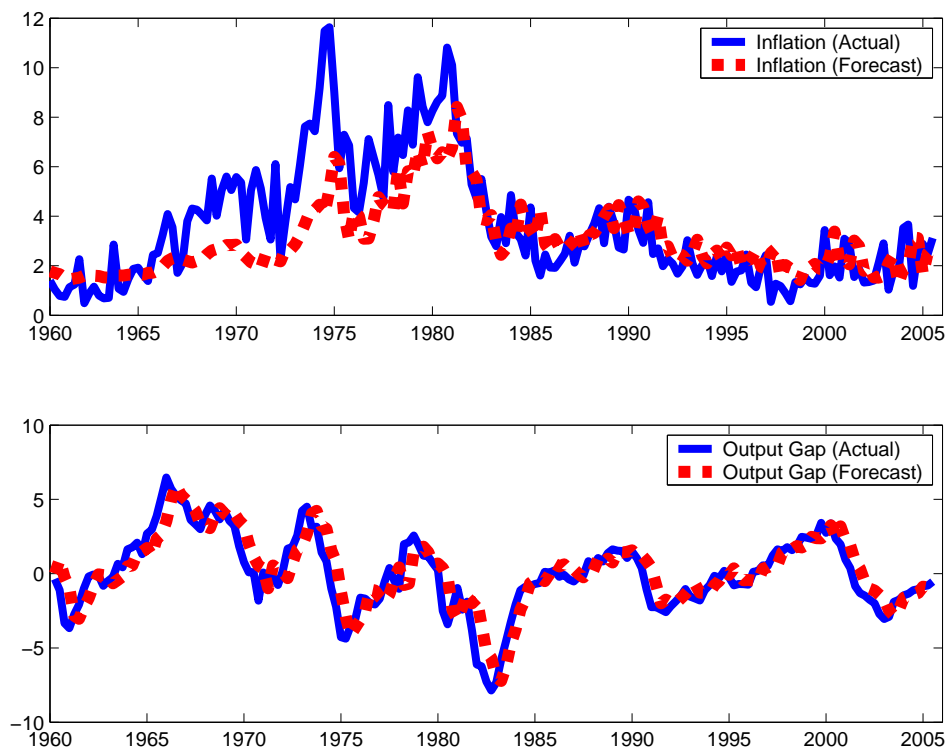


Figure 3: Inflation and Output Gap Forecasts from Economic Agents' Perceived Law of Motion.



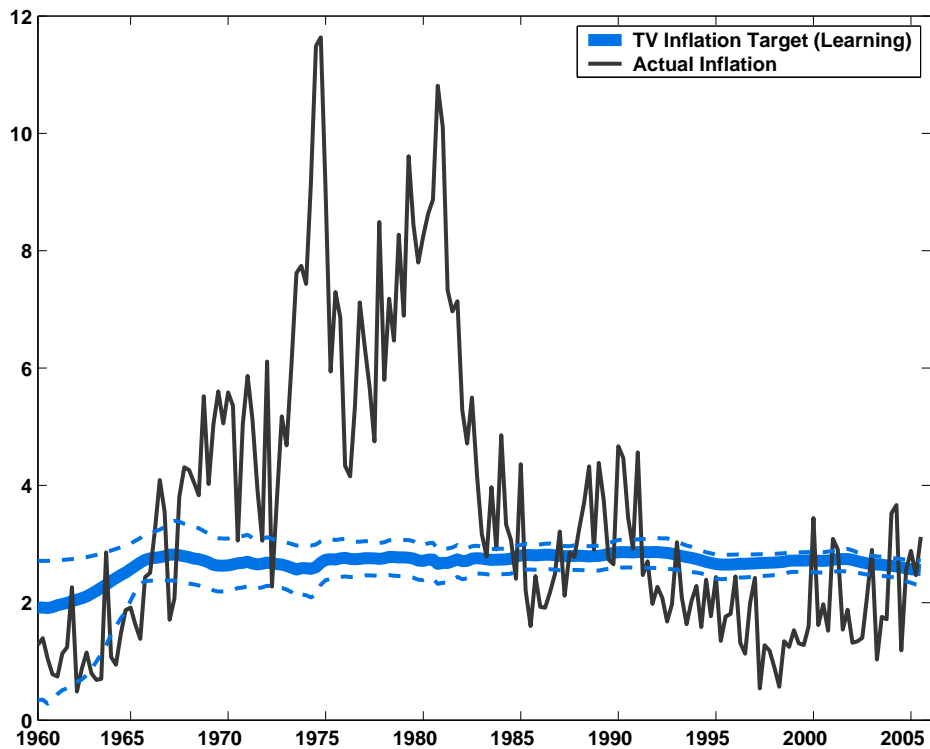


Figure 4: Actual Inflation and Inferred Time-Varying Inflation Target in a Model with Subjective Beliefs and Constant-Gain Learning.

