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THE LAAKSO–TAAGEPERA INDEX IN A MEAN AND VARIANCE FRAMEWORK

Scott L. Feld and Bernard Grofman

ABSTRACT

The Laakso–Taagepera index (Laakso and Taagepera, 1979) has become the most commonly used measure to specify the ‘effective’ number of political parties in a party system where parties vary substantially in their vote and/or seat shares. It is well known that the Laakso–Taagepera index is the inverse of the even more widely used Herfindahl–Hirschman index of concentration (Hirschman, 1945; Herfindahl, 1950; cf. Taagepera and Grofman, 1981). Drawing on little known work by Feld and Grofman (1977, 1980) on the so called ‘class-size paradox’, it can also be shown that both indices may be re-expressed as simple functions of a distribution’s mean and variance. As far as we can judge, these latter relationships appear to be unknown in the party and electoral systems literatures. By expressing the Herfindahl–Hirschman index and the Laakso–Taagepera index in terms of means and variances we can see that each index has a ‘natural’ interpretation in terms of well known statistical parameters which allows their fundamental mathematical properties to be more clearly revealed.

KEY WORDS • electoral rules • Laakso–Taagepera Index • proportionality • seats and votes • voting

The Laakso–Taagepera index (Laakso and Taagepera, 1979) was invented to provide a non-arbitrary way to ‘count’ the ‘effective’ number of political parties in situations where parties vary substantially in their vote and/or seat shares. Consider a five-party constellation with party seat shares of (.42, .37, .13, .07, .01). Is this, effectively, a two-party system, since two of the parties are so much larger than the other three? Is this, effectively a three-party system, since the third largest party can form a majority by joining with either the largest or the second largest party, or because we can safely disregard parties with, say, less than 10 per cent of the vote? Is this, effectively, a four-party system because the only parties that we can safely disregard are those, with say, 2 per cent of the vote or less? It is not obvious how best to answer this seemingly simple ‘counting’ question. Laakso and Taagepera propose to answer this question by calculating

$$L-T = 1/\sum p_i^2, \quad (1)$$

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where p_i is the proportion (here of seats) of the i th party, $i = 1, n$. In this example, the Laakso–Taagepera ($L-T$) index gives us an ‘effective number’ of seat-winning parties of 2.98.

The rationale Laakso and Taagepera (1979) give for their index is that it satisfies a number of desirable properties, including taking on appropriate values at the boundaries (i.e., if all parties are of equal size then $L-T = n$; while if all components except one are zero, $L-T = 1$), being invariant if components of zero weight are added; and responding monotonically to changes in component shares. While it is not the only index to satisfy these properties it is, conceptually, arguably the simplest. Moreover, for data on Western European party systems, Laakso and Taagepera compare values for indices in the same family as $L-T$ and argue that the index of effective size they advocate is best at distinguishing clearly certain types of important differences, for example between moderate and heavily polarized systems. It is also easy to see, as they point out, that the denominator of the Laakso–Taagepera index, $\sum p_i^2$, is simply the very well known Herfindahl–Hirschman ($H-H$) index of concentration (Hirschman, 1945; Herfindahl, 1950; cf. Taagepera and Grofman, 1981). Thus the Laakso–Taagepera index is the inverse of the Herfindahl–Hirschman index.

Various authors (e.g., Wildgen, 1971) have proposed indices that can be adapted to become alternatives to the Laakso–Taagepera index as a measure of the number of ‘effective’ parties. However, thanks to its ease of calculation, its attractive theoretical properties (e.g., its link to the Herfindahl–Hirschman index, and the fact that, when all the parties are of the same size, the effective number of parties equals the actual number of parties), and its adoption by a number of senior scholars in the comparative politics area, including Arend Lijphart (see e.g., Lijphart, 1984; Grofman and Lijphart, 2002), virtually all scholars who study party systems or electoral competition make use of this index to specify the ‘effective’ number of political parties within a given district, or for a given region, or for the national party system as a whole, for a given election or set of elections.¹

While the straightforward relationship between the Herfindahl–Hirschman index of concentration and the Laakso–Taagepera index is well known, it does not appear to be known in the electoral systems or party literatures that both indices may be re-expressed as simple functions of a distribution’s mean and variance. Feld and Grofman (1977) appears to be the first paper in which the means-variance interpretation of the Herfindahl–Hirschman index is given, but because

1. However, even in the original Laakso–Taagepera work there are acknowledgements of cases in which the $L-T$ index may be misleading, for example, a situation in which one party has more than a majority of the seats. Recently, Taagepera (2005) has offered a new index of party balance to deal with some of the limitations of the $L-T$ index. Also, Grofman (forthcoming) and Dumont and Caulier (2003) have independently proposed what we might call the ‘ $L-T$ Banzhaf index’, in which we first calculate normalized Banzhaf power scores for the parties based on their seat (or vote) shares and then apply the $L-T$ index to the resulting proportions. The Banzhaf index (Banzhaf, 1965; cf. Brams, 1975) is a game-theory measure of power which calculates the likelihood that a group of voters which is voting as a bloc will have its votes be pivotal in converting a losing issue into a winning one (or a winning issue into a losing one) if it changes the directionality of its vote from no to yes (or, conversely, yes to no).

