Heterogeneous Expectations, Indeterminacy, and Postwar US Business Cycles

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Abstract

This paper estimates a New Keynesian model extended to include heterogeneous expectations: consumers and firms form either rational or boundedly-rational expectations. The inclusion of heterogeneous expectations alters the determinacy properties of the model, with the details of expectations potentially becoming more influential than the Taylor principle for equilibrium stability.

The model is estimated with Bayesian techniques, using rolling windows and allowing the parameters to fall both in the determinacy and indeterminacy regions. The estimates reveal large shares of agents who depart from rational expectations. Heterogeneous expectations are decisively preferred by the data everywhere in the sample.

Finally, the paper revisits the narrative that sees postwar US macroeconomic data as consistent with indeterminacy in the pre-1979 sample, with a switch to determinacy starting in the early 1980s, and it shows that it's overall robust to inclusion of heterogeneous expectations.

Keywords: Heterogeneous Expectations in New Keynesian Model, Indeterminacy, Sunspots, Taylor Principle, Deviations from Rational Expectations, Time-Varying Parameters.

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1 Introduction

A generally accepted account of US postwar macroeconomic history views output and inflation fluctuations in the 1960-1970s as the outcome of passive monetary policy and indeterminacy, followed by a shift to active monetary policy after 1979, which induced stability in the macroeconomic environment by ensuring a determinate equilibrium.

Empirical evidence in favor of this narrative is provided, for example, in the influential papers by Clarida, Galí, and Gertler (2000), and Lubik and Schorfheide (2004). Their works, as well as the related literature, use New Keynesian frameworks that particularly highlight the role of monetary policy in managing private-sector expectations. A weak monetary policy response may cause inflation and output expectations to become unhinged, steering the economy away from the rational expectations equilibrium. Expectations may coordinate, instead, around a more volatile equilibrium, where realizations of a sunspot variable affect agents' beliefs and, ultimately, the dynamics of macroeconomic variables.

The results on the transition from indeterminacy to determinacy are, however, obtained in models that impose two strong expectational assumptions: first, that expectations are formed according to the rational expectations hypothesis, and, second, that expectations are homogeneous. All households and firms in the model are assumed to form identical expectations conditioned on the same, correct, model of the economy.

However, recent studies have provided growing evidence in favor of heterogeneous expectations. Branch (2004) uses individual-level data on inflation expectations from the Michigan Survey of Consumers to show that different shares of consumers form expectations based on alternative forecasting models, with different levels of complexity. Mankiw, Reis, and Wolfers (2004) and Branch (2014) document a significant extent of disagreement among professional forecasters. The experimental evidence has also strongly stressed the importance of heterogeneous expectations: Hommes (2011, 2013) show that expectations cluster around different groups, with some characterized by adaptive, and others by trend-following, behavior.

Branch and McGough (2009) introduce heterogeneous expectations in a New Keynesian model and analyze their microfoundations. The model retains a share of agents who form conventional rational expectations, but they allow for a share of agents whose expectations deviate from rational (RE) and can be adaptive, naïve, or extrapolative. As they stress, the inclusion of heterogeneity alters the determinacy properties of the system, and may be stabilizing or destabilizing depending on the nature of the expectations. If expectations have an adaptive component, that is they respond less than one-to-one with respect to the lagged value of the endogenous variable, a larger share of non-RE agents will be stabilizing. The determinacy region is expanded, and the equilibrium can be determinate even with weak responses to inflation. On the other hand, if expectations are extrapolative, by responding more than one-to-one to past values, indeterminacy will be more likely. For large enough shares of non-RE agents, conventional active monetary policies become powerless in trying to steer the economy toward a stable equilibrium. Hence, the Taylor principle is neither necessary nor sufficient for equilibrium uniqueness. As a consequence, it is possible that determinacy exists in the 1970s in conjunction with passive monetary policy, or that indeterminacy continues to provide a better explanation even of the post-Volcker data, as long as beliefs are extrapolative. The details of expectations, overall, may be more important than monetary policy for determinacy of the equilibrium. The traditional narrative of postwar shifts from indeterminacy to determinacy, therefore, may not be robust to expectational heterogeneity.

In this paper, we estimate the New Keynesian model with heterogeneous expectations and allow for both determinacy and indeterminacy. We use the techniques developed in Bianchi and Nicoló (2019) to solve the model and obtain the likelihood under indeterminacy. We also recognize that monetary policy coefficients and the formation of expectations may have changed over time. Therefore, we adopt a rolling-window estimation from 1954 to the end of the sample. We use different window sizes: in the benchmark estimation, our rolling window includes twenty years of observations, but we also repeat the estimation with a ten-year window. The model is re-estimated every year, and we calculate how the probability of determinacy versus indeterminacy varies over time. We use a rolling sample because, while there may be evidence of a structural break for monetary policy coefficients in correspondence of the appointment of Paul Volcker as the Federal Reserve's Chairman in 1979, we cannot realistically assume a one-time structural break in the expectation parameters, or in other structural coefficients.

Results. The estimation results strongly indicate that heterogeneous expectations provide a better fit of the data than homogeneous expectations do, as measured by the marginal likelihoods. Significantly, heterogeneous expectations outperform homogeneous expectations decisively in every rolling sample from 1954 to 2007. We estimate large shares of boundedly rational agents: at points in the sample, they reach up to 80% for output expectations and more than 60% for inflation expectations.

We document ample evidence of time-variation in the structural and expectation parameters. Monetary policy is passive toward inflation in the earlier samples, as expected, and it becomes more reactive to the output gap for windows that are centered around the 1970s. The shares of boundedly rational agents agents vary over time. They are low in the earlier windows and for some of the samples that begin in the second half of the 1970s, but, in general, they indicate that the vast majority of agents do not conform with rational expectations. We also document that expectations about the output gap display backward-looking coefficients close to one over most of the sample. In contrast, inflation expectations are typically formed adaptively, with coefficients ranging from below 0.5 to 0.7, but with the exception of windows that start in the late 1970s, when the coefficients rise above one and expectations display extrapolative or trend-chasing behavior.

Our approach allows us to test whether the conventional narrative about monetary policy and indeterminacy, based on Clarida, Galí, and Gertler (2000) and Lubik and Schorfheide (2004), is robust to more realistic assumptions about expectations. The conventional story regarding post-war switches from indeterminacy to determinacy is largely confirmed, and is, therefore, lent additional credibility. The early years in our sample are best explained as coming from a data-generating process characterized by passive monetary policy and multiple equilibria. For samples starting after the early 1980s, there's a definite shift to determinacy. But there are other periods in which the Taylor principle is usually considered as satisfied in the literature, but where we find expectations that become extrapolative. In these periods, the probability of indeterminacy rises considerably, up to 70%. These additional episodes of potential indeterminacy are usually absent in estimations under homogeneous expectations. Under indeterminacy, the economy is susceptible to the action of sunspots. Based on our estimates, sunspot shocks typically don't explain a large share of fluctuations in output and inflation, but they can account for up to 20% in some episodes. Sunspots also account for a similar share of the variability in inflation in those samples where expectations become extrapolative.

Related Literature. This paper aims to contribute to multiple literatures. More directly, it builds empirical evidence in favor of modeling heterogeneous expectations in the New Keynesian model. The microfoundations of heterogeneous expectations were developed in Branch and McGough (2009), who also document the conditions for equilibrium determinacy and show that heterogeneity can expand or shrink the region of determinacy depending on the nature of non-RE expectations. Massaro (2013) adds heterogeneous expectations in the microfoundations of a New Keynesian model with long-horizon expectations instead. Gasteiger (2017) develops alternative microfoundations in a stochastic version of the New Keynesian model. Beqiraj et al. (2018) estimate shares of agents that depart from rational expectations ranging from 13% to 46%; they don't consider the implications of heterogeneous expectations for indeterminacy in the estimation, and don't allow for time variation in the parameters. Our empirical estimates suggest that non-RE shares are even larger. Elias (2021) estimates a New Keynesian model with heterogeneous expectations, where different shares of agents use correctly-specified or misspecified adaptive learning models. He finds large shares of agents that form expectations from the more parsimonious, misspecified, models, which could be suggestive of cognitive or computational limitations on their part. Other papers have studied the implications of heterogeneous expectations on the amplification of technology shocks on output (Branch and McGough, 2011), optimal monetary policy (Gasteiger, 2014, Di Bartolomeo et al., 2017, Beqiraj et al., 2019), and monetary-fiscal policy interactions (Gasteiger, 2018). While we provide evidence on the importance of expectation heterogeneity at the macro level, other papers have documented its importance on micro data (Branch, 2004, 2014, Pesaran and Weale, 2006, Dovern et al., 2012, Cole and Milani, 2020), or in laboratory experiments (Hommes, 2011, 2013). Cole and Milani (2019) use a DSGE-VAR approach and reveal that a major misspecification in the New Keynesian model lies in the way expectations are modeled: when heterogeneous expectations, chosen to mirror lab evidence, replace homogeneous rational expectations, the data become more supportive of DSGE restrictions. Calvert Jump and Levine (2019) show that the model with heterogeneous expectations can account for some empirical regularities (excess kurtosis, stochastic volatility, and departures from rational expectations) that cannot be explained by the benchmark New Keynesian model.