*Notes:* the inflation target series shown in the figure is the mean across MCMC draws (solid line); the dashed lines represent the corresponding 5 and 95% percentiles.

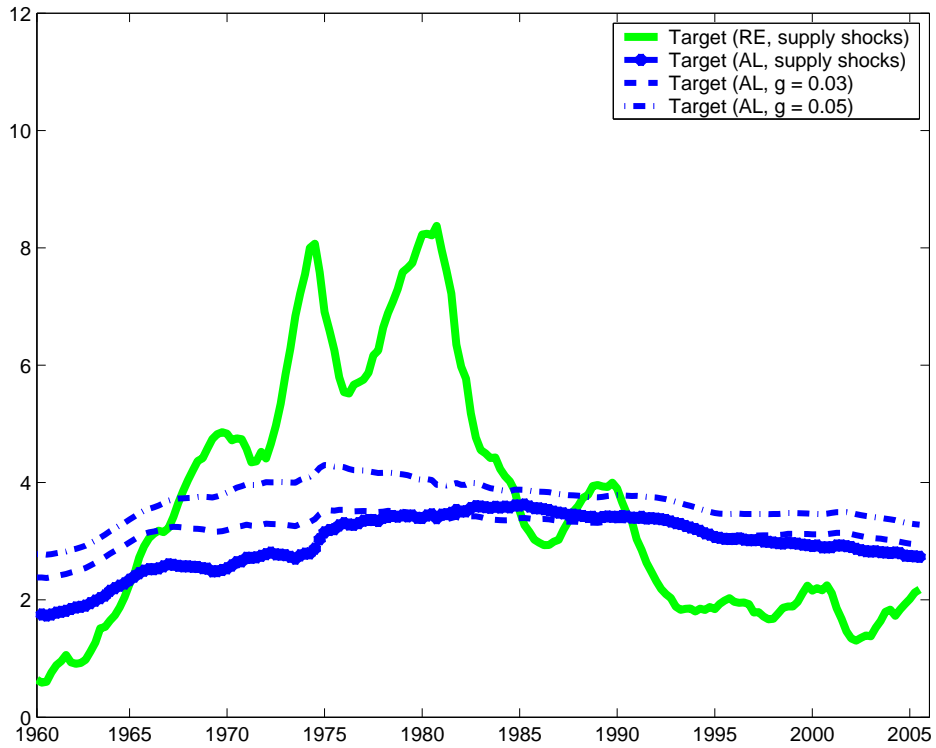


Figure 5: Sensitivity Analysis: Time-Varying Inflation Targets under alternative assumptions (response to supply shocks, higher gain coefficients).

