

Expectational Data in DSGE Models

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Abstract

This chapter surveys the literature that exploits data on expectations, typically from surveys, in the estimation of macroeconomic DSGE models.

Expectational data can be used to test whether the model-implied expectations, obtained under the assumption of rational expectations, are consistent with the empirical evidence. They can point to sources of misspecification in the model and shed light on the importance of alternative theoretical mechanisms, assumptions, and frictions.

Moreover, data on expectations are increasingly used in behavioral models that consider departures from rational expectations. Survey data can help put discipline on the behavioral features that are added to the model, and help discern those that are more successful in simultaneously explaining macroeconomic expectations and realizations.

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1 Introduction

One of the main developments in the field of macroeconomics in recent years has been its increased reliance on empirical work to guide and evaluate theories. Structural models, built from microfoundations, are now routinely and meticulously tested against the data: if necessary, the models are revised to include additional frictions or mechanisms that can improve their fit. State-of-the-art macroeconomic DGSE models, as those built on the works by Christiano et al. (2005) and Smets and Wouters (2003, 2007), can successfully match a large set of observable time series. This approach marks a clear shift with respect to previous decades in which model evaluation was based on a more indirect strategy: first, the models were calibrated using some accepted parameter values, and, then, a range of moments from simulated series were informally compared to those obtained in the data.

In both theoretical and empirical work in macroeconomics, expectations play a central role. Optimizing decisions by households and firms are, in large part, influenced by expectations about future economic conditions, future inflation rates, and future policies. In the vast majority of cases, DSGE models are estimated under the assumption that all economic agents' expectations are formed according to the rational expectations hypothesis. Economic agents in the model form expectations that are, therefore, model-consistent, being formed from a mathematical conditional expectation based on the same model that is assumed to be generating the data, and using all the available information.

When estimating the models, researchers conventionally use data on realized macroeconomic variables (real output, consumption, investment, inflation, interest rates, and so forth). But it has been far less common to use direct data on expectations, which can be obtained from surveys, or extracted from market prices. Expectations are treated, instead, more dogmatically, as they are simply assumed to be equal to the values implied by rational expectations.

Many mechanisms of interest in the model, however, heavily rely on the response of expectations. Monetary policy acts largely through its effects on expectations: this channel has become even more important with the shift of central banks toward transparency and increased communication. Consumers' and firms' decisions are driven by expectations of future real interest rates, future inflation rates, and marginal costs. The effects of fiscal policy depend on the anticipated responses of wages and interest rates. Any misspecification in the modeling of expectations would, therefore,

severely taint the conclusions that can be obtained from a model.

Given how widespread the assumption of rational expectations has been, it is natural to ask whether rational-expectations DSGE models produce expectation series that can accurately approximate observed expectations from surveys. One way to answer the question is to directly incorporate information from survey expectations in the estimation of DSGE models. Data on expectations should be used not only as a means of external validation; rather, they should be routinely included as observable variables that estimations have to match. The importance of incorporating survey expectations has been similarly emphasized in Coibion, Gorodnichenko, and Kamdar (2018).

First, expectational data can be useful to discriminate among competing theories, to evaluate alternative assumptions, and to test the importance of different frictions. When responses to shocks in the model hinge on behaviors of expectations that are found to be unrealistic in the data, the models can be revised accordingly.

Observed expectations can also provide useful information about latent states, which are unobservable for the econometrician, yet play a key role in many economic theories. For example, expectations, particularly for long-horizon variables, can be used to infer economic agents' perceptions about low-frequency movements in the economy (such as movements in the inflation target, in the long-term growth rate of the economy, or in the natural rate of interest).

In this chapter, I discuss the research to date that uses expectational data in DSGE models. First, I present models that retain the assumption of rational expectations. Later, I move to research that examines deviations from rational expectations. In the latter case, observed expectations can be particularly valuable to put structure on the forms of non-rationality that should be inserted in the models.

2 Expectational Data in Rational Expectations DSGE Models

2.1 Do DSGE Models Generate Expectations that Fit Observed Data?

A major step forward in the empirical macroeconomics literature in recent years has been to show that general equilibrium models, based on microfoundations, and including various frictions and a menu of different shocks, can fit macroeconomic data well, often outperforming reduced-form alternatives such as VARs and Bayesian VARs.

The DSGE models under rational expectations that have been built on the frameworks of Christiano, Eichenbaum, and Evans, (2005), Smets and Wouters (2003, 2007), have been particularly successful. They match very closely the responses of endogenous variables to identified monetary policy shocks, as obtained from structural VARs. Moreover, they are able to capture a variety of empirical moments, leading to auto-correlation and cross-correlation profiles that track closely those in the data.

To facilitate their analysis and estimation, log-linearized DSGE models are typically cast in state-space form. The state transition equation is then given by the model solution under rational expectations:

$$\xi_t = F(\theta)\xi_{t-1} + G(\theta)\epsilon_t, \quad (1)$$

where the vector $\xi_t = [Y_t, E_t Y_{t+1}, w_t]'$ includes the model's endogenous variables Y_t (some observed and some potentially unobserved), the corresponding expectations terms $E_t Y_{t+1}$, and the exogenous disturbances w_t , the vector ϵ_t contains the exogenous *i.i.d.* innovations, and $F(\theta)$, $G(\theta)$, are coefficient matrices of appropriate dimensions, which are functions of the vector of structural parameters θ . The variables ξ_t in the model are then linked to the available data series through a set of observation equations, given by

$$Y_t^{obs} = H_0 + H\xi_t (+ o_t). \quad (2)$$

The observation equations typically select a subset of the endogenous state variables in Y_t , through the selection matrix H , and matches them to a vector of observables Y_t^{obs} ; the vector H_0 can contain, instead, steady-state values. Observation equations in the DSGE literature usually abstract from measurement error terms, but those can be easily added through o_t if necessary. Under the assumption that the exogenous innovations ϵ_t are Normally distributed, the state-space system (1)-(2) is both linear and Gaussian, and its likelihood function can be conveniently evaluated using the Kalman filter.

DSGE models are often estimated using sets of observables in Y_t^{obs} that include the growth rates of real output, consumption, investment, wages, levels of hours worked, inflation, and nominal interest rates. In larger models, the set of observables is expanded to include financial variables, such as credit spreads and stock price indexes, labor variables, as the unemployment rate, fiscal variables, related to government debt, deficits, and taxes, and many others. Expectations, on

the other hand, are almost universally modeled as rational, that is, $E_t Y_{t+1}$ simply equals the implied rational expectations value. Unlike other parts of the model, they are treated dogmatically and typically not required to conform to available time series observations, even though those are potentially available and easy to incorporate in the estimation. This would simply require extending the set of observation equations to include observed expectations, denoted by $E_t^{obs} Y_{t+1}$, as follows:

$$\begin{bmatrix} Y_t^{obs} \\ E_t^{obs} Y_{t+1} \end{bmatrix} = H_0 + H \xi_t (+ o_t). \quad (3)$$

For example, inflation expectations $E_t \pi_{t+1}$ in the model can be related to observations on survey inflation expectations $E_t^{obs} \pi_{t+1}$, through the relation $E_t^{obs} \pi_{t+1} = \pi^* + E_t \pi_{t+1} (+ o_t)$, where π^* can denote the value of inflation in steady-state. The addition of *i.i.d.* measurement error recognizes that survey expectations can be noisy measures of actual expectations in the economy. In lieu of measurement error, researchers can alternatively add a new structural shock to the original model, such as a time-varying inflation target. A potential pitfall, however, is that the shock may not be fully structural, since it can spuriously embody deviations between survey and model-consistent expectations.

Recently, a number of papers are making efforts to move beyond the neglect of expectational data in the literature, as they start to explicitly test whether model-implied expectations are consistent with observations from surveys.

Most of the interest in the literature so far has, perhaps unsurprisingly, fallen on inflation expectations. Survey inflation expectations were already used in the context of single-equation equation estimations in the past. Roberts (1997), Brissimis and Magginas (2008), and Adam and Padula (2011), for example, use survey forecasts as proxies for the expectation term in the New Keynesian Phillips curve, and they find that the equation with this modification can fit inflation dynamics well. More recently, Coibion and Gorodnichenko (2015a) use an expectations-augmented Phillips curve, with household inflation expectations from the Michigan Survey, and show that this specification can account for the missing drop in inflation in 2009 and beyond.