Feld and Grofman do not refer to the Herfindahl–Hirschman index by name, the algebraic identity they demonstrate has never, as far as we are aware, been picked up by subsequent authors.²

Here we repeat the analyses they provide of means and variance formulas, but now we tie the results explicitly to both the Herfindahl–Hirschman index and the Laakso–Taagepera index. By expressing the Herfindahl–Hirschman index and the Laakso–Taagepera index in terms of means and variances we can see that each index has a ‘natural’ interpretation in which its fundamental mathematical properties are more clearly revealed. We also show the link of both indices to another theoretical literature, that on the class-size paradox, the puzzle which is the central focus of Feld and Grofman (1977, 1980).

Explicating the Mathematical Linkages Between Means and Variances, and the H–H Index, the L–T Index, and the ‘Class-size’ Paradox

We begin by explaining the idea of what Feld and Grofman (1977, 1980) refer to as the ‘class-size’ paradox.³

Assume, for simplicity, that, at a given university, each of its m faculty teach the same number of courses, say k , and that the total enrollment in all courses taught in some semester is E . Clearly we have n , the number of courses, equal to mk . The average class taught by faculty, \bar{S} , thus contains E/n students. It must also be the case that, if the i th class is of size s_i , then

$$\bar{S} = \sum s_i/n \quad (2)$$

But, how large is the class size experienced by the average student? Well, if the i th class is of size s_i , then exactly s_i students experience a class of that size. Thus, the average class size experienced by students is the ‘student-weighted’ class average, that is,

$$\sum s_i^2/\sum s_i \quad (3)$$

2. Feld and Grofman (1977), in effect, reinvent the Herfindahl–Hirschman index without realizing that they are doing so, but they use their equivalent measure only to study differences between weighted and unweighted means, and do not link their results to work on indices of concentration or dispersion.

3. However, we would note that we now know that this paradox had been known in the statistics literature, under various names, for many decades before the Feld and Grofman papers were published – perhaps most commonly under the name of the ‘paradox of stocks and flows’. Here we explicate the paradox in the class-size context, but the paradox is one that apparently is subject to continual rediscovery, since it emerges in numerous quite different substantive contexts. For example, imagine cars on a road. We can find the average number of drivers on the road by counting how many drivers are on the road at any given time and then looking at the average number of cars on the road over the various time intervals. Alternatively, we can look at the number of drivers on the road as experienced by other drivers. In this latter approach, since many (probably most) drivers will be on the road (during rush hours) when there are lots of other drivers on the road, the driver-weighted experience of ‘road crowdedness’ will tend to be an experience involving very crowded roads.

Feld and Grofman (1977) show that the value of the formula given in Equation 1 must always be less than or equal to the value of the formula shown in Equation 2. In particular, they show that R , the ratio of Equation 2 to Equation 1 is given by

$$R = 1 + \sigma^2/\mu^2, \quad (4)$$

where σ is the standard deviation, and μ the mean, of the s_i values. Since both σ and μ are greater than or equal to zero (with σ equal to zero only if all classes are of exactly the same size), it is obvious that R must be greater than or equal to one.

As we can see from Equation 3 the greater the variance in class size, the greater will be the difference between the mean class size experienced by students taking the courses and the mean class size experienced by faculty teaching the courses. In particular, if there are a number of large lecture classes and also a number of small seminars, the variance in class size will be high and thus students will experience classes that are much larger, on average, than those experienced by faculty. Feld and Grofman (1977) use data from the classes at the State University of New York at Stony Brook to analyze this ratio for several different majors and for the university as a whole. For the university as a whole, classes had a mean size of 40.5 with a standard deviation of 65.8.⁴ Plugging those numbers into Equation 3 we get a ratio of roughly 3.74, that is, while faculty thought (correctly) that they were teaching classes with an average of just under 41 students in each, students thought (correctly, as well) that they were taking classes which averaged over 150 students!

While the Feld and Grofman (1977) calculations are in terms of raw numbers, it is easy to convert their formulae to percentages. In particular, if we divide through by \bar{E} , then we get the average class size as a proportion of total class enrollment, \bar{p} , as being given by

$$\bar{p} = \Sigma p_i/n = 1/n. \quad (5)$$

Similarly, the class-size proportion experienced by the average student is given by

$$\Sigma p_i^2/\Sigma p_i = \Sigma p_i^2 \quad (6)$$

R , the ratio of Equation 2 to Equation 1, remains, of course, as

$$R = 1 + \sigma^2/\mu^2, \quad (7)$$

Now we wish to move from the class-size paradox context to that of effective number of parties. The first important thing to note is that we can see from

4. Note that the reported data with the standard deviation being larger than the mean is not a typo.

Equation 2 that the class-size proportion experienced by the average student is simply the Herfindahl–Hirschman index. But, then, from Equations 1 and 3, we must also have

$$H-H = (1 + \sigma^2/\mu^2)/n. \quad (8)$$

Or, in other words,

$$H-H = (1 + n^2 \sigma^2)/n = \mu + n\sigma^2, \quad (9)$$

where $\mu = 1/n$.

