Heterogeneity in Individual Expectations, Sentiment, and Constant-Gain Learning

Stephen J. COLE*
Marquette University

Fabio MILANI†
University of California, Irvine

Abstract

This paper uses adaptive learning to understand the heterogeneity of individual-level expectations. We exploit individual Survey of Professional Forecasters data on output and inflation forecasts. We endow all forecasters with the same information set that they would have as economic agents in a benchmark New Keynesian model. Forecasters are, however, allowed to differ in the constant gain values that they use to update their beliefs and in their sentiments. The latter are defined as the degrees of excess optimism or pessimism about the economy that cannot be justified by the learning model. Our results highlight the heterogeneity in the gain coefficients adopted by forecasters. The median values of the gain coefficients occasionally jump to higher values in the 1970-80s, and stabilize in the 1990s and 2000s. Individual sentiment is also persistent and heterogeneous. Differences in sentiment, however, do not simply cancel out in the aggregate: the majority of forecasters exhibit excess optimism, or excess pessimism, at the same time.

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* *Address:* Stephen J. Cole, Assistant Professor, Department of Economics, Marquette University, P.O. Box 1881, Milwaukee, WI 53201. *Phone:* +1 (414) 288-3367. *E-mail:* stephen.cole@marquette.edu. *Homepage:* http://www.stephencoleeconomics.com.

† *Address*: Fabio Milani, Professor, Department of Economics, 3151 Social Science Plaza, University of California, Irvine, CA 92697-5100. *Phone:* 949-824-4519. *E-mail:* fmilani@uci.edu. *Homepage:* http://www.socsci.uci.edu/˜fmilani.

1 Introduction

Since the introduction of the rational expectations hypothesis in the 1970s, a number of macroeconomists have raised questions about its empirical validity, and have offered potential replacements. The approach that has emerged as the main alternative to rational expectations is probably provided by the literature on adaptive learning (Evans and Honkapohja, 2001, Sargent, 1993, 1999). Under rational expectations, economic agents are endowed with substantial knowledge about the economy: they know the structure of the model, the values of the parameters representing preferences, technology, and policy, and the processes for the exogenous disturbances. Learning relaxes these strict informational assumptions to introduce some limitations to agents' understanding. For instance, agents within the model are no longer assumed to know the magnitudes of all economic relationships; instead, they have to learn about them based on past experiences and historical data.

Various papers have already provided empirical evidence that adaptive learning matters at the macroeconomic level. Learning is an important driver of persistence and volatility in macroeconomic variables and it acts to amplify business cycle fluctuations.[1] Models with learning typically outperform models with rational expectations in their ability to fit macroeconomic time series (Milani, 2007, Slobodyan and Wouters, 2012).

Learning models have also been shown to be consistent with the formation of aggregate expectations from survey data. Branch and Evans (2006) find that constant-gain learning fits median survey expectations about inflation and output better than learning with decreasing and Kalman-filter gains.[2] Malmendier and Nagel (2016) analyze inflation forecasts from the University of Michigan Survey of Consumers; they also show that, at the aggregate level, mean expectations are closely replicated by a constant-gain learning updating rule with a gain coefficient of similar magnitude to that estimated in macroeconomics models (e.g., Milani, 2007).

The main scope of our paper is to contribute to the literature that studies learning and aspects of the formation of *individual-level* expectations. In particular, we focus on the heterogeneity of expectations across individual forecasters. Other studies have already revealed a significant degree of heterogeneity (e.g., Mankiw, Reis, and Wolfers, 2004, Coibion and Gorodnichenko, 2012, Andrade et al., 2016); Hommes (2019) reviews the theoretical, empirical, and experimental research

[1] See, for example, the papers by Orphanides and Williams (2005), Milani (2007, 2011, 2014, 2017), Branch and Evans (2007), Eusepi and Preston (2011), and Dave and Malik (2017).

[2] Markiewicz and Pick (2014) similarly find that models based on constant gain learning provide a better fit of professional forecaster's expectations concerning a wider range of macroeconomic and financial variables.

on the formation of heterogeneous expectations. In this paper, we aim to impose more structure on the heterogeneity, by examining individual-level heterogeneity through the lenses of a benchmark macroeconomic model with learning. We do so by treating individual forecasters as if they were agents in a model, and we provide them with a similar information set.

We use data on individual forecaster expectations from the Survey of Professional Forecasters (SPF). We use the same forecasts that would enter in a benchmark New Keynesian model: one-quarter-ahead forecasts for output (growth) and inflation. To minimize composition effects coming from the entry and exit of forecasters, we retain in the sample only observations for forecasters that remain in the survey for at least ten (and, as robustness, twenty) periods.

We then investigate how these individual expectations are formed. As background, we assume that the underlying economy is summarized by a canonical New Keynesian model. In a first step, we estimate the aggregate model by matching expectations in the model to the mean from our panel of forecasts, and assuming that aggregate expectations are formed under constant-gain learning. From the estimated aggregate model, we obtain the filtered structural disturbances, which are typically part of the information set for agents in the DSGE model, and that we assume to be part of the individual forecasters' information set as well.

In the second step, we then turn to the analysis of individual expectations. We assume that those expectations are formed from a perceived linear model of the economy (PLM). Our assumption is that forecasters are given the same model and the same information set that they would have as economic agents in a benchmark New Keynesian model. Therefore, they use a PLM that is equal to the Minimum State Variable (MSV) solution of the corresponding macro model under rational expectations, and they are assumed to observe the disturbances (for robustness, we will also relax this assumption, endowing agents with knowledge of the lagged endogenous variables, but not of disturbances). For each forecaster, we minimize the distance between their observed forecasts and the expectations formed from the learning PLM. As a result, we obtain the best-fitting constant gain for each individual forecaster. The gain governs their speed of learning for the sample during which they are in the survey: it can be interpreted as their perceived probability that the variable they are forecasting will be subject to a structural break, as well as their memory of past observations.

We denote the difference between observed expectations and the portion that is explained by the learning model as 'excess optimism and pessimism', or 'sentiment'. Those optimism and pessimism terms may be serially correlated and represent an individual-level version of the aggregate sentiment analyzed in Milani (2011, 2017), which show that sentiment shocks are responsible for about half of business cycle fluctuations.

Main Results. We document a substantial heterogeneity in the learning approach of individual forecasters. Their gain coefficients are heterogeneous: in many periods, forecasters who are largely unresponsive to new information coexist with forecasters who employ gains around 0.1 or higher. The gains vary over time: they are often higher in the 1970s and 1980s, with averages that rise to values of 0.03-0.05, and they decline in the second part of the sample, stabilizing around 0.015, and with a lower dispersion. The micro evidence is, therefore, consistent with switches in the gain as identified in Milani (2014), who also proposed time variation in the gain as a potential driver of stochastic volatility in output and inflation.

Beliefs about macroeconomic relationships estimated at the individual level also reveal substantial heterogeneity across forecasters and changes over time. On average, perceptions about the persistence of inflation increase over the sample, before reverting back later on. Forecasters also significantly revise their beliefs about the effectiveness of monetary policy: the perceived sensitivity of output to interest rates fall to values between -1.3 and -2 for most of the 1970s, and it moves upward with Volcker's disinflation. At the end of the sample, the perceived sensitivity has been reduced to a coefficient of -0.5. Individual beliefs can also affect the dynamics of the aggregate model: impulse responses and the role played by different shocks can be very different depending on which beliefs prevail in the population.

Moreover, we provide estimates of sentiment series at the individual level. Excesses of optimism and pessimism by single forecasters do not cancel out in the aggregate, rather they typically move in herd. The evolution of mean sentiment mirrors the series that is estimated at the aggregate level. Sentiment is persistent, and it has a volatility that is comparable to that of other structural disturbances.

Related literature. The paper can be seen in connection to the broader literature that analyzes the formation of expectations. The surveys by Mansky (2004, 2017) discuss how data on expectations should be used to test economic models and assumptions about expectations. The empirical study of expectations has, for a long time, being critical of the rational expectations hypothesis (Pesaran, 1987). Various recent literatures have, therefore, proposed adjustments. Mankiw and Reis (2002), Sims (2003), Woodford (2003a), and Mackowiak and Wiederholt (2009), assume that agents have sticky, or noisy, information about economic variables, as a result of limited attention and costs of updating information. Coibion and Gorodnichenko (2012, 2015) analyze survey data to test sticky information theories. In a different application, Gennaioli, Ma, and Shleifer (2016) use actual data on expectations to understand corporate investment decisions. Their work indicates that expectations are more extrapolative than rational. Fuster, Laibson, and Mendel (2010) model agents as using expectations based on simple prediction models: beliefs display extrapolation bias and may be too optimistic or pessimistic relative to rational expectations.

The formation of heterogeneous expectations is reviewed at length in the survey by Hommes (2019). Heterogeneity can be modeled following the classical works of Brock and Hommes (1997, 1998). The heterogeneity of expectations is a robust feature of the data that arises both in experimental research (Hommes, 2011, Anufriev and Hommes, 2012) and in surveys (Mankiw, Reis, and Wolfers, 2004).

Our paper can be inserted in this broader literature, but it studies heterogeneity from the lens of adaptive learning models. The literature on adaptive learning in macroeconomics has historically been more focused on the formation of expectations at the aggregate level. Evans and Honkapohja (2001) and Sargent (1993, 1999) review the foundations of the adaptive learning approach. Adaptive learning has important implications for which monetary policy strategies are desirable (as described, for example, by Orphanides and Williams, 2005, Preston, 2006, Gaspar, Smets, and Vestin, 2006, Eusepi and Preston, 2010), for fiscal policy (Evans, Honkapohja, and Mitra, 2009), and for the effects of 'forward guidance' (Cole, 2020a,b). In addition, previous research has examined how learning behavior can explain fluctuations in the macroeconomy. Milani (2007, 2014) shows that learning is successful in capturing the persistence and volatility of macroeconomic data. Eusepi and Preston (2011) find that learning helps explain the propagation of shocks over the business cycle. Prior literature also provides evidence that learning accurately captures the formation of aggregate survey expectations (Orphanides and Williams, 2005, Branch and Evans, 2006, Markiewicz and Pick, 2014, Bräuning and van der Cruijsen, 2019).

Fewer papers delve, instead, into the the formation of *individual expectations*, which is the main objective of this paper. In particular, we provide evidence on the importance of heterogeneity at the microeconomic level by exploiting individual survey expectations. Therefore, our work is more closely connected to Branch (2004), Pfajfar and Santoro (2010), and Malmendier and Nagel (2016). Branch (2004) describes how the forecasting models used by agents are not necessarily constant, but they can shift over time. Different shares of agents may switch between models in forming expectations, generating heterogeneity.[3] Pfajfar and Santoro (2010) study heterogeneity by examining the time series of different percentiles from the cross-sectional distribution of inflation forecasts. They consider percentiles of a distribution (which not necessarily represent the same individual forecasters), while we track the same individual forecasters over time. Malmendier and Nagel (2016) use an adaptive learning model to argue that consumers discount the past differently based on their age. Prior literature has also shown heterogeneity in individual level expectations can emanate from the financial sector. For instance, Chiang et al. (2011) and Kaustia and Knüpfer (2008) show that previous experience in IPO auctions can be a determinant of individual forecasts.

A number of papers provide theoretical foundations and interpretation for learning gain coefficients. Evans and Honkapohja (2001) examine and discuss the gain coefficient in terms of convergence of the learning model to its rational expectations counterpart. Barucci (1999) and Honkapohja and Mitra (2006) describe that gain coefficients can also be interpreted as the degree of memory forecasters attach to past observations. Berardi and Galimberti (2017a) document appropriate approaches for calibrating and interpreting gain coefficients. Berardi (2019) offers a Bayesian framework for interpreting the gain coefficient as the probability of estimated parameters changing every period. Our paper adds to this literature by shedding light on realistic values for gains at the individual level, and it reveals both heterogeneity in the cross-section of forecasters and time-variation over the sample.

Finally, our results regarding the importance of sentiment in individual expectations also provide an important rationale at the microeconomic level for the type of aggregate sentiment that has been recently introduced in a variety of macroeconomic models. Milani (2011, 2017) utilizes adaptive learning and aggregate survey expectations from the SPF and finds that sentiment shocks explain a significant portion of business cycle fluctuations.[4] Angeletos, Collard, and Dellas (2018) also describe the importance of sentiment (or "confidence") shocks for explaining the business cycle. When firms are faced with a signal extraction problem for their goods, Benhabib, Wang, and Wen (2015) show that sentiment can lead to equilibria away from a standard rational expectations solution. Our modeling of sentiment shocks differs along the following dimensions. The confidence shocks of Angeletos et al. (2018) denote autonomous variations in expectations regarding short-

[3] Model uncertainty is also considered in Baranowski (2014), who studies the type of adaptive learning rule that best fits individual forecasts of inflation, unemployment, and real GDP growth, using various combinations of regressors.

[4] The papers by Charalampidis and Milani (2020), Chatterjee and Milani (2020), and Cole and Milani (2019) also highlight the empirical importance of sentiment in different frameworks.

term economic outcomes, which arise in an environment with coordination frictions: sentiments create a gap between first-order and higher-order beliefs. Benhabib et al. (2015) defines sentiment as a view held by all agents about aggregate demand, which they perceive through noisy signals, and that is represented as a normally distributed random variable. They both retain the assumption of rational expectations. Our paper, instead, defines sentiment shocks as the difference between observed expectations from SPF forecasters and learning-implied expectations, where agents form expectations as an econometrician utilizing an adaptive learning model. We also show that sentiment shocks are not necessarily *iid*, but exhibit persistence over time. Moreover, our results add to the previous research by providing evidence of sentiment shocks as an additional source of heterogeneity at the microeconomic/individual level.

2 Individual Survey Expectations Data

We use individual expectations data from the SPF, hosted by the Federal Reserve Bank of Philadelphia. We focus on forecasts about future real GDP growth and the future inflation rate calculated from the GDP Implicit Price Deflator. The series of reference are *'RGDP'* and *'PGDP'*, and specifically we use *'RGDP2'*, *'RGDP3'*, *'PGDP2'*, and *'PGDP3'*: they refer to expectations formed by forecasters at time t, while being able to observe the published values of the same variables up to $t-1$, about the value of the variables at the end of the current period t and of the next period $t+1$. Expectations about $t+1$, hence, have the same horizon as those that would enter in a benchmark New Keynesian model.[5]

Mansky (2011) has highlighted the potential composition effects that can arise due to the entry and exit of forecasters. Therefore, we keep in the panel only those forecasters that remain in the survey and submit forecasts for at least ten periods (in the robustness section, we consider a sample with those that remain twenty periods), and for which we have both output and inflation forecasts. After constructing this data set, our sample includes 204 individual forecasters that participate in the survey at different points, and for a number of periods above the threshold, between the last quarter of 1968 and the third quarter of 2016.

[5] The SPF documentation from FRB of Philadelphia (2019b) states "The identification numbers are consistent over time, allowing you to trace a given forecasters responses from one survey to the next" (p. 33). However, it points out a caveat for the portion of the survey that was conducted by the NBER/ASA. The documentation states that there are some cases in which a forecaster participates in the survey, then drops, and then reappears only after several periods. It is impossible to know whether, in those instances, the identification numbers actually correspond to the same individuals or are wrongly assigned to new entrants.

Given our interest in inferring the learning process and any excess optimism/pessimism in real-time, it is crucial that we try to capture the actual information set that was available to forecasters at the time the forecasts were produced. We do so by exploiting the real-time data series that the SPF provides in correspondence of each forecast (obtained through the Real-Time Data Set for Macroeconomists, also hosted by the Federal Reserve Bank of Philadelphia). Therefore, we use the corresponding real-time data for output and inflation (the series with acronym ROUTPUTQvQd and PQvQd) as our observables for the realized variables. For each series, we use the first-vintage observation.

Figure 1 shows the evolution of individual expectations over the sample, along with the implied mean and the actual realized series.[6] At the individual level, expectations of output growth and expectations of inflation often display a negative correlation (see Figure 2), with a median value equal to -0.28. Whether the unconditional correlation is positive or negative depends on whether forecasters expect demand or supply shocks to be dominant, with demand shocks implying a positive correlation, and supply shocks implying a negative correlation.

An interesting aspect of Figure 1 is the behavior of inflation forecasts after the introduction of formal inflation targeting in 2012. Intuitively, the forecasts of inflation should move closer to its realized value; however, Figure 1 displays that they do not. This characteristic may be due to private sector forecasters not fully believing the Fed will achieve 2% inflation. Indeed, Binder, Janson, and Verbrugge (2019) show SPF forecasts of inflation at the individual level become unanchored. They suggest that forecasters may disagree about the Fed's ability to fulfill its promises.

We will assume that individual expectations are formed in a way that is consistent with a typical Perceived Law of Motion (PLM) from a canonical macroeconomic model. We turn to the presentation of the model and the expectation formation assumptions next.

3 New Keynesian Model

We assume that our individual forecasters are endowed with the same information set that economic agents would have in a benchmark New Keynesian model. Therefore, in forming their macroeconomic forecasts, they use information from past realizations of the endogenous variables (output gap, inflation, and interest rates). Agents also utilize information about structural disturbances to

[6] We find that difference between the mean and median are trivial (with a correlation above 0.99), once we have cleaned the sample to include only 'long'-term participants to the survey.

demand and supply (autoregressive natural-rate and cost-push shocks). The set of variables that they use in their forecasting models, therefore, corresponds to the same variables that appear in the MSV solution of the model under rational expectations.