Moving to DSGE settings, Del Negro and Eusepi (2011) include (one-year ahead) inflation expectations as an observable in the estimation of the New Keynesian model, in addition to conventional variables related to output growth, hours worked, labor share, inflation, and the interest rate. They consider different variants of the model, depending on whether agents know or have to learn about the inflation target.

First, they find that, without expectations data, it is hard to disentangle the perfect versus imperfect information versions of their model. The four-quarter-ahead expected inflation series generated from the different model specifications remain all far from the median four-quarter-ahead survey forecast. The survey indicates higher expected inflation in the 1980s until the early 1990s (with model-implied expectations that are sometimes half of the survey value), and much lower expected inflation than the model during the dot-com boom in the second half of the 1990s. The correlations between survey and model-implied expectations are modest, ranging around 0.25 and less. With the addition of inflation expectations, the new information is, instead, useful to discriminate among the different specifications.

Moreover, the use of inflation expectations matters also for inference regarding latent variables. When information on expectations is omitted, the estimates for the time-varying inflation target look almost identical between the perfect and imperfect information cases. When expectations are included, the inferred targets are different: under perfect information, the target estimates increase toward the values reached by inflation expectations. The model is unable to match the new observable; therefore, the unobserved variable moves to capture the unexplained difference. Overall, it's possible to conclude that none of the model versions are able to capture the dynamics of observed inflation expectations. That suggests a misspecification that needs to be resolved in the model. Inflation expectations contain extra information, not present in realized series, that can be exploited to revise some of the modeling assumptions.

An earlier work by Schorfheide (2005) also compares inflation expectations from two versions of the New Keynesian model (full information and learning about the monetary policy regime, which can be subject to switches in the inflation target) with survey estimates for average one-year-ahead and ten-year-ahead inflation expectations. Those are not included as observables in the estimation, though, but only used as external validation for the model predictions, an approach that has been probably more common in the literature to date. The inflation expectation series generated from different versions of the model resemble each other, but are not always close to the survey measures. The model-implied short-run expectations are more volatile than the corresponding survey expectations, whereas model-implied long-run expectations are even more sluggish than the survey forecast.

Data on inflation expectations are also used in a paper by Del Negro, Giannoni, and Schorfheide (2015), which aims to analyze the performance of a DSGE model with financial frictions around the

Great Recession and shed light on the behavior of inflation during this period. Many interpret the missing deflation in those years as a failure of models based on the Phillips curve. Del Negro et al. use ten-year inflation expectations from the Blue Chip Economic Indicators survey and from the Survey of Professional Forecasters as observables in the estimation, to better capture low-frequency movements in inflation. The behavior of inflation predicted by the model is reasonable, although it misses higher-frequency movements as the deflation in 2009. They don't include, however, a broader comparison between the implied rational expectation series and the survey evidence. A larger set of forecasts is analyzed in Del Negro and Schorfheide (2013). Their main focus is not on whether the model can generate expectations that match those from surveys. Instead, they show that adding external information, incorporating survey forecasts by professional forecasters for example, into conventional DSGE models, can significantly improve their out-of-sample forecasting performance. The standard set of observables in the estimation is augmented to include long-run inflation expectations (the average CPI inflation rate expected over the next ten years, obtained using data from both the Blue Chip Economic Indicators survey and the Survey of Professional Forecasters), long-run output growth expectations (expected ten-year average, from the Blue Chip Economic Indicators survey), and k -quarters ahead Federal funds rate forecasts (from the Blue Chip Financial Forecasts survey), as well as a number of nowcasts. To avoid singularity, the model requires additional structural shocks that can account for the discrepancies between expectations in the model and the data: here, this is achieved by adding time-varying inflation target shocks, technology growth shocks, and anticipated monetary policy shocks. The strong forecasting performance of survey forecasts is already well known in the literature (e.g., Bekaert, Cho, and Moreno, 2006, Ang, Bekaert and Wei, 2007). Adding expectational data can help DSGE models benefit from their informational advantages. The results in the paper indicate that long-run survey inflation forecasts are particularly useful in improving forecasting accuracy, while output growth forecasts are less effective. The forecasting performance is also analyzed in Monti (2010), who presents an alternative way to combine judgmental forecasts from surveys with model-based forecasts: the survey forecasts can be seen as an optimal estimate of a variable, which is possibly produced using a different, richer, information set than the one available in the model.

The role of expectations data in enhancing the models' forecasting performance is not limited to the academic literature, but it has proven valuable to policymaking institutions as well. The Federal Reserve Bank of New York's DSGE model (see Del Negro et al., 2013), for example, is

estimated using Bayesian methods and including data on ten-year-horizon survey inflation expectations and on future Federal Funds rate expectations based on Overnight Index Swap (OIS) rates. Model-based forecasts are routinely disseminated and compared with professional forecasts. Therefore, innovations in the theoretical modeling of expectations, tested against the data, can now rapidly spill over into policy models used by central banks. They can prove useful for forecasting applications and help policymakers more accurately simulate the economy's response to different policy paths.

Other papers explore the option of directly replacing rational expectations with expectations measured from surveys (without adding expectations to the observation equation). Hence, expectations are no longer model-consistent. Instead, they are assumed to equate the values from the survey at each point in time (with the measured value $E_t^{meas}Y_{t+1}$ entering the model equations directly, in place of RE). This approach, therefore, generalizes similar exercises in a single-equation context (e.g., Roberts, 1997, Adam and Padula, 2011) to a system setting, and it is used, for example, in Paloviita (2007). In some cases, expectations are allowed to be a weighted average of the survey forecast and the rational expectation (Kortelainen et al., 2016, Nunes, 2010). It would be an important drawback, however, to assume that survey expectations can be simply taken as exogenous. Fuhrer (2017) employs a semi-structural model and, in fact, endogenizes expectations by iterating the model equations forward, and replacing infinite sequences of expected terms with long-run expectations from the Survey of Professional Forecasters. The system estimation reveals that the use of direct expectations reduces the need for lagged endogenous variables in the model equations. Inertia hence arises from the sluggishness of expectations, not from intrinsic features of the economy. The paper also performs a horse race between the rational expectations and survey expectations versions. The results are stark, with rational expectations receiving a weight that is not significantly different from zero and survey expectations receiving a weight of one. Interestingly, the estimates show a significant degree of partial adjustment of survey expectations compared to rational expectations: this seems to indicate that individual forecasters adjust their expectations to the mean value of forecasters' expectations in the previous period, as also documented in Fuhrer (2015).

Finally, the information from expectational data is of obvious importance also in the macro-finance literature. De Graeve, Emiris, and Wouters (2009) consider a macro-finance DSGE model, which includes information on the term structure of interest rates. The predictions for inflation from

the model are compared, again as an external validation tool, with survey inflation expectations. As some of the previous papers, they conclude that an inflation target shock is essential to match observed expectations, as well as longer-term yields. Expectations in relation to the term structure of interest rates are also discussed in Chapter 17, *The Term Structure of Expectations*.

2.2 Survey Expectations to Evaluate Alternative Frictions

DSGE models assume optimizing behavior by agents, who also have perfect information about their environment, and form rational expectations. In their stripped-down versions, the models have trouble fitting the data because they imply exceedingly rapid adjustments to shocks, which are at odds with real-world observations. Therefore, researchers need to introduce various frictions to generate persistence: habit formation in consumption, adjustment costs in investment, indexation to past inflation in price setting, and so forth. Using only a subset of available data, it is difficult to identify whether agents' decisions may be due to alternative preferences, frictions, or to beliefs. Data on expectations can be used to reject assumptions that are imposed about expectations, and to better identify whether empirical observations can be attributed to specific preferences or expectation formation models. The role of data on expectations has been recognized for some time also outside the macroeconomic literature, see for example Manski (2004).

We have already discussed the works by Del Negro and Eusepi (2011) and Schorfheide (2005), who use inflation expectations to compare model versions with perfect or imperfect information (about the inflation target). A number of papers exploit additional information contained in survey expectations to test a number of alternative, more pervasive, frictions, spanning from monetary frictions, to dispersed information, learning, inattention, and ambiguity aversion.

