Adaptive Learning and Macroeconomic Inertia in the Euro Area*

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Abstract

This article aims to study the determinants of macroeconomic inertia in the euro area. To this end, it estimates a simple monetary DSGE model with private-sector learning, but which also includes more structural sources of inertia, such as habit formation in consumption and inflation indexation. Economic agents are assumed to form near-rational expectations and to learn the model parameters over time. Likelihood-based Bayesian methods are used to estimate the agents' beliefs jointly within the system and to provide evidence on the fit of alternative learning rules. The results show that European macroeconomic inertia has only moderately changed over the sample. The evidence is consistent with a small gain coefficient and low degrees of habits and indexation, although some uncertainty remains after the estimation.

Introduction

Macroeconomic variables display considerable inertia. It is well known that the baseline general equilibrium models that are used to study monetary policy are problematic in matching the observed inertia. These models assume rational expectations and fully optimizing economic agents: the agents take optimal decisions knowing the correct model of the economy, the model parameters and other agents' expectations.

To match the inertia in the data, however, it is often necessary to consider some departures from the baseline model. One possibility implies the modification of consumers' preferences to include habit formation in consumption. Consumers are assumed to derive utility not only from the current level of consumption, but also from its deviation from an index of their own past consumption or of the aggregate past consumption. Moreover, to match the inertia in inflation, it has become common to extend the conventional Calvo pricing model to include an indexation term, as proposed by Christiano *et al.* (2005). In this way, the firms that are not allowed to set the price optimally in a given period can still revise their price by indexing it to the past aggregate inflation rate. An alternative approach to induce inertia, proposed by Galí and Gertler (1999), consists, instead, of allowing for a fraction of non-rational agents, who, rather than solving optimization problems, follow a simple rule of thumb to choose their consumption or to set prices.

All of these extensions have the effect of making lagged terms of the endogenous variables appear in the model equations, thus helping the model in capturing the inertia in the data.

The cited sources of persistence are needed in the model if one assumes rational expectations (Giannoni and Woodford, 2003; Smets and Wouters, 2003). But several authors question the strong informational assumptions required by rational expectations. Marcet and Sargent (1989) study a class of models in which the law of motion perceived by agents is affected and affects the actual law of motion of the economy, and show how the two may converge. Evans and Honkapohja (2001), similarly, consider a minimal deviation from rational expectations, by assuming that economic agents know the correct model of the economy, but need to learn the model parameters over time. To do so, agents behave as econometricians, that is they estimate simple linear models and update their estimates as soon as new data become available.

Therefore, it becomes interesting to study whether, once the rational expectations assumption is relaxed in favour of learning, large degrees of structural sources of persistence are still needed to fit the data. For any welfare analysis based on the model would depend on the chosen microfoundations. The performance of alternative monetary or fiscal policy rules, for example, would depend on whether learning, habits, indexation or rule-of-thumb behaviour represent the crucial mechanisms that induce inertia in the economy.

This article aims to disentangle the sources of inertia in the euro area by estimating a simple monetary model, which incorporates both learning and structural sources of persistence as habits and indexation. Milani (2007) has studied this issue with US data, showing that habits and indexation may

become redundant under learning. This article focuses on the euro area and, compared with Milani (2007), it tries to extrapolate more deeply the best-fitting learning rule from the estimated model. Not only, in fact, does the article estimate the learning gain coefficient jointly with the other parameters, but it also jointly estimates the initial beliefs, and tests alternative versions of the learning rule, encompassing the case of different included regressors and different gains.

As Marcet and Nicolini (2003) have emphasized, in fact, models with learning require researchers to make a number of possibly arbitrary choices. There are various degrees of freedom that can render the model hard to falsify. Examples are the choice of the gain coefficient, the initial values of agents' beliefs and, more generally, the structural form of the learning rule. This is why the article tries jointly to extrapolate the learning rule together with the deep parameters of the economy. In the simplest specification, the constant gain coefficient is jointly estimated with the rest of the model coefficients. Subsequently, the article also considers the joint estimation of the constant gain, initial beliefs and structural parameters of the economy. The Bayesian methods used in the estimation simplify this task. I also evaluate different learning rules: one that corresponds to the Minimum State Variable solution under rational expectations, a simpler rule represented by a three-variables VAR, and I consider both constant-gain and recursive least squares learning. The most appropriate learning rule may be chosen in terms of its fit to the data.

Other articles have shown the importance of learning to generate persistence in macroeconomic variables. Orphanides and Williams (2003, 2005) and Adam (2005) have shown that adaptive learning may work as a propagation mechanism in the economy, creating persistence in output and inflation. This article aims to provide additional evidence on the role of learning and it tests learning against popular devices that are used to create inertia in rational expectations models. The estimation follows Schorfheide (2000), Lubik and Schorfheide (2004), as well as Smets and Wouters (2003), among others, who employ a similar Bayesian procedure. They assume rational expectations, whereas this article exploits similar techniques to estimate a simple DSGE model with near-rational expectations and learning. Different versions of euro area models, instead, have been estimated by Smets and Wouters (2003), Coenen and Wieland (2005) and Andrés *et al.* (2006), who all work under the assumption of rational expectations.