The paper then adds to the literature on changes in monetary policy and indeterminacy to explain postwar US economic history. The paper generally confirms the shift from indeterminacy to determinacy over the sample found in Clarida, Galí, and Gertler (2000) and Lubik and Schorfheide (2004), but it uncovers additional periods when indeterminacy may arise because of extrapolative expectations. Other papers relax rational expectations or add behavioral elements to revisit the empirical evidence in favor of indeterminacy, but they impose homogeneous expectations (Milani, 2008, Ilabaca, Meggiorini, and Milani, 2020).

Benhabib and Farmer (1999) and Farmer (2019) review the broader literature on indeterminacy in macroeconomics. Empirical studies have been few in the past due to difficulties in solving and estimating models with multiple equilibria. Lubik and Schorfheide (2004), Farmer, Khramov, and Nicoló (2015), and Bianchi and Nicoló (2019) provide key methodological contributions that make it easier to take these models to the data. Our paper adds to the literature by revisiting the empirical evidence for indeterminacy versus determinacy in a model that studies the interaction between monetary policy and expectations, by allowing for heterogeneous expectations and parameter variation.

Furthermore, the paper is connected to the literature that deals with parameter instability in DSGE models. Fernandez-Villaverde and Rubio-Ramirez (2008) estimate a model with time-varying parameters to judge whether parameters can be interpreted as 'structural'. Canova (2009), Canova and Ferroni (2012), and Castelnuovo (2012), use a closer approach to ours, by employing a rolling window estimation. Compared to models with Markov-Switching, time-varying parameters, or stochastic volatilities, a rolling Bayesian estimation allows us to consider time variation in all parameters at the same time in a computationally tractable way (the other approaches typically only allow a minor subset of parameters to shift, while forcing others to remain constant). It's particularly suitable for a situation in which there is uncertainty over which parameters are subject to instability, and what form that instability takes. Our rolling-window approach doesn't require the specification of evolution equations for each parameter that is assumed to be time-varying, and it doesn't require the estimation of its corresponding autoregressive and variance coefficients. Moreover, it allows us to retain the simplicity of estimating a model in linearized form, rather than dealing with nonlinearities and higher-order approximations. The previous papers still impose determinacy and work with models with homogeneous expectations, while we emphasize parameter variation related to heterogeneous expectations, and we show how parameters may fall in regions that are conducive to indeterminacy.

Finally, the paper is more broadly connected to the literatures that model deviations from rational expectations, such as the literature on adaptive learning in macroeconomics (e.g., Evans and Honkapohja, 2001). We don't directly model agents' learning here, but we show how the parameters related to expectations change over time through our rolling window estimation. Both under adaptive learning and under our framework, expectations have backward-looking components that can induce additional persistence in the economy, amplify the effects of shocks, and improve the fit of the model to the data. Under learning, agents can adopt a perceived law of motion to form expectations that includes the same variables that would appear in the solution under rational expectations (a correctly-specified model). But they wouldn't know the reduced-form parameters of the rational expectations solution and, therefore, they would attempt to learn them over time using the most updated sample of data they have available. An alternative would be to assume that agents use underspecified models, which include only some of the relevant variables. As discussed before, Elias (2021) estimates a model that merges these approaches. Our heterogeneous expectations framework assumes, instead, that agents form backward-looking expectations based on the lagged value of the variable they are trying to forecast. They correspond, therefore, to a stronger departure from rationality.

2 New Keynesian Model with Heterogeneous Expectations

We use a version of the New Keynesian model extended to include two types of agents who are identical except in the way they form expectations: one forming rational, and the other boundedly-rational, expectations.

Introducing heterogeneity of expectations into a dynamic general equilibrium may complicate the aggregate dynamics, as aggregation into IS and AS relations is not obvious. Branch and McGough (2009) provide the microfoundations for heterogeneity and discuss the axioms that are needed to ensure that the laws of motions for aggregate variables in the model remain tractable and are easily comparable to the case of rational expectations. Most importantly, restrictions on expectations need to ensure that the law of iterated expectations holds both at the individual and the aggregate level (so that higher-order beliefs don't matter), and that agents have the same expectations about limiting wealth (so that there's no wealth heterogeneity in the long run and no need to track the wealth distribution as a state variable).[1] In their original paper, Branch and McGough model heterogeneous expectations in a deterministic yeoman-farmer economy, with no exogenous shocks.

Gasteiger (2017) extends the heterogeneous expectations setting to the case of a stochastic New Keynesian model, which includes shocks to aggregate demand and supply. Instead of assuming a yeoman farmer, his model includes households and firms that interact in decentralized markets. We use this version of the model for our analysis.[2]

[1] The full list of axioms can be found in the original paper.

[2] The log-linearized equations remain similar to those in Branch and McGough (2009), with the addition of the

Thus, the evolution of the economy can be summarized by the following log-linearized equations:

$$x_t = \tilde{E}_t x_{t+1} - \sigma^{-1}(i_t - \tilde{E}_t \pi_{t+1}) + g_t \tag{1}$$

$$\pi_t = \beta \tilde{E}_t \pi_{t+1} + \kappa x_t + u_t \tag{2}$$

$$i_t = \rho_i i_{t-1} + (1 - \rho_i)(\psi_\pi \pi_t + \psi_x x_t) + \varepsilon_t \tag{3}$$

$$g_t = \rho_g g_{t-1} + \varepsilon_t^g \tag{4}$$

$$u_t = \rho_u u_{t-1} + \varepsilon_t^u, \tag{5}$$

where x_t denotes the output gap, π_t denotes inflation, and i_t denotes the short-term nominal interest rate, which serves as the monetary policy instrument. All variables are in log deviations from steady-state. The parameter σ denotes the intertemporal elasticity of substitution, β the household's discount factor, and $\kappa \equiv \frac{(1-\xi_p\beta)(1-\beta)}{\xi_p} \frac{\omega+\sigma}{1+\omega\epsilon}$ is a composite parameter that captures the slope of the Phillips curve; κ is an inverse function of the degree of price stickiness (the Calvo parameter ξ_p), and it depends on the elasticity of marginal costs to income (denoted by ω), and on the steady-state markup of prices over marginal costs (ϵ).

Equation (1) is the Euler equation, which expresses the output gap as a function of one-period-ahead expectations, and of the ex-ante real interest rate; g_t denotes a demand disturbance (for example, due to exogenous shifts in consumer tastes or government spending). Equation (2) is a New Keynesian Phillips curve: inflation depends on expected inflation, on the current output gap, and on the exogenous cost-push disturbance u_t. Both g_t and u_t evolve as AR(1) processes. Monetary policy in the model is described by the Taylor rule in Equation (3): the policy instrument changes only gradually (with smoothing parameter ρ_i) in reaction to fluctuations in inflation or the output gap, with reaction coefficients given by ψ_π and ψ_x. The term ε_t denotes an *i.i.d.* monetary policy shock.

Expectations $\tilde{E}_t$ denote aggregate expectations in the economy, and are obtained as a convex combination of heterogeneous expectation operators. Fractions n_x and n_π of agents are assumed to form conventional rational expectations, denoted here by the mathematical expectation operator E_t. The remaining fractions $(1 - n_x)$ and $(1 - n_\pi)$ of agents form expectations that deviate from

exogenous disturbances, although the microfoundations for heterogeneous expectations are different.

rational expectations and are denoted by $\hat{E}_t$:

$$\tilde{E}_t x_{t+1} = n_x E_t x_{t+1} + (1 - n_x)\hat{E}_t x_{t+1} \tag{6}$$

$$\tilde{E}_t \pi_{t+1} = n_\pi E_t \pi_{t+1} + (1 - n_\pi)\hat{E}_t \pi_{t+1}. \tag{7}$$

Boundedly-rational expectations $\hat{E}_t$ are formed as

$$\hat{E}_t x_{t+1} = \theta_x^2 x_{t-1} \tag{8}$$

$$\hat{E}_t \pi_{t+1} = \theta_\pi^2 \pi_{t-1}. \tag{9}$$

These expectations can be adaptive (mean-reverting), if $0 \leq \theta < 1$, naïve, if $\theta = 1$, or extrapolative (trend-chasing), when $\theta > 1$.

Therefore, an implication of introducing heterogeneous expectations in the model is that the New Keynesian Phillips Curve and the IS equation become "hybrid": they incorporate both forward-looking and backward-looking terms. As a result, they help the model match the inertia typically observed in macroeconomic data and the hump-shaped responses of endogenous variables to shocks.

2.1 Determinacy and Indeterminacy Regions

In the benchmark, and nested, New Keynesian model, with $n_x = n_\pi = 1$, the determinacy condition is given by the "Taylor principle": $\psi_\pi + \frac{(1-\beta)}{\kappa}\psi_x > 1$ (Bullard and Mitra, 2002, Woodford, 2003). The aggressiveness of monetary policy toward inflation (and, to a lower extent, toward the output gap) is the main driver of stability in the system. In the New Keynesian model with heterogeneous expectations, however, the state of beliefs has the potential to significantly alter the stability of the equilibrium. The determinacy properties have been analyzed in detail in Branch and McGough (2009) and Gasteiger (2011) and they are similar in our case. We report them here in Figures 1 and 2 to make the paper self-contained.[3]

When $\theta \leq 1$, the determinacy region expands as the share of adaptive agents increases. On the other hand, when $\theta > 1$ indeterminacy becomes much more likely. Even relatively modest shares of non-RE agents are enough to alter the determinacy properties considerably.