Aruoba and Schorfheide (2011) develop a DSGE model that allows them to merge monetary frictions (based on search theory models) and New Keynesian frictions, such as price stickiness, and to measure their relative importance. The model has a two-sector structure and is composed by a decentralized and a centralized market. In the decentralized market, agents meet and engage in bilateral trades; the double coincidence problem and their anonymity create a motive for holding money and using it as a medium of exchange in equilibrium. When agents leave the decentralized market, they move to a centralized market, which resembles a standard New Keynesian model, with price rigidities à la Calvo. The authors treat the inflation target in the model as observable and assume that it can be interpreted as the agents' long-run inflation expectations. They combine

three different inflation measures: bandpass-filtered GDP deflator, one-year- and ten-year-ahead inflation expectations; a common factor across the series is extracted using the Kalman filter. The target joins output, inflation, the interest rate, and the velocity of money, as variables to be matched in the estimation. The data used to identify the inflation target play an important role, since the paper studies, among other things, the effects of changes in the target on welfare. The results indicate that for some target inflation rates, distortions caused by monetary frictions may be as important as distortions due to price stickiness.

Melosi (2017) develops a DSGE model with dispersed information among price-setting firms. Firms have noisy private signals about aggregate conditions. The model includes a signalling channel of monetary policy: a change in the policy rate signals to the private sector the views that the central bank holds on the state of the economy. For example, an increase in the short-term rate might signal that the central bank has knowledge about higher than expected inflation; therefore, the information may influence private sector's beliefs and it may counteract the contractionary effects of the initial policy decision. Melosi estimates the model using a Bayesian approach. The observables include data on one-quarter-ahead and four-quarters-ahead inflation expectations from the SPF, along with the real-time output gap, and real-time inflation from the Federal Reserve's Greenbook. As benchmark, Melosi considers a monetary VAR. The VAR impulse responses show that inflation expectations barely respond on impact after a monetary policy shock, while actual inflation adjusts more rapidly. The resulting inflation forecast errors are, therefore, very persistent and can last for around five years, meaning that inflation expectations can remain unanchored for a long time. Turning to the DSGE model, the paper shows that conventional models with perfect information fail to match these facts: the conditional forecast errors are zero by construction. The Smets and Wouters model, under almost any possible parameterization, doesn't come close to matching the response of inflation expectations to a monetary shock that is found in the VAR. The model with dispersed information and signaling, instead, matches the responses of inflation expectations and forecast errors quite well. Moreover, the model matches much more closely the run-up of inflation rates in the 1970s.

Nimark (2014), instead, investigates the business cycle implications of “man-bites-dog” news, that is, signals that are likely to be observed in correspondence of extraordinary, tail, events. He estimates the model using the entire cross-section of individual survey responses from the SPF, in addition to standard macro variables. The cross-sectional dispersion can help identify man-bites-

dog episodes in the sample. He finds that the model with man-bites-dog events fits the data better than a baseline specification that omits public signals. These papers well exemplify the importance of incorporating information from expectations: empirical evidence on the response of aggregate or individual expectations, and of the corresponding forecast errors, to shocks is used to help decide among alternative theoretical frameworks.

Milani (2017) estimates a model that includes both econometric learning by economic agents and endogenous sources of persistence, such as habit formation, inflation and wage indexation, adjustment costs in investments, as well as a number of serially-correlated disturbances. Data on expectations are used in the estimation to evaluate, among other things, whether macroeconomic persistence is driven by endogenous features, or by the sluggishness of agents' beliefs. The results suggest that learning can match survey expectations closely. Moreover, the empirical importance of the structural sources of persistence and the degree of autocorrelation that are estimated for structural disturbances are considerably reduced under learning. The paper by Fuhrer (2017) discussed in the previous section points in the same direction: replacing rational expectations with survey expectations reduces the need for lagged variables in the model equations and simplifies the properties of exogenous disturbances.

Acuña Armenta (2021) integrates survey forecasts in the estimation of a model that merges different expectational frictions: non-rational expectations based on learning models and sticky information. She finds that both frictions improve the model's fit to the data compared with the rational expectations version. However, the estimated level of information stickiness is sensitive to the modeling of expectations: it is substantially reduced when learning replaces rational expectations. Hajdini (2020) tests an expectation formation process based on misspecified forecasting rules and myopia against rational expectations. The model's testable implications are evaluated using survey inflation expectations. Again, departures from rational expectations improve fit and reduce the need for real rigidities as sources of persistence.

Survey data can also be used to evaluate alternative preference structures. The literature on ambiguity, for example, implies a sizable departure from conventional assumptions about expected utility preferences in macroeconomic models. They assume households, who are averse to ambiguity, or Knightian uncertainty. Agents behave as if they maximize utility under worst-case beliefs. A number of papers in this literature (that will be discussed in detail in Chapter 24, *Ambiguity and Uncertainty*) exploit survey expectations, including measures of their dispersion, to discipline

agents' beliefs and identify confidence or ambiguity shocks (Ilut and Schneider, 2014, Ilut and Saijo, 2021, Bianchi et al., 2018, Bhandari et al., 2016, Rossi et al., 2016).

2.3 Survey Expectations & News Shocks

Beaudry and Portier (2006) estimate a bivariate SVAR that includes a measure of total factor productivity and a stock price index, as an example of a variable that contains forward-looking information about the economy. They consider two identification schemes: one that imposes a short-run restriction, often used to identify demand shocks, and one that imposes long-run restrictions, often used to disentangle demand versus supply, or technology, shocks. Their results reveal a correlation close to one between the two shocks identified under the different strategies. This finding motivates them to offer a news-based interpretation: one shock is a conventional technology shock, with long-run effects, while the other represents news shocks about future technology. The news is reflected into stock prices, before it can materialize into actual TFP. Barsky and Sims (2011) employ a different strategy to identify news shocks about future productivity. The news shock is assumed to be orthogonal to the TFP innovation, and it is obtained as the shock that can explain the largest share of the TFP's forecast error variance. The resulting news shock is responsible for a significant fraction of medium-term output fluctuations. Barsky and Sims (2012), instead, test whether innovations in consumer confidence reflect the action of "animal spirits" or news about future productivity. They conclude that the relationship between confidence and economic activity can be mostly attributed to news.

Similar ideas are inserted in DSGE settings, which imply that news about the future can cause macroeconomic fluctuations. In the earlier papers (Beaudry and Portier, 2004, Jaimovich and Rebelo, 2009), news relates to future technology and is embedded in neoclassical models. The approach has later expanded to consider news about a wide variety of disturbances besides technology, and within different modeling frameworks.¹

In the DSGE literature, fluctuations are usually attributed to shocks that are entirely unanticipated by economic agents. The news view literature extends the shock structure to include both anticipated, the "news", and unanticipated, or surprise, components. Anticipated components are indeed pervasive in the economy. They can capture monetary policy announcements, which have

¹Chapter 23, *Expectations and Incomplete Markets*, covers the response of the economy to news shocks in HANK set-ups with search and matching in the labor market.

become progressively more important with the growing reliance on communication and forward guidance by central banks. They are also essential to understand the impact of fiscal policies: tax changes are legislated, announced, and incorporated into private-sector's expectations many quarters before they become official. News may also incorporate expectations by economic agents that may or may not materialize later on: for example, anticipations about future productivity, or demand, improvements may be immediately expansionary, but become recessionary in the future when the actual improvement is smaller than expected.

Therefore, the news literature assigns a central role to shifts in expectations, possibly due to optimism and pessimism, while keeping expectations rooted in the rational expectations hypothesis.

The paper by Schmitt-Grohe and Uribe (2012) estimates a DSGE model with real rigidities and news shocks, but they don't use expectational data. The standard deviations of news shocks are well identified, in their case, even if the number of shocks far exceeds the number of observable variables. One of the reasons is that they use extra variables that provide information on the disturbances themselves: their observable list includes the relative price of investment, which in the model is related to the inverse of the investment-specific technology shock, and total factor productivity, which corresponds to the neutral technology shock in the model. Their exercise consists in comparing the shares of output fluctuations that are due to news versus unanticipated shocks. They find that news shocks can explain almost half of business cycle fluctuations, but news about technology is quite unimportant. Sims (2016) delves deeper into the results to separate news about future fundamentals that has yet to materialize and news that has already materialized in the past. He finds that the quantitative importance of news is mostly due to the latter.