This article finds that near-rational expectations and learning may limit the degree of structural forms of persistence necessary to fit the data. The best-fitting specification indicates slow learning and small degrees of habits and indexation. But a lot of uncertainty remains on the main sources of inertia in the euro area, since European data appear not very informative on the choice

Table 1: Euro Area Output and Inflation Data: Empirical Auto- and
Cross-Correlations

	Empirical Facts
$corr(x_t, x_{t-1})$	0.87
$corr(x_t, x_{t-4})$	0.52
$corr(\pi_t, \pi_{t-1})$	0.98
$corr(\pi_t, \pi_{t-4})$	0.89
$corr(\Delta\pi_t, \Delta\pi_{t-1})$	0.50
$corr(\Delta\pi_t, \Delta\pi_{t-4})$	−0.05
$corr(x_t, \pi_t)$	−0.06
$corr(x_t, \pi_{t-4})$	−0.28
$corr(x_t, \pi_{t+4})$	0.07
$corr(x_t, \Delta\pi_t)$	0.43
$corr(x_t, \Delta\pi_{t-4})$	0.27
$corr(x_t, \Delta\pi_{t+4})$	0.21

Source: Author's own data.

between learning and indexation. Overall, the data are suggestive of slower
learning and a larger degree of structural persistence in inflation in Europe
than in the US.

Empirical Facts: The Persistence of Output and Inflation

Before turning to the substance of this article, it is essential to discuss the
main empirical properties of euro area output and inflation series. Table 1
presents the empirical autocorrelation and cross-correlation functions derived
from estimating a monetary VAR(3) on euro-wide data using the output gap,
inflation and nominal interest rates as endogenous variables.[1]

Both inflation and the output gap are highly persistent: the first-order
autocorrelations are 0.98 for inflation and 0.87 for output gap, while the
fourth-order autocorrelations are 0.89 and 0.52. The cross-correlations
between the two variables are small and there is a positive relation only
between current inflation and lags of the output gap. Turning to the properties
of $\Delta\pi_t$, the first-difference of inflation, the table shows that this still has a
substantial first-order autocorrelation (0.50), but the autocorrelation function
decays to 0 much more rapidly. The first-difference of inflation is positively
correlated with the output gap, both contemporaneously and with the gap lags
and leads.

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[1] The estimation of the autocorrelation function via a VAR model, rather than being based directly on the
data, is motivated by Coenen and Wieland (2005) and Coenen (2005), who discuss how a parametric
assumption is helpful in estimating higher-order correlations, which would be, otherwise, unreliable due to
the finite samples.

I. The Model

This section introduces a simple New Keynesian model, which is often used as a framework to study monetary policy. The model is derived from the optimizing choices of consumers and firms, it is characterized by sticky prices, monopolistic competition, and in the version considered here it includes features that can help matching the observed inertia in macroeconomic variables, such as habit formation in consumption and inflation indexation in price-setting.

Consumers

To match the observed inertia of consumption and output, it is typically necessary to assume habit formation in consumption. Here, I assume that consumers' utility positively depends on the deviation of current consumption from a stock of internal habits, and negatively on the hours of labour supplied h_t. Each household maximizes

$$E_t \left\{ \sum_{T=t}^{\infty} \beta^{T-t} \left[U(C_T - \eta C_{T-1}; \zeta_t) - \int_0^1 v(h_T(j); \zeta_t) dj \right] \right\} \tag{1}$$

subject to the period budget constraint

$$M_t + B_t = (1 + i_{m,t-1}) M_{t-1} + (1 + i_{t-1}) B_{t-1} + P_t Y_t - T_t - P_t C_t \tag{2}$$

where $\beta \in (0,1)$ is the discount factor, C_T is an index of the household's consumption of each of the differentiated goods supplied in t, $h_T(j)$ is the amount of labour supplied for the production of each good j, ζ_T is a vector of exogenous aggregate preference shocks, the parameter $0 \leq \eta \leq 1$ measures the degree of habit formation, M_t denotes money holdings, B_t riskless bond holdings, $i_{m,t-1}$ and i_t denote nominal interest rates on money and bonds, and T_t are lump sum taxes and transfers, Y_t is household's real income in period t and P_t denotes the aggregate price level. Notice that E_t here denotes model-consistent rational expectations. I shall relax this assumption later in the article.

Producers

A continuum of monopolistically competitive firms populates the economy. Following Calvo (1983), firms are allowed to set their prices optimally with probability $(1 - \alpha)$ each period. When they cannot optimize, they follow the indexation rule proposed by Christiano *et al.* (2005).

$$\log p_t = \log p_{t-1} + \gamma \pi_{t-1} \tag{3}$$

where $0 \leq \gamma \leq 1$ represents the degree of indexation to past inflation π_{t-1}. When γ is positive, then, each firm resets its price p_t every period. Firms maximize the discounted flow of future profits

$$E_t \left\{ \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T} \left[\Pi_T \left(p_t (P_{T-1}/P_{t-1})^{\gamma} \right) \right] \right\} \tag{4}$$

where $Q_{t,T} = \beta^{T-t}(P_t \,/\, P_T)(U_C(C_T - \eta C_{T-1}; \zeta_T) \,/\, U_C(C_t - \eta C_{t-1}; \zeta_t))$ is the stochastic discount factor and $\Pi_T(\cdot)$ denotes period T firm's nominal profits.

Central Bank

Monetary policy is described by the following Taylor rule with partial adjustment

$$i_t = \rho i_{t-1} + (1-\rho)[\chi_{\pi} \pi_t + \chi_x x_t] + \varepsilon_t, \tag{5}$$

where ρ denotes the degree of interest rate smoothing, χ_{π} and χ_x are feedback coefficients to inflation and output gap and the policy shock ε_t accounts for unanticipated deviations from the systematic monetary policy rule.