[3] The determinacy and indeterminacy regions in the Figures are obtained by fixing the structural parameters at their prior means shown in Table 1, and by varying the monetary policy and expectation parameters. For ease of exposition, we assume $n_x = n_\pi$ and $\theta_x = \theta_\pi$. The Figure shows the cases for n_x and n_π equal to 0.5, 0.8, 0.9, 0.99, and 1.

We can also investigate the impact of different parameters in driving the system's stability and determinacy properties by using Monte Carlo Filtering techniques. This approach has been proposed by Ratto (2008) to map and study the stability regions of DSGE models, and it has been used by Beqiraj et al. (2018) in models with heterogeneous expectations based on short- or long-horizon forecasts.

With Monte Carlo Filtering, a multi-parameter simulation is performed, by sampling a set of parameters $(\Theta_1, ..., \Theta_k)$ from their prior distributions and propagating their values through the model. The resulting Monte Carlo realisations fall either in the stability or instability region. After N Monte Carlo draws, there are two subsamples of realizations $(\Theta_i|S)$ and $(\Theta_i|I)$, where S and I denote the stable and unstable/indeterminate regions, coming from different probability density functions $f_{n_s}(\Theta_i|S)$ and $f_{n_i}(\Theta_i|I)$, where n_s and n_i denote the number of Monte Carlo runs consistent with each subset.

To compare whether the two unknown distributions are the same, the cumulative distribution functions of Θ_i, denoted by $F_{n_s}(\Theta_i|S)$ and $F_{n_i}(\Theta_i|I)$, can be compared using the Kolmogorov-Smirnov test. The d_{n_s,n_i} statistic is given by

$$d_{n_s,n_i} = \sup ||F_{n_s}(\Theta_i|S) - F_{n_i}(\Theta_i|I)||, \tag{10}$$

with a null hypothesis $F_{n_s}(\Theta_i|S) = F_{n_i}(\Theta_i|I)$. Larger values of d_{n_s,n_i} signify a larger impact of the parameter in driving the stability or instability of the corresponding DSGE model.

Table 2 reports the outcomes of the test. It shows the values of the d_{n_s,n_i} statistic for all the estimated parameters in the model, and the corresponding p-values; the values are shown for the cases of existence of a unique solution, indeterminacy, and instability. The results clarify that the key parameters that drive uniqueness or indeterminacy are the shares of boundedly-rational agents (n_π and n_x), the expectation parameters (θ_π and θ_x), and the policy reaction coefficient to inflation (ψ_π). For these parameters, the null of equality between the two CDFs is rejected at the 5% significance level. The expectation parameters also drive model departures into instability.

Therefore, whether monetary policy is passive or active (by satisfying or not the Taylor principle) may cease to be the main determinant of macroeconomic stability. It's an empirical question whether the existence of heterogeneous expectations changes the conclusions about monetary policy and macroeconomic stability that are typically documented in the literature.

3 Bayesian Estimation under Determinacy/Indeterminacy

We estimate both the benchmark New Keynesian model with homogeneous expectations and the extended model with heterogeneous expectations, using a rolling-window approach that allows us to provide evidence on structural parameter instability. For both models, we estimate specifications that accommodate either determinacy or indeterminacy of the equilibrium.

We consider two window sizes. The benchmark results are presented for a rolling window lasting twenty years; in the robustness section, we show the results obtained with a ten-year window. The model is, therefore, re-estimated every year beginning in 1954:Q3 and until 2007:Q3 (we stop before the Great Recession to avoid the effective-lower-bound nonlinearity). We use quarterly time series for output growth, inflation, and the Federal Funds rate as observables. Output growth and inflation are computed as the log first-differences of Real GDP and the GDP Implicit Price Deflator, respectively. The Federal Funds rate is transformed into a quarterly rate. All variables are obtained from the Federal Reserve Economic Database (FRED).

The observation equations are given by:

$$\begin{bmatrix} \Delta Real\ GDP_t \\ \Delta GDP\ Deflator_t \\ FFR_t \end{bmatrix} = \begin{bmatrix} \gamma \\ \bar{\pi} \\ \bar{r} + \bar{\pi} \end{bmatrix} + \begin{bmatrix} x_t - x_{t-1} \\ \pi_t \\ i_t \end{bmatrix} \tag{11}$$

where γ, $\bar{\pi}$, and $\bar{r}$ denote the steady-state growth rate of output, steady-state inflation and real interest rate, respectively.

The model is estimated using Bayesian methods. We use the approach proposed by Bianchi and Nicoló (2019) to estimate the model under indeterminacy. The approach consists of appending to the model state equations an auxiliary variable ω_t, which follows an autoregressive process:

$$\omega_t = \frac{1}{\alpha}\omega_{t-1} + \zeta_t - \eta_t^i, \tag{12}$$

where α is a positive coefficient, ζ_t is a sunspot shock, and η_t^i, with $i = x, \pi$, is one of the expectational errors[4] (in our model, either related to the output gap or inflation).[5] When the model is already determinate, the autoregressive coefficient for the auxiliary process is below one ($\alpha > 1$),

[4] As shown by Bianchi and Nicoló (2019), the choice of expectational errors to include in (10) does not affect the solution. In this paper, we will make the modelling assumption that the expectational error included in (10) corresponds to inflation.

[5] We write the equation for the case of indeterminacy of order 1, which is the empirically relevant case here. Therefore, the dimensions of ω_t, ζ_t, and η_t^i are 1×1 in our case. We have run estimations allowing for indeterminacy of order 2, but those cases are rejected by the data.

and the auxiliary variable is redundant, as it doesn't affect any other variable in the system. When the model is indeterminate, the autoregressive coefficient goes above one ($\alpha < 1$) to restore the necessary number of explosive eigenvalues. The auxiliary process ω_t again doesn't influence the remaining parts of the system, but it permits the inclusion of a sunspot shock by inducing a mapping between the expectational errors η_t^i and the sunspot ζ_t.[6]

For each year, we estimate observations in our rolling window, by using a Metropolis-Hastings algorithm to generate draws from the posterior distribution.[7] Therefore, we estimate 34 (44) windows of observations under the 20-year (10-year) window.

Our prior choices are shown in Table 1. The intertemporal elasticity of substitution σ follows a Gamma prior with mean 2 and standard deviation 0.75. The Calvo parameter ξ_p follows a Beta distribution with mean 0.7 and standard deviation 0.1.[8] The monetary policy reaction coefficients for inflation and the output gap follow $N(1.1, 0.35)$ and $N(0.25, 0.12)$ distributions. We choose a mean just above one, with a sufficiently large standard deviation, for the inflation response, to let the data speak freely on the probability of determinacy versus indeterminacy. All the autoregressive parameters follow Beta distributions to maintain their support between 0 and 1.

For the parameters related to heterogeneous expectations, we assume a Beta prior with mean 0.7 and standard deviation 0.2 for the shares of non-RE agents n_x and n_π; these priors slightly favor the benchmark hypothesis of rational expectations, but they are sufficiently diffuse to allow large deviations.[9] We assume a Gamma prior with mean equal to 1 and standard deviation 0.5 for θ_x and θ_π: we center the prior at one, given the importance of this value for determinacy/indeterminacy, and we allow for a large standard deviation, since the parameter may be expected, a priori, to be closer to zero, at least in some parts of the sample.

Finally, we assume inverse Gamma distributions for the fundamental and sunspot shocks; the sunspot is assumed to be potentially correlated with the fundamental shocks. We use a $U(-1, 1)$ prior for the correlation coefficients. As discussed in Bianchi and Nicoló (2019), leaving the correlations unrestricted guarantees that the results don't depend on which expectational error (related

[6] Bianchi and Nicoló's methodology is a simplification over previous approaches proposed by Lubik and Schorfheide (2004) and Farmer et al. (2015); in all cases, however, the solutions under indeterminacy remain equivalent.

[7] For each year in the sample, we use 300,000 draws and discard the first 33% as burn-in.

[8] The slope of the Phillips curve κ depends also on the elasticity of marginal costs to income ω and on the steady-state markup of prices over marginal costs ϵ. These parameters are not separately identified; therefore, we fix $\omega = 2$ and $\epsilon = 11$ in the estimation.