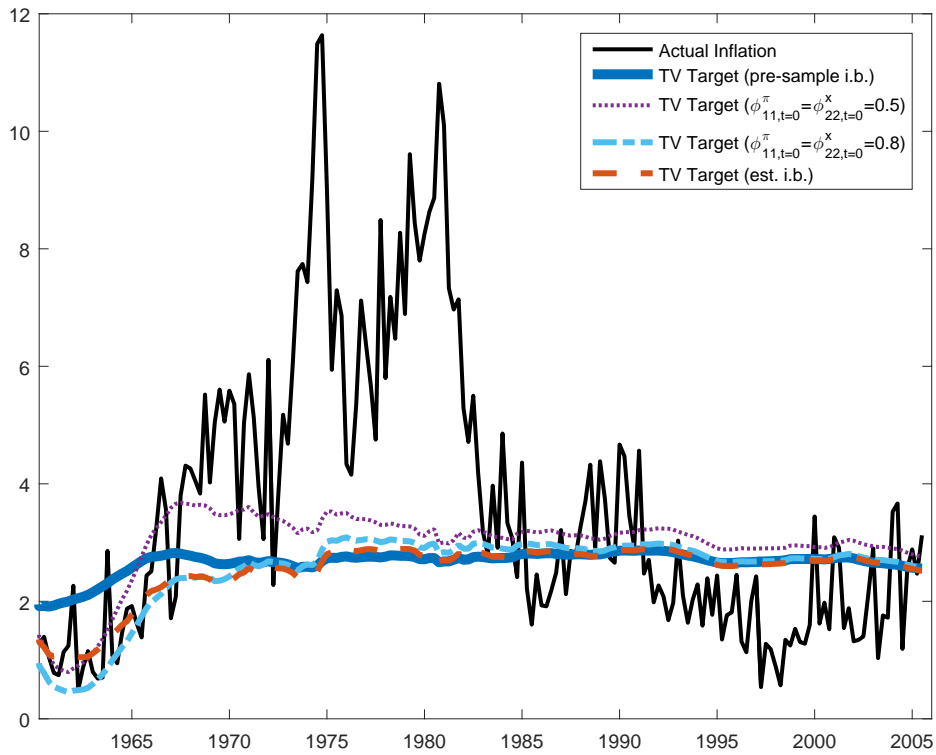


Figure 6: Sensitivity Analysis: Time-Varying Inflation Targets under alternative assumptions about initial beliefs (based on pre-sample estimation, with perceived AR coefficients equal to 0.5, with perceived AR coefficients equal to 0.9, or jointly estimated).

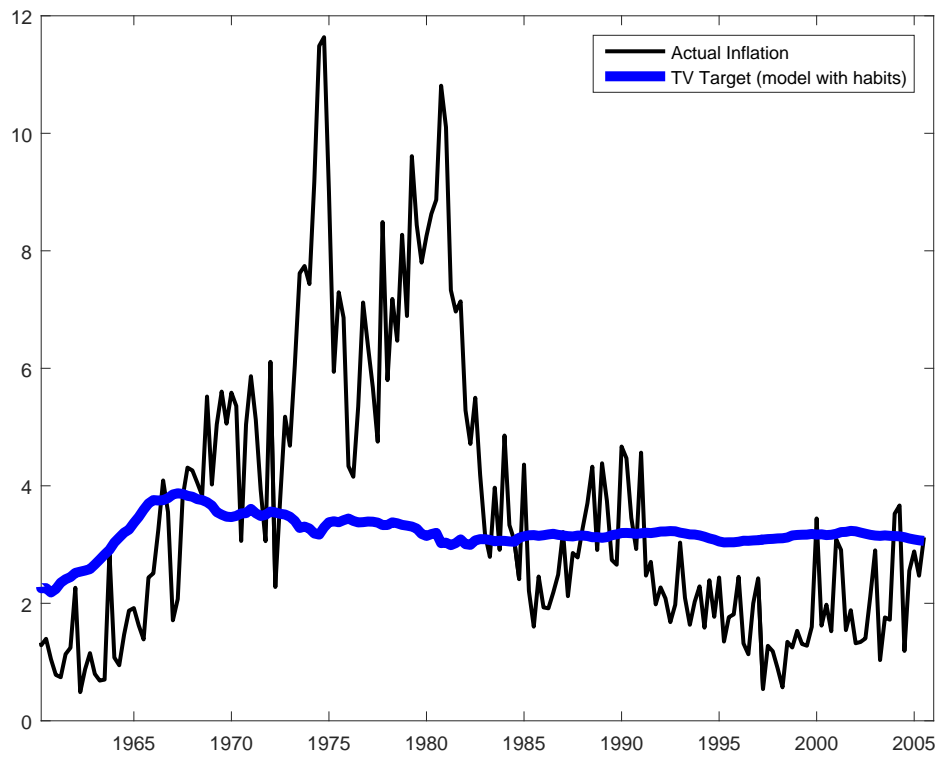


Figure 7: Sensitivity Analysis: Time-Varying Inflation Target in the Model with Habit Formation.