In general, however, the results on the empirical contribution of news shocks in the papers that estimate DSGE models without expectational data remain mixed. Fujiwara, Hirose, and Shintani (2011) find that technology news explains less than 10% of output fluctuations, Khan and Tsoukalas (2012) find that news shocks explain less than 15%, with non-technology news playing a role, but technology news accounting for an almost nil share. These papers differ in their model choice from Schmitt-Grohe and Uribe (2012), as they use a sticky-price/sticky-wage framework.²

The identification of news in a microfounded model is entirely driven by its impact on forward-looking expectations, which, in turn influences economic agents' optimal choices of consumption,

²Milani and Treadwell (2012) also estimate a New Keynesian model with news, and focus on the importance of surprise versus news monetary policy shocks; Gomes et al. (2017) also document the importance of monetary policy news shocks in a medium-scale model. Born, Peter, and Pfeifer (2011) focus, instead, on anticipated tax shocks.

investment, and labor supply. Unanticipated shocks are, instead, by construction, unforecastable. While possible in theory, the identification is complicated in practice. Structural shocks are unobserved to the econometrician: with news, these unobserved variables contain multiple terms, some unanticipated, some anticipated, that are all equally unobserved. Disentangling them often relies on a very limited number of observables (with shocks outnumbering observables by many factors). The use of expectation data can, therefore, be crucial for the identification of news shocks.

The papers by Hirose and Kurozumi (2021) and Milani and Rajbhandhari (2020) take this route and add a large set of survey expectations, about different variables and at different horizons, as observables; the number of shocks and observables can now coincide (which is not necessary, but can help identification). Hirose and Kurozumi estimate a small-scale New Keynesian model, using forecasts on output growth, inflation, and the nominal interest rate from the Survey of Professional Forecasters. The paper finds that news shocks about technology represent the main source of U.S. output fluctuations. When expectations data are not used, unanticipated shocks are predominant, mirroring the results in Fujiwara, Hirose, and Shintani (2011) and Khan and Tsoukalas (2012). They show that the model estimated with expectational data is much more successful in replicating the existing cross-correlations between output growth and inflation expectations than the estimation that omits survey expectations. The estimates of news shocks and other structural parameters also become more precise, judging by the tighter posterior probability intervals. Milani and Rajbhandhari (2020) consider, instead, a medium-scale DSGE model, based on Smets and Wouters (2007). To the original observable series, they add information from the term structure of survey expectations, including SPF forecasts about output growth, consumption growth, investment growth, government spending growth, inflation, and interest rates, at horizons ranging from one-quarter ahead to five-quarters ahead. In total, the set of observables has eight realized variables and thirty expectation series. The link between expectation series and expectations in the model is provided by observation equations, which allow for *i.i.d.* measurement error terms. The paper evaluates the strength of identification using the tests proposed by Iskrev (2010). The parameters related to news shocks are shown to be poorly or non-identified when the model is estimated using the same observable variables as in Smets and Wouters, without expectations. In many cases, the priors for news standard deviations are not updated in light of the data, or slightly updated toward zero (Gamma priors with equal mean and standard deviation are used to assign higher probability to zero and progressively lower probability to larger values). Moreover, news plays a

limited role, again as in Fujiwara, Hirose, and Shintani (2011) and Khan and Tsoukalas (2012). When the model is instead estimated by exploiting data on expectations, news shocks are more clearly identified. The estimates of other parameters also change: for example, the importance of real frictions, such as habits and adjustment costs, and nominal frictions, as price and wage stickiness, are reduced in the model with observed expectations and news shocks. The news series thus obtained largely differ from their counterparts that are estimated using only data on realized variables. The identified news shocks explain roughly 40% of business cycle fluctuations, with news about investment-specific technology and risk premium shocks accounting for the largest share.

Miyamoto and Nguyen (2020) also estimate a medium-scale New Keynesian model, augmented with news referring to horizons from one-quarter to eight-quarters ahead. Their empirical results indicate that longer-horizon news plays a more limited role for fluctuations, while short-horizon news shocks are confirmed as an important determinant of business cycles.

All the papers that use expectations data demonstrate that they are helpful in improving the identification of news. Overall, they also suggest that the implied role of news over business cycles increases.

One possible drawback, on the other hand, is that observed expectations may reflect biases that are not reflected in the assumption of rational expectations. In that case, news may spuriously incorporate the effects of these biases, in addition to actual anticipations. Hirose and Kurozumi (2021), however, consider this possibility in a robustness exercise and conclude that, even though the model with biased expectations provides a better fit, the main conclusions remain unchanged.

2.4 Survey Expectations & Sunspots

A long-standing literature in macroeconomics views economic systems as inherently unstable, potentially characterized by multiple equilibria, and subject to sunspot-driven fluctuations. Benhabib and Farmer (1999), and Farmer (2019) review the early and the more recent literature. The first phase in the literature shows how equilibrium indeterminacy can arise in real business cycle models when production functions exhibit increasing returns to scale or external effects, and in New Keynesian models, typically as a result of passive monetary policies, or, in extended environments, of coordination failures between monetary and fiscal policies. In these cases, the models retain the assumptions of market clearing and rational expectations. In the most recent, second phase, the model, instead, is revised to include a belief function, which is modeled separately and represents

a departure from rational expectations. For example, in Farmer and Nicolò (2018), it is assumed that expectations for nominal output growth equal contemporaneous nominal output growth plus a random shock. The literature provides one possible approach to model the idea of “animal spirits” popularized by Keynes in the *General Theory*. When the equilibrium is not determinate, forecast errors are no longer simply a function of fundamental innovations, but they are also influenced by a non-fundamental sunspot shock (the animal spirit). Models with indeterminacy, therefore, often exhibit extra volatility and they can solve some of the propagation and persistence issues that characterize models relying on a unique equilibrium.

The existence of indeterminacy, however, has for a long time rendered full-information estimation of the corresponding models more challenging. Lubik and Schorfheide (2004) developed the techniques to estimate a New Keynesian model, in which parameter draws can be allowed to fall either in regions characterized by determinacy or indeterminacy. They can then test whether particular samples of data (for example, U.S. time series before and after Volcker’s appointment as Federal Reserve’s Chairman) are better explained by a determinate or indeterminate system. In their empirical application, they find that pre-1979 data are consistent with a passive monetary policy and indeterminate equilibria, while post-1982 data are characterized by active monetary policy, which is conducive to a determinate equilibrium. Lubik and Schorfheide’s approach has been used in Benati and Surico (2009), Hirose (2008), Hirose, Kurozumi, and Van Zandweghe (2020), and others. Recently, Farmer, Khramov, and Nicolò (2015) and Bianchi and Nicolò (2021) have provided alternative approaches that can substantially reduce the estimation burden. Farmer and Nicolò (2018), using those techniques, run a horse race of the three-equation New Keynesian model, either closed using a standard Phillips curve or the proposed autonomous belief function. Cuba-Borda and Singh (2019), Ilabaca, Meggiorini, and Milani (2020), Ilabaca and Milani (2021), Dai, Weder, and Zhang (2020) estimate models with determinacy and indeterminacy using Bianchi and Nicolò’s approach.

With its emphasis on forecast errors and on fluctuations in beliefs that can be triggered by sunspots, the indeterminacy literature places the dynamics of expectations decidedly in a central role. So far, however, estimations have resorted to conventional sets of observable variables. It is possible, and probably natural, to start exploiting the extra information contained in observed expectations to inform the estimation of DSGE models with indeterminacy as well.