Since, the Laakso–Taagepera is simply the inverse of the H–H index, we now have

$$L-T = n / (1 + n^2\sigma^2) = 1 / (\mu + n\sigma^2), \quad (10)$$

where again $\mu = 1/n$.

We believe that it is useful for students of comparative politics to be aware of these important links between the H–H and L–T indices and the mean and variance calculations familiar to all students of elementary statistics. The reasons for the importance of knowing these links are twofold.

On the one hand, the L–T index (and related indices of concentration such as the H–H index) are commonly viewed as *sui generis*, taught to students of comparative politics as completely disconnected from the standard statistics that every social scientist is taught. Yet, it is obvious from Equation 8 that these indices can be reformulated as simple combination of the two most basic properties of a distribution, its mean and its standard deviation.⁵

On the other hand, when we express the Laakso–Taagepera index in terms of means and variances some features of the index now become completely transparent. For example, from Equation 8 it is apparent that, as must be the case, when the variance is zero (i.e., all parties are of equal size), the L–T index is simply equal to n . Moreover, we can see from Equation 8 that the L–T index increases inversely with the variance of the distribution of party shares and, it is also apparent that the L–T index is a non-linear function of n .⁶

5. Even knowledgeable authors may not realize that the L–T index is linked to both the mean and the variance of a distribution. For example, in an essay providing an approach to complement the acknowledged limitation in the L–T index that it makes most sense for the situation in which all parties are of equal size, Taagepera (2005: 283) asserts that the additional measure of ‘balance’ he proposes is akin to ‘supplementing the mean with the standard deviation’. But, as we have seen, the L–T index already is a function of a distribution’s standard deviation.

6. Arguably, the L–T index responds in a ‘natural’ way to the mean and variance of the distribution whose properties it is attempting to model. But that is a question beyond the scope of this short research note (Dumont and Caulier, 2003; Siaroff, 2003; Taagepera, 2005).

REFERENCES

- Banzhaf, John (1965) 'Weighted Voting Doesn't Work: A Mathematical Analysis', *Rutgers Law Review* 19: 317–43.
- Brams, Steven J. (1975) *Game Theory and Politics*. New York: Free Press.
- Dumont, Patrick and Jean-Francois Caulier (2003) *The 'Effective Number of Relevant Parties': How Voting Power Improves Laakso-Taagepera's Index*. Unpublished manuscript, Comparative Politics Center, Catholic University of Louvain, Belgium.
- Feld, Scott L. and Bernard Grofman (1977) 'Variation in Class Size, the Class Size Paradox, and Some Consequences for Students', *Research in Higher Education* 6(3): 215–22.
- Feld, Scott L. and Bernard Grofman (1980) 'Conflict of Interest between Faculty, Students and Administrators: Consequences of the Class Size Paradox', *Frontiers of Economics* 3: 111–16.
- Grofman, Bernard (forthcoming) 'The Impacts of Electoral Laws on Political Parties', in Barry R. Weingast and Donald Wittman (eds) *The Oxford Handbook of Political Economy*. New York: Oxford University Press.
- Grofman, Bernard and Arend Lijphart (eds) (2002) *The Evolution of Electoral and Party Systems in the Nordic Countries*. New York: Agathon Press.
- Herfindahl, O. C. (1950) *Concentration in the Steel Industry*. Unpublished PhD Dissertation, Columbia University.
- Hirschman, A. O. (1945) *National Power and Structure of Foreign Trade*. Berkeley, CA: University of California Press.
- Laakso, M. and Rein Taagepera (1979) "'Effective" Number of Parties: A Measure with Application to West Europe', *Comparative Political Studies* 12: 3–27.
- Lijphart, Arend (1984) *Democracies: Patterns of Majoritarian and Consensus Government in Twenty-one Countries*. New Haven, CT: Yale University Press.
- Siaroff, Alan (2003) 'Two-and-a-half Party Systems and the Comparative Role of the Half', *Party Politics* 9(3): 267–90.
- Taagepera, Rein (2005) 'Conservation of Balance in the Size of Parties', *Party Politics* 11(3): 283–98.
- Taagepera Rein and Bernard Grofman (1981) 'Effective Size and Number of Components', *Sociological Methods & Research* 10(1): 63–81.
- Wildgen, J. K. (1971) 'The Measurement of Hyperfractionalization', *Comparative Political Studies* 4: 233–43.

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