We assume that the underlying aggregate economy is characterized by a canonical New Keynesian model (e.g., Woodford, 2003b), extended to include endogenous sources of persistence as habit formation and inflation indexation. The model is summarized by the following equations:

$$\tilde{y}_t = \widehat{E}_t \tilde{y}_{t+1} - \psi(i_t - \widehat{E}_t \pi_{t+1} - r_t^n) \tag{1}$$

$$\tilde{\pi}_t = \beta \widehat{E}_t \tilde{\pi}_{t+1} + \kappa \left(\omega y_t + \psi^{-1} \tilde{y}_t \right) + u_t \tag{2}$$

$$i_t = \rho_t i_{t-1} + (1 - \rho_t) \left[\chi_t^\pi \pi_t + \chi_t^y y_t \right] + \sigma_{\varepsilon,t} \varepsilon_t \tag{3}$$

where $\tilde{y}_t \equiv y_t - \eta y_{t-1}$, $\tilde{\pi}_t \equiv \pi_t - \gamma \pi_{t-1}$, $\psi \equiv \sigma(1-\eta)$, $\kappa \equiv (1-\alpha\beta)(1-\alpha)/\alpha$, and where y_t denotes the output gap, π_t denotes inflation, and i_t denotes the short-term nominal interest rate, which serves as the monetary policy instrument. The coefficient σ denotes the elasticity of intertemporal substitution, ω the inverse of the Frisch elasticity of labor supply, β the household's discount factor, α the Calvo price stickiness parameter, and ρ_t, χ_t^π, and χ_t^y are Taylor rule coefficients that denote the inertia of interest rate decisions, and the monetary policy reaction to inflation and the output gap. The degree of (external) habit formation in consumption is measured by η and the extent of indexation to past inflation in price setting by γ.

The model includes three exogenous disturbances: the demand (real natural rate) disturbance r_t^n, the supply (cost-push or price markup) disturbance u_t, which are assumed to evolve as AR(1) processes with autoregressive coefficients ρ_t^j and standard deviations σ_t^j, with $j = r, u$, and the monetary policy shock, which, following the convention in the literature, is assumed to be *i.i.d.*:

$$r_t^n = \rho_t^r r_{t-1}^n + \sigma_t^r \varepsilon_t^r \tag{4}$$

$$u_t = \rho_t^u u_{t-1} + \sigma_t^u \varepsilon_t^u. \tag{5}$$

To improve the fit of the model to postwar data, we allow some of the coefficients to vary over time (and they are denoted with a t subscript). The Taylor rule coefficients, including the volatility of monetary policy shocks, are allowed to assume different values in the pre-1979 sample, in the non-borrowed-reserve targeting experiment years between 1979 and 1982, and in the post-1982 period:

$$\begin{cases} \left[\rho_{pre79}, \chi^{\pi}_{pre79}, \chi^{y}_{pre79}, \sigma_{\varepsilon,pre79}\right] & t \leq 1979:3 \\ \left[\rho_{79-82}, \chi^{\pi}_{79-82}, \chi^{y}_{79-82}, \sigma_{\varepsilon,79-82}\right] & 1979:4 \leq t \leq 1982:4 \\ \left[\rho_{post82}, \chi^{\pi}_{post82}, \chi^{y}_{post82}, \sigma_{\varepsilon,post82}\right] & t \geq 1983:1 \end{cases}$$

Moreover, substantial evidence points toward a break in the volatility of the macroeconomic shocks around 1984 (e.g., McConnell and Perez-Quiros, 2000). Hence, we allow the remaining disturbance parameters (both the autoregressive coefficients and standard deviations) to potentially switch between the pre-1984 and post-1984 samples:

$$\begin{cases} \left[\rho^{j}_{pre84}, \sigma^{j}_{pre84}\right] & t < 1984:1 \\ \left[\rho^{j}_{post84}, \sigma^{j}_{post84}\right] & t \geq 1984:1 \end{cases}$$

for $j = r, u$. Expectations in the model are denoted by $\widehat{E}_t$ and they are measured by the mean of expectations from our sample of individual forecasters: $\widehat{E}_t = \int_j \widehat{E}_{t,j} dj$.

3.1 Near-Rational Expectations

Following the literature on adaptive learning in macroeconomics (Evans and Honkapohja, 2001, 2013, Sargent, 1999), we assume that agents do not enjoy a knowledge advantage compared with the modeler, and they try to infer relationships among variables by analyzing historical data, as econometricians would. To produce forecasts about future variables (e.g., output or inflation), they employ a linear perceived model, estimated using standard techniques (e.g., OLS or WLS). As new information arrives every period, they update forecasts accordingly, thus continuously learning about the economy.

Therefore, aggregate expectations in the model are assumed to be formed as in Milani (2011, 2017), i.e., from the following Perceived Law of Motion (PLM):

$$Y_t = a_{t-1} + b_{t-1} Y_{t-1} + c_{t-1} \epsilon_t + \nu_t \tag{6}$$

where $Y_t = [y_t, \pi_t, i_t]'$, $\epsilon_t = [r^n_t, u_t]'$, and a_t, b_t, and c_t, are vectors and matrices of coefficients of appropriate dimensions. The term ν_t denotes an econometric error term. As common in the adaptive learning literature, economic agents are assumed to use a correctly-specified model to generate their forecasts: the model corresponds to the Minimum State Variable (MSV) solution of the system under rational expectations. Agents, hence, use the correct set of endogenous variables in their perceived model, for which they observe data up to $t-1$, and they are assumed to observe the contemporaneous disturbances. The model contrasts with rational expectations, since agents are assumed to lack knowledge about the reduced-form coefficients in the PLM: therefore, they do not know the magnitude of the relationships among variables. For example, they do not know how sensitive output and inflation are to interest rate changes or to demand and supply shocks, or the persistence of output and inflation, or the slope of the Phillips curve. This approach is still typically interpreted as a minimal deviation from rational expectations.

Given their imperfect knowledge, agents attempt to learn the magnitudes of the relationships over time, based on the realizations of macroeconomic data that they observe. They update their beliefs at each t according to the constant-gain learning formula:

$$\widehat{\phi}_t = \widehat{\phi}_{t-1} + \mathbf{\overline{g}} R_t^{-1} X_t (Y_t - \widehat{\phi}_{t-1}' X_t)' \tag{7}$$

$$R_t = R_{t-1} + \mathbf{\overline{g}} (X_t X_t' - R_{t-1}) \tag{8}$$

where $X_t \equiv [1, Y_{t-1}, \epsilon_t]'$, and $\widehat{\phi}_t = [a_t, b_t, c_t]'$. The key coefficient of interest is $\mathbf{\overline{g}}$, the constant-gain coefficient. The gain governs the speed at which agents learn and adjust their beliefs to new information. The gain can also be interpreted as the degree of memory that agents have, given that they discount past information more heavily than recent observations (at the rate $(1-\mathbf{\overline{g}})^j$ for observations falling j periods in the past). Given the PLM and the updated beliefs $\widehat{\phi}_t$, the aggregate expectations entering in equations (1)-(3) are formed as

$$\widehat{E}_t Y_{t+1} = \left(I + \widehat{b}_{t-1}\right) \widehat{a}_{t-1} + \widehat{b}_{t-1}^2 Y_{t-1} + (\widehat{c}_{t-1}\rho + \widehat{b}_{t-1}\widehat{c}_{t-1})\epsilon_t + ds_t. \tag{9}$$

The expectation formation mechanism includes two components: one endogenous and due to learning about the economy and responding to observed conditions, and the second (ds_t, where d is simply a selection matrix), exogenous. The latter represents the components of expectations that cannot be justified by the near-rational learning model. These terms, denoted by s_t define, as in Milani (2011, 2017), "sentiment", or waves of excess optimism and pessimism, in the model.

Sentiments about output and inflation are assumed to evolve as

$$s_t = \rho_t^s s_{t-1} + \Sigma_t^s \zeta_t,$$

where $s_t = [s_t^y, s_t^\pi]'$, $\rho_t^s = [\rho_t^r, 0; 0, \rho_t^\pi]$, and $\Sigma_t^s = [\sigma_{r,t}, 0; 0, \sigma_{\pi,t}]$, with autoregressive coefficients and standard deviations allowed to switch before and after 1984, as for the other disturbances.

We believe it is appropriate to model aspects of the expectations formation process of SPF forecasters with constant-gain learning. First, this framework is motivated by a special survey conducted by the Federal Reserve Bank of Philadelphia that asked SPF participants how they construct their forecasts. In particular, the panelists reported that they overwhelmingly utilize mathematical models (akin to equation (13)) in their formation of macroeconomic forecasts (see Stark, 2013). The survey also found that "panelists update their projections frequently, suggesting that their projections incorporate the most recent information available on the economy around the survey's deadline" (Stark, 2013, p. 5). These findings suggest that SPF respondents form expectations in a way that can be approximated by the constant-gain learning approach used in this paper. Indeed, prior literature demonstrates that a constant-gain learning framework provides the best fit for expectations from the SPF, outperforming various alternatives (e.g., Branch and Evans, 2006).

We acknowledge, though, that there are other factors, besides recency bias, that can create differences across the expectations of forecasters. For instance, Clements (2015), Ehrbeck and Waldmann (1996), Lamont (2002), and Laster et al. (1999) demonstrate biased private sector forecasts along other dimensions (e.g., herding, reputational factors, and publicity). However, our paper focuses on a parsimonious model of heterogeneity to keep it as consistent as possible with the theoretical DSGE literature.

3.2 Bayesian Estimation of New Keynesian Model

The previous model, with aggregate expectations formed as in (9), can be expressed in state-space form as:

$$OBS_t = H_0 + H\mathbf{Y}_t \tag{10}$$

$$\mathbf{Y}_t = A_t + F_t\mathbf{Y}_{t-1} + G\widetilde{\epsilon}_t \tag{11}$$

where OBS_t collects the observable variables to be matched in the estimation, $\mathbf{Y}_t$ collects the endogenous variables, the expectations, and the exogenous disturbances, and $\widetilde{\epsilon}_t$ collects the exogenous innovations.

We use real-time data (first-vintage) in the estimation of the DSGE model and in trying to match the individual forecasters' expectations. The real-time series are obtained from the Real-Time data set for Macroeconomists, hosted on the Federal Reserve Bank of Philadelphia's website.

As explained there, quarterly vintages correspond to the real time data available to forecasters in February, May, August, November of each year. For this reason, when downloading the Federal Funds Rate, we also use the corresponding values in the same months.

The following example details real-time data for Real GDP in 2012:II. The first vintage corresponds to the Bureau of Economic Analysis' first release of Real GDP for 2012:II. Data regarding the periods before 2012:II include any relevant revisions.[7] Thus, when calculating Real GDP growth from 2012:I to 2012:II, the value for 2012:I has been revised. Additional information on the construction of real-time dataset from the SPF can be found in Croushore and Stark (2001) and SPF documentation on real-time dataset (see FRB of Philadelphia, 2019a).

Inflation is obtained as log first difference of the GDP Implicit Price Deflator; we use the log first difference of Real GDP for output growth. Expectations are given by the mean across our set of forecasters of one-period-ahead output growth and inflation expectations, using the series described in the previous section. In the estimation, we assume a piecewise-linear trend for output, following the evidence in Perron and Wada (2009). They find that when changes in the slope of the trend function are accounted for, there is no longer evidence of stochastic trends, and the resulting cyclical component aligns well with NBER recession dates. We allow for changing slopes between the 1954:III-1973:III, 1973:IV-1994:IV, 1995:I-2007:I, and 2007:II-2016:III subperiods, based on the growth facts presented in Jones (2016). Therefore, the vectors and matrices in the observation equation (10) are given by

$$\begin{bmatrix} Output\ Growth_t \\ Inflation_t \\ FFR_t \\ Expected\ Output\ Growth_t \\ Expected\ Inflation_t \end{bmatrix} = \begin{bmatrix} \delta_t \\ \bar{\pi} \\ \bar{i} \\ \delta_t \\ \bar{\pi} \end{bmatrix} + \begin{bmatrix} y_t - y_{t-1} \\ \pi_t \\ i_t \\ \widehat{E}_t(y_{t+1} - y_t) \\ \widehat{E}_t \pi_{t+1} \end{bmatrix}. \tag{12}$$

The changing slopes implied by the piecewise-linear trend are introduced through the parameter δ_t, which is subject to the structural breaks over the sample outlined above.[8]

A potential issue in the estimation regards the existence of a binding zero-lower bound (ZLB) starting in 2009, which could introduce a nonlinearity into our model. We solve this issue, as others have done, by using data on the 'shadow' short rate. Unlike the short-term US nominal interest rate, the shadow short rate is allowed to have negative values to capture a more accommodating stance

[7] Berardi and Galimberti (2017a) describe that estimates should reflect the fact that historical data are revised over time.

[8] The parameters $\bar{\pi}$, $\bar{i}$, and the values of δ_t are simply identified from the corresponding sample means.

of monetary policy at the ZLB (for example, due to unconventional monetary policy interventions). We utilize the shadow short rate made available by Krippner (2013) in place of the FFR from 2009:Q1 to 2016.

The last choice before estimating the New Keynesian model with adaptive learning concerns initial beliefs $\hat{\phi}_{t=0}$ and $R_{t=0}$. To obtain values for these coefficients, we utilize a presample estimation. We start from uninformative initial beliefs[9] at the beginning of the pre-sample period, i.e. 1954:Q3, which is the first quarter of availability of Federal Funds rate data. We next run the estimation from 1954:Q3 to 1968:Q3 without expectations data to give us initial beliefs for our main estimation, which includes the expectations series starting in 1968:Q4. The likelihood is then computed for the 1968:Q4-2016:Q3 sample.

The model is estimated using Bayesian methods as in Milani (2007, 2011). Table 1 shows the chosen prior distributions, along with the posterior estimates for our vector of structural, disturbance, and learning, parameters. With survey expectations and learning, we estimate lower degrees of habit formation and inflation indexation ($\eta = 0.366$, $\gamma = 0.088$) than in corresponding models under rational expectations. The estimated response of monetary policy to inflation, as well as the degree of interest rate inertia, are higher during the 1979-1982 experiment, than in other periods. Cost-push shocks are close to *iid*, while natural rate shocks display significant persistence. Sentiment shocks, both related to output and inflation, are also persistent and they have comparable volatility to that of fundamental disturbances. In line with the Great Moderation literature, the standard deviations of most shocks fall in the second part of the sample. Finally, we provide an estimate of the best-fitting constant gain coefficient in a macroeconomic model with expectations matched to aggregate survey expectations. The posterior mean for the gain equals 0.015.

4 Individual Expectations

4.1 Perceived Model and Constant-Gain Learning

The previous section estimated the New Keynesian model at the aggregate level, which allowed us to obtain mean estimates of the model parameters and the filtered structural disturbances. We now turn to examining expectations at the individual level. We impose structure on our forecasters' expectation formation process by assuming that they form expectations from a near-rational model

[9] All elements of $\hat{\phi}_{t=0}$ are initialized at 0, and the precision matrix $R_{t=0}$ as the identity matrix.

that allows for learning. Agents have a correctly-specified PLM, which has the same endogenous variables as the solution under rational expectations, and the same aggregate disturbances.

Hence, the PLM is the same as (6) for each individual forecaster j:

$$Y_t = a_{t-1}^j + b_{t-1}^j Y_{t-1} + c_{t-1}^j \epsilon_t + \nu_t^j \tag{13}$$

where $Y_t = [y_t, \pi_t, i_t]'$, $\epsilon_t = [\varepsilon_t^r, \varepsilon_t^u]'$, and a_t, b_t, and c_t are vectors and matrices of coefficients; ν_t is the usual regression error term. The estimation of the New Keynesian macroeconomic model allows us to include the filtered structural disturbances in the individual agents' information sets.[10]

While forecasters are assumed to base their expectations on a correctly-specified model, they may differ in their beliefs. Each forecaster j updates beliefs through constant-gain learning:

$$\widehat{\phi}_t^j = \widehat{\phi}_{t-1}^j + \overline{\mathbf{g}}^j (R_t^j)^{-1} X_t (Y_t - \widehat{\phi}_{t-1}^{j\,\prime} X_t)' \tag{14}$$

$$R_t^j = R_{t-1}^j + \overline{\mathbf{g}}^j (X_t X_t' - R_{t-1}^j) \tag{15}$$

where $X_t \equiv [1, Y_{t-1}, \epsilon_t]'$, and $\widehat{\phi}_t^j = \left[a_t^j, b_t^j, c_t^j\right]'$. Expectations for each individual forecaster are, therefore, assumed to be formed as

$$\widehat{E}_t^j Y_{t+1} = \left(I + \widehat{b}_{t-1}^j\right) \widehat{a}_{t-1}^j + (\widehat{b}_{t-1}^j)^2 Y_{t-1} + (\widehat{c}_{t-1}^j \rho + \widehat{b}_{t-1}^j \widehat{c}_{t-1}^j)\epsilon_t + \widehat{d} s_t^j, \tag{16}$$

where ϵ_t collects the structural AR(1) disturbances and s_t denotes sentiment, or unjustified optimism and pessimism (that is, unjustified based on the state of the economy and the updated beliefs). Therefore, equation (16) provides a way to study and extract excess optimism and pessimism at the micro level.

Following most of the learning literature, we utilize a correctly specified PLM for individual forecasters in this section and at the aggregate level in Section 3.1 for our benchmark case. Hence, the structural shocks r_t^n and μ_t are assumed to be known. However, in Section 6.1, we study the results with a VAR(1) plus constant PLM, which assumes no knowledge of the shocks.