The study of interactions between expectations and sunspots can be inspired, for instance, by

related work by Canova and Gambetti (2010). They use theoretical restrictions from the New Keynesian model, which they test using expectations data and including them in a VAR. Expectations would be a new state variable under indeterminacy: under the commonly accepted story of a policy switch from indeterminacy to determinacy in the early 1980s, there should be structural breaks in the predictive power of expectations. Canova and Gambetti employ different measures of expectations, obtained from survey data, Greenbook forecasts, and from bond markets. They find that the role of expectations is unchanged over time, casting doubts on the switch from indeterminacy to determinacy narrative. However, in their case, they don't run a full estimation of the New Keynesian model. Here, we are not taking a stand on the indeterminacy versus determinacy debate, but simply argue that there is a clear direction for the literature to use expectational data more regularly to assess the empirical importance of sunspots. Ilabaca et al. (2020) do estimate, in one of their cases, the model with indeterminacy using survey inflation expectations as observable, but fitting expectations is not a main focus of the paper. They find that the evidence supports determinacy rather than indeterminacy even before 1979, in a Behavioral New Keynesian model with myopic agents.

2.5 Misspecification of Expectations

One of the possible uses of expectational data is to uncover misspecification in DSGE models. The DSGE-VAR approach, proposed by Del Negro and Schorfheide (2004), provides a powerful tool to detect where the sources of misspecification lie in a model. DSGE-VARs allow researchers to entertain all models in a continuum spanning between two extremes: the rational expectations DSGE model, with all the resulting cross-equation restrictions, and an unrestricted VAR, where no DSGE restrictions are imposed. A key parameter in this approach is given by λ , which measures the tightness of the DSGE-restrictions prior used in the VAR estimation. The coefficient λ can be estimated: if it's close to zero, it means that DSGE restrictions are better ignored; if it's large, it suggests that DSGE restrictions carry useful information that improve over VARs. In general, the techniques are applied to models that impose rational expectations. Hence, conditional on expectations being formed according to the rational expectations hypothesis, they can reveal whether misspecification is due to the modeling of inflation, consumption, or other variables, by comparing the corresponding impulse responses when the DSGE restrictions are imposed, to those obtained in a best-fitting specification that partially relaxes those restrictions. Discrepancies in the

impulse responses may indicate to researchers that modifications are needed in the Phillips curve, the Euler equation, or in other relationships, for the model to become consistent with the data.

But misspecification may also arise from the expectation formation side of New Keynesian DSGE models. Del Negro and Eusepi (2011) compare the fit for inflation expectations obtained by the DSGE model with the fit from a VAR, following Del Negro and Schorfheide's (2004) DSGE-VAR approach. They find that all model versions are inferior to VARs in capturing the dynamics of inflation expectations: when the tight DSGE cross-equation restrictions are loosened, the fit for inflation expectations gets better.

Cole and Milani (2019) investigate the ability of New Keynesian models to match the dynamic interactions between macroeconomic variables and the corresponding expectations that are observed in the data. The interactions between macroeconomic realizations and expectations represent a central channel for monetary policy transmission, which is largely based on the 'management of expectations'. The paper estimates a DSGE-VAR on output growth, inflation, interest rate, as well as expectations about one-period-ahead real output growth, two-period-ahead real output growth, and one-period-ahead inflation. The results reveal serious misspecification in the modeling of expectations in the New Keynesian model. When the model is estimated under rational expectations, and including expectational data, the posterior estimate for the prior tightness parameter λ declines closer to zero, indicating that the data are more favorable toward the unrestricted VAR version. The impulse responses of the model-implied expectations often show the wrong sign and magnitude compared with the responses of expectation series in the data. The DSGE restrictions achieve, instead, a higher weight when the model with rational expectations is spared the requirement to match the data on expectations. The paper then proposes alternative models of expectation formation that relax rational expectations, including models that resemble agents' laws of motion under learning, or forecasting rules that are heterogeneous and consistent with the evidence from the experimental literature. The latter are based on the results in Hommes (2011) and model expectations by grouping them in three clusters: trend-following expectations, adaptive expectations, and anchor-and-adjustment expectations. The DSGE-VAR estimations show an overall fit that improves under the alternative expectation formation mechanisms: DSGE restrictions are now more valuable, with estimates for λ that increase significantly compared with rational expectations. The fit of the DSGE model, however, still remain far from that of the best-fitting DSGE-VAR specification.

These results suggest that expectational data should be used more frequently to uncover the main sources of misspecification in DSGE models, particularly when they relate to the key interaction between macroeconomic expectations and outcomes. The findings also point researchers to rethink the modeling of expectations, in favor of models that are able to match the available observations. In the next sections, we will discuss papers that take this route by introducing deviations from rational expectations.

3 Expectational Data & Deviations from Rational Expectations

There is a long tradition of research that uses survey expectations to test the rational expectations hypothesis using micro-level data from surveys (see Pesaran and Weale, 2006). In most cases, the tests lead to rejection of rational expectations. Coibion and Gorodnichenko (2012, 2015b), Bordalo et al. (2020), and Angeletos et al. (2020), present ample, updated, evidence documenting the existence of frictions and incomplete information in the formation of expectations. In what follows, we consider papers that depart from rational expectations and use survey expectations in structural models.

3.1 Adaptive Learning

Probably the main alternative to the dominant paradigm of rational expectations has been offered over the years by the literature on adaptive learning in macroeconomics (e.g., Sargent, 1999, Evans and Honkapohja, 2001). Initially, agents' learning was introduced to justify the plausibility of rational expectations: agents were learning during a transition period, but, under some conditions, the system would converge to the same equilibrium that would exist under rational expectations (Lucas, 1986). A large literature subsequently studied the conditions under which convergence to the Rational Expectations Equilibrium could take place, in a variety of model settings. Researchers, however, also started to propose learning as an alternative model of expectation formation, which could replace rational expectations. In an economy potentially subject to large degrees of structural change (due to fundamental factors, such as changes in technology, political institutions, globalization, or due to shifts in monetary policy or exchange rate regimes, and so forth), economic agents may live almost permanently in a period of transition and, as a result, they need to continuously engage in learning about their economic environment.

The first papers to estimate DSGE models with adaptive learning did so by matching the same

observable variables that were used under rational expectations. Milani (2007) estimates a New Keynesian model, augmented to include endogenous sources of persistence, such as habit formation in consumption and price indexation to past inflation. The model with adaptive learning fits the data better than the model with rational expectations does, according to marginal likelihood comparisons. Learning introduces additional inertia in the system and it provides a parsimonious way to account for time-variation in the formation of beliefs. Several other papers insert learning in versions of the benchmark New Keynesian model and estimate them.³ In such estimations, the best-fitting learning process is extracted to allow the model to match as closely as possible the dynamics of realized inflation, output gap, and interest rate. The learning process in these cases is not required to match empirical counterparts for the expectations themselves. Slobodyan and Wouters (2012) extend the analysis to a medium-scale DSGE models based on Smets and Wouters (2003, 2007).⁴ Again, they select similar sets of variables as the original papers as observables.

More recently, the literature has started to incorporate expectational data to impose more discipline in the estimation of the learning process. Previous evidence, based on minimum-distance exercises, already suggested that survey expectations were closely matched by constant-gain learning models. Orphanides and Williams (2005) calibrate constant gains to the values that minimize the distance between the learning expectations and those obtained from the Survey of Professional Forecasters. Branch and Evans (2006) show that constant-gain learning provides the best fit of SPF expectations for output growth and inflation, among a set of competing models, outperforming also Kalman-filter learning.

A new strand of papers estimate DSGE models with adaptive learning, which are now required to match both realized macroeconomic series and observed expectations from surveys. Ormeño and Molnar (2015) start by adding survey data on inflation expectations to estimate the Smets and Wouters' model. They compare its performance under rational expectations and learning. They find that adaptive learning does substantially better than rational expectations in fitting, at the same time, realized macro series and the corresponding survey expectations.

A larger number of expectation series are incorporated in Milani (2011, 2017) to help infer the best-fitting learning process. The first paper assumes a small-scale New Keynesian model, esti-

³For example, Milani (2008, 2009, 2014, 2020), Berardi and Galimberti (2017), Chevillon et al. (2010), Gaus and Ramamurthy (2019), Best (2017), Meggiorini and Milani (2021).