It should be noted that it is not clear that monetary policy in the euro area could be simply described by a Taylor rule as (5) for the whole sample. In the pre-euro period, monetary policy was independently decided by individual central banks in each country: the expression (5) estimates a sort of 'average' Taylor rule, which might conceal more complicated interactions among national monetary policies. There is, for example, some evidence that the *Bundesbank* was acting as leader and other central banks as followers in setting interest rates.[2]

Various papers, however, have found that simple versions of the Taylor rule fit quite well the *Bundesbank*'s actual policy (Clarida *et al.*, 1998) and they are also successful in capturing the behaviour of average interest rates in the EMU area before 1998 (Gerlach and Schnabel, 2000; Gerdesmeier and Roffia, 2004; Peersman and Smets, 1999). Although clearly an approximation, the choice of estimating a pre-euro common monetary policy rule is typical in recent empirical studies (Smets and Wouters, 2003).

[2] Giavazzi and Giovannini (1987) present evidence that Germany was the anchor country during the European monetary system (EMS). The hypothesis that monetary authorities in EMS countries were following the *Bundesbank*, the so-called 'German dominance' hypothesis, has been tested by Von Hagen and Fratianni (1990), among others, but the empirical evidence has generally been controversial.

Aggregate Macroeconomic Dynamics

The aggregate dynamics of the economy can be summarized by the following equations, which are derived by solving the optimization problems previously outlined and log-linearizing the implied first-order conditions around a zero-inflation steady-state:[3]

$$\hat{x}_t = E_t\hat{x}_{t+1} - (1-\beta\eta)\sigma\left[i_t - \hat{E}_t\pi_{t+1} - r_{t,n}\right] \tag{6}$$

$$\hat{\pi}_t = \xi_p\left[\omega x_t + [(1-\eta\beta)\sigma]^{-1}\hat{x}_t\right] + \beta\hat{E}_t\hat{\pi}_{t+1} + u_t \tag{7}$$

$$i_t = \rho i_{t-1} + (1-\rho)[\chi_\pi\pi_t + \chi_x x_t] + \varepsilon_t, \tag{8}$$

where

$$\hat{\pi}_t \equiv \pi_t - \gamma\pi_{t-1} \tag{9}$$

$$\hat{x}_t \equiv (x_t - \eta x_{t-1}) - \beta\eta\hat{E}_t(x_{t+1} - \eta x_t), \tag{10}$$

and where σ denotes the intertemporal elasticity of substitution (in the absence of habit formation), ξ_p is a composite parameter that negatively depends on the degree of price stickiness α, ω is the elasticity of marginal costs to changes in income, $r_{t,n}$ is the natural real interest rate and u_t is a cost-push shock that can arise endogenously in the model.

The model departs from the conventional rational expectations assumption: $\hat{E}_t$ now denotes subjective (possibly non-rational) expectations.[4]

When the microfoundations of the model with learning are taken seriously, agents are assumed to know: (1) their own preferences; (2) their own constraints; (3) how to solve their optimal decision problems. Agents, however, do not know other agents' preferences. To solve their decision problems, economic agents need to form expectations about future aggregate conditions, i.e. future inflation rates, and future output gaps, in equations (6) and (7). Since they do not know other agents' preference parameters, they need to infer the reduced-form coefficients of the economy from the history of aggregate data. I start by assuming that the agents adopt the following forecasting rule to form their expectations

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[3] The details of the derivation, which can be found in Woodford (2003).
[4] Preston (2005) shows that when learning is introduced from the 'primitives' of the model, the derived linearized equations one obtains differ from those under rational expectations, since long-horizon expectations also matter (this case has been empirically analysed in Milani, 2006). The linearized equations under learning, however, reduce to those obtained under rational expectations under the conditions described by Honkapohja *et al.* (2003), i.e. that agents need to recognize that the market clearing condition $C_t = Y_t$ holds in every period.

$$Z_t = a_t + b_t Z_{t-1} + c_t u_t + d_t r_{t,n} + \varepsilon_t \tag{11}$$

where $Z_t \equiv [\pi_t, x_t, i_t]'$, and a_t, b_t, c_t, d_t are coefficient vectors and matrices of appropriate dimensions. Therefore, the agents use a Perceived Law of Motion (PLM) that corresponds to the Minimum State Variable solution of the system under RE. Their PLM has the same structural form of the RE solution, but possibly different parameter values.

Agents use historical data to infer the PLM coefficients (a_t, b_t, c_t, d_t) and they update their estimates according to the constant-gain learning (CGL) formula

$$\Phi_t = \Phi_{t-1} + \overline{g}\left(R_t\right)^{-1} X_t \left(Z_t - X_t'\Phi_{t-1}\right) \tag{12}$$

$$R_t = R_{t-1} + \overline{g}\left(X_{t-1} X_{t-1}' - R_{t-1}\right) \tag{13}$$

where Φ_t describes the updating of the learning rule coefficients, $\Phi_t = (a_t',$ $vec(b_t, c_t, d_t)')'$, and R_t denotes the matrix of second moments of the stacked regressors $X_t \equiv \{1, Z_{t-1}, u_t, r_{t,n}\}$.[5]

Using their PLM and the updated parameter estimates, agents can form expectations for any horizon $T > t$ as

$$\begin{aligned}
\hat{E}_t Z_T &= \left(I_5 - b_{t-1}\right)^{-1}\left(I_5 - \left(b_{t-1}\right)^{T-t}\right)a_{t-1} + \left(b_{t-1}\right)^{T-t}\hat{E}_t Z_t + \\
&\quad \Phi_u u_t \left(\Phi_u I_5 - b_{t-1}\right)^{-1}\left(\left(\Phi_u\right)^{T-t} I_5 - \left(b_{t-1}\right)^{T-t}\right)c_{t-1} + \\
&\quad \Phi_r r_{t,n} \left(\Phi_r I_5 - b_{t-1}\right)^{-1}\left(\left(\Phi_r\right)^{T-t} I_5 - \left(b_{t-1}\right)^{T-t}\right)d_{t-1},
\end{aligned} \tag{14}$$

which substituted into (6) and (7) yields the Actual Law of Motion of the economy. The ALM can be written in state-space form and its likelihood simply evaluated using the Kalman filter.