[9] We have also experimented with Uniform and Beta(0.5,0.25) priors with similar results.

to inflation or the output gap) is included in the appended equation for the auxiliary variable ω_t.[10]

We also acknowledge a potential criticism, raised in Pagan and Robinson (2020) and Canova and Ferroni (2019), which applies to estimations that include more shocks than observables, and which also applies for our estimation under indeterminacy. In the case of indeterminacy, we have four disturbances (including the sunspot) and three observables. The previous papers show that, in such situations, the shocks may end up being correlated (even if they are ex-ante assumed as uncorrelated), thus potentially muddling their interpretation as structural shocks and the interpretation of impulse responses. In our case, we already assume an unrestricted correlation structure between the sunspot and the remaining fundamental shocks. When we compute impulse responses, we follow the same approach as Bianchi and Nicoló (2019), by orthogonalizing the shocks using a Cholesky decomposition, with the sunspot treated as the most exogenous shock.[11]

4 Empirical Results

4.1 Homogeneous vs. Heterogeneous Expectations

Before turning to the analysis of time variation in the model parameters and determinacy of the equilibrium, we provide evidence on the importance of heterogeneous expectations. Figure 3 reports the marginal likelihoods for model specifications with different expectational assumptions over all windows. The model with heterogeneous expectations outperforms the rational, homogeneous, expectations specification unambiguously and everywhere in the sample. According to Jeffreys' scale of evidence, the strength of evidence is to be considered as 'decisive', with Bayes factors abundantly above 100 in each window. This result emphasizes the importance of considering heterogeneous expectations more broadly in the models that are used to study business cycles and monetary policy decisions.

4.2 Parameter Variation

Most DSGE model estimations force the parameters to remain constant over the whole sample of interest. Here, we allow all parameters to vary over time, through our rolling window estimation.

Figures 4, 5, and 6, show the evolution of the estimated posterior means, along with the associated 90% credibility bands, for three sets of parameters: structural, including policy, parameters,

[10]Bianchi and Nicoló (2019) argue that the two representations would deliver the same fit to the data if the correlations are left unrestricted, and suggest setting uniform prior distributions over the (-1,1) interval.

[11]The main findings are robust to alternative orderings of the sunspot shock.

expectation parameters, and parameters related to exogenous shocks.

Monetary policy responds to inflation quite passively in the early samples spanning the years between the 1950s and the mid-1970s, with coefficient between 0.4 and 0.6. The response is slightly lower than 1 later on, and it jumps above one in correspondence of the window starting in 1978 (it reverts back occasionally afterwards). The response to the output gap ψ_x is lower in windows that include the 1950s-1960s and 1980s-2000s, but it goes up as a result of the 1970s. Therefore, the pre-Volcker estimates regarding the inflation response are in line with those found in Clarida, Galí, and Gertler (2000) and Lubik and Schorfheide (2004); our post-Volcker estimates, instead, remain on lower ranges.[12] The higher response to the output gap around the 1970s is also consistent with the historical evidence, as it is documented that the Federal Reserve believed that there was an exploitable trade-off between inflation and unemployment (Cogley and Sargent, 2001). As a result, they didn't impede the run-up of inflation to try to reduce unemployment. This behavior is also reflected in our rolling estimates of the steady-state inflation rate, which follows the same trajectory, by rising in the 1970s and falling after the 1980s. The intertemporal elasticity of substitution and the degree of price stickiness increase somewhat over the sample; increases in prices stickiness are consistent with the progressive reduction in the slope of the Phillips curve that has been documented in various studies. The estimates of γ capture the productivity slowdown in the 1970s, with posterior means falling below 0.6, down from the highs around 0.8 in the 1960s and 1990s.

The paper also provides evidence on time variation regarding the importance of heterogeneous expectations. The shares of RE agents are high over the initial windows (with values around 0.8 or above), but they subsequently fall around 0.2 for output expectations and 0.4 for inflation expectations. It's possible that backward-looking expectations become more relevant in the samples centered in the 1970s, when the persistence of inflation increases, and when the economy becomes more unstable. The share of boundedly-rational expectations in inflation jumps back to 0.8 in correspondence of the 1978-1998 window. The shares point to evidence of large deviations from rationality: backward-looking expectations are more prevalent than rational expectations for vast periods of time. The parameters related to expectations also drift over the sample. For output, θ_x fluctuates in the neighborhood of 1 for most of the sample. For inflation, expectations are often adaptive, with $\theta_\pi < 1$, but they become extrapolative for the windows starting in 1977-1978.[13]

[12] Likely, our lower post-Volcker estimates are the result of a less informative prior than the N(1.5,0.25) typically assumed in the DSGE literature.

[13] A rise in the perceived persistence of inflation in the 1970s is also estimated in models where agents form

Our results can be compared to those in Elias (2021), who estimates the model under a different set of heterogeneous expectations (and maintaining determinacy everywhere). In his model, agents can either form expectations based on a correctly-specified learning model (which can converge asymptotically to the rational expectations equilibrium) or on misspecified models, which include only the aggregate demand or the aggregate supply disturbance. He finds that around 60-70% of agents use the misspecified models, and that the shares are not too different in the pre- and post-Great Moderation periods. With a different set of expectation formation mechanisms, our estimates similarly highlight the key role of bounded rationality, with shares of adaptive expectations around 60-80% in the corresponding samples.

We recognize that the model with heterogeneous expectations has similarities with DSGE models that introduce lagged components in the Phillips and IS curves through the assumptions of price indexation and habit formation in consumption. Several papers estimate the relative importance of backward versus forward-looking behavior in hybrid equations, often focusing on the New Keynesian Phillips curve (Galí and Gertler, 1999, Sheedy, 2010). Single-equation estimation usually find a dominant role for forward-looking terms. However, Lindé (2005), shows, using Monte Carlo simulations, that single-equation GMM estimates are biased and found to overestimate the true degree of forward-lookingness. He argues, instead, for a full-information likelihood approach, as the one we use here. Under a full-information estimation, backward-looking terms for inflation and output are more precisely estimated and usually more important. Our estimates are in line with this finding, as we show that backward-looking terms in expectations are predominant for most of the sample.

Finally, as widely documented in the literature related to the 'Great Moderation' (e.g., McConnell and Perez-Quiros, 2000, Sims and Zha, 2006, Justiniano and Primiceri, 2008), the standard deviations of the exogenous shocks appear to decrease in the later samples: from 0.6 to 0.3 for demand shocks, from 0.3 to 0.18 for cost-push shocks, and from 0.32 to 0.13 for monetary policy shocks.

4.3 Heterogeneous Expectations & Equilibrium Determinacy/Indeterminacy

In the model with heterogeneous expectations, determinacy of the equilibrium depends on the interaction between monetary policy and expectation formation parameters. As shown in Figures 1 and

expectations through adaptive learning, as in Milani (2007, 2011).

2, when beliefs are characterized by $\theta < 1$, larger shares of boundedly-rational agents make determinacy much more likely, and, under some parameter configurations, almost guaranteed. When $\theta > 1$, the Taylor principle does not suffice, and the economy may be much more prone to sunspot-driven fluctuations.

We let the data speak about the likelihood of unique versus multiple equilibria. Figure 7 shows the posterior model probability for the model specifications with determinacy versus indeterminacy over the sample. Indeterminacy prevails for estimation windows between 1955-1975 until 1961-1981, with model probabilities reaching up to 1, while the data prefer determinacy starting from samples beginning in 1962/1963. Despite evidence of quite passive monetary policy in this period (as shown in Figure 4), adaptive behavior in the formation of expectations, especially with inflation expectation coefficients θ_π falling below 0.5 and share of non-RE agents above 50%, restores determinacy in the system. The data then reject indeterminacy clearly, until encountering windows starting in 1974, when the probability of indeterminacy rises to 0.3. As seen in Figure 5, expectations show signs of extrapolative behavior in the late 1970s, in a period when monetary policy starts reacting more aggressively toward high inflation (windows that start around Volcker's appointment have Taylor rule coefficients to inflation above one). Extrapolative expectations about output (in 1974) and, more importantly, about inflation (between 1976 and 1979) increase the probability of indeterminate equilibria, which rises to values around 0.5 and 0.7. In the absence of heterogeneous expectations, indeterminacy would be ruled out in these samples. Finally, post-1983 windows are consistent with determinate equilibria. Even though large shares of non-RE agents populate the economy, their expectations are formed in an adaptive manner, and, therefore, they represent a stabilizing force in the system.

As a caveat, we need to point out that our paper follows the same approach of Clarida, Gal, and Gertler (2000), and Lubik and Schorfheide (2004), in focusing exclusively on monetary policy, at the expense of fiscal policy. The potential importance of fiscal-monetary policy interactions for indeterminacy has been documented in Leeper (1991), Davig et al. (2007), and Bianchi and Ilut (2017), all assuming rational homogeneous expectations. Gasteiger (2018), instead, investigates policy interactions in a model with bounded rationality and heterogeneous expectations, and he examines their implications for equilibrium determinacy. It's an open question whether the increased importance of extrapolative expectations that we uncover in the 1970s remains a feature of the data in an extended model with a more extensive fiscal policy block.

4.4 Propagation of Shocks

Passive monetary policy and/or extrapolative expectations may cause the economy to be driven by sunspot shocks. Figure 8 shows the share of output gap and inflation variance that can be explained by sunspots over time. Sunspot-driven fluctuations are important during the early windows, with sunspot shocks accounting for 15-20% of the variability in output and inflation. They become relevant again in the samples around the late 1970s and very early 1980s, when they are driven by extrapolative expectations in inflation. In the same periods, however, they don't cause large fluctuations in the output gap.

The propagation of fundamental shocks in the economy is also considerably different in the cases of determinacy versus indeterminacy, and under homogeneous versus heterogeneous expectations.