⁴Medium and larger-scale DSGE models with learning are also estimated in Milani (2017), Vázquez and Aguilar (2021), Rychalovska (2016), Bassanin and Maldonado (2021), among others.

mated on observables for output growth, inflation, nominal interest rate, and on the corresponding one-period-ahead expectations for the same three variables. The second extends the analysis to the Smets and Wouters' model: the observable list is augmented to include expectations about consumption, investment, and inflation, from the Survey of Professional Forecasters (mean of expectations across forecasters, about the $t + 1$ value of the variable). Given that the paper tries to identify the learning process of economic agents in real-time, it becomes essential to match as closely as possible the information set that was actually available to agents at each point in time. Therefore, it uses real-time data in the estimation. Real-time data are available through the *Real Time Data Set for Macroeconomists*, provided by the Federal Reserve Bank of Philadelphia. The data consist of different vintages of macroeconomic variables, which capture the information that would have been available to researchers, policymakers, and the public, at each point in the past (see Croushore and Stark, 2001, for more discussion of the data set).

Carvalho et al. (2020) estimate a New Keynesian model using data on inflation and the following short-term inflation expectations: one- and two-quarter ahead CPI inflation forecasts from the SPF, and two measures from the the Livingston Survey, based on the expected CPI inflation six months ahead. The model aims to explain low-frequency movements in inflation expectations and relate them to the short-term forecasting performance. They assume a state-dependent learning rule, which follows Marcet and Nicolini (2003). Even though the model is estimated only using short-term expectations, they show that it can explain long-term survey inflation expectations very well. Those are again explained through an endogenous learning process, rather than resorting to an exogenous target shock. Chapter 17, *The Term Structure of Expectations*, expands on models that can explain the joint dynamics of short-term and long-term forecasts.

Gáti (2020) presents a model in which the degree of expectations unanchoring depends on a continuously time-varying endogenous gain function. She estimates the functional form of the gain using a Simulated Method of Moments approach, targeting the autocovariance properties of a set of observables that include the 12-month ahead CPI inflation forecast from the SPF. Her results, along with those in Carvalho et al. (2020) and Milani (2014), suggest that the sensitivity of expectations to new information, as measured by the value of gain coefficients, increases when forecast errors are larger by historical standards.

Finally, turning to the literature studying issues at the intersection between macro and finance, the paper by Dewachter and Lyrio (2008) integrates information on the term structure of interest

rates, average inflation expectations over the next year, and over the next ten years (as the full term structure of inflation expectations is unavailable) in their observation equations. The microfounded macro-finance model, extended with learning, successfully explains both variations in the yield curve and the dynamics of inflation expectations.

Given the central role that the adaptive learning literature devotes to expectations, we can expect that in the future it will become more popular, and perhaps standard, to use expectational data in empirical work. In addition to comparisons with rational expectations, the data can be fruitfully used to discriminate among different perceived laws of motion that agents might use, and to refine the algorithms used to describe their updating of beliefs.

3.2 Survey Expectations and Sentiment

Many of the papers that include information about expectations and that we have discussed so far adopt a similar approach. They relate expectations obtained from the model to the corresponding survey data, up to some exogenous measurement errors, which need to be introduced in the observation equation to avoid stochastic singularity. The measurement errors do not receive any structural interpretation.

In systems under rational expectations, expectational errors are obtained entirely as a function of fundamental shocks: they can be solved out and removed from the system as independent sources of fluctuations. There is no role for excesses of optimism or pessimism that go beyond the actions of structural innovations.

With the addition of expectational data, however, such assumptions can be relaxed. This permits researchers to reassign a role to psychological factors, which were seen as central in the writings of Pigou (1927), Keynes (1936), and Haberler (1937). When data on expectations are used, expectations in the model should still be treated as endogenous. Under learning, for example, they can be assumed to be formed from a near-rational perceived model of the economy (the Perceived Law of Motion, or PLM), given by:

$$Y_t = a_{t-1} + b_{t-1}Y_{t-1} + c_{t-1}w_t + \varepsilon_t. \quad (4)$$

Agents adopt the PLM to form their forecasts (here shown for a $t + 1$ horizon), as:

$$\widehat{E}_t^{obs} Y_{t+1} = \left(I + \widehat{b}_{t-1} \right) \widehat{a}_{t-1} + \widehat{b}_{t-1}^2 Y_{t-1} + (\widehat{c}_{t-1} \rho + \widehat{b}_{t-1} \widehat{c}_{t-1}) w_t + s_t \quad (5)$$

$$= \widehat{E}_t^{PLM} Y_{t+1} + s_t. \quad (6)$$

Milani (2011, 2017) follows this approach to model sentiment in DSGE models and to investigate the role that sentiment shocks play over the business cycle. The observed expectations are assumed to be formed, on average, as the outcome of the learning model that agents are using. The agents' perceived model is linear and is assumed to have the same structural form of the system solution under rational expectations (that is, it shares the same variables). Expectations are hence formed in reaction to values assumed by past endogenous variables Y_{t-1} (assuming information on those up to $t-1$ rather than t), the contemporaneous realization of exogenous disturbances w_t (if assumed to be observed), with the most recently updated beliefs by agents denoted by \hat{a}_{t-1} , \hat{b}_{t-1} , and \hat{c}_{t-1} (ρ refers instead to the disturbances' persistence parameters). This represents the endogenous component of expectations that arises from the learning model, and can be more compactly written as $\hat{E}_t^{PLM} Y_{t+1}$. However, agents can, in every period, form expectations that deviate from the point forecasts arising from the learning model. It is these deviations of actual, observed, expectations from the expectations that can be rationalized by the learning model that define the "sentiment" terms in the model, denoted above by the vector s_t . Sentiments are meant to capture exogenous waves of undue optimism or undue pessimism, which are not justified based on the state of contemporaneous and past fundamentals. In the estimation, it is the dynamic interactions among observed expectations and realized macroeconomic time series that are exploited to extract sentiment shocks.

The estimation is performed using realized macroeconomic series and survey expectations. Real-time data are used to increase the credibility of the identification of endogenous learning versus exogenous sentiment in each period.

The results show that the expectations from the learning model can closely approximate the survey expectations. Moreover, the empirical evidence shows that sentiment shocks, typically omitted in DSGE models with rational expectations, explain a sizable portion (about half) of U.S. business cycle fluctuations. Different sentiments can be identified: excess optimism/pessimism related to consumption, investment, or inflationary pressures.

The resulting sentiment series are obtained without using any data on consumer or investor confidence, only expectations. The papers, however, show that the identified sentiment is strongly correlated with purified measures (purified by regressing them on a vector of contemporaneous and lagged macro variables) from popular surveys, such as the University of Michigan Consumer Sentiment Index, the Business Confidence Indicator obtained from the OECD's Business Tendency Surveys for Manufacturing, and the Duke Fuqua School of Business' CFO Expectations Index,

which computes the shares of respondents feeling more optimistic or more pessimistic about the U.S. economy. Sentiments, on the other hand, do not appear to spuriously reflect extraneous factors, such as credit spreads, oil prices, total factor productivity, or news shocks.

The approach used in these papers treats the modeling of expectations more symmetrically with respect to the way other variables are treated. In modeling consumption, for example, the dynamics of the variable is governed by the Euler equation; unexplained deviations are captured by exogenous disturbances, which are given a structural interpretation, as preference disturbances, or sometimes disturbances to bonds' risk-premia. Along the same lines, expectations are obtained endogenously based on a model (in this case, a near-rational perceived law of motion with learning), and the deviations are interpreted as an exogenous disturbance, here denoted as 'sentiment'.

The work on sentiment in learning models has points of connection with other recent studies, which have introduced sentiment, although modeled in a variety of ways. For example, in Benhabib et al. (2015), sentiment arises because of imperfect information about expected demand and it can lead to self-fulfilling fluctuations. Angeletos, Collard, and Dellas (2018) introduce a belief shock, interpreted as a shift in confidence, which can emerge as a result of coordination frictions based on Angeletos and La'O (2013). Chapter 22, *Dampening General Equilibrium: Incomplete Information and Bounded Rationality* surveys this line of research in more depth. They discuss specifications that relax the assumption of common knowledge, but maintain rational expectations, and specifications that replace rational expectations with Level-k Thinking. In all cases, they show that general equilibrium effects are attenuated. A key testable implication is that expectations should under-react to shocks, a feature that appears consistent with survey evidence. Various other papers have investigated the impact of confidence in more empirical frameworks (e.g., Barsky and Sims, 2012, Bachmann and Sims, 2012, Enders et al., 2021, Bianchi, Ludvigson and Ma, 2021).