Initial beliefs for each forecaster also need to be specified. Instead of fixing them at ad hoc values, we proceed in the following way. In the SPF dataset, forecasters may enter at different time periods, and thus, each individual may happen to utilize a different length of histories when producing an initial forecast. Therefore, we jointly estimate the initial beliefs, using the relevant

[10] In the robustness Section 6.1, we will address the sensitivity of the results to the exclusion of information about disturbances: in that case, agents only use a VAR(1) plus constant in the observable variables as their PLM.

pre-sample data for each forecaster, which would have been obtained by the forecaster using the same constant gain learning approach as

$$\widehat{\phi}_{\tau}^{j} = \left[\sum_{i=1}^{\tau}\left(1-\overline{\mathbf{g}}^{j}\right)^{(i-1)} X_{\tau-i}X'_{\tau-i}\right]^{-1}\left[\sum_{i=1}^{\tau}\left(1-\overline{\mathbf{g}}^{j}\right)^{(i-1)} X_{\tau-i}Y'_{\tau-i+1}\right] \tag{17}$$

where $X_t \equiv [1, Y_{t-1}, \epsilon_t]'$ denotes the regressors in forecaster j's PLM and $\overline{\mathbf{g}}^j$ is the constant gain parameter *specific* to individual j. The symbol τ represents the length of histories each forecaster utilizes when forming initial beliefs. We assume $R_\tau = c * I$, a scaled identity matrix, with $c = 0.1$ to allow for a larger degree of uncertainty characterizing the initial beliefs.

The following example will help to clarify how forecasters form initial beliefs. If an individual enters the survey in 1979:Q4 and stays for twenty periods, we would assume that she can observe the presample data from 1954:Q3 to 1979:Q3, discount them (we obtain the best-fitting gain in the estimation, which governs this discounting) and then update beliefs and form expectations for the subsequent years.[11] In this way, the forecasters' learning speed and initial beliefs are both estimated, but in a parsimonious way.[12]

Overall, forecasters are assumed to have both similarities and differences. Each individual j produces expectations using an adaptive learning model with the same set of variables, based on the correctly-specified solution under rational expectations. They all have access to the same set of presample data. However, forecasters expectations can vary based on the endogenous learning component influenced by $\overline{\mathbf{g}}$, and the exogenous sentiment shocks s_t displayed in (16).

4.2 Estimation

For each individual forecaster, we compare observations on their one-period ahead forecasts for output growth and inflation to their counterparts implied by the adaptive learning model. We find the best-fitting gain coefficient by minimizing the loss function implied by the mean squared errors for these series

$$\underset{\overline{\mathbf{g}}^{\mathbf{j}}}{\operatorname{argmin}} \left[E_t^{j,obs} Z_{t+1} - \widehat{E}_t^j Z_{t+1}\right]' \left[E_t^{j,obs} Z_{t+1} - \widehat{E}_t^j Z_{t+1}\right] \tag{18}$$

[11] In this example, the value of τ, the number of pre-sample observations, would be 26.

[12] By parsimonious, we mean that initial beliefs are not assumed to be formed by special or unique assumptions, but by a simple and conventional method (i.e., WLS in equation (17)), and adding only one free parameter, the constant gain.

where $E_t^{j,obs} Z_{t+1}$ denotes the observed survey forecast from the SPF for forecaster j, with $Z = [\Delta y, \pi]'$ and $\widehat{E}_t^j Z_{t+1}$ denotes the implied expectations obtained from the learning model. Thus, the best-fitting gain is the one that minimizes the mean squared forecast error for each forecaster. The unexplained component of each expectations series, that is the part that is not explained by the learning model with the best-fitting gain, is denoted as sentiment: forecasters are either more or less pessimistic than their near-rational learning model implies.

The heterogeneity across forecasters, therefore, can stem from two sources: different learning speeds, as measured by different constant gains (motivated by different agent's perceptions about incoming structural change or by different memories), and different sentiment.

5 Empirical Results

We now examine the two potential sources of heterogeneity in individual forecasters' expectations, as well as the implications these differences have for the economy.

5.1 Best-Fitting Constant Gain Parameter

The source of heterogeneity in individual respondents' forecasts can stem, first, from their constant gain parameters $\overline{\mathbf{g}}^j$. As explained above, this parameter can have two interpretations. First, it governs the speed at which agents adjust their beliefs to new information about the economy, possibly because they are concerned about future structural breaks of unknown form. It can also be interpreted as the degree of memory agents have about past data (Malmendier and Nagel, 2016, provide a behavioral explanation for differences in discounting, showing that older agents assign more weight to observations in the more distant past). To understand how the best-fitting constant gain parameter can be a source of heterogeneity in SPF respondents, we examine the $\overline{\mathbf{g}}^j$ that minimizes equation (18) for each forecaster j.

Figure 3 displays the value of the estimated best-fitting constant gain for each forecaster for each period she submits a response.[13] The vertical axis denotes the value of the constant gain parameter from 0.0001 to 0.2. The horizontal axis indicates the date. In each time period, we represent the distribution of the best-fitting gain of forecasters with a boxplot. The target signifies the median, the edges of the box the 25^{th} and 75^{th} percentiles, the whiskers the extreme values not

[13] We assume that the $\overline{\mathbf{g}}^j$ that minimizes equation (18) is the same across the entire time period that forecaster j is in the sample.

considered outliers, and the '+' symbol outliers.

The results show heterogeneity in individual expectations that stems from different values of the best-fitting constant gain across SPF forecasters. Before the beginnings of the Great Moderation (i.e., prior to the mid to late 1980s), there exists larger dispersion in the value of $\overline{\mathbf{g}}$. For instance, the interquartile range is much larger during this period relative to after the Great Moderation. The median gains during these decades often fluctuate and occasionally rise to values around 0.05. Several forecasters place a large degree of weight on new information about incoming structural change in the economy and they substantially discount the past. As the sample period moves into the 1990s and concludes in 2016, the distribution of constant gains tightens and steadily coalesces around smaller values of $\overline{\mathbf{g}}$. The upper limits of the outliers and extreme values tend to take on smaller numbers; the interquartile range is also generally decreasing over this time period. The median value of the constant gain also centers around lower numbers (0.01-0.02) of $\overline{\mathbf{g}}$, with a value of 0.0146 in the final period of our sample (i.e., 2016:Q3). Forecasters do not perceive a high prospect of structural change in the economy implying lower values of the best-fitting $\overline{\mathbf{g}}$. Another reason why the dispersion of the best-fitting constant gain varies over time regards the time period when each forecaster entered the survey. Respondents with lower ID numbers tend to enter our dataset at the start of our sample, in 1968:Q4. Since economic conditions were relatively more volatile during this period, higher dispersion in $\overline{\mathbf{g}}$ across forecasters is witnessed. However, those forecasters in our dataset with ID numbers between 404 and 579, started entering the survey in 1990:Q3. The period after 1990:Q3 included the Great Moderation when macroeconomic variables were less volatile. Forecasters tended to not place as high a weight on recent economic activity, and, thus, $\overline{\mathbf{g}}$ across respondents had less dispersion.

Expectations are well approximated by the learning model for each forecaster. The first column of Figure 4 displays histograms of the correlation coefficient between expectations from the learning model and the SPF counterparts across each forecaster. The histograms for expected output and inflation is centered over positive values and towards one. Sentiment appears to drive inflation forecasts more than those for output. But, overall, the assumption that SPF forecasters utilize the learning model to construct forecasts seems to match well the SPF data on expectations. We can also investigate to what extent the heterogeneity across forecasters is due to heterogeneity in gains versus sentiments. Table 2 displays a fixed effects panel regression of observed SPF output and inflation expectations on the corresponding expectations from the learning model (considering only the endogenous component from the PLM, without the exogenous portion due to sentiment). Heterogeneity in the learning forecasts, arising from heterogeneous gains, explains a high proportion of the variation in expectations across forecasters, with $R^2_{between}$ values equal to 92.83% for output expectations and 90.80% for inflation expectations. Overall, instead, PLM forecasts explain 76.32% and 48.31% for output and inflation expectations, with the remaining variation attributed to sentiment.

5.1.1 Beliefs

Differences in constant gains can affect agents' beliefs, that is, the elements of $\widehat{\phi}_t = [a_t, b_t, c_t]'$. Thus, a natural question regards the implications for the economy when there exists heterogeneity in forecasters' estimates of a, b, and c. To attempt to answer this question, we analyze selected beliefs of SPF respondents.

Figures 5 – 7 display the results. In each figure, we represent the distribution of the belief coefficient each period with a boxplot as before. Figure 5 shows the slope of the Phillips Curve parameter, that is, $b_{2,1}$. The inflation persistence parameter (i.e, $b_{2,2}$) is displayed in Figure 6. Figure 7 presents the policy parameter $b_{1,3}$, which governs the sensitivity of the output gap to changes in the interest rate.[14] In addition, the belief coefficients correspond to the best-fitting gain of each forecaster.

Three important takeaways emerge after examining Figures 5 – 7. First, looking at the evolution of the slope of the Phillips Curve $b_{2,1}$, there are notable shifts in the values of the coefficients during recessions: the curve appears to steepen during recessions. Second, the median value of the perceived inflation persistence parameter (i.e., $b_{2,2}$) increases in the middle of the sample and then declines again at the end. Finally, forecasters perceive the influence of policy on output gap (i.e., $b_{1,3}$) to be less effective as time elapses. Figure 7 shows that the median value of $b_{1,3}$ stays negative the entire period, but moves in an upward trajectory towards zero. The median coefficients are close to -2 in the 1970s, but they are revised closer to -0.5 after the Great Recession. This result is consistent with the VAR evidence from (Boivin and Giannoni, 2006) on the reduced effectiveness of monetary policy over a post-1960 sample.

[14]Note that $b_{2,1}, b_{2,2}$, and $b_{1,3}$ correspond to elements in each individual forecaster's b matrix in equation (13). For instance, $b_{2,1}$ refers to the element found in the second row first column of b, that is, the slope of the Phillips Curve.

5.1.2 Evolving Beliefs, Heterogeneity, and Responses to Shocks

The response of the economy to shocks will be substantially different depending on the state of private-sector beliefs. In Figure 8, we show the responses of output, inflation, and interest rates, to the structural shocks (natural rate, cost-push, and monetary policy), which would exist if the aggregate beliefs in the New Keynesian model were assumed to be equal to those held by each forecaster j. We show the responses (in terms of median and 5^{th}-95^{th} percentiles) in the early part of the sample (1971:Q1) and in the late part, before the Great Recession (2006:Q1).

As the figure shows, the heterogeneity is more pronounced in 1971, as the ranges of responses are usually wider. For example, a portion of forecasters believes that positive cost-push shocks have only mild (and possibly even positive) effects on the economy, while others believe that cost-push shocks have extremely large recessionary effects on output. The majority of forecasters lay in between these two extremes. In a self-referential system, individual forecasters' beliefs can be partially self-fulfilling: if agents perceive supply, or any other, shocks to be particularly effective, they will indeed play a larger role than in an economy in which their effects are perceived as trivial in the formation of expectations. The responses for 2006 indicate that significant dispersion still exists regarding the magnitude of the effects, but without major disagreement on the sign and overall shape of the responses.

The belief parameters regarding the response of output to structural shocks also provide explanation for the heterogeneity in the impulse responses. Figure 9 displays the histograms of the parameters $c_{1,1}$ and $c_{1,2}$ across forecasters. The belief parameters $c_{1,1}$ and $c_{1,2}$ are found in the matrix c in equation (13) and represent that perceived response of output to demand and supply shocks, respectively. The top row of Figure 9 denotes the parameters for 1971:Q1 and the bottom row for 2006:Q1. As displayed in the first column, the effect of demand shocks on output is more dispersed with 1971:Q1 beliefs than 2006:Q1 beliefs. The cost-push shocks also show a larger dispersion in beliefs during 1971:Q1 as some forecasters believe supply shocks have large recessionary effects while others do not. Analogously to the impulse responses, there exists heterogeneity in the 2006:Q1 estimates of $c_{1,2}$, but with a more subdued level of disagreement, and with SPF respondents who anticipate smaller effects of cost-push shocks on the economy. These results could be due to the second row of beliefs occurring during the Great Moderation, when macroeconomic variables exhibit less volatility, and, thus, less of a perceived response to economic shocks relative to pre-Great Moderation period.[15]

Figure 10 shows, instead, the contribution of each shock to the forecast error variance of output, when, as before, the aggregate beliefs are assumed to be fixed, in turn, to exactly match the beliefs of each single forecaster j.[16] It is apparent from the figure that the state of beliefs plays a central role for the transmission and importance of shocks: the natural rate shock can explain anywhere from close to zero to almost all output fluctuations. Depending on forecasters' perceptions, monetary policy shocks can end up explaining between few percentage points and 60% of business cycle movements.

It is important to discuss how the heterogeneity in beliefs described in Section 5.1.1 propagates to the real economy. In particular, Figure 7 shows that forecasters perceive the influence of policy on output gap to be less effective later in our sample period. The bottom-right graph Figure 10 shows that the initial forecast error variance decomposition agrees with the previous results. However, an interesting result regards the increased relevance of monetary policy shocks in 2006:Q1 at medium-to-longer horizons. A potential reason could be the implementation of forward guidance by the Federal Reserve in response to the Great Recession. Since our model does not explicitly incorporate central bank communication about the future path of policy, forward guidance shocks could then be captured by the monetary policy shock in equation (3), causing relatively larger changes in output.

5.2 Sentiment

In this section, we examine a potential second source of heterogeneity in expectations: sentiment shocks. These shocks are defined as the difference between observed expectations and model-implied expectations for one-period ahead output and inflation:

$$s_t^{j,y} = E_t^{j,obs} y_{t+1} - \hat{E}_t^j y_{t+1} \tag{19}$$

$$s_t^{j,\pi} = E_t^{j,obs} \pi_{t+1} - \hat{E}_t^j \pi_{t+1} \tag{20}$$

As stated in Milani (2011, 2017), these shocks can be defined as waves of excess optimism and pessimism by agents about the economy in a particular time period.

[15] In addition to changing beliefs, it is also important to note that the model parameters can affect the response of macroeconomic variables to structural shocks. Since the two time periods used in this subsection represent two distinct regimes, different values of the model's deep parameters can exist and lead to different impulse responses.

[16] Notice that the shares don't sum to one, since the remaining portion is explained by sentiment shocks.

The sentiment shocks of each individual forecasters over time corresponding to the best-fitting gain are displayed in Figures 11 and 12. As in the previous section, we represent the distribution each period with a boxplot. Figure 11 corresponds to the sentiment shock for expectations of one-period ahead output. Figure 12 shows the sentiment shock for expectations of one-period ahead inflation.

Figures 11 and 12 show that an additional source of heterogeneity in forecasts can stem from different waves of optimism and pessimism for each individual forecaster. These disparities are apparent when examining pre- and post-Great Moderation periods in the U.S. In the former, the distribution of sentiment shocks is very wide reflecting the higher volatility of macroeconomic variables during this time period. After mid-1980s, forecasters' sentiment shocks are still different, but not as volatile and much tighter. They tend to cluster together during the Great Moderation era. In addition, the output sentiment shock tends to align with downturns in the U.S. economy. For instance, during the 2001 and 2007-2009 recessions in the U.S., the median value for the output shock turns negative. This result is not surprising as agents are becoming more pessimistic about the economy and matches well with Milani (2011, 2017) who shows these sentiment shocks can explain greater than 40% of business cycle fluctuations in the U.S.

The influence of sentiments shocks on economic activity occurs via multiple channels. As discussed in Stark (2013), SPF forecasters use subjective factors to construct expectations, which we interpret as sentiment shocks. Consumer sentiment about the future state of the economy affects their spending decisions today, which affects output.[17] In addition, the sentiment shocks play an important role for the conduct of monetary policy. As explained in Croushore and Stark (2019), two key macroeconomic relationships monetary authorities utilize are the Phillips Curve and a monetary policy rule. Both of these relationships are (partly) driven by expectations of future variables, which are dependent on sentiments of forecasters as shown above.

An important question relates to the properties of the sentiment process for individual forecasters. To investigate this issue, we fit the sentiment shocks to a VAR(1) plus constant model:

$$s_t^j = \Phi_0^j + \Phi_1^j s_{t-1}^j + \varepsilon_t^{j,\Phi} \tag{21}$$

[17] Although we do not explicitly model this sector, sentiment about investment and financial variables influences economic fluctuations. For example, since physical capital construction entails long-term projects, optimism or pessimism about the future can be a key factor for firms deciding to take out a loan and invest in physical capital today. Indeed, Milani (2017) shows that investment sentiment shocks are a main source of fluctuations in output and the business cycle frequency.

where $s_t^j = [s_t^{j,y}, s_t^{j,\pi}]'$, $\varepsilon_t^{j,\Phi}$ is a 2×1 vector of usual white noise error terms and the coefficient matrices are given by

$$\Phi_0 = \begin{bmatrix} \phi_{0,1}^j \\ \phi_{0,2}^j \end{bmatrix} \tag{22}$$

$$\Phi_1 = \begin{bmatrix} \phi_{1,1}^j & \phi_{1,2}^j \\ \phi_{2,1}^j & \phi_{2,2}^j \end{bmatrix}. \tag{23}$$

The VAR(1) plus constant model allows for the possibility of correlation across output and inflation sentiment shocks. For each forecaster and model, we estimate the coefficients using OLS. The results are presented in Figure 13, which displays the histograms of the coefficients across SPF respondents.

Three important takeaways emerge. First, the sentiment shocks seem to be (slightly) biased. The histograms of the estimated constants are skewed right towards positive values. Biased sentiment shocks can also imply evidence that the learning expectations are biased estimates of actual forecasts of agents. Second, the sentiment shocks of SPF forecasters exhibit persistence. The histograms of the estimated values of the autoregressive terms are centered over positive values. Finally, the shocks do not seem to be highly (if at all) correlated across output and inflation sentiment shocks, as evidenced by the estimates of off-diagonal coefficients (i.e., $\phi_{1,2}$ and $\phi_{2,1}$). The histograms of these estimated coefficients are slightly skewed left. However, they are centered around zero, suggesting minimal to no correlation across sentiment shocks. This result is consistent with prior literature. For instance, Milani (2011) estimates a New Keynesian model with U.S. data in which output and inflation sentiment shocks are uncorrelated. Furthermore, a motivating instance in the U.S. of uncorrelated shocks regards the period post-Great Recession through pre-COVID-19. During this era, the unemployment rate was gradually sinking to historic lows indicating optimism in the expectations of output by agents. However, this optimism did not correlate to higher amounts of inflation. The U.S. inflation rate, overall, was below its 2% target the majority of the time since the introduction of formal inflation targeting.