II. Empirical Results

The model is estimated using likelihood-based Bayesian methods to fit quarterly series for euro area inflation, output gap and nominal interest rates, from 1980:I to 2005:IV. The data are taken from the euro area data set developed and described in detail in Fagan *et al.* (2001). The estimation procedure follows Milani (2007), who extends the techniques reviewed in An and Schorfheide (2007) to account for near-rational expectations and learning by economic agents. I use the Metropolis-Hastings algorithm to generate draws

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[5] The learning assumptions are not unreasonable. Branch and Evans (2006) provide evidence that constant-gain learning outperforms alternatives in forecasting output and inflation. Adam (2007) presents some experimental evidence that economic subjects tend to use simple linear rules in forecasting future inflation.

Table 2: Prior Distributions

Description	Coeff.	Distr.	Range	Prior Mean	95% Prior Int.
Habit Form.	η	U	[0,1]	.5	[0.025,0.975]
Discount Factor	β	–	–	.99	–
IES	σ	Γ	R^+	.25	[.03,.71]
Indexation	γ	U	[0,1]	.5	[0.025,0.975]
Fcn. Price Stick.	ξ_p	Γ	R^+	.125	[0.014,0.35]
Elast. mc	ω	–	–	.8975	–
Int. Rate-Smooth.	ρ	U	[0,0.97]	0.485	[0.024,0.946]
Feedback Infl.	χ_π	N	R	1.5	[1.01,1.99]
Feedback Output	χ_x	N	R	.5	[0.01,0.99]
Autoregr. supply	Φ_u	–	–	.9	–
Autoregr. demand	Φ_r	–	–	.9	–
Std. ε	σ_ε	Γ^{-1}	R^+	.5	[0.06,1.39]
Std. r	σ_r	Γ	R^+	.5	[0.06,1.39]
Std. u	σ_u	Γ	R^+	.5	[0.06,1.39]
Constant Gain Output	g_x	Γ	R^+	.0341	[0.004,0.087]
Constant Gain Infl.	g_π	Γ	R^+	.0341	[0.004,0.087]

Source: Author's own data.
Notes: B = Beta, Γ = Gamma, N = Normal, U = Uniform.

from the posterior distribution: I run 300,000 draws discarding the initial 60,000 as burn-in. The parameters to be estimated are collected in

$$\theta = \{\eta, \beta, \sigma, \gamma, \xi_p, \omega, \rho, \chi_\pi, \chi_x, \Phi_r, \Phi_u, \sigma_\varepsilon, \sigma_r, \sigma_u, \Phi_{1|0}, \ \overline{g}_x, \overline{g}_\pi\}.$$

Table 2 presents information about the priors, which are assumed independent. I fix $\beta = 0.99$, $\omega = 0.8975$ as in Milani (2007) and the autoregressive parameters of the shocks to 0.9. To minimize the influence of the priors, I assume uniform distributions between 0 and 1 for the habits and indexation coefficients.

I start by estimating the model under the learning rule that corresponds to the MSV solution. This case corresponds to the smallest deviation from RE. The constant gain coefficient is also estimated. For now, instead, I fix the initial beliefs coefficients. These are assumed to be equal, for simplicity, to 0.9 for the autoregressive coefficients of the endogenous variables, and to 0 for the other coefficients.[6]

The estimation results are reported in Table 3. I find a low degree of habit formation ($\eta = 0.132$) and a slightly larger degree of inflation indexation

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[6] The rationale for this choice comes from noticing that agents in 1980, when the sample starts, have already experienced the 1970s and they have learned that output and inflation are characterized by large autoregressive terms. This assumption will be later relaxed when the initial beliefs will also be estimated from the data.

Table 3: Posterior Estimates. MSV Learning Rule, Constant-Gain Learning

Description	Coefficients	Posterior Mean	95% Post. Prob. Int.
Habit Form.	η	.132	[.003,.45]
Discount Factor	β	.99	–
IES	σ	.314	[.15,.63]
Indexation	γ	.249	[.006,.87]
Fcn. Price Stick.	ξ_p	.024	[.003,.06]
Elast. mc	ω	.8975	–
Int. Rate-Smooth.	ρ	.932	[.87,.97]
Feedback Infl.	χ_π	1.408	[1.01,1.87]
Feedback Output	χ_x	.576	[.1,1.01]
Autoregr. supply	Φ_u	.9	–
Autoregr. demand	Φ_r	.9	–
Std: ε	σ_ε	.565	[.48,.66]
Std. r	σ_r	2.55	[1.57,4.27]
Std. u	σ_u	.375	[.32,.43]
Constant Gain Output	g_x	.004	[.0003,.012]
Constant Gain Infl	g_π	.006	[.0008,.015]

Source: Author's own data.