3.3 First Moment vs. Second Moment Shocks

The information contained in observed expectations allows empirical researchers to more credibly measure the transmission mechanisms of interest, and to better identify the properties of structural shocks. The simplest, and most common, approach is to use mean or median forecasts from surveys. Higher moments, instead, generally don't enter the model or the formation of expectations.

An even more ambitious direction, however, would be to also exploit those higher moments available from the surveys. Some surveys (the Survey of Professional Forecasters, for example) also

provide data that reflect the forecasters' whole subjective probability distributions. Forecasters are asked to communicate their probability assessments for inflation, output growth, and other variables, falling within specified intervals.⁵

Subjective probability distributions can be used to extract forecasters' real-time beliefs about second moments, for example, to gauge their degree of uncertainty toward future inflation or output developments. Typically, uncertainty is included in DSGE models through the assumption of stochastic volatility: the exogenous shocks have a variance that changes, exogenously, over time. The innovation to the variance is treated as an uncertainty shock, and, when the model is solved using third-order or higher-order approximations, it affects consumers and firms' optimizing decisions.

Chatterjee and Milani (2020) offer a different approach to study the importance of first and second moments in the context of a microfounded model, and to estimate the importance of uncertainty over the business cycle. The paper uses information from the Subjective Probability Density forecasts about inflation and the GDP growth rate that are contained in the Survey of Professional Forecasters. Uncertainty is extracted from the subjective distributions using the approach by D'Amico and Orphanides (2008). It represents here the expected future variance of output and inflation by economic agents.

The model includes several behavioral features. Agents are allowed to form non-fully rational expectations, they have finite, rather than infinite, horizons, and they are learning over time. In forecasting, they have a potentially asymmetric loss function, which allows them to weigh differently positive and negative forecast errors and which creates a direct channel for expected variances to enter the model equations. As a result of finite horizons, expectations up to L -periods ahead matter for current dynamics in the Euler equation and in the New Keynesian Phillips curve. Agents' beliefs are also subject to excesses in optimism and pessimism and they may be affected by perceptions about future uncertainty. The estimation uses twelve observed survey expectation series (mean across forecasters): expected growth rates of real GDP at horizons from $t + 1$ to $t + 4$, expected inflation from $t + 1$ to $t + 4$, and expected nominal interest rates from $t + 1$ to $t + 4$. In addition, the list of observables includes also the perceived uncertainty series, given by the approximate $t + 4$ uncertainty (expected variance) for output growth and the approximate $t + 4$ uncertainty

⁵The value of probabilistic questions is discussed in Manski (2017), and Potter et al. (2017). Chapter 18, *Inference on Probabilistic Surveys in Macroeconomics*, discusses the challenges and approaches connected with extracting information from probabilistic surveys on macroeconomic variables.

for inflation. The use of a large set of expectation series, allows the identification of sentiment related to the short-run, longer-run (up to $t + 4$ in the main exercise of the paper), and perceived uncertainty (at $t + 4$ horizon).

The empirical results show that private sector's beliefs are impacted by uncertainty: higher perceived uncertainty about output leads to lower output expectations, whereas higher uncertainty about inflation leads to higher inflation expectations. Through such confidence channel, perceived uncertainty shocks have recessionary effects on output, which are comparable to the effects estimated in models with stochastic volatility. However, it is first-moment, sentiment, shocks, particularly those at longer horizons, that are responsible for the largest share of business cycle fluctuations.

In principle, the same subjective density forecasts can be used to assess tail risk (Andrade et al., 2015) and possibly integrated with DSGE models that give a role to beliefs about rare events and disasters (example of DSGE models that consider tail risks are Kozlowski, Veldkamp, and Venkateswaran, 2019, Orlik and Veldkamp, 2014, Gourio, 2012). On the negative side, it can be argued that estimates of tail risk by forecasters are potentially less accurate than estimates of first moments, as they require computation of probabilities related to events that happen extremely rarely.

A related literature has emphasized the role of disagreement, a concept that is usually discussed in relation to uncertainty, but which is distinct. Mankiw, Reis, and Wolfers (2003) document substantial disagreement among survey respondents about expected future inflation. Novel stylized facts about disagreement from survey data are reported in Andrade et al. (2016). The construction of measures of disagreement and uncertainty, and their relationship, is discussed in more depth in Chapter 3, *Survey of Professionals*.

The disagreement that can be extrapolated from surveys can also, in principle, be used as an observable in an estimation of a model that allows for significant heterogeneity. We discuss in the next section papers that deal with heterogeneity, but the use of disagreement as an observable has not become customary yet.

Measures of uncertainty extracted from surveys could also be used in the estimation of DSGE models with stochastic volatility (the literature is reviewed in Fernández-Villaverde and Guerrón-Quintana, 2020, but it doesn't use expectational data). Treating subjective uncertainty as an observable can provide additional restrictions that can help the identification of the volatility series.

A potential issue is that uncertainty modeled as stochastic volatility and obtained from particle filter estimations looks quite different from the perceived uncertainty series from the SPF. The discrepancies may be due to data limitations, or, possibly to behavioral biases by forecasters in their estimates of uncertainty.

4 Heterogeneity in Survey Expectations

The majority of papers in the DSGE literature assume expectations that are homogeneous: all consumers and firms share exactly the same expectation formation model as everybody else.

A growing literature in macroeconomics is, however, documenting the substantial heterogeneity that exists in observed survey expectations. Branch (2004) analyzes inflation expectations data at the individual level from the Michigan Survey of Consumers and finds different shares of consumers that form expectations from models of varying degrees of complexity. Several other studies (e.g., Mankiw, Reis, and Wolfers, 2004, Coibion and Gorodnichenko, 2012, Andrade et al., 2016, Pesaran and Weale, 2006, Doovern et al. 2012, Cole and Milani, 2021) similarly reveal large degrees of heterogeneity in micro-level survey data. Heterogeneity also consistently arises as an outcome of laboratory experiments focused on forecasting behavior. Hommes (2011, 2013), for example, demonstrate that expectations can be approximated by different types of forecasting behaviors: some are adaptive, others are trend-following, or based on anchor-and-adjustment heuristic.

Branch and McGough (2009) models expectation heterogeneity in a New Keynesian setting and analyzes the corresponding microfoundations. In the simplest case, a fraction of agents are assumed to form rational expectations, while the remaining agents form backward-looking expectations based on lagged endogenous variables, which can be either adaptive (mean-reverting), naïve, or extrapolative (trend-chasing). The literatures on heterogeneous expectations are reviewed in detail in Hommes (2021) and Branch and McGough (2018).

A number of papers take the Heterogeneous Expectations New Keynesian (HENK) model to the data. Massaro (2013) estimates a model that includes long-horizon expectations, Beqiraj et al. (2018) estimate shares of agents that depart from rational expectations that vary and can reach up to almost half of the forecasters. Ilabaca and Milani (2021) estimate the HENK model using rolling windows. They find even larger shares of agents who depart from rational expectations. Moreover, the model with heterogeneous expectations is preferred by the data at each point in the sample.

Elias (2021) also estimates a New Keynesian model with heterogeneous expectations: agents in his model, however, use either a correctly-specified or an under-specified learning model. A large share of agents employ the simpler model, suggesting significant deviations from rational expectations.

The heterogeneity can arise from different sources. The previous papers typically rest it on the existence of rational versus backward-looking forecasters. More generally, heterogeneity may originate from model uncertainty, with agents using a set of models and possibly switching among them based on past performance, as in Brock and Hommes (1997) and Branch (2004). Alternatively, agents can have different priors or initial beliefs (introduced as in Suda, 2018, for example), which may also arise as a result of different ages or life experiences (Malmendier and Nagel, 2016). In Cole and Milani (2021), instead, the heterogeneity stems from different degrees of recency bias, captured by different constant-gain values across forecasters. Chapter 16, *The Epidemiology of Economic Expectations* offers another explanation for heterogeneity, focusing on how beliefs spread in the society, using an epidemiological framework.