Overall, the heterogeneity across forecasters can stem from two sources: (1) different learning speeds, as measured by different constant gains (motivated by different agent's perceptions about incoming structural change or by different memories); and (2) different degrees of excess optimism and pessimism. The distribution of $\overline{\mathbf{g}}^j$ across forecasters tends to be more dispersed pre-Great Moderation and tightens up around lower values towards the end of the sample. In addition, the individual sentiment shocks track the U.S. business cycle fairly well. These shocks also seem to be biased upwards and exhibit persistence across time. In many periods, particularly at the beginning of the sample, forecasters who display excess optimism for either output or inflation coexist with others who display excess pessimism. But this is mostly due to forecasters in the tails of the distribution. Overall, sentiments don't cancel out: the 25-75% interquantile ranges show that many forecasters move in herd, tending to be overly optimistic or pessimistic at the same time.

6 Robustness

6.1 Alternative Forecasting Model

Our benchmark results were obtained under an adaptive learning PLM that included a constant, lagged endogenous variables, and knowledge of structural disturbances, that is, equation (13). But how would the heterogeneity across forecasters change if their forecasting model assumed more limited knowledge about the economy, for example by not including exogenous disturbances in their information set?

We attempt to answer the previous question by comparing the benchmark results to a PLM that consists of a VAR(1) plus constant. Equation (13) is modified to the following:

$$Y_t = a^j + b^j Y_{t-1} + \epsilon_t^j. \tag{24}$$

We repeat the benchmark exercise of Section 5.1 and display the outcomes of the best-fitting gains in Figure 14. As before, we represent the distribution of individual forecasts with a boxplot.

With a misspecified forecasting model, the best-fitting $\overline{\mathbf{g}}^j$ across SPF respondents tend to cluster over smaller values, compared with the benchmark case in Figure 3. The extent of time variation is also more modest.

Our choice of a PLM that resembles the MSV solution as benchmark case in the paper, however, is motivated by a number of factors. First, from a theoretical perspective, it corresponds to the model and information set that agents would be endowed with in a baseline New Keynesian model. Moreover, it appears to be empirically supported in our data set. We compute each forecaster's mean squared error based on equation (18) under both MSV and VAR(1) plus constant PLMs. We find that for 70.64% of forecasters the MSE is lower under our benchmark PLM than under the alternative VAR(1) PLM. Finally, by utilizing a correctly-specified forecasting model, we can better identify the sentiment shocks and reduce the probability that they spuriously capture the effects of omitted demand and supply disturbances.

We also examine if the observed persistence in the sentiment shocks are artificially affected by the inclusion of structural disturbances in the benchmark PLM. Figure 15 shows that the results are similar to the benchmark case of Section 5.2. Thus, the persistence of the sentiment shocks does not seem to be influenced by the inclusion or exclusion of structural shocks from the PLM.

6.2 Alternative Initialization of R_τ

Our benchmark results of Section 5 were obtained under the assumption that the initial precision matrix (i.e., R_τ) of each individual forecaster was equal to $c * I$, a scaled identity matrix, with $c = 0.1$. This specification allowed for a larger degree of uncertainty characterizing the initial beliefs of individual forecasters. However, it is natural to investigate the results under an alternative parameterization of R_τ for each forecaster. The importance of properly accounting for uncertainty about initial beliefs has been emphasized in Galimberti (2020, 2021).

Therefore, we proceed by estimating the initial beliefs about R for each forecaster as an additional extension exercise. Similar to the assumption for the initial $\widehat{\phi}_\tau$, each forecaster is assumed to use the relevant pre-sample data, and the same constant-gain learning approach, to estimate the initial second-moment matrix R_τ:

$$R_\tau \quad = \quad \overline{\mathbf{g}}^j \sum_{i=1}^{\tau} \left(1 - \overline{\mathbf{g}}^j\right)^{(i-1)} X_{\tau-i} X'_{\tau-i}. \tag{25}$$

In this case, we also avoid introducing any rescaling aimed to render the initial beliefs more diffuse. The precision matrices are simply equal to the available pre-sample estimates for each forecaster.

Figure 16 displays the individual best-fitting $\overline{\mathbf{g}}^j$ when utilizing equation (25). The median values across the sample are overall at higher levels, between 0.04 and 0.06, which align with Berardi and Galimberti (2017b) standard estimation approach under OLS-based initials. The remaining baseline qualitative results (e.g., belief parameters, sentiments) do not noticeably change.

7 Conclusions

We analyze aspects of the formation of expectations at the individual forecaster level. We treat forecasters in the same way as we would treat agents in a benchmark New Keynesian model. They are assumed to have a perceived model of the economy that resembles the MSV solution under rational expectations, and they have the same information set: therefore, they have a correctly-specified model, and they observe the same endogenous variables and disturbances that they would in the RE solution.

Their expectations can, however, be heterogeneous since different forecasters are allowed to have different gain coefficients. The best-fitting gain coefficient for each individual forecaster in the sample is estimated by minimizing the mean squared errors between the actual forecast and the forecast implied by the corresponding PLM. Moreover, each forecaster may be subject to sentiment, i.e., waves of excess optimism and pessimism, identified as in Milani (2011, 2017).

Our results reveal gain coefficients at the micro level that are, on average, of similar values to those estimated on aggregate data for macro models. The gains are, however, heterogeneous, with a dispersion that is higher in the 1970s and 1980s and much smaller by the end of the sample. The median gains are occasionally higher in the 1970s and 1980s. As a consequence, beliefs about key economic magnitudes are also heterogeneous and vary over time: for example, perceptions are consistent with a declining effectiveness of monetary policy over time.

Finally, we provide evidence at the micro level of the kind of sentiment shocks that have been shown in the recent literature to be important determinants of business cycles. Individual excesses of optimism and pessimism do not cancel out in the aggregate, but they are instead consistent with aggregate contagion or herd behavior.

In future research, it will be important to start from the evidence of heterogeneity at the micro level and investigate more thoroughly the implications of heterogeneous beliefs and sentiment for the macroeconomy.

References

[1] Andrade, P., Crump, R.K., Eusepi, S., and Moench, E. (2016). “Fundamental disagreement,” *Journal of Monetary Economics*, 83(C), 106-128.

[2] Angeletos, G. M., Collard, F., and Dellas, H. (2018). “Quantifying confidence,” *Econometrica*, 86(5), 1689-1726.

[3] Anufriev, M., & Hommes, C. (2012). “Evolutionary selection of individual expectations and aggregate outcomes in asset pricing experiments.” *American Economic Journal: Microeconomics*, 4(4), 35-64

[4] Baranowski, R. (2014). “Individual heterogeneity and adaptive learning in survey data,” mimeo, Coe College.

[5] Barucci, E., (1999). “Heterogeneous beliefs and learning in forward looking economic models,” *Journal of Evolutionary Economics*, 9(4), 453-464.

[6] Benhabib, J., Wang, P., and Wen, Y. (2015). “Sentiments and aggregate demand fluctuations,” *Econometrica*, 83(2), 549-585.

[7] Berardi, M. (2019). “A probabilistic interpretation of the constant gain algorithm,” mimeo, University of Manchester, U.K.

[8] Berardi, M., and Galimberti, J.K. (2017a). “Empirical calibration of adaptive learning,” *Journal of Economic Behavior and Organization*, 144(C), 219-237.

[9] Berardi, M., & Galimberti, J. K. (2017b). “On the initialization of adaptive learning in macroeconomic models,” *Journal of Economic Dynamics and Control*, 78, 26-53.

[10] Binder, C., Janson, W., & Verbrugge, R. (2019). “Thinking outside the box: Do SPF respondents have anchored inflation expectations?,” *Federal Reserve Bank of Cleveland Working Papers*, (WP 19-15).

[11] Boivin, J., and Giannoni, M.P. (2006). “Has Monetary Policy Become More Effective?,” *Review of Economics and Statistics*, 88(3), 445-462.

[12] Branch, W.A. (2004), “The theory of rationally heterogeneous expectations: Evidence from survey data on inflation expectations,” *The Economic Journal*, 114(497): 592-621.

[13] Branch, W.A. and Evans, G.W., (2006). “A simple recursive forecasting model,” *Economics Letters*, 91(2), 158-166.

[14] Branch, W. A., and Evans, G. W. (2007). “Model uncertainty and endogenous volatility,” *Review of Economic Dynamics*, 10(2), 207-237.

[15] Bräuning, C., and van der Cruijsen, C. (2019). “Learning dynamics in the formation of European inflation expectations,” *Journal of Forecasting*, 38(2), 122-135.

[16] Brock, W. A., & Hommes, C. H. (1997). “A rational route to randomness.” *Econometrica: Journal of the Econometric Society*, 1059-1095.

[17] Brock, W. A., & Hommes, C. H. (1998). “Heterogeneous beliefs and routes to chaos in a simple asset pricing model.” *Journal of Economic Dynamics and Control*, 22(8-9), 1235-1274.

[18] Charalampidis, N., and Milani, F. (2020). “Evolving beliefs and animal spirits in the euro area,” mimeo, University of California Irvine.

[19] Chatterjee, P., and Milani, F. (2020). "Perceived uncertainty shocks, excess optimism-pessimism, and learning in the business cycle," *Journal of Economic Behavior & Organization*, 179, 342-360.

[20] Chiang, Y. M., Hirshleifer, D., Qian, Y., & Sherman, A. E. (2011). "Do investors learn from experience? Evidence from frequent IPO investors," *The Review of Financial Studies*, 24(5), 1560-1589.

[21] Clements, M. P. (2015). "Do US macroeconomic forecasters exaggerate their differences?," *Journal of Forecasting*, 34(8), 649-660.

[22] Coibion, O., and Gorodnichenko, Y. (2012). "What can survey forecasts tell us about information rigidities?," *Journal of Political Economy*, 120(1), 116-159.

[23] Coibion, O., and Gorodnichenko, Y. (2015). "Information rigidity and the expectations formation process: a simple framework and new facts", *American Economic Review*, 105, 2644-2678.

[24] Cole, S.J., (2020a). "Learning and the effectiveness of central bank forward guidance," forthcoming, *Journal of Money, Credit, and Banking.*

[25] Cole, S.J., (2020b). "The limits of central bank forward guidance under learning," forthcoming, *International Journal of Central Banking.*

[26] Cole, S., and F. Milani (2019). "The misspecification of expectations in new keynesian models: a DSGE-VAR approach," *Macroeconomic Dynamics*, 23(3), 974-1007.

[27] Croushore, D., & Stark, T. (2001). "A real-time data set for macroeconomists," *Journal of Econometrics*, 105(1), 111-130.

[28] Croushore, D., & Stark, T. (2019). "Fifty years of the survey of professional forecasters," *Economic Insights*, 4(4), 1-11.

[29] Dave, C., and Malik, S. (2017), "A tale of fat tails," *European Economic Review*, 100(C), 293-317.

[30] Ehrbeck, T., & Waldmann, R. (1996). "Why are professional forecasters biased? Agency versus behavioral explanations," *The Quarterly Journal of Economics*, 111(1), 21-40.

[31] Eusepi, S., and Preston, B. (2010). "Central bank communication and expectations stabilization," *American Economic Journal: Macroeconomics*, 2(3), 235-71.

[32] Eusepi, S., and Preston, B. (2011). "Expectations, learning, and business cycle fluctuations," *American Economic Review*, 101(6), 2844-72.

[33] Evans, G. W. and Honkapohja, S. (2001). *Learning and expectations in macroeconomics.* Princeton University Press, Princeton, NJ.

[34] Evans, G. W., Honkapohja, S., and Mitra, K. (2009). "Anticipated fiscal policy and adaptive learning," *Journal of Monetary Economics*, 56(7), 930-953.

[35] Evans, G. W., and Honkapohja, S. (2013). "Learning as a rational foundation for macroeconomics and finance," *Rethinking expectations: The way forward for macroeconomics*, 68.

[36] FRB of Philadelphia (2019a). "Documentation: First-, Second-, and Third-Release Values Philadelphia Fed's Real-Time Data Set for Macroeconomists." *Last Update: December 23, 2019.*

[37] FRB of Philadelphia (2019b). "Survey of Professional Forecasters: Documentation." *Last Update: May 6, 2020.*

[38] Fuster, A., Laibson, D., and B. Mendel, (2010). "Natural Expectations and Macroeconomic Fluctuations", *Journal of Economic Perspectives*, 24(4), 67-84.

[39] Galimberti, J. K. (2020). "Information weighting under least squares learning", CAMA Working Papers 2020-46, The Australian National University.

[40] Galimberti, J. K. (2021). "Initial Beliefs Uncertainty and Information Weighting in the Estimation of Models with Adaptive Learning", Working Papers 2021-01, Auckland University of Technology, Department of Economics.

[41] Gaspar, V., Smets, F., and Vestin, D. (2006). "Adaptive learning, persistence, and optimal monetary policy," *Journal of the European Economic Association*, 4(2-3), 376-385.

[42] Gennaioli, N., Ma, Y., and A. Shleifer, (2016). "Expectations and Investment, *NBER Macroeconomics Annual*, 2015, ed. M. Eichenbaum and J. Parker, Chicago: University of Chicago Press.

[43] Hommes, C. (2011). "The heterogeneous expectations hypothesis: Some evidence from the lab." *Journal of Economic Dynamics & Control*, 35(1), 1-24.

[44] Hommes, C. (2019). "Behavioral & Experimental Macroeconomics and Policy Analysis: a Complex Systems Approach," forthcoming, *Journal of Economic Literature.*

[45] Honkapohja, S., and Mitra, K. (2006). "Learning stability in economies with heterogeneous agents," *Review of Economic Dynamics*, 9(2), 284-309.

[46] Jones, C. (2016). "The Facts of Economic Growth," Chapter 1, *Handbook of Macroeconomics*, 2016, vol. 2, 3-69, Elsevier.

[47] Kaustia, M., & Knüpfer, S. (2008). "Do investors overweight personal experience? Evidence from IPO subscriptions," *The Journal of Finance*, 63(6), 2679-2702.

[48] Krippner, L. (2013). "Measuring the stance of monetary policy in zero lower bound environments," *Economics Letters*, 118(1), 135-138.

[49] Lamont, O. A. (2002). "Macroeconomic forecasts and microeconomic forecasters," *Journal of Economic Behavior & Organization*, 48(3), 265-280.

[50] Laster, D., Bennett, P., & Geoum, I. S. (1999). "Rational bias in macroeconomic forecasts," *The Quarterly Journal of Economics*, 114(1), 293-318.

[51] Mackowiak, B., and M. Wiederholt, (2009). "Optimal sticky prices under rational inattention", *American Economic Review*, 99, 769-803.

[52] Malmendier, U., and Nagel, S. (2016). "Learning from inflation experiences," *The Quarterly Journal of Economics*, 131(1), 53-87.

[53] Mankiw, N. G., & Reis, R. (2002). "Sticky information versus sticky prices: a proposal to replace the New Keynesian Phillips curve." *The Quarterly Journal of Economics*, 117(4), 1295-1328.

[54] Mankiw, N.G., Reis, R., and Wolfers, J. (2004). "Disagreement about inflation expectations," NBER Chapters, in: *NBER Macroeconomics Annual*, Volume 18, pages 209-270, National Bureau of Economic Research, Inc.

[55] Mansky, C.F. (2004). "Measuring Expectations, *Econometrica*, 72, 1329-76.

[56] Mansky, C.F. (2011). "Interpreting and Combining Heterogeneous Survey Forecasts," in M. Clements and D.Hendry (editors), *Oxford Handbook on Economic Forecasting*, Chapter 16, 457-472, Oxford: Oxford University Press.

[57] Mansky, C.F. (2017). "Survey measurement of probabilistic macroeconomic expectations: progress and promise", NBER Working Paper No. 23418.

[58] Markiewicz, A., and Pick, A. (2014). "Adaptive learning and survey data," *Journal of Economic Behavior & Organization*, 107(PB), 685-707.

[59] McConnell, M. M., and Perez-Quiros, G. (2000). "Output fluctuations in the United States: What has changed since the early 1980's?," *American Economic Review*, 90(5), 1464-1476.

[60] Milani, F. (2007). "Expectations, learning and macroeconomic persistence," *Journal of Monetary Economics*, 54(7), 2065-2082.

[61] Milani, F. (2011). "Expectation shocks and learning as drivers of the business cycle," *Economic Journal*, 121(552), 379-401.

[62] Milani, F. (2014). "Learning and time-varying macroeconomic volatility," *Journal of Economic Dynamics and Control*, 47, 94-114.

[63] Milani, F. (2017). "Sentiment and the US business cycle," *Journal of Economic Dynamics and Control*, 82, 289-311.

[64] Orphanides, A., and Williams, J.C. (2005). "Imperfect knowledge, inflation expectations and monetary policy." In *Inflation Targeting*, edited by Ben S. Bernanke and Michael Woodford. University of Chicago Press.

[65] Perron, P., and Wada, T. (2009). "Let's take a break: Trends and cycles in US real GDP", *Journal of Monetary Economics*, 56(6), 749-765.

[66] Pesaran, H., (1987). "The Limits to Rational Expectations", Oxford: Blackwell.

[67] Pfajfar, D., and Santoro, E. (2010). "Heterogeneity, learning and information stickiness in inflation expectations," *Journal of Economic Behavior & Organization*, 75(3), 426-444.

[68] Preston, B. (2006). "Adaptive learning, forecast-based instrument rules and monetary policy," *Journal of Monetary Economics*, 53(3), 507-535.

[69] Sargent, T.J., (1993). *Bounded Rationality in Macroeconomics.* Oxford University Press: Oxford, U.K.

[70] Sargent, T.J., (1999). *The Conquest of American Inflation.* Princeton: Princeton University Press.

[71] Sims, C.A. (2003). "Implications of rational inattention", *Journal of Monetary Economics*, 50, 665-690.

