Available online at www.sciencedirect.com
ScienceDirect
Journal of Monetary Economics 54 (2007) 2065–2082
Journal of MONETARY ECONOMICS
www.elsevier.com/locate/jme

Expectations, learning and macroeconomic persistence ☆

Fabio Milani*

Department of Economics, 3151 Social Science Plaza, University of California, Irvine, CA 92697-5100, USA

Received 28 September 2005; received in revised form 17 November 2006; accepted 21 November 2006
Available online 17 January 2007

Abstract

Monetary DGSE models under rational expectations typically require large degrees of features as habit formation in consumption and inflation indexation to match the inertia of macroeconomic variables.

This paper presents an estimated model that departs from rational expectations and nests learning by economic agents, habits, and indexation. Bayesian methods facilitate the joint estimation of the learning gain coefficient together with the 'deep' parameters of the economy.

☆I am greatly indebted to Michael Woodford for his guidance and advice. I am grateful to the editor of this journal, Robert King, an anonymous referee, Lars Svensson, Chris Sims, Klaus Adam, Francesco Belviso, Pierpaolo Benigno, Jean Boivin, Bill Branch, Jon Faust, Carlo Favero, Marc Giannoni, Michael Kiley, Athanasios Orphanides, Bruce Preston, Giorgio Primiceri, Ricardo Reis, John Roberts, Andrea Tambalotti, Noah Williams, Luis-Felipe Zanna, my discussants Enrico Berardi and Eric Swanson, and all seminar participants at Columbia University, ECB-University of Frankfurt-Bundesbank, Federal Reserve Board, Federal Reserve of Cleveland Conference on "Empirical Methods and Applications for Dynamic Stochastic General Equilibrium Models and Factor Models", Federal Reserve of New York, North Carolina State University, Princeton University, SUNY Stony Brook, University of Bologna IV Workshop on "Dynamic Macroeconomics: Theory and Applications", University of California—Irvine, University of California—Riverside, University of Pennsylvania IUSC, and the 2005 World Congress of the Econometric Society, for comments and useful discussions. I would also like to thank Columbia University and the Federal Reserve Board for their hospitality during part of this project.

*Tel.: +1 949 824 4519; fax: +1 949 824 2182.
E-mail address: fmilani@uci.edu.
URL: http://www.socsci.uci.edu/~fmilani.

0304-3932/$ - see front matter
doi:10.1016/j.jmoneco.2006.11.007

The empirical results show that when learning replaces rational expectations, the estimated degrees of habits and indexation drop closer to zero, suggesting that persistence arises in the model economy mainly from expectations and learning.

JEL classification: C11; D84; E30; E50; E52

Keywords: Persistence; Constant-gain learning; Expectations; Habit formation; Inflation inertia; Bayesian econometrics; New-Keynesian model

1. Introduction

Dynamic stochastic general equilibrium (DSGE) models have become a popular tool for the analysis of the monetary transmission mechanism.[1] These models are built under the hypothesis of *rational expectations* and assume intertemporal optimizing behavior by economic agents. Being derived from explicit microeconomic foundations, they facilitate policy evaluation in terms of the welfare of private agents. Unfortunately, the canonical monetary models with rational expectations often cannot match the observed behavior of macroeconomic variables, and, in particular, they fail to match the *persistence* of aggregate output and inflation.

Economists have therefore proposed a number of extensions to the standard framework by embedding potential sources of endogenous persistence. They have incorporated features such as habit formation in consumption, indexation to lagged inflation in price-setting, rule-of-thumb behavior, or various adjustment costs. Christiano et al. (2005) incorporate several of these extensions and can account for the inertia in the data. Smets and Wouters (2003, 2005) estimate similar models by Bayesian methods, incorporating a mix of frictions and persistent structural shocks, and obtain a remarkable fit of the data. Also, Boivin and Giannoni (2006) and Giannoni and Woodford (2003), in smaller models, but which still incorporate additional sources of persistence, derive impulse responses that approximate those derived from VARs.

The cited extensions essentially improve the empirical fit by adding lags in the model equations. Researchers estimating these rich models under the assumption of rational expectations typically find that substantial degrees of habit persistence and inflation indexation are supported by the data. Those additional sources of persistence appear, therefore, necessary to match the inertia of macroeconomic variables.

1.1. Contribution of the paper

This paper suggests a different direction, by revisiting the expectations formation of the agents. The paper departs from the conventional rational expectations assumption. Agents in the model form expectations using correctly specified economic models, but they do not have knowledge about the model parameters. They use historical data to learn those parameters over time, updating their beliefs through constant-gain learning (CGL). The paper then evaluates the potential for *learning* as a mechanism that can endogenously

[1] Clarida et al. (1999), Goodfriend and King (1997); McCallum and Nelson(1999) and Woodford (2003) are standard examples describing dynamic general equilibrium models for monetary policy analysis.

generate persistence in the economy and improve the fit of current monetary DSGE models. More in detail, the paper aims to disentangle the role of learning versus 'mechanical' sources of persistence,[2] such as habits and indexation, in generating persistence in macroeconomic variables.

The paper starts by taking an agnostic view. The model *nests* different sources of persistence: learning by private agents along with the 'mechanical' sources of persistence, such as habit formation in consumption and indexation to past inflation in price-setting, which are essential under rational expectations to account for the observed persistence. It is left to the data to disentangle the role of the various sources. The scope is to test whether those mechanical sources of persistence are still necessary to match the data when the assumption of rational expectations is relaxed in favor of learning.

The model is estimated using likelihood-based Bayesian methods. The econometric approach allows me to *jointly* estimate the coefficients describing agents' learning, such as the gain coefficient (indicating their learning speed), together with the 'deep' parameters of the economy. This strategy responds to a potential criticism of models with learning, in which the results might depend on the parameters that need to be chosen by the researcher. Here the learning speed is, instead, jointly estimated with the rest of the system.

Orphanides and Williams (2005a, b) In providing an empirical analysis of the importance of learning, the paper builds on previous literature on adaptive learning in macroeconomics. Not many studies have analyzed the empirical implications of adaptive learning. At the earlier stages, this literature was mainly theoretical and focused on convergence of the models to the rational expectations equilibrium (REE).[3] More recently, a number of papers[4] have employed learning to analyze the evolution of U.S. inflation and monetary policy. These papers share the use of learning as a tool that can help in understanding some particular historical episodes, which are often harder to explain under rational expectations.

The present paper tries, instead, to provide a more general empirical study of the effects of learning. Its scope is akin to the work by Williams (2003), who studies the implications of learning for persistence and volatility in simple calibrated real and monetary business cycle models. The present paper shares his scope of studying the effects of learning, but it exploits, instead, actual time series data. This allows me to verify if learning is supported by the empirical evidence and to compare the model with learning with alternative descriptions of the economy. The paper is also related to the recent work by Adam (2005), who likewise assumes that economic agents use simple econometric models to forecast macroeconomic variables and shows how deviations from rational expectations may strengthen the internal propagation mechanism of a simple business cycle model.

[2]The paper refers to them as 'mechanical' since in the case of habits, researchers need to alter the consumers' utility function to imply dependence on lagged consumption, and in the case of indexation, they posit a rule to induce inertia through the assumption that a fraction of firms simply adjust prices automatically, according to the past observed inflation rate.

[3]Evans and Honkapohja (2001), Bullard and Mitra (2002), and Preston (2005) are examples that verify the learnability of the REE in monetary models.