($\gamma = 0.249$). There is a lot of uncertainty, however, on the indexation param-
eter: the 95 per cent posterior probability interval lies between 0.006 and 0.87.
For the other parameters, I estimate $\sigma = 0.314$, $\xi_p = 0.024$ and, for the policy
rule, $\rho = 0.932$, $\chi_\pi = 1.408$, $\chi_x = 0.576$. The article estimates the constant gain
coefficients, allowing for different gains for output and inflation. The gains
equal 0.004 for output and 0.006 for inflation, which indicate rather slow
learning about both variables.

Those results may be dependent on the specific choice of initial beliefs.
Therefore, I re-estimate the system, but now estimating also the initial beliefs
together with the gain and the other model parameters. Regarding the initial
beliefs coefficients that now need to be estimated, I assume Beta priors with
mean 0.8 and standard deviation 0.1 for the autoregressive parameters in the
perceived equations for inflation and output (that is, for $b_{11,t=0}$ and $b_{22,t=0}$) and
Normal priors with mean 0 and standard deviation 0.5 for all the other
parameters in $b_{t=0}$, $c_{t=0}$, $d_{t=0}$.

Table 4 shows the results, which remain similar. I find $\eta = 0.082$ and
$\gamma = 0.22$. The gain coefficients now equal 0.0078 for output and 0.004 for
inflation. The estimated ξ_p has increased to 0.12. For the initial beliefs, the
data indicate that agents in 1980 perceived an autoregressive coefficient in
output equal to 0.35 and in inflation equal to 0.69.

Suppose now that the agents do not use the MSV solution, but a more
empirically oriented model, such as a VAR in the three endogenous variables.

Table 4: Posterior Estimates. MSV Learning Rule with Estimated Initial Beliefs, Constant-Gain Learning

Description	Coefficients	Posterior Mean	95% Post. Prob. Int.
Habit Form.	η	.082	[.003,.29]
Discount Factor	β	.99	–
IES	σ	.44	[.16,.75]
Indexation	γ	.22	[.006,.75]
Fcn. Price Stick.	ξ_p	.12	[.018,.27]
Elast. mc	ω	.8975	–
Int. Rate-Smooth.	ρ	.934	[.89,.97]
Feedback Infl.	χ_π	1.414	[1,1.87]
Feedback Output	χ_x	.535	[.03,1.01]
Autoregr. supply	Φ_u	.9	–
Autoregr. demand	Φ_r	.9	–
Std: ε	σ_ε	.51	[.42,.61]
Std. r	σ_r	1.03	[.58,1.71]
Std. u	σ_u	.39	[.27,.61]
Est. Init. Beliefs	$b_{11,t=0}$	.35	[.14,.56]
"	$b_{12,t=0}$	−.005	[−.13,.11]
"	$b_{13,t=0}$	.52	[.03,1.26]
"	$c_{1,t=0}$	.1	[−.05,.26]
"	$d_{1,t=0}$	−.03	[−.21,.15]
"	$b_{21,t=0}$	.02	[−.12,.16]
"	$b_{22,t=0}$	.69	[.5,.84]
"	$b_{23,t=0}$	.2	[.03,.36]
"	$c_{2,t=0}$	−.03	[−.16,.09]
"	$d_{2,t=0}$	−.08	[−.26,.11]
Constant Gain Output	g_x	.0078	[.001,.021]
Constant Gain Infl	g_π	.004	[.0005,.012]

Source: Author's own data.

I am therefore assuming that the agents now do not observe the shocks when forming their expectations. Therefore, the agents' model is misspecified. The results in Table 5 are again comparable: the estimated level of indexation, however, is now larger ($\gamma = 0.376$) and the 95 per cent posterior probability interval wide.

So far, I have assumed that the agents learn using a constant gain. This choice is desirable when agents are concerned about potential structural breaks at unknown dates in the economy. Since it is assumed here that they know the correct model of the economy, however, they may be confident that their estimates will converge to the true coefficients over time. Therefore, they may use a decreasing gain (equal to t^{-1}), instead. I estimate the model under the alternative assumption of a decreasing gain (Recursive Least Squares

Table 5: Posterior Estimates. VAR as PLM, Constant-Gain Learning

Description	Coefficients	Posterior Mean	95% Post. Prob. Int.
Habit Form.	η	.088	[.003,.35]
Discount Factor	β	.99	–
IES	σ	.29	[.16,.54]
Indexation	γ	.376	[.012,.94]
Fcn. Price Stick.	ξ_p	.028	[.003,.07]
Elast. mc	ω	.8975	–
Int. Rate-Smooth.	ρ	.944	[.88,.97]
Feedback Infl.	χ_π	1.3	[.84,1.78]
Feedback Output	χ_x	.55	[.07,1]
Autoregr. supply	Φ_u	.9	–
Autoregr. demand	Φ_r	.9	–
Std: ε	σ_ε	.56	[.48,.65]
Std. r	σ_r	2.4	[1.46,3.8]
Std. u	σ_u	.39	[.33,.45]
Constant Gain Output	g_x	.0076	[.004,.011]
Constant Gain Infl	g_π	.0072	[.001,.023]

Source: Author's own data.

learning). This learning rule implies larger variation in agents' beliefs, particularly at the beginning of the sample, when the gain is larger. The estimation results, reported in Table 6, now indicate strong indexation in the data ($\gamma = 0.74$). The estimated IES parameter σ is also considerably higher ($\sigma = 1.1$).