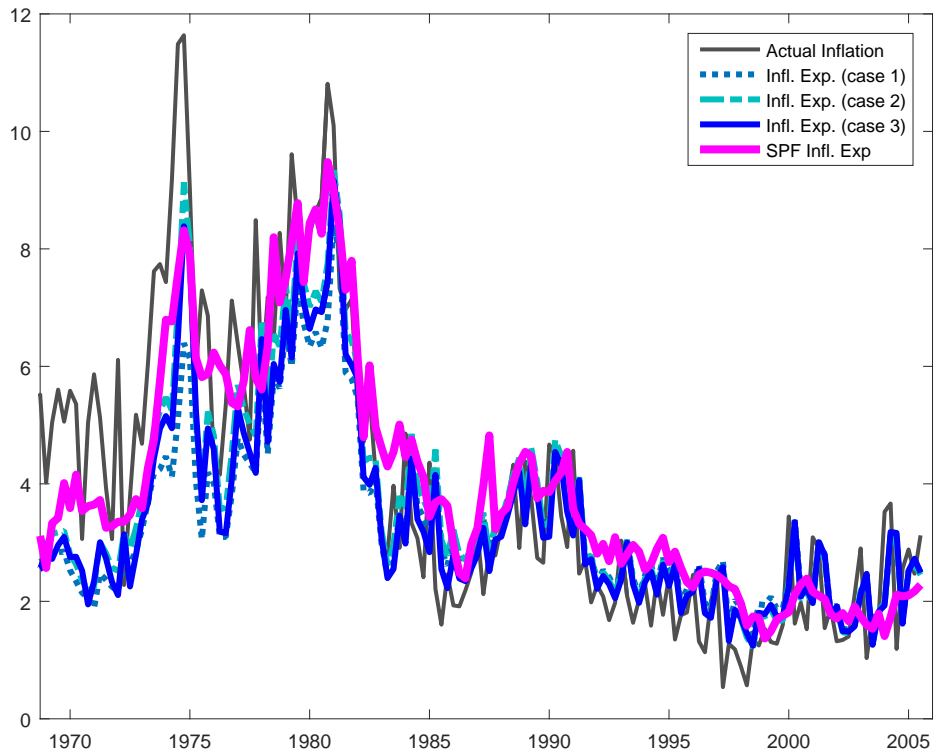


Figure 8: Inflation Expectations from PLM versus Survey Inflation Expectations.

*Notes:* Case 1 refers to the inflation expectations from the benchmark estimation with pre-sample beliefs; case 2 to the inflation expectations from the model with a higher gain coefficient ( $\bar{g} = 0.03$ ); case 3 to the inflation expectations from the PLM with initial beliefs for the autoregressive coefficients equal to 0.5.

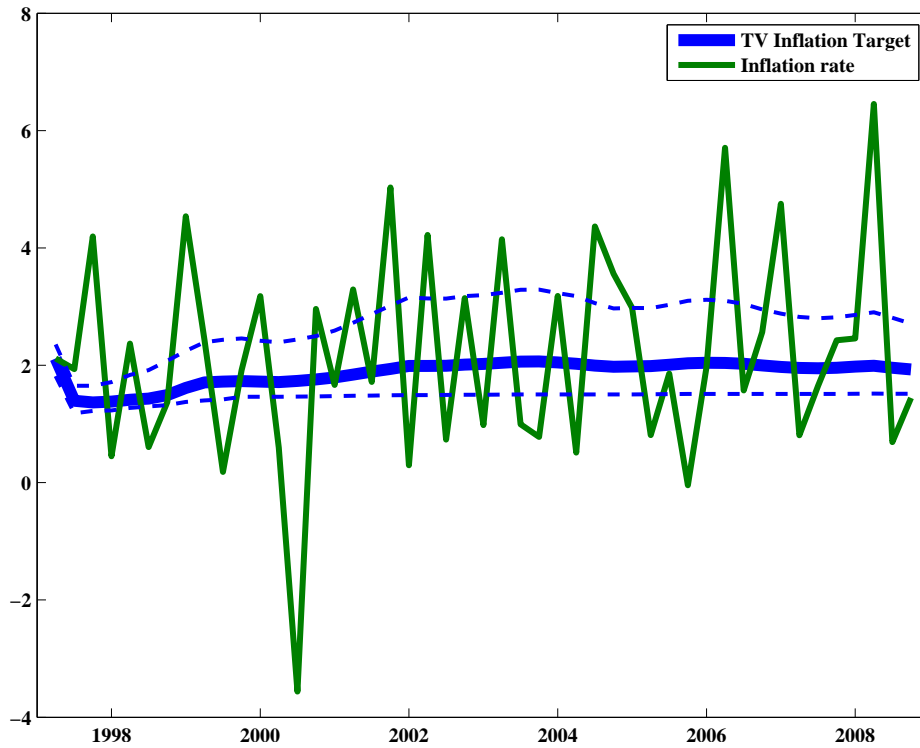


Figure 9: Actual Inflation and Inferred Time-Varying Inflation Target: Estimation for United Kingdom.

*Notes:* the inflation target series shown in the figure is the mean across MCMC draws (solid line); the dashed lines represent the corresponding 5 and 95% percentiles.