[4]Branch et al. (2004), Bullard and Eusepi (2005), Orphanides and Williams (2003, 2005b), Primiceri (2006), Sargent (1999), and Sargent et al. (2006), among others.

Similarly to recent empirical papers in macroeconomics,[5] this paper adopts Bayesian methods in the estimation. The techniques are similar to those used by Schorfheide (2000, 2005) and Lubik and Schorfheide (2004, 2007), among others. But Schorfheide (2000), as well as several papers that share the same techniques, estimate DSGE models under rational expectations.[6] The current paper, instead, provides the first example of the use of Bayesian methods to estimate a DSGE model with non-fully rational expectations and learning. This represents a methodological contribution of the paper. Bayesian methods are appealing in this context because they facilitate the joint estimation of the learning parameters together with the rest of the system.

A potential criticism of models with adaptive learning, also discussed in Marcet and Nicolini (2003), emphasizes the arbitrary choices, often available to the researcher, which render the model hardly falsifiable. Milani (2004a), for example, shows how estimates strongly vary over the range of possible gain coefficients. In the present paper, instead, the gain coefficient is also estimated, leaving less room for arbitrariness.

More generally, by estimating a DSGE model with learning, the paper provides an example of a 'non-rational expectations econometrics', which Ireland (2003) judged as missing from the branch of the literature that studies, usually theoretically, the impact of learning in macroeconomics.

1.2. Results

The empirical results show that the essential role of mechanical sources of persistence (habits, indexation) in DSGE monetary models rests on the assumption of rational expectations. When agents are allowed to learn the true parameters of the economy over time, habits and indexation are no longer essential, being estimated at values close to zero in the data. This finding suggests that learning can represent an important source of persistence in the economy. Indeed, learning might represent a *single* mechanism capable of creating persistence, replacing the features needed in various sides of the conventional rational expectations model to improve its empirical properties. Furthermore, the posterior model probabilities show that the specification with learning fits better than the specification with rational expectations.

2. A simple model with learning and structural sources of persistence

The aggregate dynamics of the model is given by the following specification, nesting learning and structural sources of persistence as habit formation and inflation indexation[7]

$$\widetilde{x}_t = \widehat{\mathrm{E}}_t \widetilde{x}_{t+1} - (1-\beta\eta)\sigma[i_t - \widehat{\mathrm{E}}_t \pi_{t+1} - r_t^n], \tag{1}$$

$$\widetilde{\pi}_t = \xi_p[\omega x_t + [(1-\eta\beta)\sigma]^{-1}\widetilde{x}_t] + \beta\widehat{\mathrm{E}}_t\widetilde{\pi}_{t+1} + u_t, \tag{2}$$

$$i_t = \rho i_{t-1} + (1-\rho)[\chi_\pi \pi_t + \chi_x x_t] + \varepsilon_t, \tag{3}$$

[5] An and Schorfheide (2007) provide a first review of this literature.

[6] Schorfheide (2005) assumes an incomplete information model in which agents need to update their beliefs about the inflation target using a Bayesian learning rule. In his model, however, agents still form fully rational expectations.

[7] The reader is referred to Milani (2004b) for a full derivation of the model. As in most papers in the adaptive learning literature (see Evans and Honkapohja, 2001 for a general treatment), the loglinearized equations are

where

$$\widetilde{\pi}_t \equiv \pi_t - \gamma\pi_{t-1}, \tag{4}$$

$$\widetilde{x}_t \equiv (x_t - \eta x_{t-1}) - \beta\eta\widehat{\mathrm{E}}_t(x_{t+1} - \eta x_t) \tag{5}$$

and where x_t denotes the output gap, π_t denotes inflation, i_t denotes the nominal interest rate, and r_t^n, u_t, and ε_t denote demand, supply, and monetary policy shocks. Eq. (1) is the log-linearized Euler equation that arises from households' consumption decisions under (internal) habit formation; $\beta \in (0,1)$ is the household's discount factor, $\sigma > 0$ is the elasticity of intertemporal substitution of consumption in the absence of habits, and $0 \leqslant \eta \leqslant 1$ measures the degree of habit formation. Current output gap depends on lagged and expected one-period and two-period ahead output gaps, and on the *ex ante* real interest rate. Eq. (2) is the Phillips curve that arises from optimal Calvo price-setting, when firms that cannot re-optimize are allowed to follow an indexation rule, as proposed by Christiano et al. (2005). Coefficient ω denotes the elasticity of the marginal disutility of producing output with respect to an increase in output, ξ_p is a parameter that is inversely related to the degree of price stickiness, and $0 \leqslant \gamma \leqslant 1$ represents the degree of indexation to past inflation. Current inflation depends on lagged and one-period ahead inflation, and on current, lagged, and one-period ahead output gap (with habit formation, in fact, the log marginal utility of real income entering the Phillips curve is a linear function of x_t and $\widetilde{x}_t$ rather than a linear function of x_t alone). Monetary policy is described by Eq. (3), which is a Taylor rule with partial adjustment, where ρ is the interest-rate smoothing term, and χ_π and χ_x are the feedback coefficients to inflation and output gap.

In the model, $\widehat{\mathrm{E}}_t$ indicates subjective (possibly non-rational) expectations, while the usual mathematical expectation operator E_t is left to denote model-consistent rational expectations.

The natural real interest rate and the cost-push shocks evolve according to univariate AR(1) processes

$$r_t^n = \phi^r r_{t-1}^n + v_t^r, \quad v_t^r \sim \mathrm{iid}(0, \sigma_r^2) \tag{6}$$

$$u_t = \phi^u u_{t-1} + v_t^u, \quad v_t^u \sim \mathrm{iid}(0, \sigma_u^2). \tag{7}$$

2.1. Expectations formation: constant-gain learning

As made clear by Eqs. (1) and (2), agents need to form forecasts of future macroeconomic conditions. Following recent learning literature, the agents are assumed to behave as econometricians, employing an economic model and forming expectations from that model.

Agents estimate

$$Z_t = a_t + b_t Z_{t-1} + c_t u_t + d_t r_t^n + \varepsilon_t \tag{8}$$

(*footnote continued*)
similar to those obtained under rational expectations, but with a different expectations operator. For a different approach of considering learning, see Preston (2005, 2006), who introduces learning directly from the primitive assumptions of multi-period decision problems. Preston's approach is followed in Milani (2004b), leading to similar estimation results.