[72] Slobodyan, S., and Wouters, R. (2012). "Learning in a Medium-Scale DSGE Model with Expectations Based on Small Forecasting Models", *American Economic Journal: Macroeconomics*, 4(2), 65-101.

[73] Stark, T. (2013). SPF panelists forecasting methods: A note on the aggregate results of a November 2009 special survey. *Federal Reserve Bank of Philadelphia.*

[74] Woodford, M. (2003a). "Imperfect common knowledge and the effects of monetary policy", in *Knowledge, Information, and Expectations in Modern Macroeconomics: In Honor of Edmund Phelps*, edited by Aghion, P., Frydman, R., Stiglitz, J., and Woodford, M., Princeton, NJ: Princeton Univ. Press.

[75] Woodford, M. (2003b). *Interest and Prices: Foundations of a Theory of Monetary Policy.* Princeton, NJ: Princeton University Press.

A SPF Respondents Submit Forecasts for at Least 20 Periods

A potential issue with using a survey dataset such as the SPF is the entry and exit of respondents as described by Mansky (2011). To remedy this issue, our baseline exercise included only those respondents that submit forecasts for at least ten periods. We chose this number partly so that we had a sufficient number of data points per respondent. However, given that this observation requirement is somewhat arbitrary, we analyze the benchmark results for those respondents that submit forecasts for at least twenty periods.

The results of this sensitivity analysis show that the baseline outcomes are largely unchanged. Figure 17 reports the best-fitting constant gains for each forecasters with a boxplot representing the distribution of forecasters as in Section 5.1. As in the baseline case, there exists larger dispersion in the value of $\overline{\mathbf{g}}$ before the Great Moderation relative to after this time period. Thus, the results of this section indicate that our benchmark requirement of ten observations for a respondent to remain in our dataset allows sufficient information about the distribution of best-fitting constant gains.

	Prior Distribution	Posterior Mean	95% Posterior Interval
IES	$\sigma \sim \Gamma(1, 0.75)$	0.694	[0.454,1.040]
Calvo	$\xi_p \sim B(0.6, 0.05)$	0.889	[0.864,0.911]
Habits	$\eta \sim B(0.5, 0.2)$	0.366	[0.229,0.531]
Indexation	$\gamma \sim B(0.5, 0.2)$	0.088	[0.016,0.212]
IRS	$\rho_{pre79} \sim B(0.7, 0.2)$	0.807	[0.679,0.916]
	$\rho_{79-82} \sim B(0.7, 0.2)$	0.929	[0.887,0.969]
	$\rho_{post82} \sim B(0.7, 0.2)$	0.613	[0.411, 0.961]
Resp. Infl.	$(\chi_{\pi,pre79} - 1) \sim \Gamma(0.5, 0.25)$	1.350	[1.099,1.741]
	$(\chi_{\pi,79-82} - 1) \sim \Gamma(0.5, 0.25)$	1.644	[1.146,2.425]
	$(\chi_{\pi,post82} - 1) \sim \Gamma(0.5, 0.25)$	1.457	[1.111,1.904]
Resp. Output	$\chi_{y,pre79} \sim \Gamma(0.25, 0.15)$	0.237	[0.118,0.372]
	$\chi_{y,79-82} \sim \Gamma(0.25, 0.15)$	0.200	[0.076,0.348]
	$\chi_{y,post82} \sim \Gamma(0.25, 0.15)$	0.209	[0.110,0.289]
AR Nat. Rate	$\rho_{r,pre84} \sim B(0.5, 0.2)$	0.727	[0.546,0.890]
	$\rho_{r,post84} \sim B(0.5, 0.2)$	0.841	[0.730,0.923]
AR Cost-push	$\rho_{u,pre84} \sim B(0.5, 0.2)$	0.066	[0.011,0.177]
	$\rho_{u,post84} \sim B(0.5, 0.2)$	0.016	[0.003,0.040]
AR Out. Sent.	$\rho_{s_y,pre84} \sim B(0.5, 0.2)$	0.559	[0.336,0.761]
	$\rho_{s_y,post84} \sim B(0.5, 0.2)$	0.694	[0.576,0.819]
AR Infl Sent.	$\rho_{s_\pi,pre84} \sim B(0.5, 0.2)$	0.854	[0.761,0.953]
	$\rho_{s_\pi,post84} \sim B(0.5, 0.2)$	0.561	[0.358,0.719]
Std. Nat. Rate	$\sigma_{r,pre84} \sim IG(0.3, 1)$	1.011	[0.697,1.417]
	$\sigma_{r,post84} \sim IG(0.3, 1)$	0.547	[0.383,0.754]
Std. Cost-push	$\sigma_{u,pre84} \sim IG(0.3, 1)$	0.447	[0.366,0.550]
	$\sigma_{u,post84} \sim IG(0.3, 1)$	0.241	[0.214,0.271]
Std. MP	$\sigma_{\varepsilon,pre79} \sim IG(0.3, 1)$	0.266	[0.215,0.324]
	$\sigma_{\varepsilon,79-82} \sim IG(0.3, 1)$	0.254	[0.226,0.287]
	$\sigma_{\varepsilon,post82} \sim IG(0.3, 1)$	0.280	[0.091,0.840]
Std. Out. Sent.	$\sigma_{s_y,pre84} \sim IG(0.3, 1)$	0.984	[0.791,1.204]
	$\sigma_{s_y,post84} \sim IG(0.3, 1)$	0.494	[0.419,0.573]
Std. Infl Sent.	$\sigma_{s_\pi,pre84} \sim IG(0.3, 1)$	0.331	[0.274,0.409]
	$\sigma_{s_\pi,post84} \sim IG(0.3, 1)$	0.183	[0.161,0.208]
Constant Gain	$\bar{g} \sim B(0.025, 0.01)$	0.015	[0.010,0.021]

Table 1: Prior and Posterior Distributions for estimated New Keynesian model coefficients.

Note: Γ refers to Gamma distribution, B to Beta, and IG to Inverse Gamma. The numbers in parenthesis refer to the chosen means and standard deviations for each distribution.

Table 2: **Panel Regression of SPF on Learning**

	$E_t^{j,obs} y_{t+1} = \alpha_i + \beta_1 \widehat{E}_t^{j,plm} y_{t+1} + \varepsilon_t^j$
R^2	0.7632
R^2_{within}	0.6923
$R^2_{between}$	0.9283

	$E_t^{j,obs} \pi_{t+1} = \alpha_i + \beta_1 \widehat{E}_t^{j,plm} \pi_{t+1} + \varepsilon_t^j$
R^2	0.4831
R^2_{within}	0.2011
$R^2_{between}$	0.9080

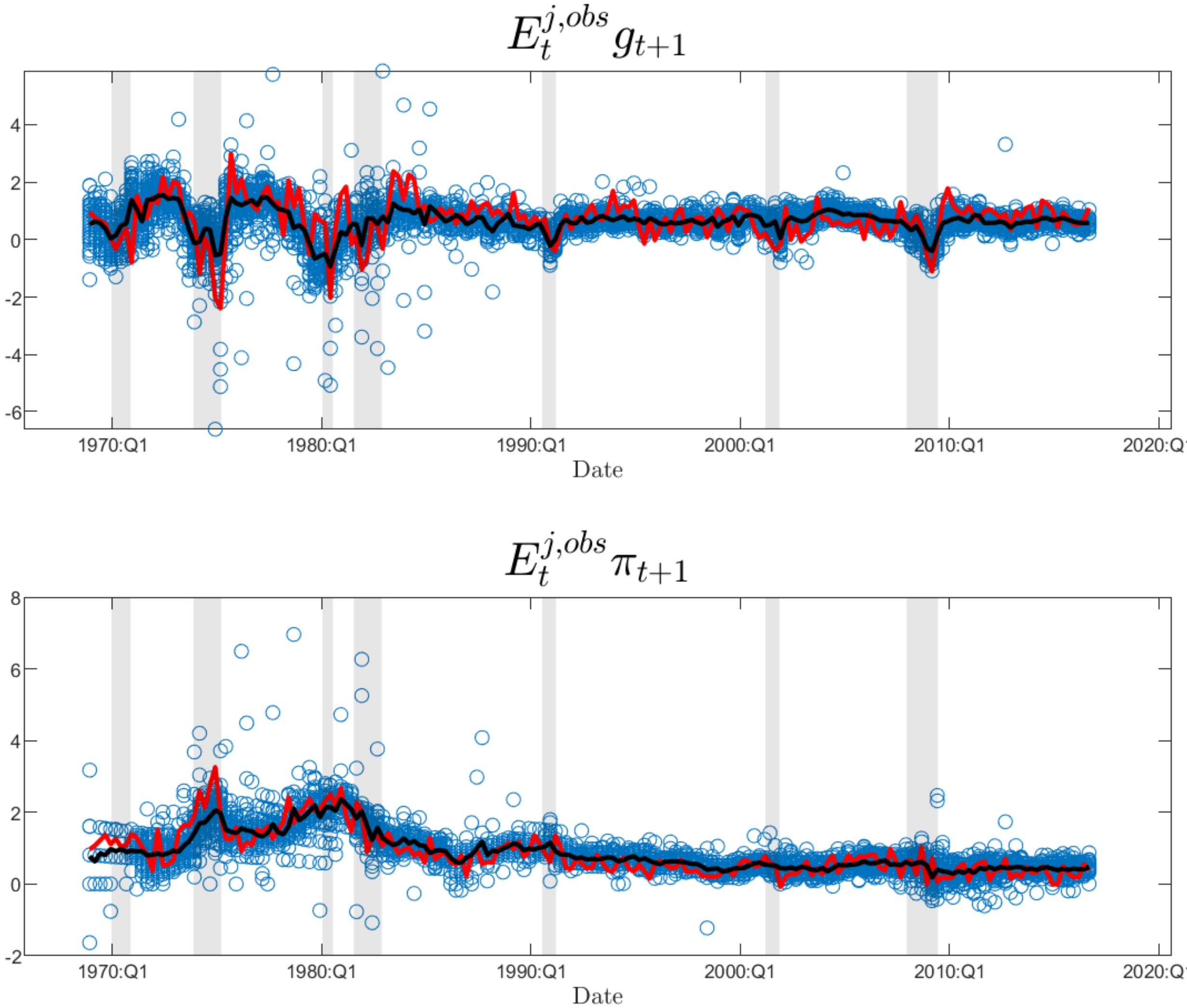

Figure 1: Evolution of Expected Output Growth and Inflation Over the Sample.

Note: 'g' denotes the growth rate of output. Blue Circles: Individual SPF Forecasters. Black Line: Implied Mean of SPF Forecasters. Red Line: Actual Realized Series.

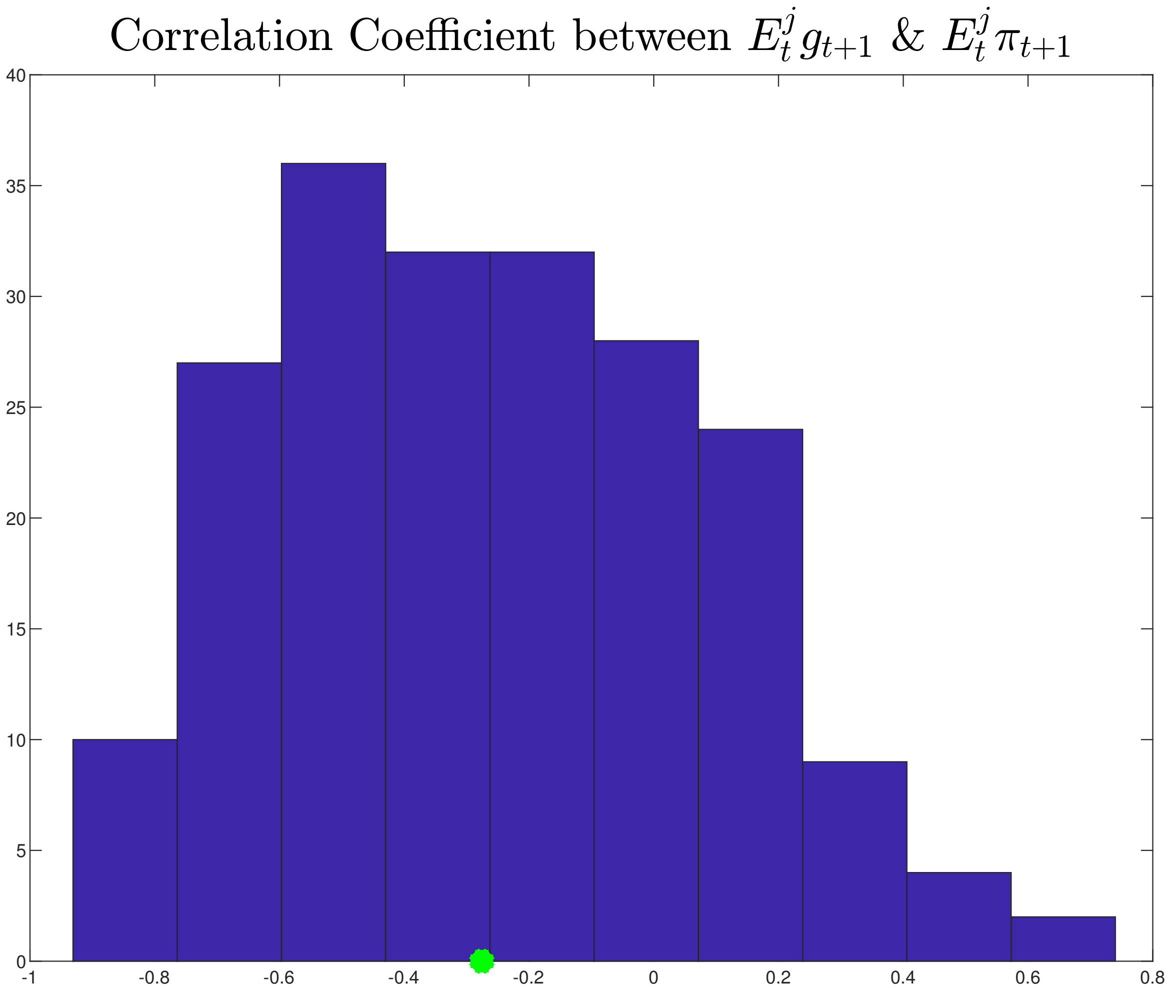

Figure 2: Histogram of Correlation Coefficient between $E_t^{j,obs} g_{t+1}$ & $E_t^{j,obs} \pi_{t+1}$ Across Forecasters

Note: Green circle denotes the median.

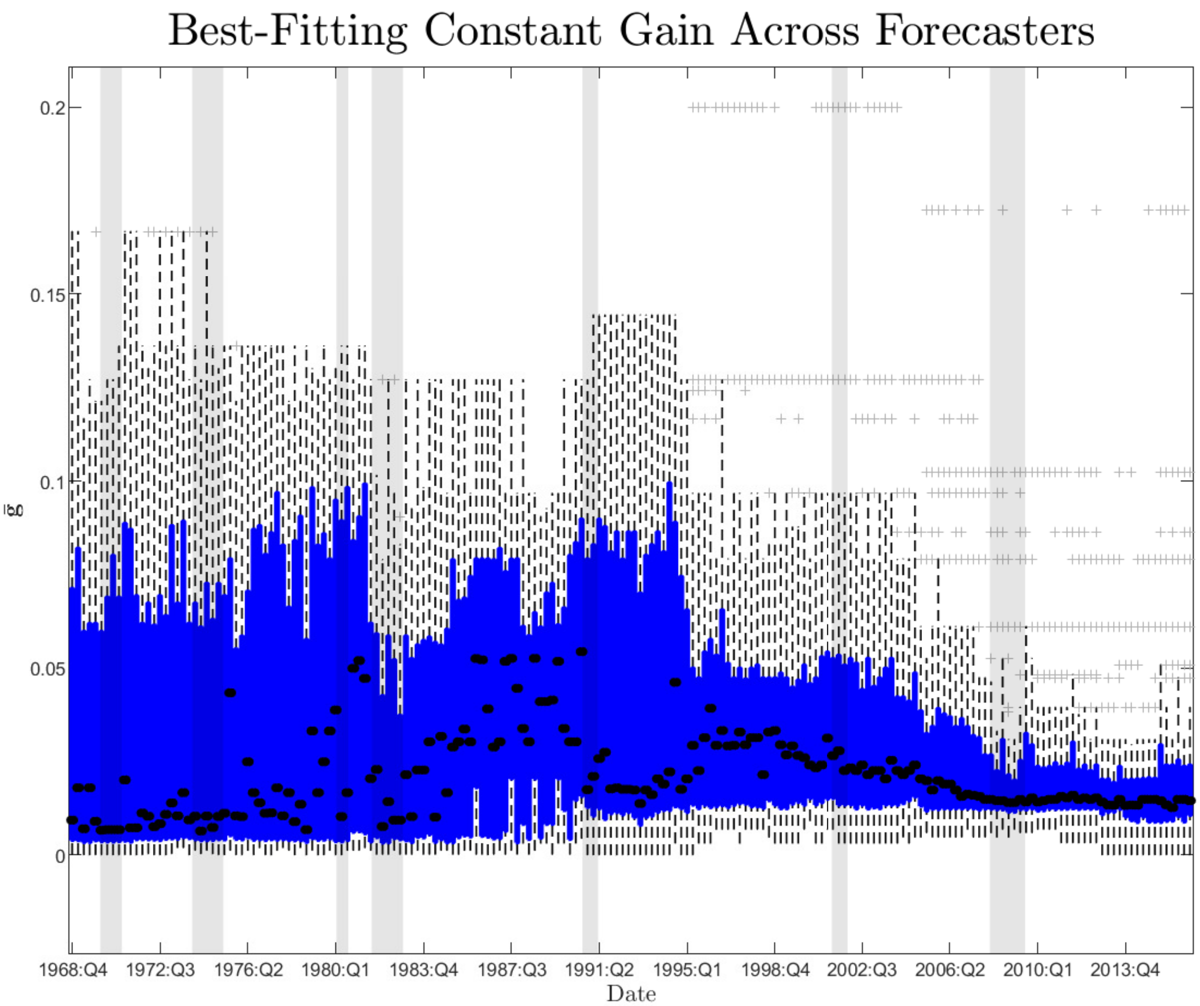

Figure 3: Distribution of the Best-Fitting Constant Gain of Individual Forecasters Each Time Period

Note: The distribution of the best-fitting constant gain of forecasters each time period is represented with a boxplot. The black circle signifies the median, the edges of the blue box the 25$^{\text{th}}$ and 75$^{\text{th}}$ percentiles, the black whiskers the extreme values not considered outliers, and the grey '+' symbol outliers.