Discussion

The evidence provided shows that it is difficult to identify whether the persistence in inflation is due to indexation or learning on European data. The results are different depending on whether constant-gain or RLS learning is assumed. Moreover, the estimated coefficient on indexation is characterized by large uncertainty as evidenced by the wide posterior probability interval. The persistence in euro data seems not to have changed a lot in the past 20 years: therefore, the data may be consistent with either automatic indexation by firms or with learning with a low gain coefficient. For habits, I have found that the degree estimated in the data is rather small (estimates between 0.08 and 0.16). Learning about output is also slow.

If compared with Milani (2007), this article's results point to a somewhat larger degree of structural persistence and slower learning in the euro area relative to the US. Similar to Milani (2007), a recent study by Vilagi (2007)

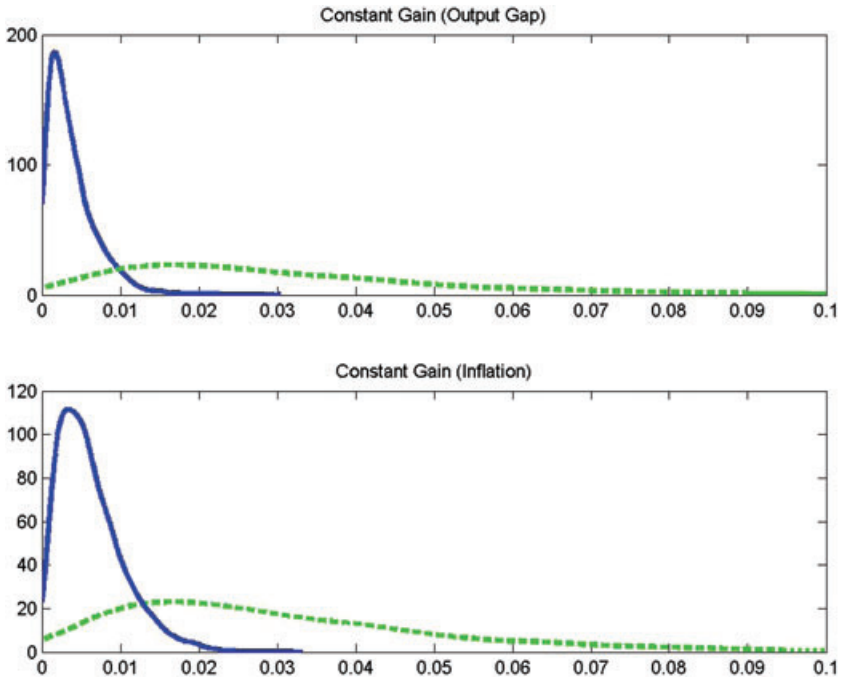Table 6: Posterior Estimates. Recursive Least Squares Learning

Description	Coefficients	Posterior Mean	95% Post. Prob. Int.
Habit Form.	η	.168	[.005,.46]
Discount Factor	β	.99	–
IES	σ	1.1	[.63,1.79]
Indexation	γ	.74	[.30,.99]
Fcn. Price Stick.	ξ_p	.04	[.005,.11]
Elast. mc	ω	.8975	–
Int. Rate-Smooth.	ρ	.82	[.70,.93]
Feedback Infl.	χ_π	1.25	[1.03,1.64]
Feedback Output	χ_x	.65	[.25,1.08]
Autoregr. supply	Φ_u	.9	–
Autoregr. demand	Φ_r	.9	–
Std. ε	σ_ε	.42	[.32,.58]
Std. r	σ_r	1	[.84,1.25]
Std. u	σ_u	.31	[.27,.36]
Gain Output	t^{-1}	–	–
Gain Infl	t^{-1}	–	–

Source: Author's own data.

shows that learning considerably improves the fit of various versions of the New Keynesian model. But learning is not enough in his case to remove the role of more structural sources of persistence in the euro area. Direct comparisons between the two papers are difficult, since both the estimated models and the samples that are considered differ. This article uses a simpler model with internal habit formation, while Vilagi estimates models with external habit formation, sticky wages and wage indexation;[7] I restrict the estimation for the post-1980 sample, while Vilagi includes the 1970s in the estimation: since the 1970s were characterized by high inflation, it is conceivable that higher levels of inflation indexation are found in empirical analyses that include that decade. The difference can also easily arise from the discussed difficulty in identifying learning versus 'mechanical' sources of persistence on European data. The main difference, however, is likely to lie in the estimated constant gain coefficients: this article finds very low values for the gain, while Vilagi's estimates support much larger gains (around 0.10). The slow learning in this article is presumably allowing the model to account for a larger degree of inertia.

[7] Under 'internal' habit formation, consumers derive utility from the deviation of current consumption from a stock of *their own* past consumption; under 'external' habit formation, instead, consumers derive utility from the deviation of current consumption from a stock of past *aggregate* consumption.

