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Conceptualizing voter choice for directional and discounting models of two-candidate spatial competition in terms of shadow candidates*

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Abstract. In contrast to the traditional modeling of voter choice based on proximity, under directional models, selection of candidates is based on the direction and/or intensity of change from a status quo or neutral point. Voter choice can also be modeled as representing both approaches, e.g., as a directional model with proximity restraint, or alternatively, in terms of proximity to discounted positions.

We provide a unified perspective for these seemingly disparate models in terms of what we call "shadow" positions. We demonstrate that voter choice in a variety of spatial models including directional components can be viewed as proximity-based choices. Voters choose the candidate whose shadow is nearer, where shadow locations are defined by a simple transformation. We apply this approach to equilibrium analysis, showing that results for a discounted proximity model can be carried over – via shadows – to a variety of directional models.

1. Introduction

The traditional approach to modeling voter choice depends on proximity. Voters are assumed to vote for the candidate who is closest to them in terms of issues – where issue preferences are represented as a location (voter ideal point) in n-dimensional issue space (Downs, l957; Davis, Hinich and Ordeshook, l970; Enelow and Hinich, l990). Matthews (1979) and Rabinowitz and Macdonald (l989) have proposed alternatives to the standard proximity model by focusing on directionality, where voters choose between candidates based on a most preferred *direction* of change.[1] We subsequently refer to the Rabinowitz-Macdonald model as the RM model. Merrill and Grofman (l996) provide a unifying geometric framework within which the standard Downsian proximity model, a discounting version of that model due

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to Grofman (l985), the RM model and the Matthews pure directional model all can be viewed as special cases.

Here we discuss an alternative way to view directional and proximity models within a unified perspective, in terms of what we call "shadow" locations. We show that voter choice in a variety of spatial models, including what are normally regarded as directional models, can be viewed as proximity decisions in which the candidate locations are replaced by shadows, where shadow locations are defined by a simple transformation, usually involving shrinking. Thus voters choose that candidate whose shadow is nearer. This approach unifies our understanding of what otherwise appear to be disparate spatial models and permits us to see relations among them.

In the one-dimensional case, we can specify Nash equilibria for a wide class of models, including directional models, in terms of these shadow positions. Except under strong symmetry assumptions about discount factors (when it also predicts convergence), the Grofman discounting model and certain directional models predict *moderate* divergence, a seemingly more realistic expectation than the strict convergence of the pure proximity model or the strong divergence of the pure RM model.[2] For higher dimensions, the shadow concept is extended via the yolk.

2. Directional, proximity, discounting, and mixed models

2.1. *Directional models*

2.1.1. *The Rabinowitz-Macdonald model*

The *RM directional model* is defined (Rabinowitz and Macdonald, 1989) via the utility function

$$U(\mathbf{V},\mathbf{C}) = \mathbf{V} \bullet \mathbf{C} = \sum_{i=1}^{n} V_i C_i. \tag{1}$$

For $i = 1,...n$, the absolute values of the coordinates, V_i and C_i, are interpreted as intensities with which a voter and a candidate hold positions on dichotomous issues. The signs (+ or –) of these coordinates reflect the positions taken. The origin, or zero vector, is interpreted as the neutral point, **N**, i.e., the point for which the voter (or candidate) is indifferent between the two positions on each issue. For the RM directional model, the indifference curve for two candidates is the (hyper)plane perpendicular to the segment joining the candidates and passing through the neutral point, **N**.

2.1.2. *Matthews directional model*

N is interpreted as a status quo point. Voter utility reflects only the direction and not the intensity of voter and candidate positions. The *Matthews directional model* is defined (Matthews, 1979) by the utility function[3]

$$U(\mathbf{V}, \mathbf{C}) = \frac{\mathbf{V} \bullet \mathbf{C}}{|\mathbf{V}||\mathbf{C}|} \tag{2}$$

where $\mathbf{V} \bullet \mathbf{C} = \sum_{i=1}^{n} V_i C_i$ is the scalar product of **V** and **C**, and $|\mathbf{V}| = \sqrt{\sum V_i^2}$ and $|\mathbf{C}| = \sqrt{\sum C_i^2}$ are the lengths of the vectors **V** and **C**, respectively. If either **V** or **C** is **0**, the utility is defined to be 0.[4] The indifference (hyper)plane for two candidates passes through **N**, bisecting the angle between the candidates.