using variables that appear in the minimum state variable (MSV) solution of the system under rational expectations (defining $Z_t \equiv [\pi_t, x_t, i_t]'$ and where a_t, b_t, c_t, d_t are coefficient vectors and matrices of appropriate dimensions). Therefore, the agents employ a correct model of the economy, but they do not have knowledge about the relevant model parameters.[8] They use historical data to learn those parameters over time. Expression (8) represents the "*Perceived Law of Motion*" (*PLM*) of the agents. As additional data become available in subsequent periods, agents update their estimates of the coefficients (a_t, b_t, c_t, d_t) according to the CGL formula

$$\widehat{\boldsymbol{\phi}}_t = \widehat{\boldsymbol{\phi}}_{t-1} + \bar{\mathbf{g}} R_{t-1}^{-1} X_t (Z_t - X_t' \widehat{\boldsymbol{\phi}}_{t-1}), \tag{9}$$

$$R_t = R_{t-1} + \bar{\mathbf{g}}(X_{t-1} X_{t-1}' - R_{t-1}), \tag{10}$$

where $\widehat{\boldsymbol{\phi}}_t = (a_t', vec(b_t, c_t, d_t)')'$ collects the learning rule coefficients, and R_t denotes the matrix of second moments of the stacked regressors $X_t \equiv \{1, Z_{t-1}, u_t, r_t^n\}_0^{t-1}$. The constant gain is expressed by the parameter $\bar{\mathbf{g}}$. Using their PLM and the updated parameter estimates, agents form expectations for any horizon $T > t$ as

$$\begin{aligned}\widehat{\mathrm{E}}_t Z_T = {} & (I_5 - b_t)^{-1}(I_5 - b_t^{\mathrm{T}-t})a_t + b_t^{\mathrm{T}-t}\mathrm{E}_t Z_t \\ & + \phi_u u_t(\phi_u I_5 - b_t)^{-1}(\phi_u^{\mathrm{T}-t} I_5 - b_t^{\mathrm{T}-t})c_t \\ & + \phi_r r_t^n(\phi_r I_5 - b_t)^{-1}(\phi_r^{\mathrm{T}-t} I_5 - b_t^{\mathrm{T}-t})d_t,\end{aligned} \tag{11}$$

where I_5 denotes a 5×5 identity matrix. The model informational assumptions are as follows: in period t, agents observe the values of the endogenous variables in $t-1$, they observe the values of the shocks in t, and they use the estimated parameters in $t-1$, to form their expectations for $t+1$ and $t+2$. An appealing feature of the learning framework is that it nests RE as a special limiting case: the asymptotic distribution of learning beliefs (for $t \to \infty$) approaches the beliefs under RE as $\bar{\mathbf{g}} \to 0$. The convergence to rational expectations holds, however, only asymptotically. In a limited sample, as the one available here, this convergence cannot be simply tested.

To summarize, the model economy is represented by the aggregate dynamics Eqs. (1), (2), monetary policy rule (3), shock processes (6), (7), and expectations formation expressions (9)–(11).

2.2. State-space form

Substitution of the expectations formed as in (11) into (1) and (2) yields the state-space form

$$\begin{aligned}\xi_t &= A_t + F_t \xi_{t-1} + G_t w_t, \\ Y_t &= H \xi_t,\end{aligned} \tag{12}$$

where $\xi_t = [x_t, \pi_t, i_t, u_t, r_t^n]$, $w_t \sim \mathrm{N}(0, Q)$, H is a matrix of zeros and ones just selecting observables from ξ_t, and A_t, F_t, G_t are time-varying matrices of coefficients, which are convolutions of structural parameters of the economy and agents' beliefs. Expression (12) is the implied "*Actual Law of Motion*" (*ALM*), of the economy (the ALM has the same structural form as the PLM, but different coefficient matrices), which will be estimated by

[8] In the estimation, agents recognize that the true mean of the variables is zero ($a_t = 0$). Allowing agents to learn also the constant term over time has no relevant effects on the results.

Bayesian methods. The scope will be to test whether persistence is due to structural characteristics, such as habits and indexation, or, instead, to learning by firms and consumers.

3. Bayesian estimation

Likelihood-based Bayesian methods are used to fit the series for US output gap, inflation, and the nominal interest rate. This paper follows a similar approach to the papers reviewed in An and Schorfheide (2007), which have also employed Bayesian methods to estimate DSGE models. But, while those papers work with traditional rational expectations models, this paper provides an example of estimation of a simple DSGE model with near-rational expectations and learning.

Using the model in state-space form in (12), the paper estimates the deep parameters and the main learning parameter, the constant gain, *jointly* within the system. The structural parameters of the model are collected in the parameter vector Ψ

$$\Psi = \{\eta, \beta, \sigma, \gamma, \xi_p, \omega, \rho, \chi_\pi, \chi_x, \phi_r, \phi_u, \sigma_\varepsilon, \sigma_r, \sigma_u, \overline{\mathbf{g}}\}. \tag{13}$$

The parameter vector Ψ includes structural parameters describing the dynamics of the economy, the policy rule coefficients, the standard deviations of the monetary policy, aggregate demand, and aggregate supply shocks, and the constant gain coefficient $\overline{\mathbf{g}}$. The estimation of the constant-gain coefficient is crucial, since, despite its increasing use in monetary policy studies, estimates of its value are missing in the literature. Ireland (2003) highlights the necessity of what he defines an "irrational expectations econometrics" and suggests estimating the gain using time series data. This is exactly what is done in this paper. The value to assign to $\overline{\mathbf{g}}$ constitutes, in fact, an important degree of freedom for the researcher and one's results may heavily depend on its choice. Indeed, Milani (2004a) shows how the estimated degree of structural persistence in inflation strongly depends on the assumed gain. This paper hence provides an estimate of $\overline{\mathbf{g}}$ to fill the gap in the literature.[9] Notice that the structural parameters and the learning speed are *jointly* estimated in the system. This is different from Milani (2004a), where the estimation of structural parameters was valid for a *given* estimated learning rule. Ideally, one would want to estimate also the initial agents' beliefs jointly in the system. Here, however, this complication is avoided to keep the number of estimated parameters tractable. I start by fixing the initial beliefs.[10] Later in the paper, the initial beliefs will be also estimated using pre-sample data.

All the information about the parameters is summarized by the posterior distribution, obtained by Bayes theorem

$$p(\Psi|Y^{\mathrm{T}}) = \frac{p(Y^{\mathrm{T}}|\Psi)p(s\Psi)}{p(Y^{\mathrm{T}})}, \tag{14}$$

[9] Orphanides and Williams (2005a, b) estimate the constant-gain coefficient as the gain the minimizes the deviation of expectations in their model from the Survey of Professional Forecasters' expectations series.

[10] The initial beliefs are $b_{11} = 0.4$, $b_{12} = 0$, $b_{13} = 0.7$, $c_1 = 0$, $d_1 = 0$, $b_{21} = 0.5$, $b_{22} = 0$, $b_{23} = 0.6$, $c_2 = 0$, $d_2 = 0$. Those values are inspired by pre-sample evidence, but do not exactly correspond to pre-sample estimates. For example, the initial autoregressive parameter for inflation b_{22} is fixed to 0, because this is consistent with the volatile inflation experience in the 1960s. I assume that agents initially perceive a high sensitivity of inflation to the output gap (coeff. b_{21}). A relatively low autoregressive coefficient b_{11} is assumed for the output gap.