Figure 1: Constant Gain Coefficients. Prior (dotted) and Posterior (solid) Distributions
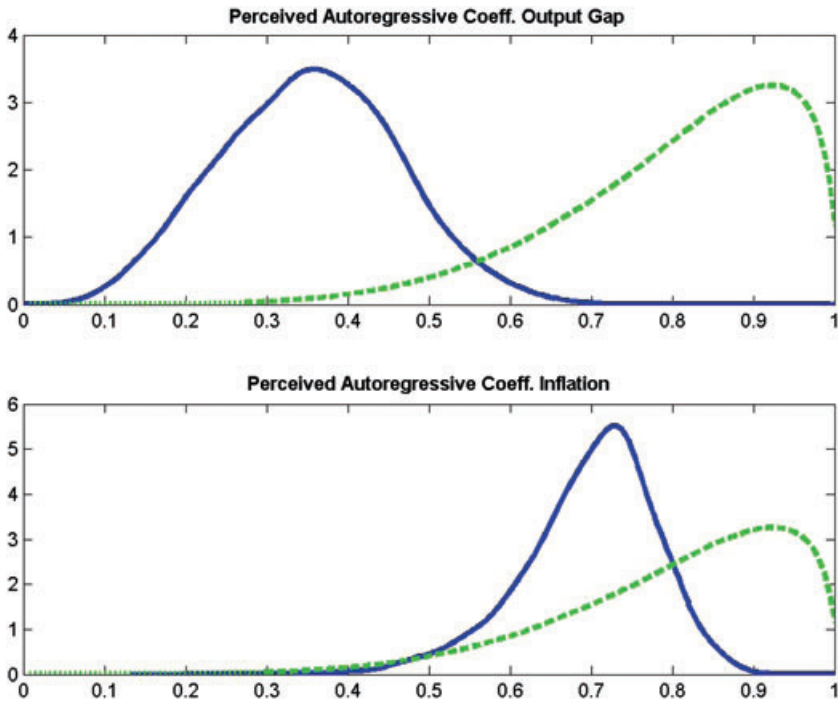

Source: Author's own data.

   Regarding the learning parameters, it appears that the constant gain coefficients are tightly estimated. Figure 1 overlaps the prior and posterior distributions of the gains. The data seem informative and suggest values of the gains close to 0. For the case in which the initial beliefs were also estimated, it seems that the beliefs' coefficients are well identified. Figure 2 shows the prior and posterior distributions for the perceived autoregressive coefficients in the output gap and inflation equations (coeff. b_{11} and b_{22}), for example.

Model Comparison

The model has been estimated under alternative learning assumptions. It is possible to evaluate the relative fit of the different learning rules by computing the corresponding marginal likelihoods (Table 7).[8]

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[8] I use Geweke's (1999) Modified Harmonic Mean approximation to compute the marginal likelihoods.

Figure 2: Agents Beliefs. Selected Prior (dotted) and Posterior (solid) Distributions
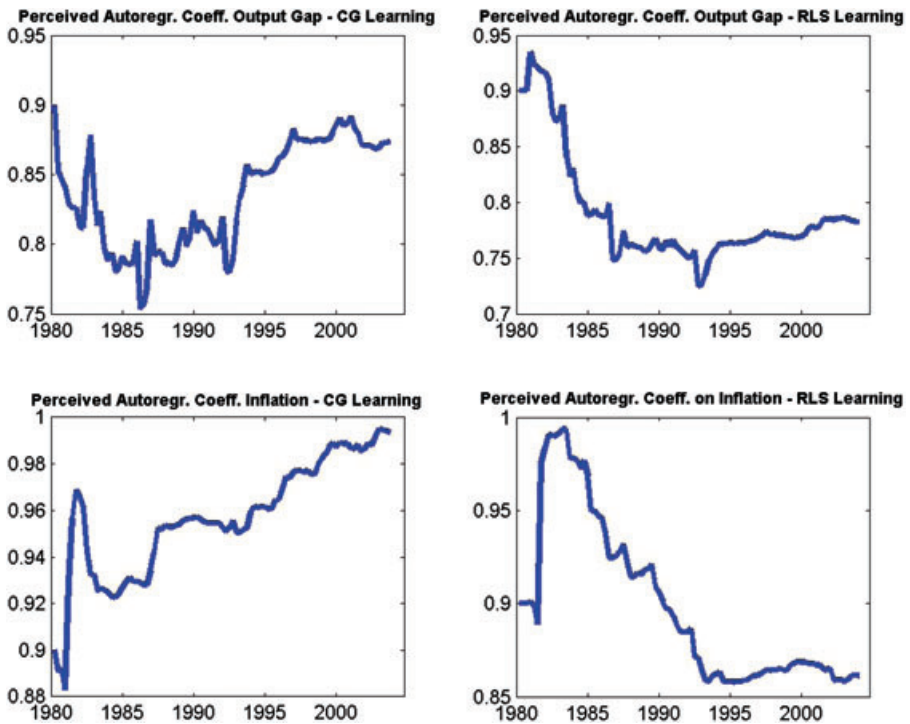

Source: Author's own data.

Table 7: Model Comparison. Marginal Likelihoods

	MSV Rule (1), CG	MSV Rule (2), CG	VAR Rule, CG	MSV Rule, RLS
MargL	234.91	200.55	223.97	−255.55

Source: Author's own data.

The model with the MSV solution as the learning rule has a marginal likelihood that equals −234.91. Estimating the initial beliefs leads to a large improvement in fit (marginal likelihood −200.55). The learning rule that resembles a VAR(1) leads to a better fit than the MSV solution (given the same initial beliefs). The data therefore indicate that it is more realistic to assume that agents have used a misspecified model, which does not allow them to observe current shocks when forming expectations.

Figure 3: Selected Agents' Beliefs under Constant-Gain and RLS Learning

Source: Author's own data.

Finally, the learning rules with constant-gain learning fit considerably better than the one with recursive least squares learning. Figure 3 shows the evolution of beliefs obtained under constant-gain and RLS learning. The dynamics of beliefs differ importantly in the two cases. The autoregressive coefficient on inflation estimated by agents, for example, increases in the early part of the sample if RLS learning is assumed, but it increases instead more toward the end of the sample under CG learning (and differences are apparent also for beliefs about the output gap). It is therefore important to let the data choose the best-fitting learning specification, as done in the article. RLS learning was the only case that led to a large indexation coefficient, but, as seen, this case does not lead to the best fit of the data.