Figure 9 shows the responses of macroeconomic variables to shocks over three different samples: the first is representative of the 1950-1970 samples, which is characterized by passive monetary policy and indeterminacy; the second refers to the later part of the sample (1985-2005), with a stable macroeconomic environment and active policy; the last one relates to the sample starting in 1978, where there's uncertainty about determinacy versus indeterminacy and expectations appear extrapolative.

In the post-1985 samples, the responses are unsurprising. Monetary policy contractions lead to declines in the output gap (due to the effects of higher real rates on private consumption operating through the Euler equation) and inflation, positive demand shocks generate sluggish increases in inflation and interest rates, and cost-push shocks have small effects on real activity. Sunspots don't matter as the equilibrium is determinate in this period. By adding lags in the system, heterogeneous expectations make the responses more inertial than they would be under purely rational expectations in this context.

The responses are substantially different in the 1958-1978 sample, chosen to represent the early windows. Under indeterminacy, contractionary monetary policy is unable to contain inflationary pressures: inflation strongly increases in response to positive monetary policy shocks. Demand shocks induce an opposite response of inflation compared with the determinate case. Negative supply shocks induce higher inflation, lower interest rates, and have much larger recessionary effects than in later windows. Indeterminacy hence substantially alters the transmission of traditional shocks. Indeterminacy is preferred by the data in this period because it allows the model to fit responses of output and inflation that are consistent with stagflation (with output and inflation responding in opposite ways to inflationary shocks), as well as episodes of rising inflation in response to contractionary monetary policy, which were both typical in the 1970s. In addition to fundamental shocks, the economy in the period is under the influence of sunspots: the sunspot ζ_t moves output gap, inflation, and interest rates in the same direction. The responses to the sunspot shock in the pre-Volcker period, therefore, have the same signs as those found in Lubik and Schorfheide (2004). The mechanism is similar: the sunspot raises inflation expectations and it reduces the ex-ante real rate. As a result, output increases, leading to further, and self-confirming, increases in inflation.

Finally, the responses for the 1978-1998 window, which has almost equal probability of determinacy/indeterminacy, and is characterized by trend-chasing expectations, fall typically closer to the case of determinacy, but with some exceptions. In particular, the responses of interest rates to demand and supply shocks are positive and larger than they are under determinacy post-1985. Sunspot shocks matter for inflation, although to a lower extent than in the previous windows, and they are quantitatively unimportant for output.

5 Robustness

Our benchmark estimation assumed a conventional timing in the Taylor rule, which allows the central bank to respond to contemporaneous values of inflation and the output gap. A possible alternative consists of assuming that the central bank uses a forecast-based, forward-looking, Taylor rule:

$$i_t = \rho_i i_{t-1} + (1 - \rho_i)(\psi_\pi E_t \pi_{t+1} + \psi_x E_t x_{t+1}) + \varepsilon_t. \tag{13}$$

Responding to forecasts is consistent with central bank parlance, but it is well known to be potentially destabilizing in rational expectations models (Batini and Haldane, 1999, Bernanke and Woodford, 1997). Policies that respond to private-sector forecasts can, in fact, make the equilibrium in the economy more sensitive to expectations than under a contemporaneous policy rule. This is no different for New Keynesian models with heterogeneous expectations. We re-estimate the model with the alternative Taylor rule in the same way using 20-year rolling windows.

Figure 10 shows how the evidence in favor of determinacy versus indeterminacy, measured by the posterior model probabilities, changes over the sample. We compare the results with those obtained for the benchmark case that was reported in Figure 7. The two cases have similar implications:

indeterminacy prevails in the early windows that have their latest observations in the early 1980s. The samples that include Volcker's disinflation, even if they start in the 1960s, are instead consistent with determinacy, as are the post-1984 samples. As expected, the forward-looking Taylor rule raises the probability of indeterminacy, which is now close to one even for windows starting in 1979 to 1983. Again, indeterminacy is then driven by extrapolative behavior in the formation of expectations.

As an additional robustness test, we analyze how the results vary when we shorten the rolling window used in the estimation to ten years. The model probabilities display more variability, as expected given the shorter windows. But the results are overall comparable: indeterminacy prevails for early windows up to those starting in 1973 and ending in 1983, although with various exceptions, suggesting much more pronounced uncertainty with short windows. The subsequent samples are generally consistent with a determinate equilibrium. The main exception is given here by the 1989-1999 sample, which can be explained equally well as the outcome of determinacy or indeterminacy.

One potential concern is that the model, particularly under indeterminacy, might imply unrealistic jumps in inflation expectations. To show that this is not the case, we first re-estimate the model, now adding inflation expectations and output expectations from the Survey of Professional Forecasters to the set of observables that need to be matched in the measurement equations (11). Figure 11 shows the model-implied inflation expectations from the baseline model, compared with the implied expectations from the estimated model with survey forecasts as observables, and along with actual inflation and observed inflation expectation series. We report the expectations for two sample windows: 1959-1979 and 1984-2004. In the first window, implied expectations from the estimated model with the additional observables are slightly closer to the survey data, but overall the differences are small. For the post-1984 sample, the model has more trouble matching survey forecasts in the 1990s, but the model-implied expectations remain similar regardless of whether they were required to match observed expectations or not.

5.1 The Role of Asymmetry in Output and Inflation Forecasts

In the baseline estimation, we have allowed the shares of boundedly-rational agents and expectation parameters to be different between inflation and output expectations. We re-estimate the model, but now forcing the parameters to be identical across forecasts of different variables.

First, as shown in 12, the evidence for heterogeneous expectations versus homogeneous expectations remain decisive over the whole sample. However, with this restriction, the evidence for indeterminacy would become much weaker: except for few early windows, in which it reaches values around 0.2, the probability of indeterminacy would be zero almost at all times. But, as shown in the lower panel of 12, the fit of the model would be much worse. As made clear in the Figure, the fit under determinacy remains similar regardless of whether we assume symmetric or asymmetric n and θ parameters (the dark blue and dashed light blue lines overlap). But the assumption of identical n and θ for output and inflation appears too restrictive under indeterminacy. The fit of the model is much lower everywhere in the sample than the fit obtained when allowing different expectation parameters. Therefore, we present the results obtained under asymmetric n and θ as our baseline findings, and show here, for robustness, the impact on the evidence of indeterminacy of forcing them to be symmetric.

6 Conclusions

We estimated a New Keynesian model with heterogeneous expectations to assess the extent of parameter variation and the probability of determinacy and indeterminacy over the sample.

Our empirical results provide substantial evidence in favor of heterogeneous expectations. The model with expectations heterogeneity dominates the benchmark with homogeneous expectations decisively and everywhere in the sample. Large shares of agents, at points reaching 80%, who are forming expectations that deviate from the rational expectations hypothesis exist in the economy. The shares of non-RE agents for output and inflation expectations, and the details regarding their expectation process change over the sample. Inflation expectations are typically adaptive, but they become extrapolative for rolling windows starting in the late 1970s, around Volcker's disinflation.

The existence of heterogeneous expectations and the degree of adaptive and extrapolative behavior may modify the determinacy conditions in the New Keynesian model and, therefore, potentially overturn well-known results in the literature. When taken to the data, the model with heterogeneous expectations suggests indeterminacy as a feature that is needed to explain observations in the early windows, while determinacy dominates the post-1984 samples. But, under heterogeneous expectations, we uncover new periods that are consistent with large probabilities of indeterminacy, such as samples that begin in the late 1970s, when inflation expectations turn extrapolative.

In light of the favorable empirical evidence for heterogeneous expectations, both at the micro, experimental, and macro level, future research should further investigate the implications of heterogeneity in a wide variety of DGSE models.

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Prior Distributions			
IES	$\sigma \sim \Gamma(2, 0.75)$	AR Demand	$\rho_g \sim B(0.5, 0.2)$
Calvo price stick.	$\xi_p \sim B(0.7, 0.1)$	AR Cost-Push	$\rho_u \sim B(0.5, 0.2)$
Int. Rate Smooth.	$\rho_i \sim B(0.7, 0.2)$	Std MP shock	$\sigma_\varepsilon \sim IG(0.5, 2)$
MP Infl	$\psi_\pi \sim N(1.1, 0.35)$	Std Demand shock	$\sigma_g \sim IG(0.5, 2)$
MP Output Gap	$\psi_x \sim N(0.25, 0.12)$	Std Cost-Push shock	$\sigma_u \sim IG(0.5, 2)$
Share non-RE in Output	$n_x \sim B(0.7, 0.2)$	Std Sunspot shock	$\sigma_\zeta \sim IG(0.5, 2)$
Param. Output Exp	$\theta_x \sim \Gamma(1, 0.5)$	Corr Sunspot & MP shock	$\sigma_{\zeta,\varepsilon} \sim U(-1, 1)$
Share non-RE in Infl	$n_\pi \sim B(0.7, 0.2)$	Corr Sunspot & Dem shock	$\sigma_{\zeta,\varepsilon^g} \sim U(-1, 1)$
Param. Infl Exp	$\theta_\pi \sim \Gamma(1, 0.5)$	Corr Sunspot & CP shock	$\sigma_{\zeta,\varepsilon^u} \sim U(-1, 1)$
Output growth	$\gamma \sim N(0.675, 0.15)$		
S.S. infl	$\bar{\pi} \sim \Gamma(0.75, 0.4)$		
S.S. real rate	$\bar{r} \sim \Gamma(0.5, 0.35)$		

Table 1: Prior Distributions for estimated coefficients.