Table 1
Prior distributions for model with learning

Description	Param.	Range	Prior distr.	Prior mean	Prior std.	95% Prior prob. int.
Habit formation	η	$[0,1]$	*Uniform*	0.5	0.289	$[0.025, 0.975]$
Discount rate	β	$[0,1]$	*Beta*	0.99	0.01	$[0.973, 0.999]$
IES	σ	$\mathbb{R}^+$	*Gamma*	0.125	0.09	$[0.015, 0.35]$
Infl. indexation	γ	$[0,1]$	*Uniform*	0.5	0.289	$[0.025, 0.975]$
Function price stick.	ξ_p	$\mathbb{R}^+$	*Gamma*	0.015	0.011	$[0.0019, 0.04]$
Elast. mc to inc.	ω	$\mathbb{R}$	*Normal*	0.8975	0.4	$[0.114, 1.68]$
Int-rate smooth.	ρ	$[0, 0.97]$	*Uniform*	0.485	0.28	$[0.024, 0.946]$
Feedback Infl.	χ_π	$\mathbb{R}$	*Normal*	1.5	0.25	$[1.01, 1.99]$
Feedback gap	χ_x	$\mathbb{R}$	*Normal*	0.5	0.25	$[0.01, 0.99]$
Autoregr. dem shock	ϕ_r	$[0, 0.97]$	*Uniform*	0.485	0.28	$[0.024, 0.946]$
Autoregr. sup shock	ϕ_u	$[0, 0.97]$	*Uniform*	0.485	0.28	$[0.024, 0.946]$
MP shock	σ_ε	$\mathbb{R}^+$	*InvGamma*	1	0.5	$[0.34, 2.81]$
Demand shock	σ_r	$\mathbb{R}^+$	*InvGamma*	1	0.5	$[0.34, 2.81]$
Supply shock	σ_u	$\mathbb{R}^+$	*InvGamma*	1	0.5	$[0.34, 2.81]$
Gain coeff.	$\bar{g}$	$\mathbb{R}^+$	*Gamma*	0.031	0.022	$[0.0038, 0.087]$

where $p(Y^{\mathrm{T}}|\Psi)$ is the likelihood function, $p(\Psi)$ the prior for the parameters, and $Y^T = [y_1, \ldots, y_T]'$ collects the data histories. The model is fitted to data on output gap, inflation, and nominal interest rates. The data are quarterly for the period 1960:I to 2004:II. Inflation is defined as the annualized quarterly rate of change of the GDP Implicit Price Deflator, output gap as the log difference between GDP and Potential GDP (CBO estimate), and the federal funds rate is used as the nominal interest rate.[11] 300,000 draws for the Markov Chain are used, discarding the first 20% as initial burn-in.

To generate draws from the posterior distribution of Ψ using the Metropolis–Hastings algorithm, the likelihood function $p(Y^{\mathrm{T}}|\Psi)$ needs to be evaluated at each iteration. Having expressed the model as a linear Gaussian system in (12), the likelihood can be recursively computed using the Kalman Filter.[12]

3.1. Specifying the prior distribution

Table 1 presents information about the priors for the parameters collected in Ψ.

Priors are assumed to be independent. The habit and indexation parameters η and γ are assumed to follow Uniform distributions in the interval $[0, 1]$. The discount factor β follows a Beta distribution, but with a tight probability around 0.99. All the autoregressive parameters (ρ, ϕ_r, ϕ_u) follow Uniform distributions. σ follows a Gamma distribution with mean 0.125 and standard deviation 0.09. This prior is in the range of estimates by Fuhrer (2000), who finds values between 0.08 and 0.16 in a New Keynesian model with habit formation.[13] There is a large disagreement in the literature, though.

[11]The series were obtained from FRED, the database of the Federal Reserve Bank of Saint Louis.

[12]The details of the algorithm are illustrated in the appendix of Milani (2004b), available online at http://www.socsci.uci.edu/~fmilani/Milani_ELMP.pdf.

[13]Rabanal and Rubio-Ramirez (2005) obtain values between 0.12 and 0.15. Dennis (2003) finds values that are closer to 0.

Table 2
Posterior estimates: model with learning

Description	Parameters	Posterior mean	95% Post. prob. interval
Habits	η	0.117	[0.006, 0.289]
Discount	β	0.99	[0.974, 0.998]
IES	σ	0.748	[0.587, 0.996]
Indexation	γ	0.032	[0, 0.11]
Fcn. price stick.	ξ_p	0.016	[0.002, 0.04]
Elast. mc	ω	0.865	[0.03, 1.61]
Int-rate smooth.	ρ	0.914	[0.875, 0.947]
Feedback Infl.	χ_π	1.484	[1.08, 1.90]
Feedback Gap	χ_x	0.801	[0.433, 1.18]
Autoregr. dem shock	ϕ_r	0.845	[0.776, 0.908]
Autoregr. sup shock	ϕ_u	0.854	[0.778, 0.93]
MP shock	σ_ε	0.86	[0.777, 0.953]
Demand shock	σ_r	1.67	[1.47, 1.91]
Supply shock	σ_u	1.15	[1.02, 1.31]
Gain coeff.	$\bar{\mathbf{g}}$	0.0183	[0.0133, 0.0231]

Therefore, the results will be also checked under a more diffuse prior, and assuming a higher prior mean as in Lubik and Schorfheide (2004, 2007). Normal distributions are assumed for the other structural parameters and inverse gamma distributions for the standard deviations of the shocks. The prior for ω is centered at the value estimated by Giannoni and Woodford (2003), with a large variance. The constant-gain coefficient is assumed to follow a Gamma distribution with prior mean 0.031 and prior standard deviation 0.022.[14]

4. Some near-rational expectations econometrics: empirical results

Table 2 presents the estimation results for the model with learning. The degree of habit formation in private expenditures, measured by the parameter η, equals 0.117. The estimated degree of inflation indexation γ equals 0.03. The reported 95% asymmetric posterior probability intervals indicate that the estimates are unlikely to be higher than 0.3 for habits and 0.1 for indexation. Habits and indexation are typically essential features in rational expectations models to match the persistence in the data and to improve fit. When learning replaces the assumption of fully rational expectations, the degrees of habits and indexation drop to values much closer to zero. Mechanical sources of persistence appear no longer essential for the empirical performance of DSGE models. The results, therefore, suggest that learning is able to generate the necessary persistence in the economy, making those additional features redundant.

The intertemporal elasticity of substitution σ equals 0.748. The monetary policy rule shows a sizeable degree of interest-rate smoothing ($\rho = 0.914$), a feedback coefficient to inflation equal to 1.484, and to the output gap equal to 0.801. The posterior distribution

[14]The results were similar under a more diffuse prior distribution for the gain, as a Uniform in the interval [0, 0.3].

for ξ_p coincides with the prior distribution: the data, therefore, appear uninformative about this parameter.

A central coefficient in my estimation is represented by the constant gain. This paper represents the first attempt to estimate the gain jointly with the rest of the parameters of the economy. The posterior mean estimate for the gain equals 0.0183. Such a value implies that private agents are learning rather slowly. The estimated value is not too dissimilar from values chosen from calibration in previous studies (often working with gains in the interval $0.015 - 0.03$) and from what found by Orphanides and Williams (2004, 2005b), exploiting data on expectations from the Survey of Professional Forecasters. To facilitate intuition, 1/gain can be interpreted as an indication of how many past observations agents use to form their expectations. A gain of 0.0183 indicates that agents make use of roughly 13–14 years of data (55 quarters). Also, looking at expressions (9) and (10), it can be noticed that with a gain of that size only a small fraction of new information is used to update the previous period coefficients' estimates.

Fig. 1 shows the evolution of selected agents' beliefs over the sample. Coefficient b_{22} in the graph represents the evolution of agents' beliefs about the persistence of inflation (the autoregressive parameter in their learning rule). Agents start with a low perceived persistence of inflation during the 1960s, but they revise their beliefs in the late 1970s and at the beginning of the 1980s. The perceived persistence drops later in the 1980s, increasing again in the second half of the 1990s. A similar dynamics, but with larger autoregressive coefficients, is found by Milani (2004a) and by Orphanides and Williams (2005b).