Taken as a whole, those results suggest that European macroeconomic variables may be best characterized by low, but positive, degrees of habits and indexation, and by private-sector learning with a small constant gain coefficient.

Output and Inflation Persistence

Table 8 shows how well the different estimated models can explain the observed empirical auto- and cross-correlations for the euro area output gap and inflation series. Each model has been simulated 1,000 times with the parameters fixed at the posterior mean estimates and the empirical correlation functions are calculated after running a VAR(3) on the implied output gap, inflation and interest rate series (this is done to be consistent with the results reported in Table 1).

All models successfully induce persistence in output and inflation, although the specifications with the VAR-PLM and RLS learning exceed the amount in the data. The specification with constant-gain learning and estimated initial beliefs, which was already preferred using the marginal likelihoods, is also the one that comes closer to the actual data in terms of matching

Table 8: Euro Area Output and Inflation: Model-Implied versus Actual Empirical Auto- and Cross-Correlations

	Data	CGL-Baseline	CGL-Est. IB	CGL-VAR	RLS
$corr(x_t, x_{t-1})$	0.87	0.93	0.89	0.95	0.93
		(0.01)	(0.04)	(0.03)	(0.02)
$corr(x_t, x_{t-4})$	0.52	0.66	0.52	0.72	0.67
		(0.11)	(0.14)	(0.12)	(0.11)
$corr(\pi_t, \pi_{t-1})$	0.98	0.98	0.97	0.99	0.99
		(0.01)	(0.01)	(0.005)	(0.005)
$corr(\pi_t, \pi_{t-4})$	0.89	0.89	0.86	0.90	0.92
		(0.07)	(0.11)	(0.03)	(0.04)
$corr(\Delta\pi_t, \Delta\pi_{t-1})$	0.50	0.70	0.51	0.84	0.83
		(0.08)	(0.10)	(0.04)	(0.06)
$corr(\Delta\pi_t, \Delta\pi_{t-4})$	−0.05	0.28	0.05	0.53	0.50
		(0.12)	(0.14)	(0.14)	(0.15)
$corr(x_t, \pi_t)$	−0.06	0.24	0.11	0.19	0.11
		(0.28)	(0.23)	(0.24)	(0.11)
$corr(x_t, \pi_{t-4})$	−0.28	0.03	0.05	0.03	−0.05
		(0.17)	(0.23)	(0.24)	(0.1)
$corr(x_t, \pi_{t+4})$	0.07	0.34	0.15	0.30	0.20
		(0.19)	(0.23)	(0.24)	(0.13)
$corr(x_t, \Delta\pi_t)$	0.43	0.41	0.38	0.60	0.62
		(0.26)	(0.19)	(0.19)	(0.12)
$corr(x_t, \Delta\pi_{t-4})$	0.27	0.21	0.13	0.47	0.43
		(0.12)	(0.10)	(0.23)	(0.16)
$corr(x_t, \Delta\pi_{t+4})$	0.21	0.34	0.29	0.57	0.68
		(0.12)	(0.09)	(0.12)	(0.11)

Source: Author's own data.

the autocorrelation functions for x_t and $\Delta\pi_t$, while the baseline CGL model does well in matching the autocorrelation in π_t. All specifications imply a positive contemporaneous correlation between inflation and output gap, while the data suggest a slightly negative relation. The models with constant-gain learning with or without estimated initial beliefs generally do a better job than the other specifications in generating cross-correlations that are closer to the ones in actual data (although the standard errors for cross-correlations are usually high). The two models match pretty well the cross-correlations between x_t and $\Delta\pi_t$, which are, instead, overestimated by the VAR-PLM and RLS cases.

Policy Implications

Understanding the main sources of inertia in European output and inflation is important not only from a positive, but also from a normative point of view. Whether the inertia is induced in the economy by structural forms of persistence, such as habit formation and indexation, or by sluggish expectations and learning, has, in fact, crucial implications for the optimal choice of monetary policy. Orphanides and Williams (2003, 2007), Gaspar *et al.* (2006) and Molnár and Santoro (2006) have consistently shown, in a variety of settings and with a variety of techniques, that the optimal monetary policy becomes *more aggressive* towards inflation when persistence is driven by the dynamics of expectations and the private sector is learning.

Gaspar *et al.* (2006) show that the aggressiveness of monetary policy strongly depends on the perceived degree of inflation persistence by the private sector. This article's estimation results, which point toward relatively moderate degrees of habit formation and indexation, but a sizeable degree of perceived persistence in the economic agents' PLM, coupled with slow learning, imply that the ECB should try to respond more aggressively to inflation fluctuations than it would in the same economy under rational expectations and no learning. The stronger feedback is motivated, in fact, by the need to keep inflation expectations anchored and ease private sector learning.

Conclusions

The article has provided evidence that allowing for near-rational expectations and learning may reduce the role of more structural sources of persistence as habits formation in consumption or inflation indexation. On euro area data, however, learning appears to have been slow, so that disentangling it from automatic indexation from post-1980 data is difficult. Overall, it seems that

persistence in the euro area has not changed much over time, somewhat differently from what happened in the US. A similar conclusion was reached by O'Reilly and Whelan (2003), who found relatively little instability in the parameters of the euro area inflation process. On the methodological side, the article has shown how to infer the agents' learning rule that seems more consistent with the data. The gain, the initial beliefs and the structural form of the learning rule have been chosen according to their fit of the data.

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