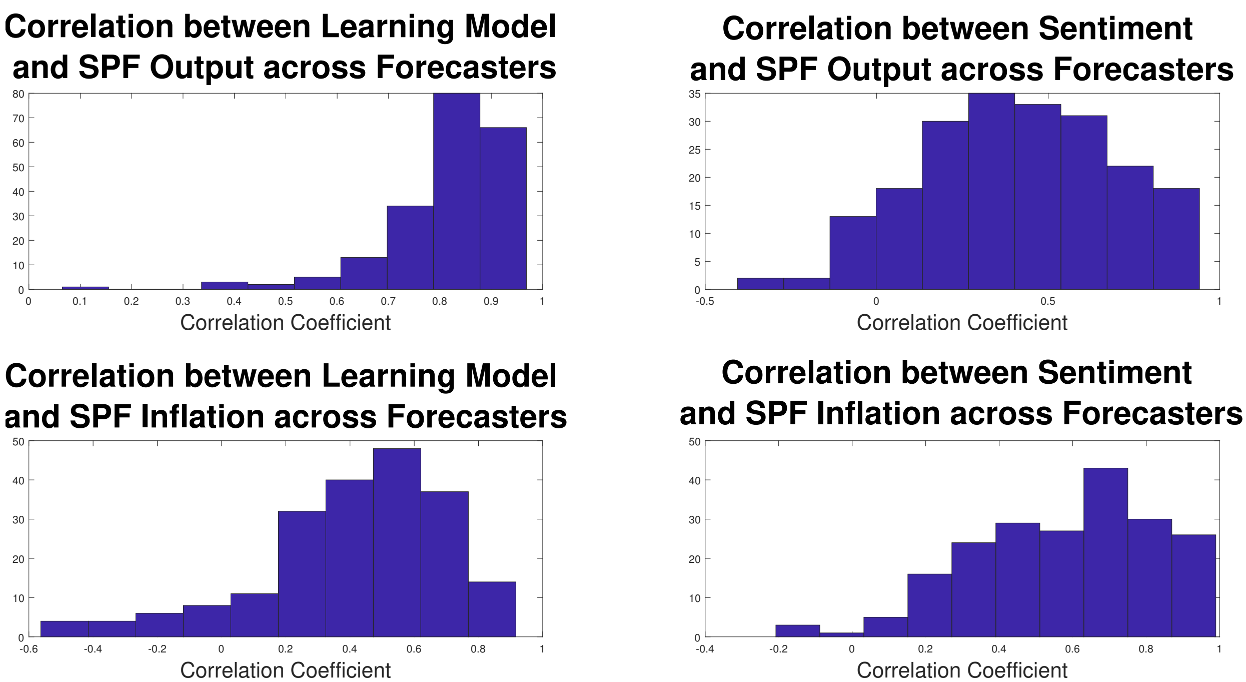

Figure 4: Histograms of Correlation Coefficients between Learning Model and SPF Expectations (left column) and Sentiment and SPF Expectations (right column).

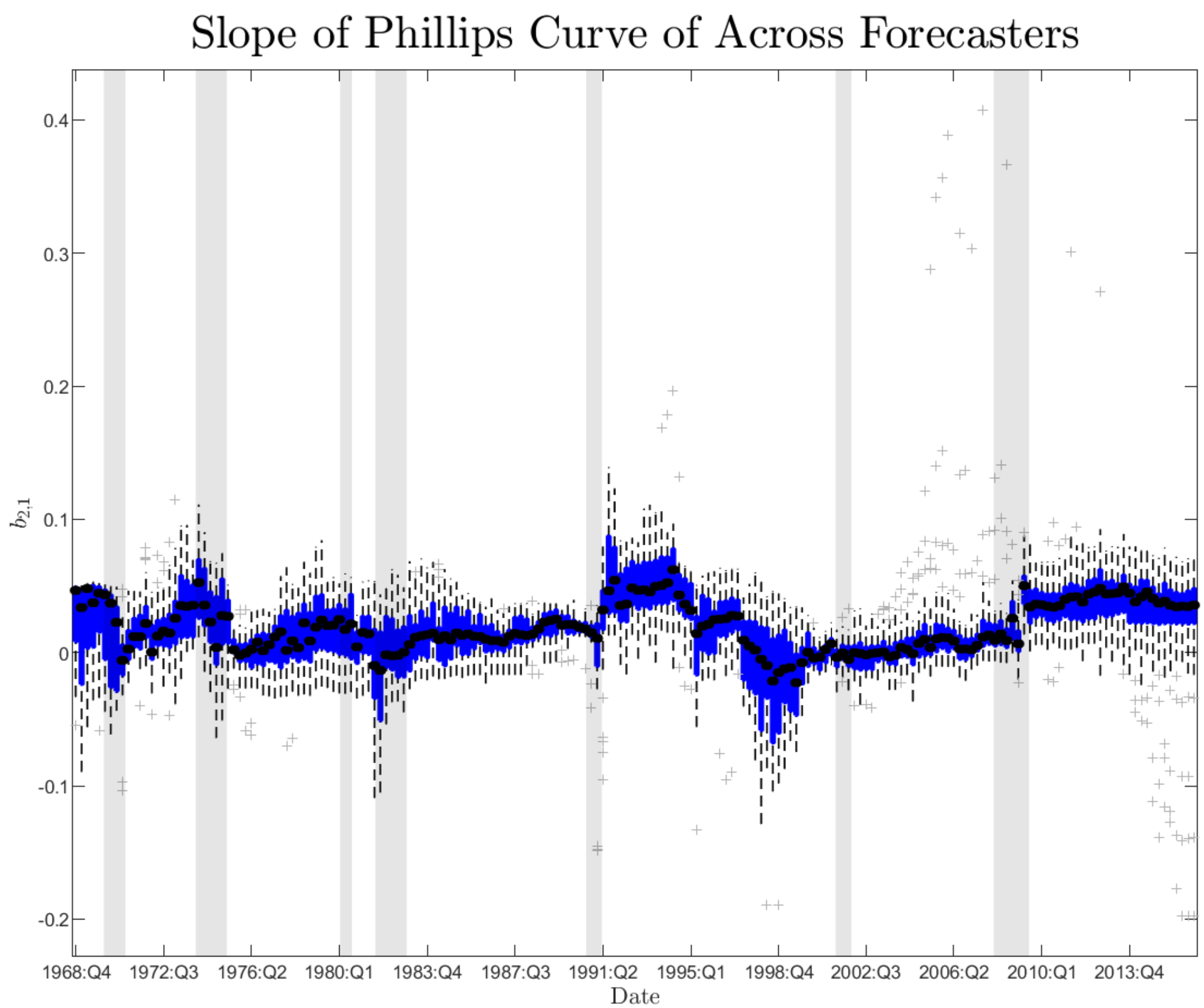

Figure 5: Estimate of Slope of Phillips Curve of Individual Forecasters Each Time Period

Note: The distribution of $b_{2,1}$ of forecasters each time period is represented with a boxplot. The black circle signifies the median, the edges of the blue box the 25$^{\text{th}}$ and 75$^{\text{th}}$ percentiles, the black whiskers the extreme values not considered outliers, and the grey '+' symbol outliers.

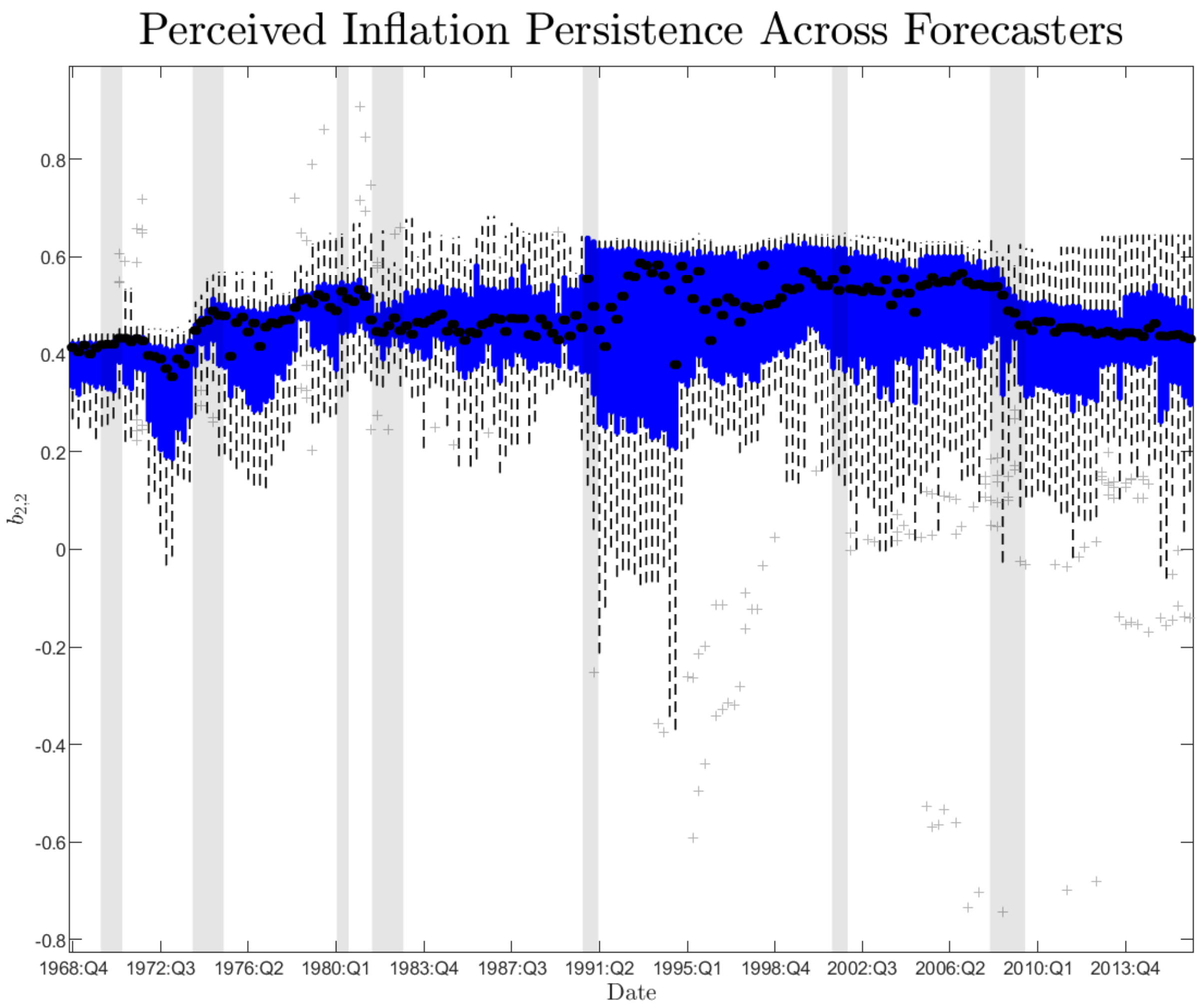

Figure 6: Estimate of Perceived Inflation Persistence Parameter of Individual Forecasters Each Time Period

Note: The distribution of $b_{2,2}$ of forecasters each time period is represented with a boxplot. The black circle signifies the median, the edges of the blue box the 25$^{\text{th}}$ and 75$^{\text{th}}$ percentiles, the black whiskers the extreme values not considered outliers, and the grey '+' symbol outliers.

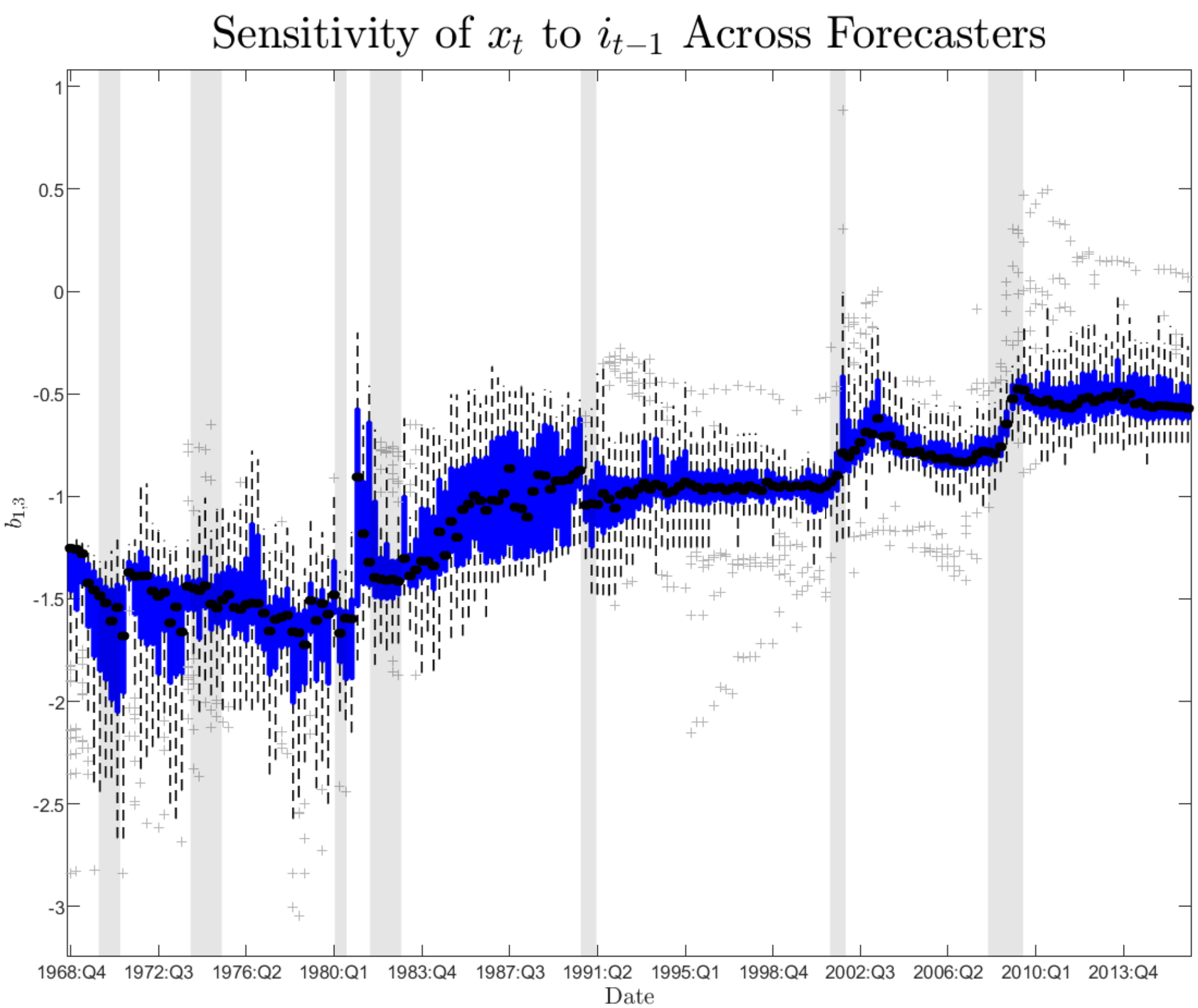

Figure 7: Estimate of Policy Parameter of Individual Forecasters Each Time Period

Note: The distribution of $b_{1,3}$ of forecasters each time period is represented with a boxplot. The black circle signifies the median, the edges of the blue box the 25^{th} and 75^{th} percentiles, the black whiskers the extreme values not considered outliers, and the grey '+' symbol outliers.

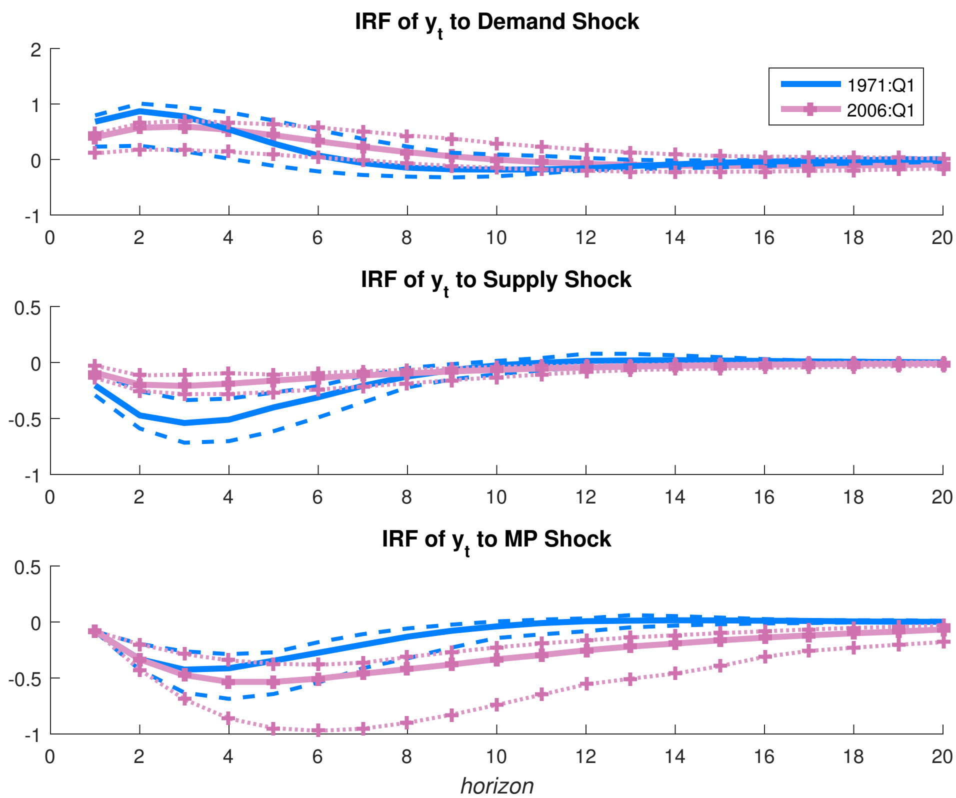

Figure 8: Impulse Response Functions of Output to Demand (Natural Rate), Supply (Cost-Push), and Monetary Policy Shocks, Across Heterogeneous Beliefs.