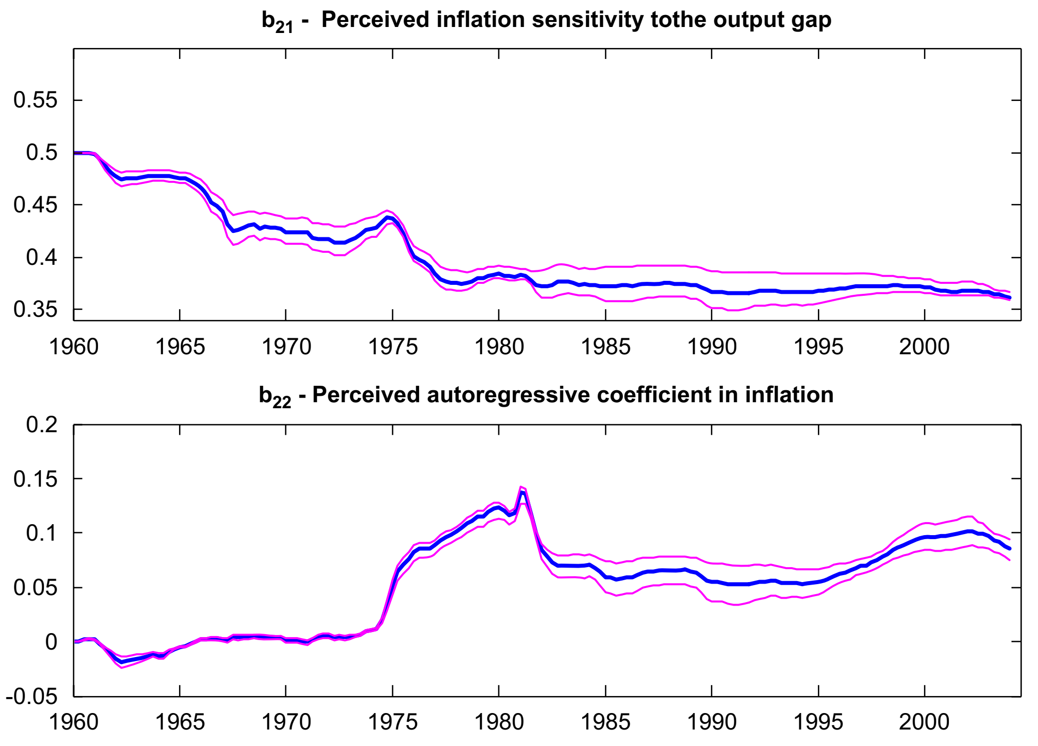

Fig. 1. Evolution of selected agents' beliefs (1960:I–2004:II).

Coefficient b_{21}, instead, indicates the estimated sensitivity of inflation to the output gap. The figure shows that in the 1970s the sensitivity was high, but it decreased in the latest two decades. This result is consistent with the recent perception of a flatter Phillips curve. The estimated 95% posterior probability bands suggest that the beliefs are tightly estimated.

Fig. 2, instead, reports the evolution of agents' inflation and output gap forecasts compared with the realized series. The figure displays prolonged periods of correlated forecast errors of inflation. Private agents underestimated inflation in the 1970s and they failed to predict its first peak in 1974–1975. They increased their inflation forecasts in the late 1970s, keeping them above realized inflation during the first quarters of Volcker's disinflation. The dynamics of inflation expectations seems consistent with actual expectations from surveys, which typically document an underestimation of inflation when it is high and an overestimation when it is low. Regarding output gap forecasts, the figure shows that private agents underestimated the depth of the 1982 recession.

Turning to the case of rational expectations, the model now consists of Eqs. (1) and (2), with E_t replacing $\widehat{E}_t$, together with (3), (6) and (7), and it is similar to the system estimated by Giannoni and Woodford (2003). They use an indirect estimation method, choosing parameters to minimize the distance between the model's implied impulse responses and those obtained from a VAR. Table 3 reports their results, together with the results I obtain by re-estimating the system with the same Bayesian procedure used for the model with learning. The priors are the same as those used for the learning model, but in the RE case I follow Giannoni and Woodford (2003) in estimating $\varphi \equiv [(1-\beta\eta)\sigma]^{-1}$ rather than σ. The prior distribution for φ is a Gamma with mean 1 and standard deviation 0.71. Estimating σ, instead, and assuming the same prior used for the learning model leads to similar results.

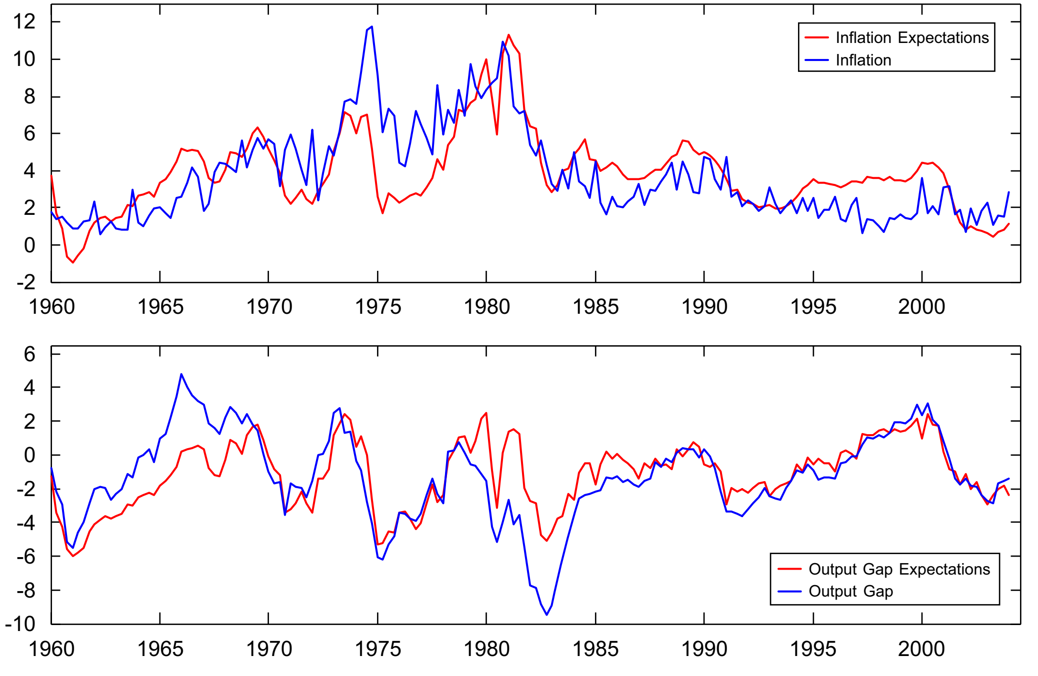

Fig. 2. Actual versus expected inflation and output gap over the sample.