Note: Γ refers to Gamma distribution, B to Beta, N to Normal, IG to Inverse Gamma, and U to Uniform. The numbers in parenthesis refer to mean and standard deviations, with the exception of Uniform priors, for which we report the lower and upper bounds.

		Kolmogorov-Smirnov Test					
		Unique Sol.		Indeterminacy		Instability	
		d_{n_s,n_i}	p-value	d_{n_s,n_i}	p-value	d_{n_s,n_i}	p-value
IES	σ	0.063	0.159	0.109	0.263	0.034	0.793
Calvo price stick.	ξ_p	0.036	0.792	0.123	0.152	0.037	0.678
Int. Rate Smooth.	ρ_i	0.063	0.154	0.087	0.530	0.047	0.377
MP Infl	ψ_π	**0.106**	**0.002**	**0.155**	**0.032**	0.058	0.158
MP Output Gap	ψ_x	0.036	0.806	0.096	0.407	0.048	0.361
Share non-RE in Output	n_x	**0.192**	**0.000**	**0.328**	**0.000**	**0.237**	**0.000**
Param. Output Exp	θ_x	**0.156**	**0.000**	**0.340**	**0.000**	**0.199**	**0.000**
Share non-RE in Infl	n_π	**0.288**	**0.000**	**0.484**	**0.000**	**0.357**	**0.000**
Param. Infl Exp	θ_π	**0.276**	**0.000**	**0.319**	**0.000**	**0.299**	**0.000**
Output growth	γ	0.058	0.236	0.107	0.275	0.040	0.588
S.S. infl	$\bar{\pi}$	0.023	0.995	0.056	0.953	0.023	0.989
S.S. real rate	$\bar{r}$	0.045	0.525	0.069	0.806	0.041	0.571
AR Demand	ρ_g	0.028	0.960	0.113	0.227	0.031	0.867
AR Cost-push	ρ_u	0.047	0.484	0.079	0.651	0.039	0.638

Table 2: Kolmogorov-Smirnov Test for estimated coefficients. Note: the table shows the d-statistic and p-values for the Kolmogorov-Smirnov test for all the estimated parameters. Entries in bold indicate a failure to reject the null of equal CDFs at the 5% significance level for the corresponding parameters. This indicates that the parameters are responsible in driving the existence of a unique solution, indeterminacy, or instability, in the model.

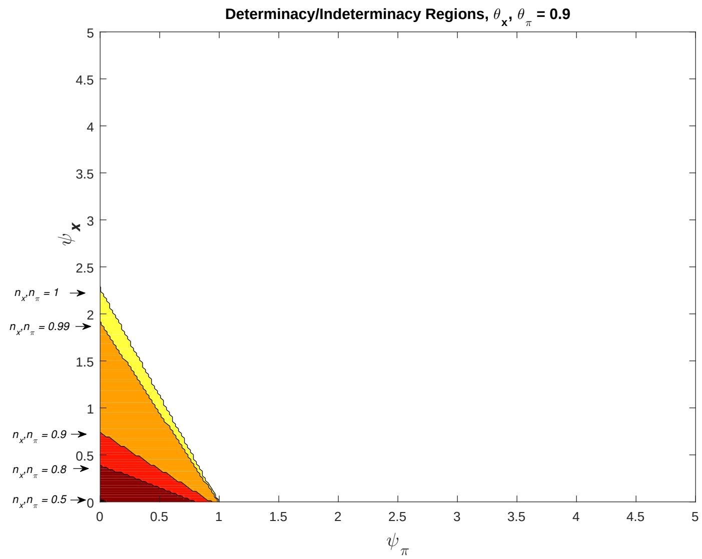

Figure 1: Determinacy and Indeterminacy Regions: case with $\theta_\pi = \theta_x = 0.9$.

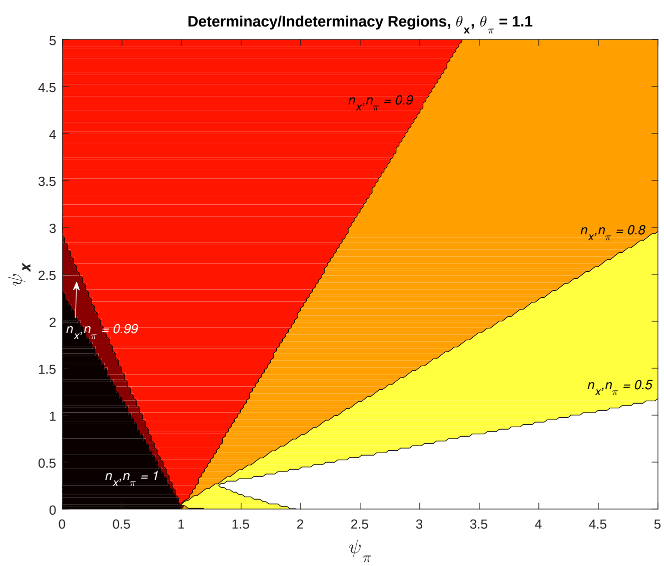

Figure 2: Determinacy and Indeterminacy Regions: case with $\theta_\pi = \theta_x = 1.1$.

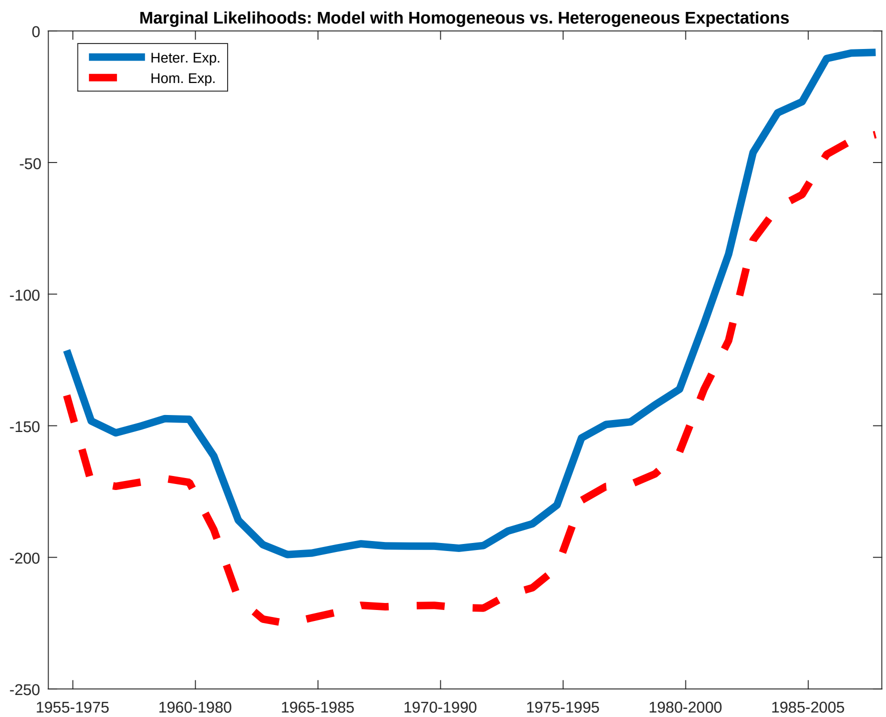

Figure 3: Bayesian Model Comparison: marginal likelihoods for the New Keynesian model under homogeneous (red, dashed) versus heterogeneous (blue, solid) expectations over time.

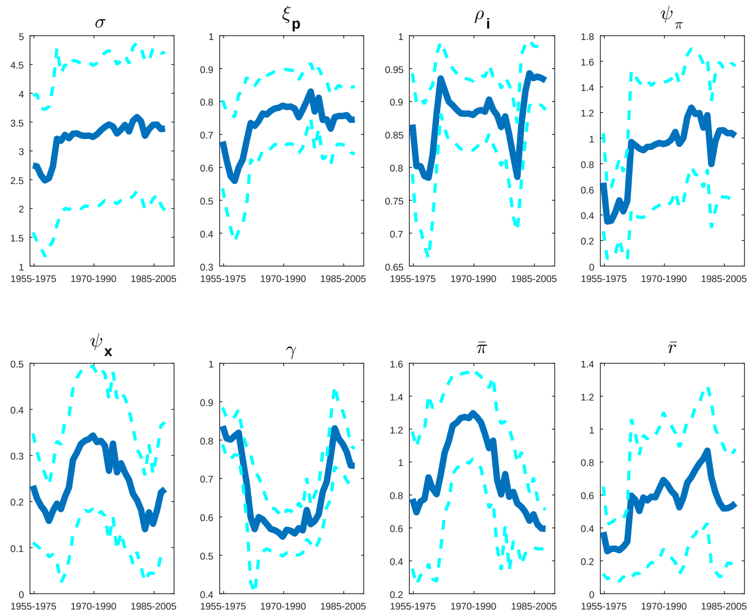

Figure 4: Parameter variation over the sample: 20-year rolling window estimation, 'structural' parameters.