Note: We fix aggregate beliefs to equal the beliefs of each forecaster j, $j = 1, ..., N$. The range of impulse responses (corresponding to the median, 5^{th} and 95^{th} percentiles, across forecasters) under heterogeneous beliefs are shown for 1971:Q1 and 2006:Q1.

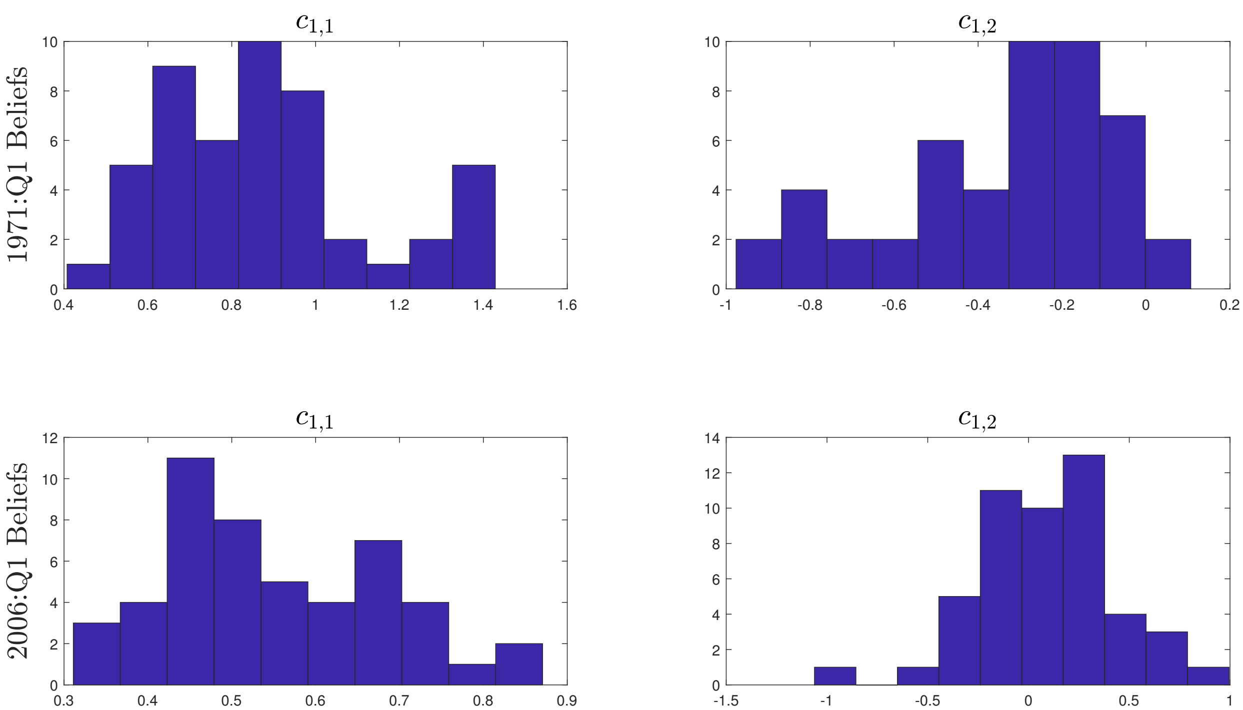

Figure 9: Histogram of the Beliefs Regarding Response of Output to Demand and Supply Shocks Across Forecasters

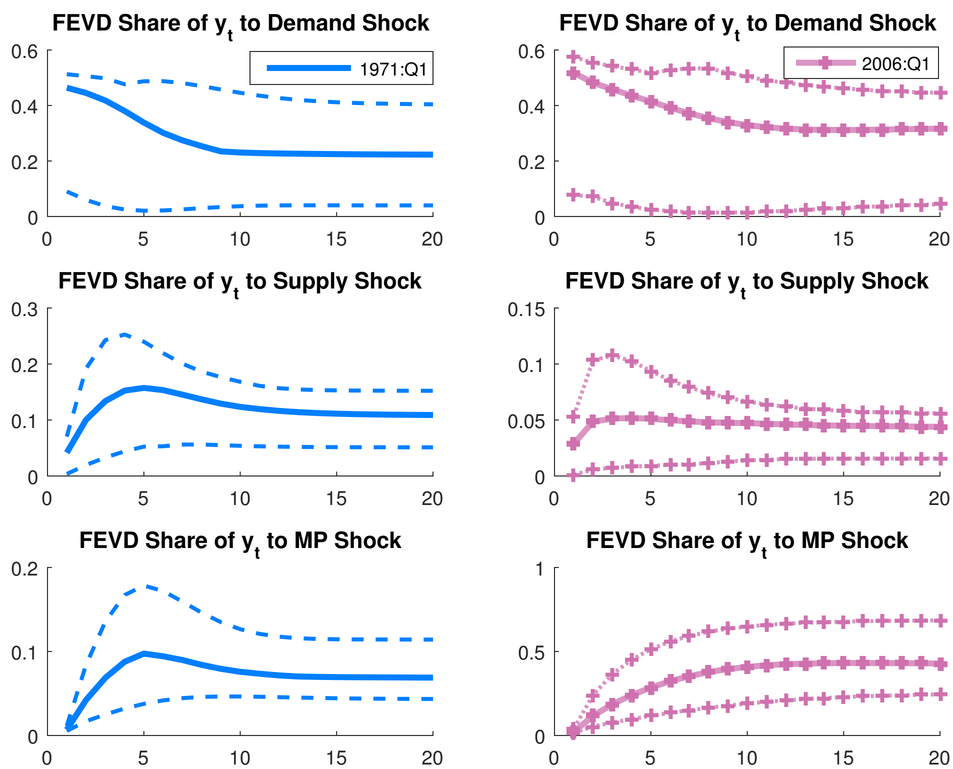

Figure 10: Forecast Error Variance Decomposition. Share of output forecast error variance explained by demand, supply, and monetary policy shocks, across heterogeneous beliefs. Aggregate beliefs in the model are fixed to match, in turn, the beliefs of each forecaster j, $j = 1, ..., N$. The figure displays median, 5^{th}, and 95^{th}, percentiles, across forecasters' responses. The left panels show the variance shares corresponding to each set of beliefs for 1971:Q1, the right panels those for 2006:Q1.

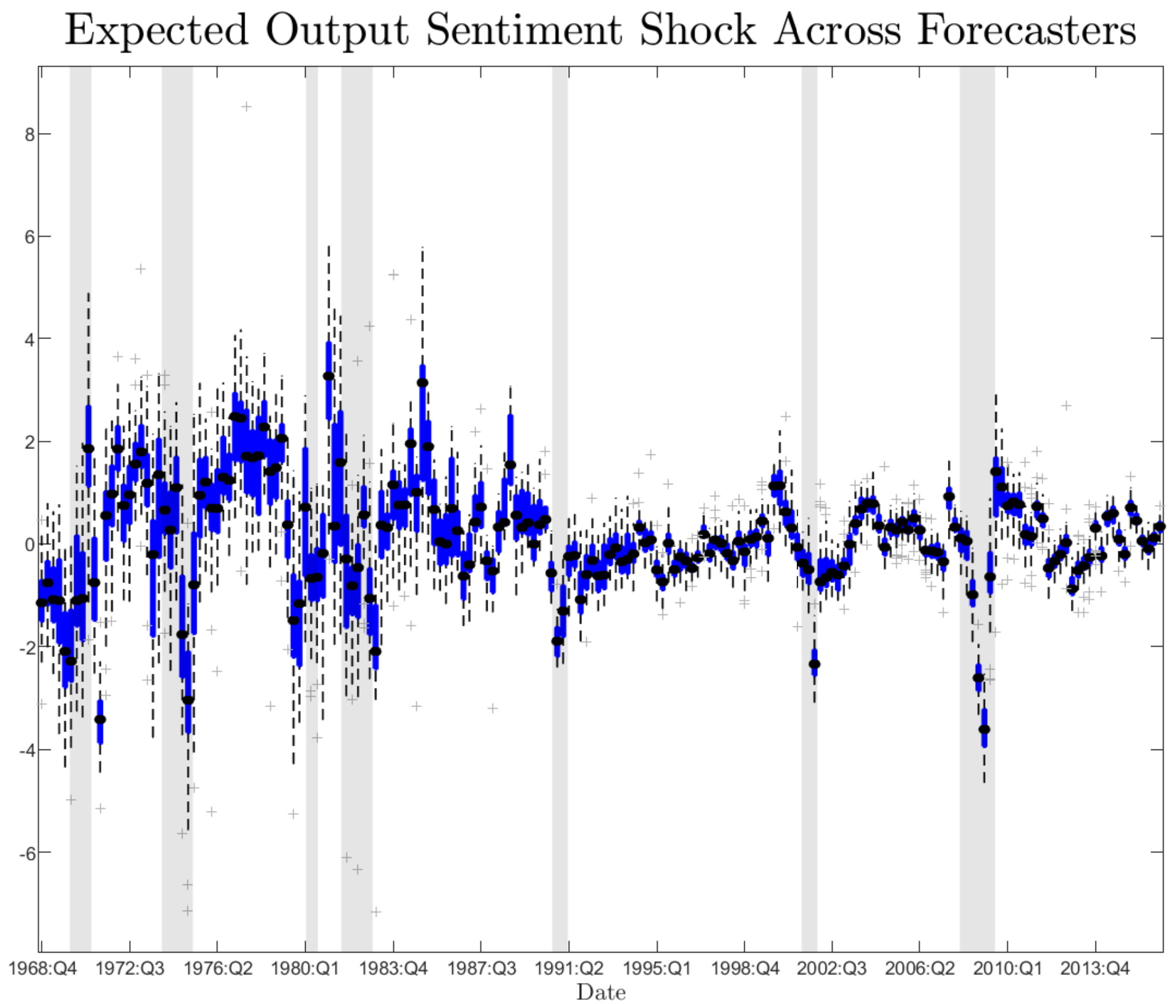

Figure 11: Expected Output Sentiment Shock of Individual Forecasters Each Time Period

Note: The distribution of expected output growth sentiment shock of forecasters each time period is represented with a boxplot. The black circle signifies the median, the edges of the blue box the 25$^{\text{th}}$ and 75$^{\text{th}}$ percentiles, the black whiskers the extreme values not considered outliers, and the grey '+' symbol outliers.

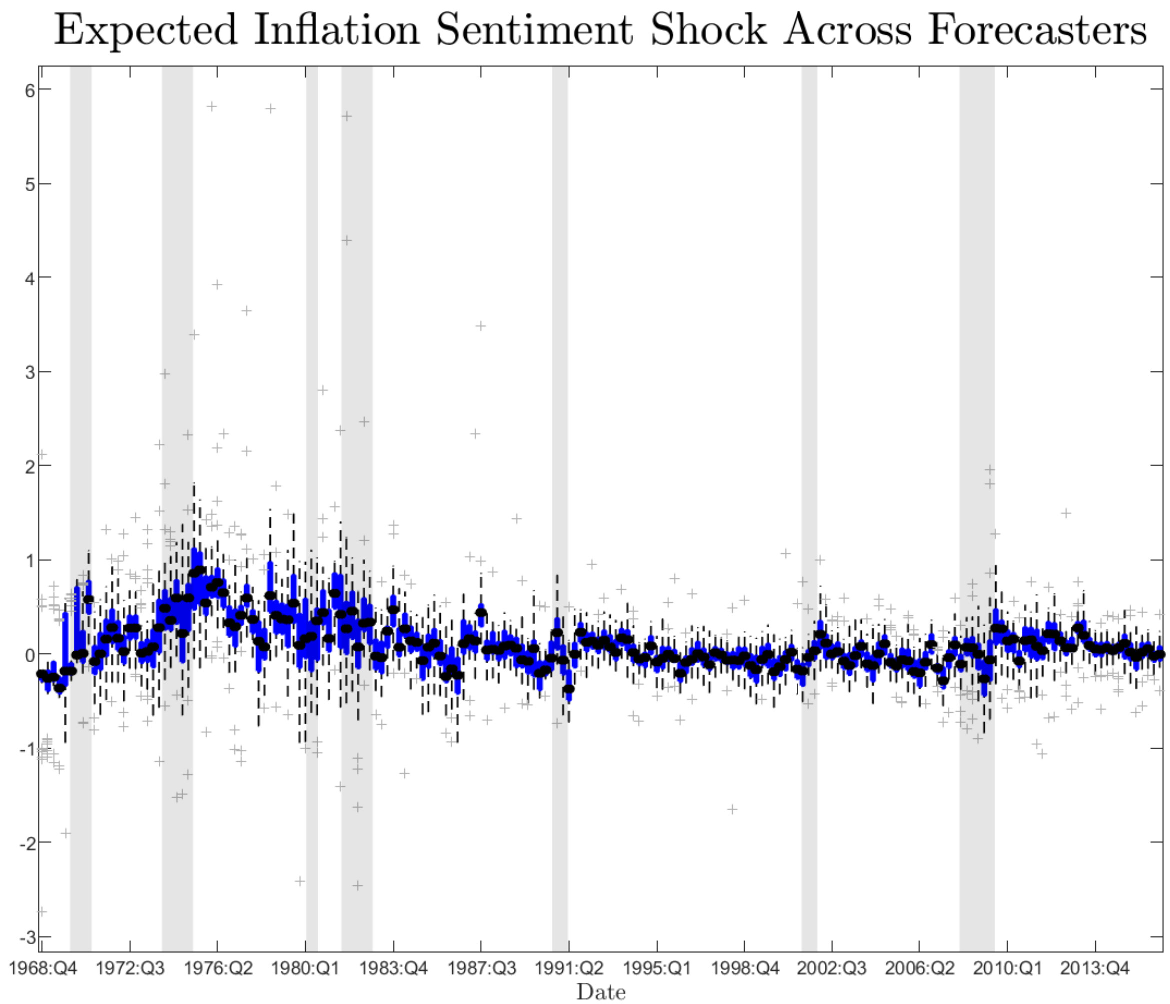

Figure 12: Expected Inflation Sentiment Shock of Individual Forecasters Each Time Period

Note: The distribution of expected inflation sentiment shock of forecasters each time period is represented with a boxplot. The black circle signifies the median, the edges of the blue box the 25$^{\text{th}}$ and 75$^{\text{th}}$ percentiles, the black whiskers the extreme values not considered outliers, and the grey '+' symbol outliers.