Table 3
Rational expectations estimates and 95% posterior probability interval

Description	Param.	GW '03	Bayesian Estimation	
		Estimate	Mean estimate	95% Post. Prob. Int.
Habits	η	1	0.911	[0.717, 0.998]
Discount	β	0.99 (fixed)	0.9897	[0.971, 0.999]
IES	φ	0.6643	3.813	[2.285, 6.02]
Indexation	γ	1	0.885	[0.812, 0.957]
Fcn. price stick.	ξ_p	0.0015	0.001	[0.0001, 0.002]
Elast. mc	ω	0.8975	0.837	[0.01, 1.63]
Int-rate smooth.	ρ	–	0.89	[0.849, 0.93]
Feedback Infl.	χ_π	–	1.433	[1.06, 1.81]
Feedback gap	χ_x	–	0.792	[0.425, 1.165]
Autoregr. dem shock	ϕ_r	–	0.87	[0.8, 0.93]
Autoregr. sup shock	ϕ_u	–	0.02	[0.0005, 0.07]
MP shock	σ_ε	–	0.933	[0.84, 1.04]
Demand shock	σ_r	–	1.067	[0.89, 1.22]
Supply shock	σ_u	–	1.146	[1.027, 1.27]

Note: 0.0187% of the draws fell in the indeterminacy region and were discarded.

The estimates indicate sizeable degrees of indexation in inflation ($\gamma = 0.885$) and habit formation in consumption ($\eta = 0.911$). The autoregressive parameter in the cost-push shock, however, is now much lower ($\phi_u = 0.02$) than it was under learning.[15] Both a large degree of habit formation and a large autocorrelation of the exogenous shock are, instead, necessary in the output gap equation.

The estimates are not far from those found by Giannoni and Woodford (2003). The biggest difference is given by the estimate of the pseudo-elasticity of intertemporal substitution parameter (denoted by φ^{-1}), which also measures the sensitivity of output to changes in the real interest rate. My estimate implies a lower sensitivity.[16]

The estimation results reaffirm what is commonly known: in rational expectations DSGE models, additional sources of endogenous persistence are essential to match the inertial behavior of economic variables and make the model fit.

4.1. Do we really need mechanical sources of persistence?

Besides Giannoni and Woodford (2003), Boivin and Giannoni (2006) also find $\eta \simeq 1$ and $\gamma = 1$. Christiano et al. (2005), instead, fix γ to 1, and estimate $\eta = 0.65$. Smets and Wouters (2005) estimate $\eta = 0.69$ and $\gamma = 0.66$ in their pre-79 sample, and $\eta = 0.44$ and $\gamma = 0.34$ in the post-82 sample. Their estimates are somewhat lower than other papers, but still surprisingly large if we consider that they are obtained in a rich model, incorporating

[15]The Working Paper version (Milani, 2004b) shows, however, that when the model is estimated under infinite-horizon learning, zero indexation becomes coupled with a low estimated autocorrelation of the cost-push shock.

[16]The difference probably arises here from the different estimation methods: the impulse responses from a VAR display a substantial response of the gap to a monetary shock. Their estimated parameter needs to match this response and is therefore bigger ($\varphi^{-1} = 1.50$). Standard estimates of this parameter by other methods are typically lower (and mine equals $\varphi^{-1} = 0.26$).

habits, sticky prices, and indexation, along with wage stickiness, capital formation, adjustment costs, and several highly autocorrelated shocks.[17] Dennis (2003) estimates a new-Keynesian model with optimal monetary policy and finds $\eta \simeq 1$ and $\gamma \simeq 0.9$. Fuhrer (2000) also obtains a strong role for habits ($\eta = 0.8$–0.9). Rabanal and Rubio-Ramirez (2005) estimate $\gamma = 0.76$.

But this paper has shown that estimated degrees of habit formation and inflation indexation close to 1 seem to hinge on the assumption of rational expectations. When this assumption is weakened by allowing agents to learn over time, the degree of persistence due to structural features (habits and indexation here, but possibly others) drops to almost zero. This result highlights the potential role of expectations and learning dynamics as sufficient sources of persistence in the economy.

4.2. *Model comparison: learning versus rational expectations*

This section compares the marginal likelihoods and posterior model probabilities of the specifications with learning and rational expectations.

Table 4 shows the model comparison between the model with learning (both with and without habits and indexation) and the model with rational expectations (with habits and indexation). The model with learning fits better than the model with rational expectations. When both models incorporate habits and indexation, the data favor the model with learning (the posterior odds ratio equals 584 in favor of learning). When the model with learning, but no sources of mechanical persistence, is compared with the model with rational expectations, the latter enriched with habit formation and indexation, the posterior odds ratio increases to 2.6764×10^6.[18]

5. Extensions

This section examines the robustness of results to the following extensions: time-varying monetary policy, different prior assumptions, different learning speeds for different variables, and estimated initial beliefs.

For the purposes of this paper, the assumption of a constant monetary policy rule over the post-war sample would be troubling only if the persistence in the economy was actually driven by the omitted evolving policy. To allow for a changing policy, suppose that the central bank has adopted, for some exogenous reasons, a time-varying inflation target over the sample. Monetary policy can be expressed by the following rule:

$$i_t = \rho i_{t-1} + (1-\rho)[\pi_t^* + \chi_\pi(\pi_t - \pi_t^*) + \chi_x x_t] + \varepsilon_t, \tag{15}$$

where the inflation target π_t^* evolves as an AR(1) process $\pi_t^* = \phi_{\pi^*}\pi_{t-1}^* + v_t^{\pi^*}$. The prior for ϕ_{π^*} follows a Uniform distribution in the $[0, 0.97]$ interval.

As shown in Table 5, the assumption of time-varying policy does not overturn the results. There is still no evidence of indexation in inflation ($\gamma = 0.035$). The degree of habit formation is positive, but low ($\eta = 0.146$). The estimated gain is now higher and equal to

[17]The variables they use are also different: they use a linearly detrended measure of inflation, for example.

[18]In results not reported, I have used bootstrapping to derive a measure of the uncertainty surrounding the marginal likelihoods and posterior odds calculations. From bootstrapping, I can compute serial-correlation-corrected standard errors of the mean of $f(\theta_j)/(\pi(\theta_j)p(Y|\theta_j))$ and use it to generate error bands for the derived marginal likelihoods and posterior odds ratios. The implied error bands are extremely narrow.

Table 4
Model Comparison: learning (with and without frictions) versus rational expectations (with frictions)

	Learning (no frictions)	Learning (+frictions)	RE (+frictions)
Log marg. Likelihood	−750.65 (1.375)	−759.08 (1.326)	−765.45 (1.316)
Posterior odds	2.6764×10^6	584.06	1
Posterior prob.	0.9998	0.00022	3.74×10^{-7}

Note: log marginal likelihoods are computed using Geweke's Modified Harmonic Mean approximation.