The initial papers in the heterogeneous expectations literature made consistent use of survey data on expectations. Those, however, have not been used in DSGE estimations to the same extent yet. Ilabaca and Milani (2021), in one of the robustness exercises, add inflation and output expectations from the Survey of Professional Forecasters to the set of observables that need to be matched, with the addition of measurement errors. They find that model-implied expectations are close to the corresponding survey data in the pre-1979 sample, whereas the model has difficulties in fitting expectation series in the 1990s. In the latter case, they appear to excessively track lagged inflation rather than the survey series.

Expectational data should ideally be used to test the ability of DSGE models to match not only mean expectations, but also the dispersion of forecasts that exists in survey data. In this respect, models with heterogeneous expectations are promising, although it remains to be seen which particular form of heterogeneity is more consistent with survey data. Identifying the main sources of heterogeneity should definitely represent a priority for future work.

Finally, expectations data can be used in models with a different kind of heterogeneity: Heterogeneous Agents New Keynesian (HANK) models (Kaplan et al., 2018). In such settings, the existence of incomplete markets and uninsurable idiosyncratic risk affects the sensitivity of the economy to expectation-driven fluctuations. Chapter 23, *Expectations and Incomplete Markets*, discusses in detail the role of expectations in HANK models. Incomplete markets induce two

changes: first, they introduce extra discounting in the Euler equation, with agents becoming less forward-looking; second, they can lead to amplification of shocks. Survey forecasts can, therefore, be exploited to evaluate the responses of expectations to shocks, and to identify the relative strength of the amplification versus myopia channels. Moreover, since the models are suitable to study wealth and income inequality, the literature could incorporate and analyze differences in the responses of expectations across the income distribution. Overall, the estimation of HANK models using survey expectations data is an important need for future research.

5 Issues and Limitations

This chapter argues in favor of a drastic increase in the use of expectational data in the estimation of DSGE models. However, we recognize here some possible limitations.

First, how representative are the surveys that are used to measure expectations? It is well known that expectations differ based on income, education levels, demographic variables, and geography. Using a limited sample of respondents may not capture all these variations. In the Survey of Professional Forecasters, central tendency measures may be affected by the entry and exit of respondents, as emphasized by Manski (2011). One potential solution is to reduce the sample to retain only those forecasters who remain in the survey for at least several quarters, but still some issues persist. For example, it's not clear that the same individuals actually remain in the SPF; sometimes the codes that they are assigned may refer to the companies employing them and may be shared by the future employees.

Researchers working with expectations data need to deal with discrepancies between the timing of information in the surveys versus the timing typically assumed in the models. Surveys are typically mailed at the beginning or at the middle of a quarter, and respondents are asked for their forecasts for the same quarter and for subsequent ones. Assuming in the model that they have time- t information is inaccurate, since they report their forecasts almost two months before the quarter is over. Assuming information up to $t - 1$, on the other hand, underestimates their knowledge when completing the survey.

The use of expectational data renders natural the use of real-time, rather than fully-revised, data in the estimation. Observed expectations are produced by forecasters based on information sets that were available to them in real time: the estimation should, therefore, try to match such

information sets as accurately as possible. If the literature moves toward a more consistent use of expectations data, it should therefore become the norm to incorporate real-time data. The use of revised data, instead, may distort the inference about best-fitting expectation formation models, learning processes, and sentiment terms. On the other hand, real-time observations bring complications connected with the treatment of data revisions. It is not obvious what vintage of data forecasters are trying to predict: it can be the first release that they encounter, the final revised data many years later (which seems highly unlikely), or one of the releases, after the first one, when many of the revisions have already been incorporated.

Many of the papers that we discussed assume that expectations in the model can be well approximated by the expectations of professional forecasters. But there's no consensus on whose expectations we should measure. Based on DSGE models, it would be natural to measure consumer and firm expectations. The most popular sources of consumer expectations data for the U.S. have been the Michigan Survey of Consumers and from the New York Fed Survey of Consumer Expectations. The elicitation of firm expectations and new related surveys are discussed instead in Chapter 4, *The Macroeconomic Expectations of Firms*. Expectations derived from surveys of professional forecasters, however, have been far more common. Chapter 3, *Survey of Professionals* discusses some of their advantages and a number of related issues.

Coibion, Gorodnichenko, and Kamdar (2018) argue that household expectations are better proxies not only for consumers in the model, but even for firm expectations, than professional forecasts. They discuss the incentives that professional forecasters may have when reporting their answers (for example, they may be averse to having their forecast fall too far from the consensus), which may skew the measurement of aggregate expectations.

On the other hand, consumer and household expectations may be affected by limitations in cognition that are quite significant (D'Acunto et al., 2019). An argument in favor of the use of professional forecasters expectations is provided by Carroll (2003). He shows evidence that household expectations are significantly affected by the the expectations of 'experts'. Another advantage is availability: professional forecasts are available for a large number of series, for a long sample, for different horizons, and for different countries. The types of forecasts are more limited for surveys at the household level, and the definitions of variable sometimes more qualitative and less immediately useful for research, due to their need of being more easily understandable.

Other measures of expectations are possible and have been used, even if more sporadically. At

least for inflation and interest rates, it is possible to use market-based expectations. For inflation, expectations can be derived as the spread between yields on nominal Treasury bonds and yields on Treasury Inflation-Protected Securities (TIPS), after some possible adjustments. Expectations about policy rates can be obtained from future and option contracts. However, such market expectations are likely to be closer to professional forecasters' expectations than to households' expectations. They may also be affected by shifts in liquidity and risk premia that are unobserved to the researcher. Finally, they can be more responsive than household expectations to asset purchases by the Federal Reserve, which may severely skew them, at least, in the most recent part of the sample.

Many of the existing issues will be solved with a better measurement of different types of expectations, which ideally would be obtained both at the household and at the firm level, to match the main sets of economic agents that populate our macroeconomic models. For the DSGE literature, ideal data on expectations should move beyond the main focus on inflation and the short term. Empirical work will benefit from expectations at a multitude of horizons, including a bigger focus on the long-run, and for a multitude of variables, including future monetary policy decisions, wages, taxes, asset returns, and so forth. A broader set of data would permit to infer the perceived models that respondents are implicitly using to form their forecasts. Forecasts data should be numerical, whenever possible, rather than qualitative. Fixed-horizon forecasts are easier to work with in macroeconomic models than fixed-event forecasts (such as the popular current year/next year forecasts), which often require arbitrary transformations. In applied work, it would be useful to have a better understanding of forecasters' real-time perceptions about trends and cycles, about developments that they perceive as transitory versus persistent or permanent. Professional forecasters' data already provide a lot of high-quality information, but it will be important to collect similar observations for consumers and firms. Ideally, the same respondents should be tracked over time, to provide researchers with panel data sets on expectations. Finally, surveys should more often try to elicit information about higher moments, capturing respondents' full subjective distributions. In this case, survey questions should be carefully crafted in ways that do not unduly influence the responses, for example, by avoiding limiting forecasters to strict pre-specified bin choices, and maybe providing them with more flexible graphical interfaces.

6 Conclusions and Future Directions

This chapter has reviewed existing and potential uses of expectational data in the estimation of DSGE models.

At this stage, it seems reasonable to predict that survey data on expectations will become more popular in the future and more routinely used to test the restrictions imposed by our theoretical models. Many papers have already used mean and median expectations, particularly those generated by professional forecasters, which are widely available. In the future, information at the micro level can be more often exploited, for example, to assess the models' ability to generate levels of disagreement that match those in reality. Expectational data can provide information on economic agents' perceptions about higher moments, such as their perceived uncertainty of economic conditions, or their estimates of tail risk. To this scope, surveys should incorporate more broadly questions that report the full subjective probability distribution that forecasters have in mind.

As in the past, data on expectations can be used to test whether rational expectations succeed or fail in matching the evidence. But different strands of the literature are recognizing that to explain observed expectations, it may be necessary to model departures from rational expectations (or at least, substantial refinements to the rational expectations assumption that still have strong behavioral connotations). The behavioral elements that are added to macroeconomic models should not be taken for granted, but they should be similarly evaluated based on their ability to jointly match both realizations and expectations.

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