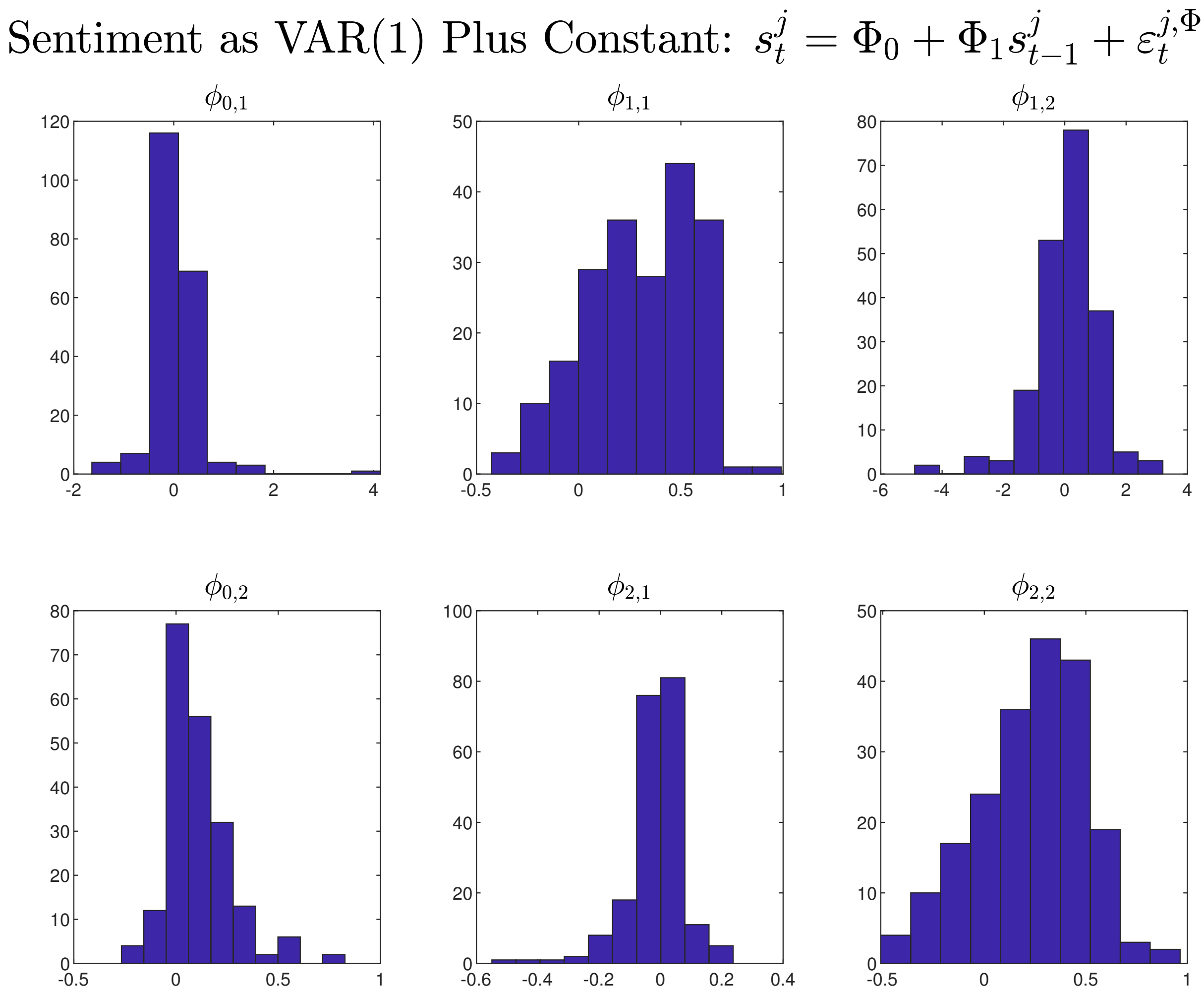

Figure 13: Histogram of Estimated Parameters Across Forecasters If Sentiment Shocks Evolve as a VAR(1) Plus Constant

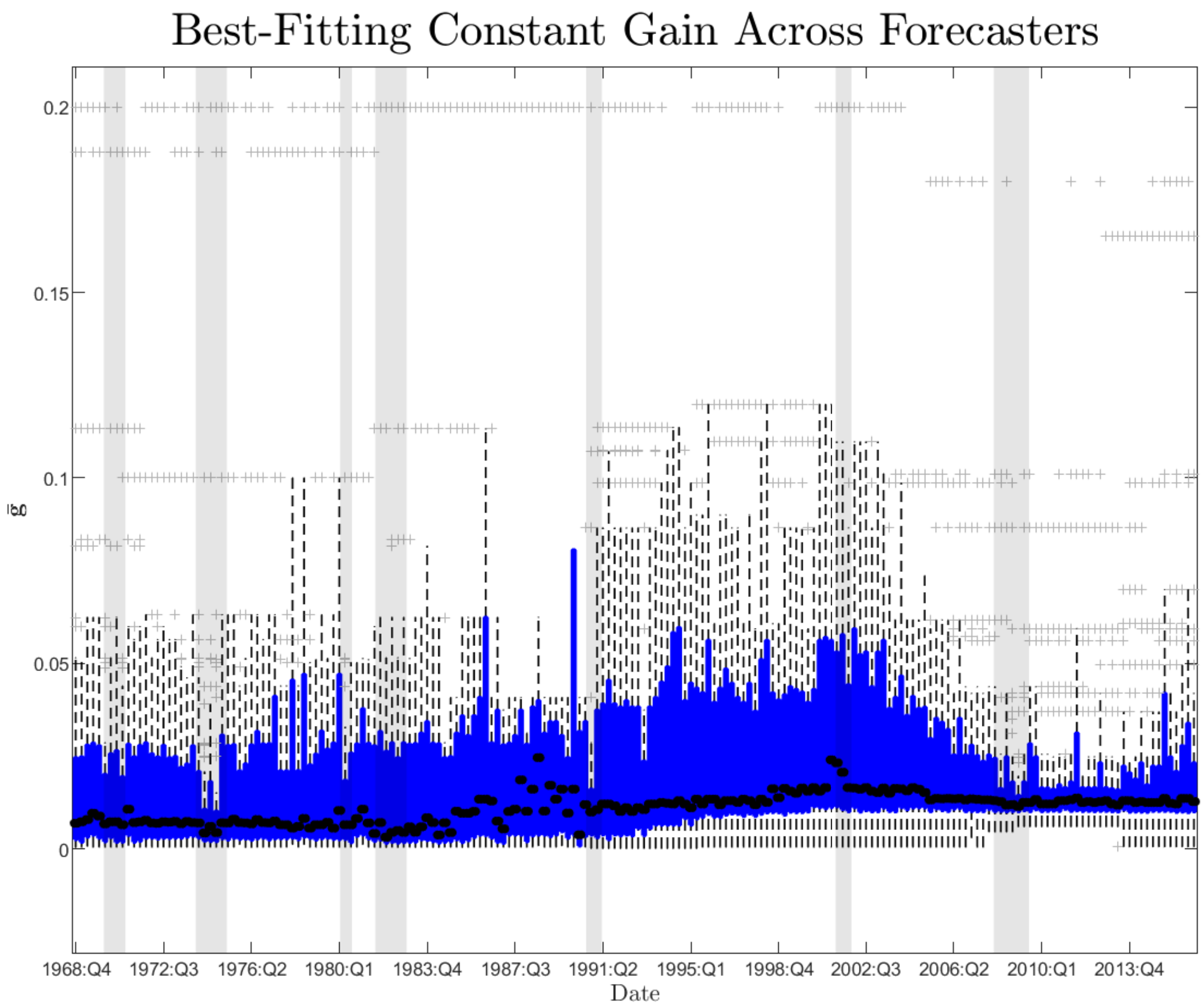

Figure 14: Distribution of the Best-Fitting Constant Gain of Individual Forecasters Each Time Period Under a VAR(1) Plus Constant PLM

Note: The distribution of the best-fitting constant gain of forecasters each time period is represented with a boxplot. The black circle signifies the median, the edges of the blue box the 25th and 75th percentiles, the black whiskers the extreme values not considered outliers, and the grey '+' symbol outliers.

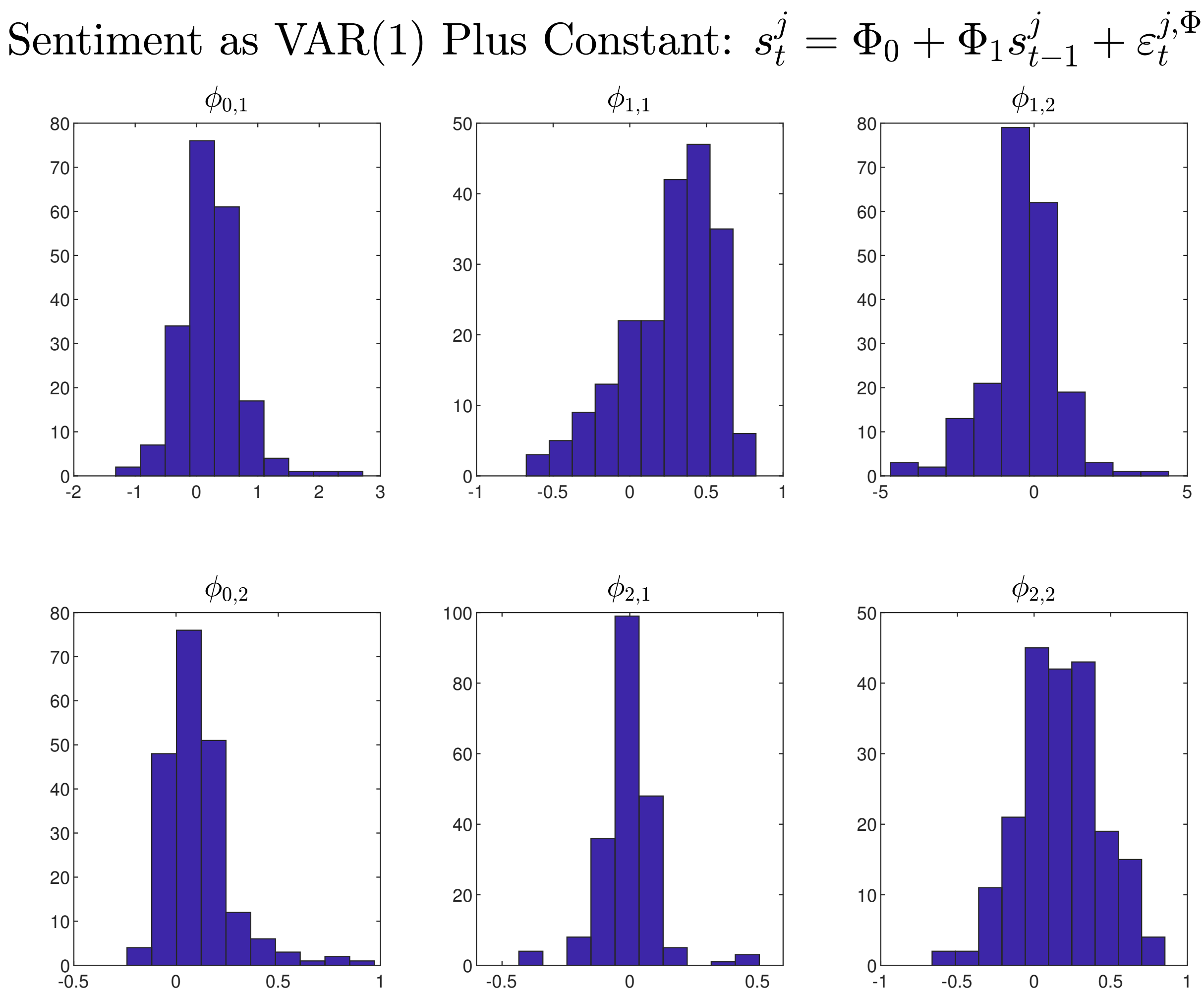

Figure 15: Histogram of Estimated Parameters Across Forecasters If Sentiment Shocks Evolve as a VAR(1) Plus Constant

Note: PLM specified as VAR(1) plus constant.

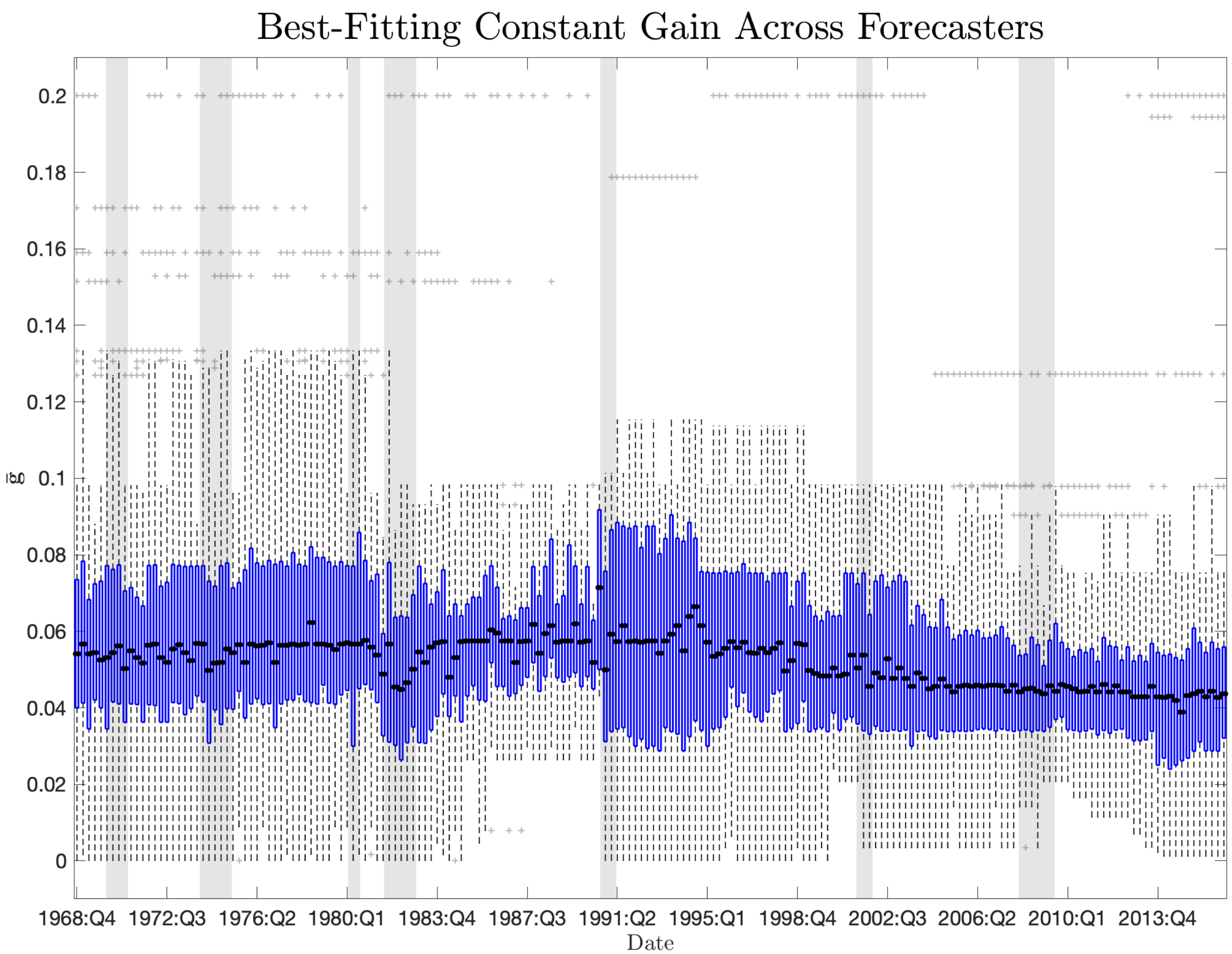

Figure 16: Distribution of the Best-Fitting Constant Gain of Individual Forecasters Each Time Period Under an Alternative Initialization of R_τ Without Rescaling

Note: The distribution of the best-fitting constant gain of forecasters each time period is represented with a boxplot. The black circle signifies the median, the edges of the blue box the 25$^{\text{th}}$ and 75$^{\text{th}}$ percentiles, the black whiskers the extreme values not considered outliers, and the grey '+' symbol outliers.

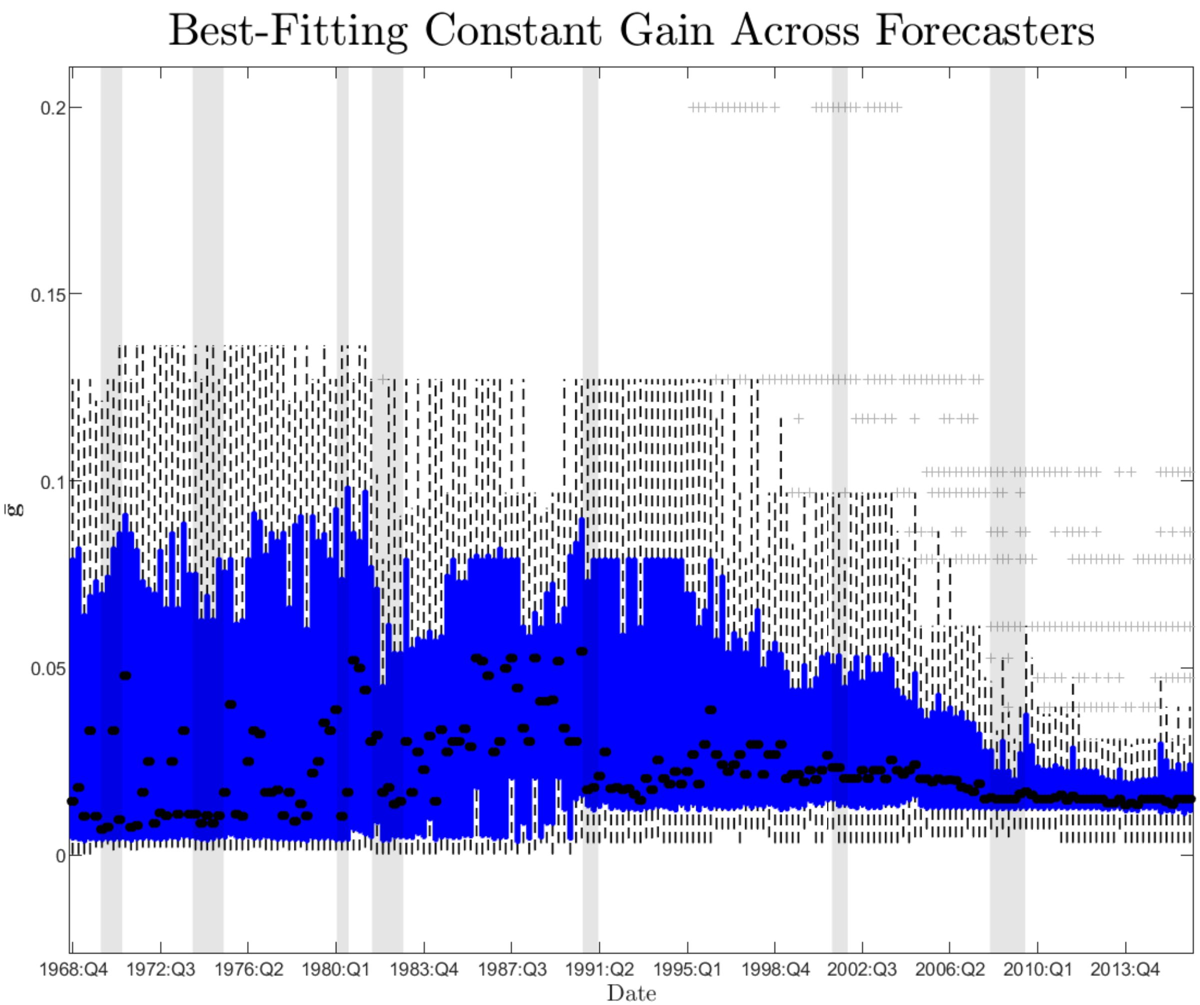

Figure 17: Distribution of the Best-Fitting Constant Gain of Individual Forecasters Each Time Period When SPF Respondents Submit Forecasts for at Least 20 Periods

Note: The distribution of the best-fitting constant gain of forecasters each time period is represented with a boxplot. The black circle signifies the median, the edges of the blue box the 25$^{\text{th}}$ and 75$^{\text{th}}$ percentiles, the black whiskers the extreme values not considered outliers, and the grey '+' symbol outliers.