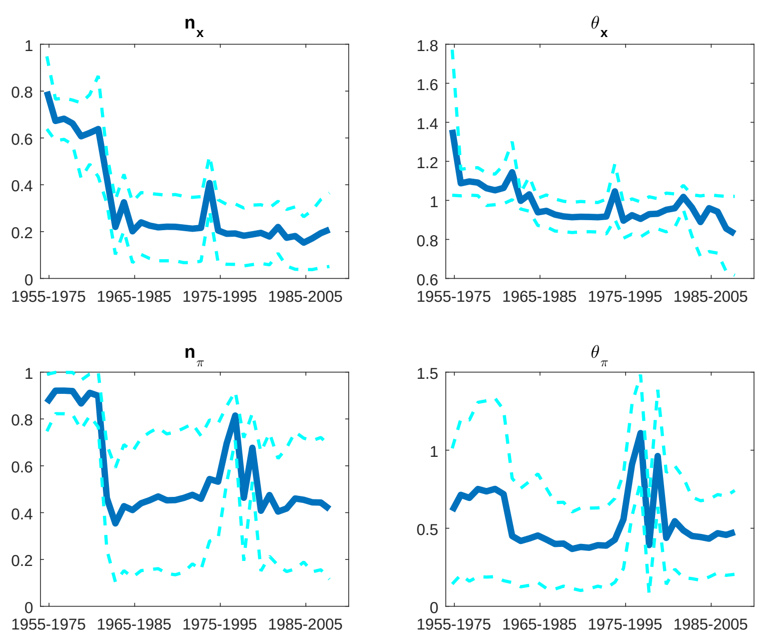

Figure 5: Parameter variation over the sample: 20-year rolling window estimation, expectation parameters.

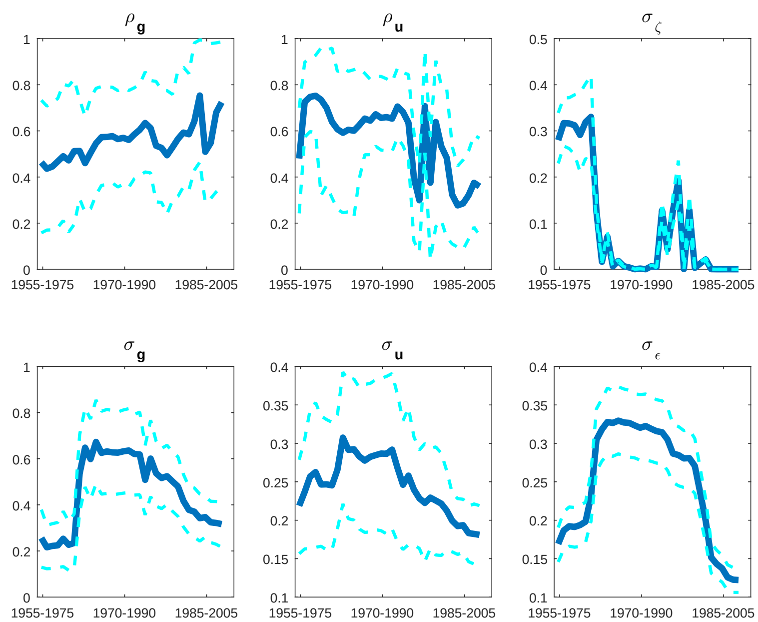

Figure 6: Parameter variation over the sample: 20-year rolling window estimation, disturbance parameters.

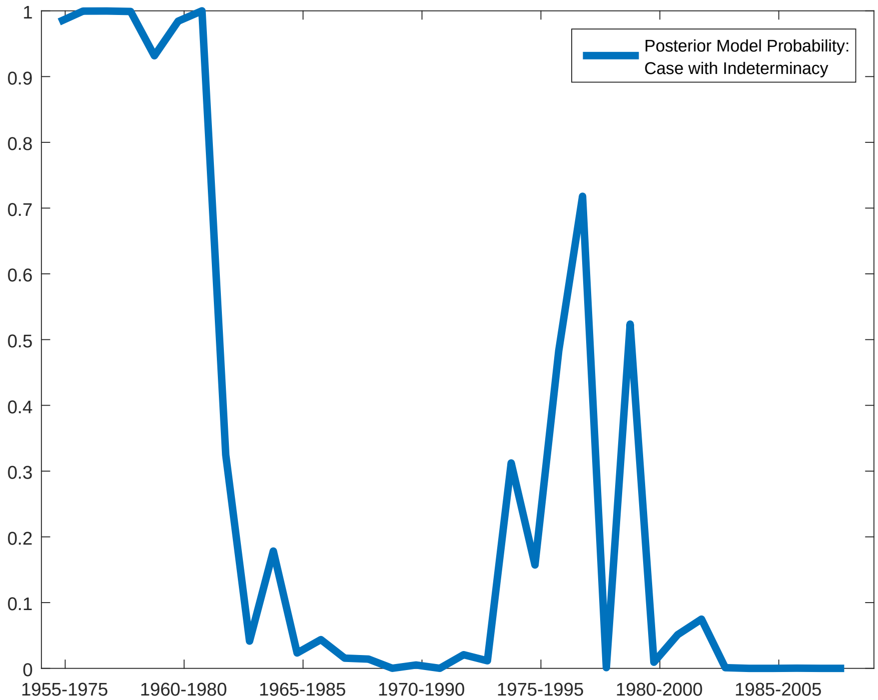

Figure 7: Posterior Model Probabilities over time: Specification with Indeterminacy versus Determinacy.

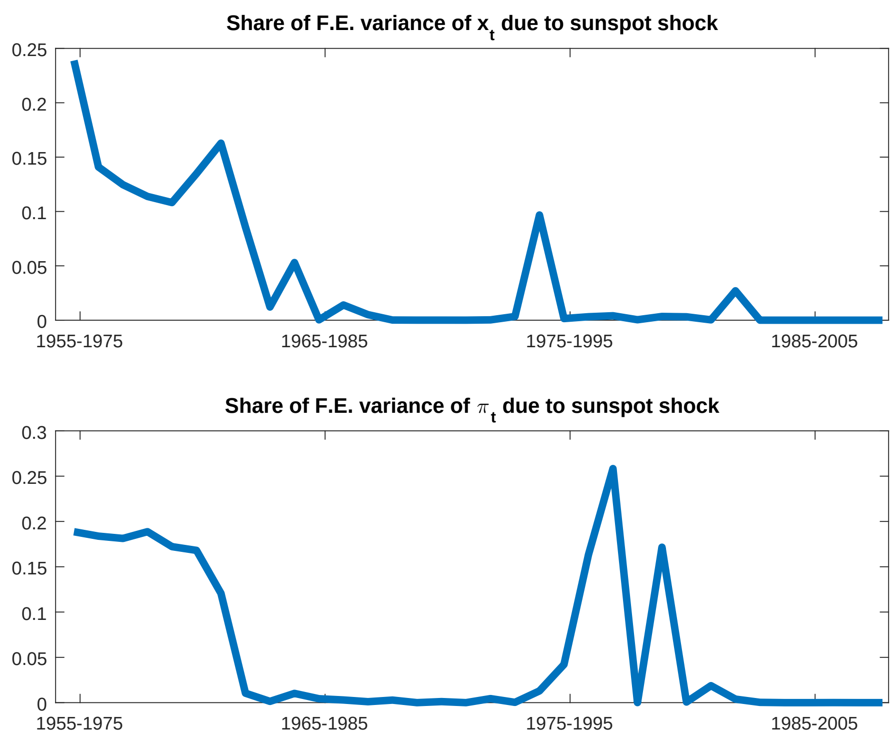

Figure 8: Forecast Error Variance Decomposition: share of variance in output gap (top panel) and inflation (bottom panel) explained by sunspot shock, over the sample.