Table 5
Posterior estimates and 95% posterior probability intervals

	T–V infl. target		Diff. prior for σ		Diff. gains		Est. init. beliefs	
Param.	Mean est.	95%P.I.	Mean	95%P.I.	Mean	95%P.I.	Mean	95%P.I.
η	0.146	[0.01, 0.36]	0.321	[0.05, 0.57]	0.106	[0, 0.25]	0.178	[0.01, 0.41]
β	0.991	[0.97, 0.99]	0.99	[0.97, 0.999]	0.991	[0.98, 0.999]	0.988	[0.97, 0.999]
σ	0.44	[0.18, 0.73]	1.19	[0.7, 2.11]	0.767	[0.60, 0.98]	0.438	[0.30, 0.71]
γ	0.035	[0, 0.14]	0.032	[0, 0.12]	0.03	[0, 0.1]	0.024	[0, 0.08]
ξ_p	0.0015	–	0.016	[0.002, 0.04]	0.0015	–	0.0015	–
ω	0.8975	–	0.91	[0.07, 1.7]	0.8975	–	0.8975	–
ρ	0.917	[0.87, 0.96]	0.906	[0.87, 0.94]	0.913	[0.88, 0.94]	0.87	[0.82, 0.91]
χ_π	1.47	[1.09, 1.78]	1.467	[1.08, 1.95]	1.46	[1.05, 1.87]	1.43	[1.1, 1.78]
χ_x	0.65	[0.4, 1]	0.813	[0.46, 1.17]	0.786	[0.43, 1.15]	0.625	[0.22, 1.03]
ϕ_r	0.838	[0.74, 0.93]	0.83	[0.75, 0.9]	0.84	[0.78, 0.90]	0.539	[0.38, 0.66]
ϕ_u	0.831	[0.77, 0.90]	0.86	[0.78, 0.93]	0.85	[0.78, 0.93]	0.83	[0.73, 0.91]
ϕ_{π^*}	0.91	[0.82, 0.98]	–	–	–	–	–	–
$\bar{\mathbf{g}}$	0.0353	[0.022, 0.047]	0.021	[0.014, 0.027]	–	–	0.035	[0.013, 0.045]
$\bar{\mathbf{g}}^x$	–	–	–	–	0.0161	[0.009, 0.023]	–	–
$\bar{\mathbf{g}}^\pi$	–	–	–	–	0.0247	[0.004, 0.04]	–	–

Note: the parameters that are not estimated are fixed at the values in Giannoni and Woodford (2003).

0.035. Moreover, when the model is re-estimated only for the Volcker–Greenspan sample 1982:IV–2004:II, the results remain similar (see Milani, 2004b). Those results suggest that learning is not simply capturing the omitted policy variation in the baseline model.

Some of the prior assumptions may affect the results. Particularly important is the prior for σ. The chosen prior puts most probability on the range of values estimated by Fuhrer (2000). Other papers, however, estimate higher coefficients. Moreover, the chosen prior may also have important effects on model comparison. The posterior distributions of the coefficients σ and φ, in fact, fall far from their prior distributions. Since the choice of a prior that puts most of the probability mass on parameter values that fit the data poorly could be a reason for a model to do poorly on model comparison, it becomes necessary to check the results under alternative priors. The best choice is to assume the same prior for both the RE and learning model. The coefficient σ is now estimated in both models assuming a Gamma prior with higher mean (the mean is 0.5 as in Lubik and Schorfheide, 2004, 2007) and higher standard deviation (0.35) than before. The estimates for the

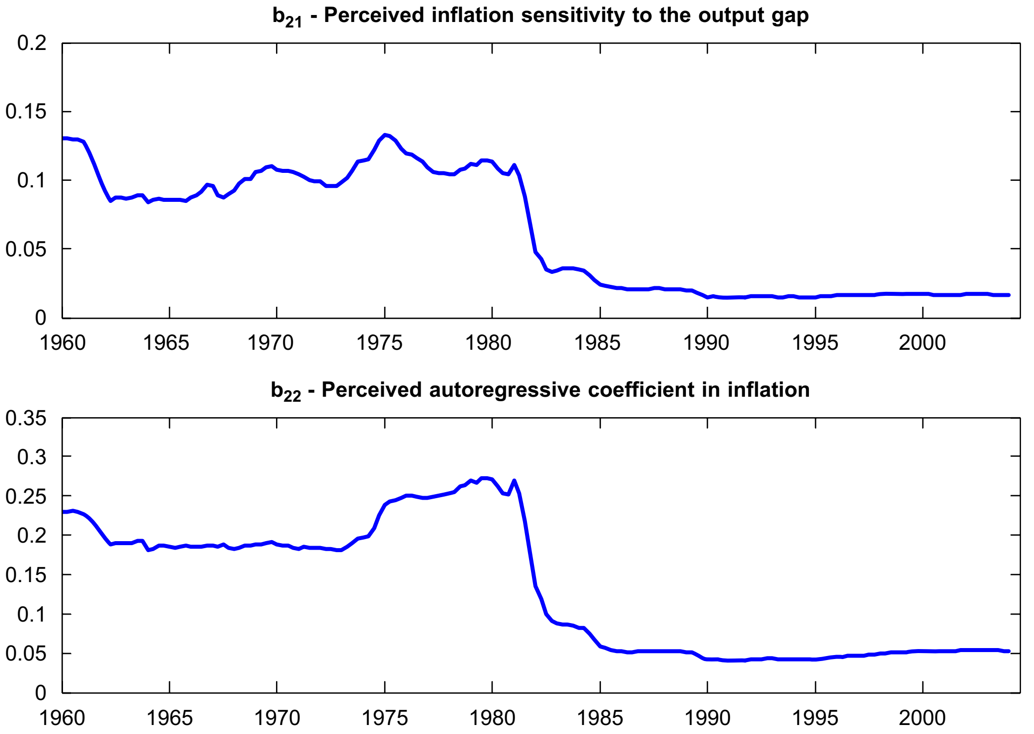

Fig. 3. Evolution of agents' beliefs with estimated initial conditions (1960:I–2004:II).

learning model imply a larger, but not excessive, degree of habit formation ($\eta = 0.32$), and still almost zero indexation ($\gamma = 0.032$). The estimates for σ and for the gain are higher ($\sigma = 1.19$, $\bar{\mathbf{g}} = 0.021$). To check the robustness of the model comparison results, the model is re-estimated under RE. The estimates are not far from those in Table 3 ($\eta = 0.78$, $\gamma = 0.89$, $\sigma = 0.42$). With the different prior, the model with learning still has a higher marginal likelihood (equal to -751.32 versus -757.64 for the model with RE, implying a posterior odds ratio equal to 553.5).[19]

So far, economic agents are assumed to learn the law of motion of different variables at the same rate. The learning process, however, can occur at different speeds when it refers to output or inflation. The results are robust to allowing different gain coefficients for the output gap (denoted by $\bar{\mathbf{g}}^x$) and inflation (denoted by $\bar{\mathbf{g}}^\pi$). Both are assumed to follow a prior gamma distribution with mean 0.031 and standard deviation 0.022. The estimated degrees of habits and indexation remain negligible ($\eta = 0.103$, $\gamma = 0.03$). The gain concerning the output gap is estimated equal to 0.0161, the gain concerning inflation, instead, equals 0.0247. The data are therefore suggestive of faster learning in the dynamics of inflation (Branch and Evans, 2006 similarly find a larger gain for inflation than for output).

As a final check, the model is re-estimated under a different set of values to initialize the learning algorithm (see Fig. 3). The initial beliefs are now estimated from pre-sample data

[19]The results are similar if priors with mean 0.125 or mean 1 are, instead, used.

(1954:III–1959:IV). The implied habit formation and indexation parameters are small ($\eta = 0.178$, $\gamma = 0.02$); the gain coefficient is now larger ($\overline{\mathbf{g}} = 0.035$).

6. Conclusions

A long-standing issue in macroeconomics has been how to endogenously generate persistence in the dynamics of economic variables to match stylized facts about aggregate data. Several extensions in various sides of the economy are typically needed to induce inertia in conventional rational expectations monetary models.

This paper has presented a simple model with learning. Agents do not know the structural parameters of the economy and use econometric models and historical data to infer parameters and form expectations over time. Realistic levels of persistence arise in the model from the updating of agents' beliefs. As a consequence, some extensions that are typically needed in rational expectations models to match the observed inertia, such as habit formation in consumption or indexation to past inflation, become redundant under learning. Learning can, therefore, represent a potential *single* mechanism, which can induce persistence without resorting to several modifications in different sides of the economy. Moreover, learning helps to improve the fit of current monetary DSGE models. The specification with learning fits better than the specification with rational expectations, according to the posterior model probabilities.

On the methodological side, the econometric approach of the paper has allowed joint estimation of the main learning rule coefficient (the constant gain), together with the structural parameters of the economy. Since the results in models with learning may be heavily dependent on the choice of the gain, this procedure avoids potentially important arbitrariness.

It is worth pointing out that other explanations of persistence are possible. Here, the paper has focused on comparing learning versus more structural sources of persistence, such as habits and indexation. It is admittedly hard to settle the issues studied in the paper using macroeconomic data alone: the results appear supportive of learning, but it should be emphasized that they need to be combined with evidence from more microeconometric studies on the relative importance of learning versus habit formation or non-expectations-based sources of persistence in inflation.[20] Ultimately, understanding the best microfoundations needed to imply inertia will be crucial, since those microfoundations will affect the welfare evaluation of alternative monetary policies.

References

Adam, K., 2005. Learning to forecast and cyclical behavior of output and inflation. Macroeconomic Dynamics 9 (1), 1–27.

An, S., Schorfheide, F., 2007. Bayesian analysis of DSGE models. Econometric Reviews, forthcoming.

Angeloni, I., Aucremanne, L., Ehrmann, M., Gali', J., Levin, A., Smets, F., 2005. Inflation persistence in the euro area: preliminary summary of findings. mimeo, ECB.

[20]The evidence is still limited: Dynan (2000), for example, finds no evidence of habits using PSID data. Angeloni et al. (2005), in a survey of preliminary results of the European Inflation Persistence Network, find that there is little evidence at the microlevel that price changes are clustered around past inflation, as would be implied by widespread automatic indexation of prices.

Boivin, J., Giannoni, M., 2006. Has monetary policy become more effective? The Review of Economics and Statistics 88 (3), 445–462.

Branch, W.A., Carlson, J., Evans, G.W., McGough, B., 2004. Monetary policy, endogenous inattention, and the volatility trade-off, mimeo.

Branch, W.A., Evans, G.W., 2006. A simple recursive forecasting model. Economics Letters 91 (2), 158–166.

Bullard, J., Eusepi, S., 2005. Did the great inflation occur despite policymaker commitment to a taylor rule? Review of Economic Dynamics 8 (2), 324–359.

Bullard, J., Mitra, K., 2002. Learning about monetary policy rules. Journal of Monetary Economics 49, 1105–1129.

Christiano, L.J., Eichenbaum, M., Evans, C.L., 2005. Nominal rigidities and the dynamic effects of a shock to monetary policy. Journal of Political Economy 113 (1).

Clarida, R., Gali', J., Gertler, M., 1999. The science of monetary policy: a new Keynesian perspective. Journal of Economic Literature 37, 1661–1707.

Dennis, R., 2003. New Keynesian optimal-policy models: an empirical assessment. Federal Reserve Bank of San Francisco Working Paper 03-16.

Dynan, K., 2000. Habit formation in consumer preferences: evidence from panel data. American Economic Review 90, 391–406.

Evans, G.W., Honkapohja, S., 2001. Learning and Expectations in Macroeconomics. Princeton University Press, Princeton, NJ.

Fuhrer, J., 2000. Habit formation in consumption and its implications for monetary policy. American Economic Review 90 (3), 367–390.

Giannoni, M., Woodford, M., 2003. Optimal inflation targeting rules. In: Bernanke, B.S., Woodford, M. (Eds.), Inflation Targeting. University of Chicago Press, Chicago.

Goodfriend, M.S., King, R.G., 1997. The new neoclassical synthesis and the role of monetary policy. NBER Macroeconomics Annual 1997, 231–283.

Ireland, P., 2003. Irrational expectations and econometric practice. Unpublished manuscript.

Lubik, T., Schorfheide, F., 2004. Testing for indeterminacy: an application to U.S. monetary policy. American Economic Review 94 (1), 190–217.

Lubik, T., Schorfheide, F., 2007. Do central banks respond to exchange rates? a structural investigation. Journal of Monetary Economics, forthcoming.

Marcet, A., Nicolini, J.P., 2003. Recurrent hyperinflations and learning. American Economic Review 93 (5), 1476–1498.

McCallum, B.T., Nelson, E., 1999. An optimizing IS-LM specification for monetary policy and business cycle analysis. Journal of Money, Credit and Banking 31 (3), 296–316.

Milani, F., 2004a. Adaptive learning and inflation persistence, mimeo. Princeton University.

Milani, F., 2004b. Expectations, learning and macroeconomic persistence. EABCN Working Paper, available at ⟨http://www.eabcn.org/research/documents/10-Milani-ELMPjul05.pdf⟩.

Orphanides, A., Williams, J., 2003. Imperfect knowledge, inflation expectations and monetary policy. In: Bernanke, B., Woodford, M. (Eds.), Inflation Targeting. University of Chicago Press, Chicago.

Orphanides, A., Williams, J., 2004. Robust monetary policy with imperfect knowledge. Paper presented at the Conference on Monetary Policy and Imperfect Knowledge, Wurzburg, October 14–15.

Orphanides, A., Williams, J., 2005a. Inflation scares and forecast-based monetary policy. Review of Economic Dynamics 8 (2), 498–527.

Orphanides, A., Williams, J., 2005b. The decline of activist stabilization policy: natural rate misperceptions, learning, and expectations. Journal of Economic Dynamics and Control 29 (11), 1927–1950.

Preston, B., 2005. Learning about monetary policy rules when long-horizon expectations matter. International Journal of Central Banking 2, September.

Preston, B., 2006. Adaptive learning and the use of forecasts in monetary policy. Journal of Monetary Economics 53 (3), 507–535.

Primiceri, G., 2006. Why inflation rose and fell: policymakers' beliefs and postwar stabilization policy. Quarterly Journal of Economics 121 (3), 867–901.

Rabanal, P., Rubio-Ramirez, J.F., 2005. Comparing new Keynesian models of the business cycle: a bayesian approach. Journal of Monetary Economics 52 (6), 1151–1166.

Sargent, T.J., 1999. The Conquest of American Inflation. Princeton University Press, Princeton.

Sargent, T.J., Williams, N., Zha, T., 2006. Shocks and government beliefs: the rise and fall of American inflation. American Economic Review 96 (4), 1193–1224.

Schorfheide, F., 2000. Loss function-based evaluation of DSGE models. Journal of Applied Econometrics 15 (6), 645–670.
Schorfheide, F., 2005. Learning and monetary policy shifts. Review of Economic Dynamics 8 (2), 392–419.
Smets, F., Wouters, R., 2003. Monetary policy in an estimated stochastic dynamic general equilibrium model of the euro area. Journal of the European Economic Association 1 (5), 1123–1175.
Smets, F., Wouters, R., 2005. Comparing shocks and frictions in US and euro business cycles: a Bayesian DSGE approach. Journal of Applied Econometrics 20 (2), 161–183.
Williams, N., 2003. Adaptive learning and business cycles, mimeo. Princeton University Press.
Woodford, M., 2003. Interest and Prices: Foundations of a Theory of Monetary Policy. Princeton University Press, Princeton.