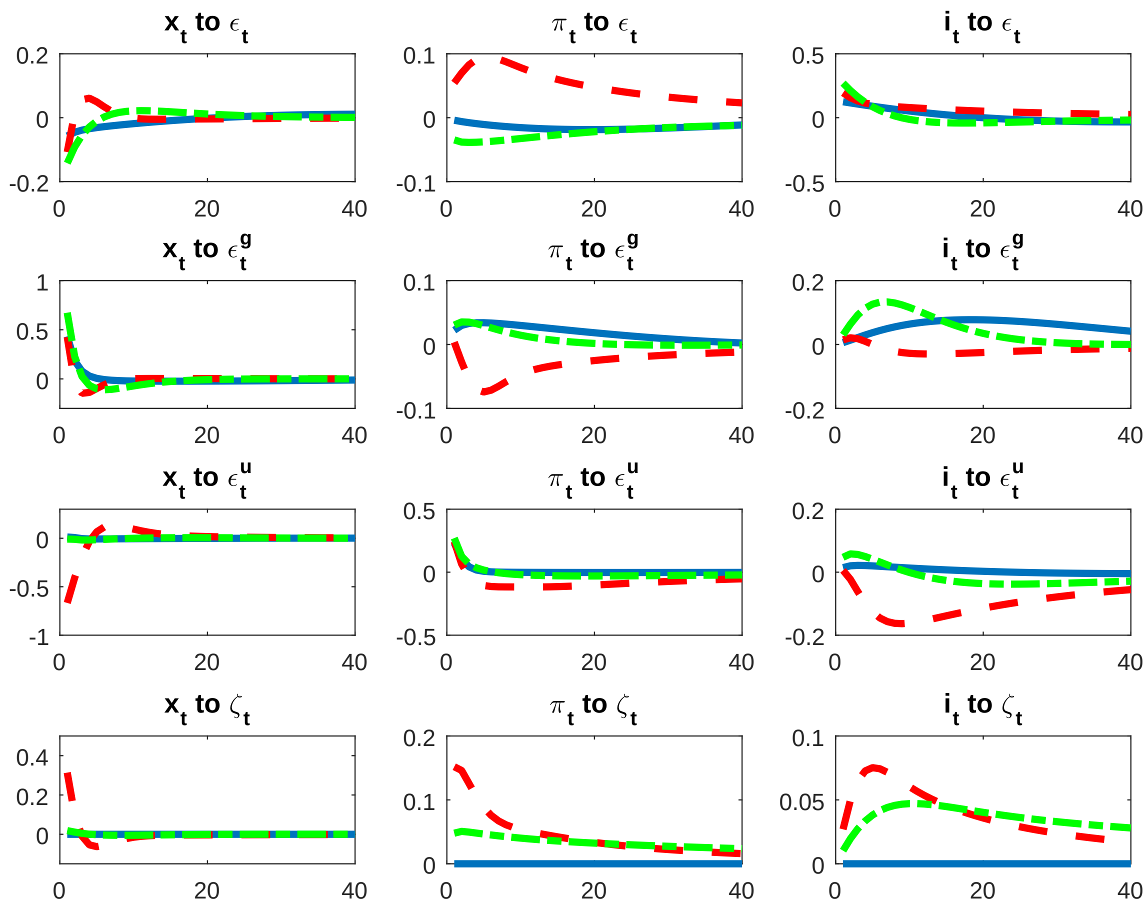

Figure 9: Propagation of Fundamental and Sunspot Shocks in different samples.

Note: The figure shows the response of the output gap, inflation, and interest rates, to positive, one-standard-deviation, monetary policy shocks (first row), demand shocks (second row), cost-push shocks (third row), and sunspot shocks (fourth row). Impulse responses are means across Metropolis-Hastings draws. The solid blue line refers to the 1985-2005 window, the dash-dot green line to the 1978-1998 window, and the dashed red line to the 1958-1978 window.

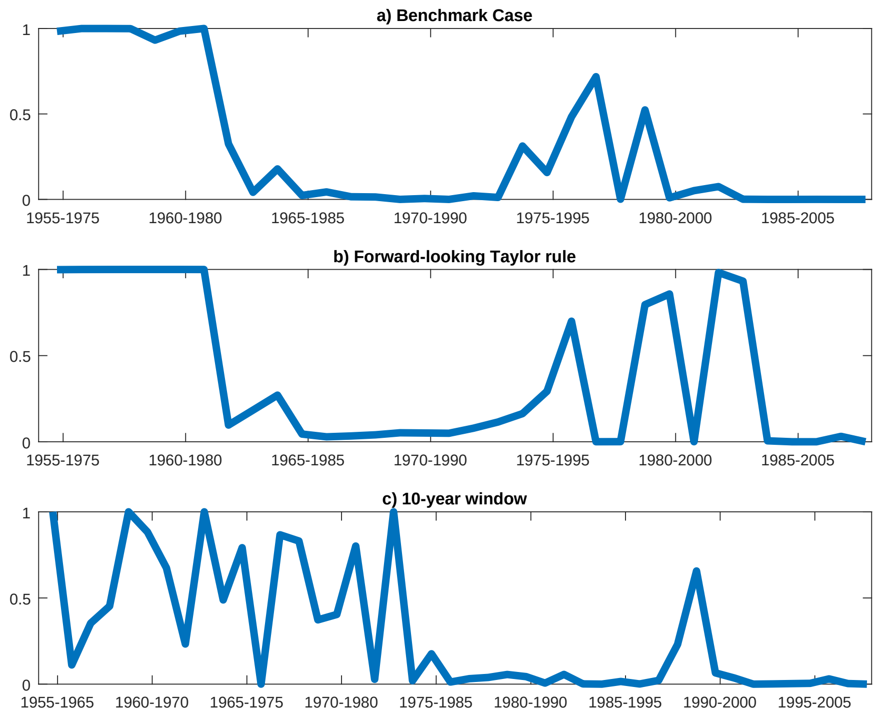

Figure 10: Robustness analysis: posterior model probabilities in favor of indeterminacy versus determinacy.

Notes: The figure shows the posterior model probabilities for three cases: a) the same benchmark case presented in Figure 7 (added for comparison) with 20-year rolling windows; b) the case with a forward-looking Taylor rule; c) the case with a 10-year rolling window.

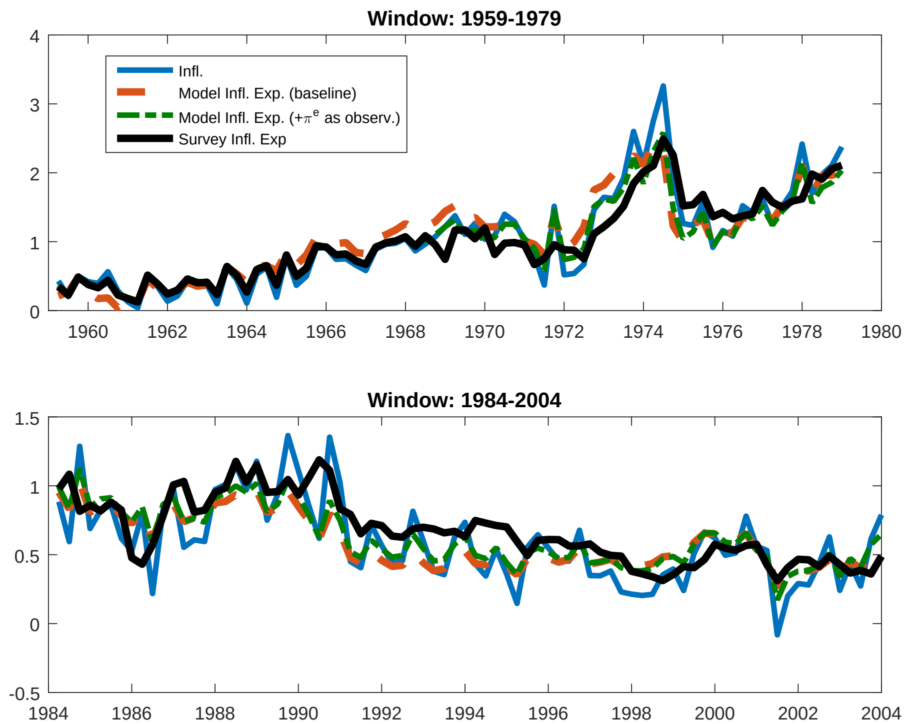

Figure 11: Model-implied inflation expectations.

Note: The figure shows model-implied inflation expectations obtained from the baseline estimation with heterogeneous expectations and from the estimation that included survey inflation and output expectations to the set of observables. The model-implied expectations are compared with the actual inflation series and survey inflation forecasts.

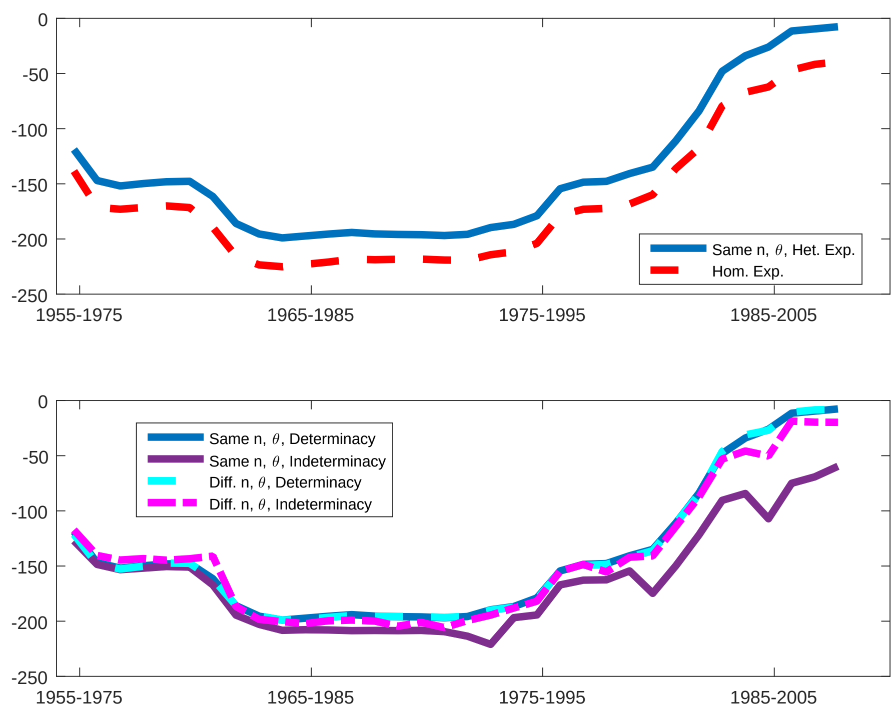

Figure 12: Marginal Likelihood Comparison: identical and different n and θ parameters across output and inflation forecasts.