2.2. *Downsian and discounted proximity models*

2.2.1. *Downsian proximity model*

The *Downsian proximity model* specifies that utility is a declining function of distance from voter to candidate. We will consider the formulation in which that function is quadratic, i.e.,

$$U(\mathbf{V}, \mathbf{C}) = -|\mathbf{V} - \mathbf{C}|^2. \tag{3}$$

The indifference (hyper)plane is the perpendicular bisector of the segment joining the candidates.

2.2.2. *A Downsian proximity model based on discounting and the status quo*

Grofman (1985) proposed a modification of the standard proximity model that incorporated two innnovations: discounting and the location of a status quo point. Suppose that spatial locations represent ideal positions in the Downsian sense and **S** is the status quo point. Grofman argues that voters are more likely to compare candidate claims not with an absolute point but with their perception of current policies, i.e., the status quo. Furthermore, he assumes that voters believe that a candidate, say **A**, will not actually move the status quo from **S** to **A** but only part way in the direction from **S** to **A.** For simplicity we may initially assume that all voters agree on a common discounting factor, d_A, so that they believe that **A** will implement policies at location $\mathbf{A}_g = d_A\mathbf{A} + (1–d_A)\mathbf{S}$ rather than at **A**. We will refer to a proximity model with both a status quo and discounting as the *Grofman discounting model*.

2.3. *Two modifications of the RM model*

2.3.1. *A directional model with proximity restraint*

Rabinowitz and Macdonald (1989) model the need for a restraint on voter utility for extreme candidates by postulating a "circle of acceptability" beyond which candidates lose utility among all voters. While we agree that there may well be parties/candidates whose locations are treated as so extreme that distances calculated to them are not meaningful,[5] the notion of a circle of acceptability appears ad hoc. The location of the circle is arbitrary and is not estimated from the data. Secondly, the implied sharp edge in utility as candidates pass across the circle seems unwarranted. Finally, the circle of acceptability ignores the fact that assessment of extremeness depends heavily on the voter's own position. What is or is not extreme to a socialist, for example, may be reversed for a right-wing voter.

Iversen (1994) has suggested that the utility function of the directional model be ideosyncratically restrained by subtracting a quantity that is small for candidates near the voter but increasingly large as candidates recede from the voter, e.g., by subtracting a multiple of the square of the distance between voter and candidate. The resulting *RM model with proximity restraint* is a mixed directional and proximity model with utility defined by

$$U(\mathbf{V},\mathbf{C}) = 2(1-\beta)\mathbf{V} \bullet \mathbf{C} - \beta|\mathbf{V} - \mathbf{C}|^2. \tag{4}$$

where β is a mixing parameter.[6] Iversen (1994) refers to this model as the *representational policy leadership model* since response to the spatial distribution of voters constitutes representation while the taking of directional positions and their intensity constitutes leadership.

This mixed model, which nests both the proximity and RM models as special cases, was introduced by Rabinowitz and Macdonald (1989) and has also been studied empirically by Platt, Poole, and Rosenthal (1992); Merrill (1993, 1994, 1995); Iversen (1994); and Dow (1995). Using survey data from the U.S. and Europe, all except Rabinowitz and Macdonald find support for a significant proximity component. For elite voting in congressional roll-calls, Platt, Poole, and Rosenthal find the proximity model superior to the directional. Westholm (1995), focusing on intrapersonal evaluation of Norwegian parties, reaches the same conclusion. Interpreted as a restrained directional model, the mixed model has a natural rationale. The Downsian proximity, Grofman discounting, RM, and Matthews model, as well as the RM model with proximity restraint, can all be nested in a three-parameter unified model

$$U(\mathbf{V},\mathbf{C}) = 2(1-\beta)\frac{\mathbf{V} \bullet \mathbf{C}}{[|\ \mathbf{V}\ ||\ \mathbf{C}\ |]^{1-q}} - \beta\ |\ \mathbf{V} - d\mathbf{C}\ |^2, \tag{5}$$

permitting statistical tests of competing models (see Merrill and Grofman, 1996).

2.3.2. *The RM model with centered restraint*

Although we prefer the proximity restraint, we can represent Rabinowitz and Macdonald's original idea of dimunition of utility as candidates recede from the origin without the deus ex machina of an arbitrary circle of acceptability by the following simpler model, which we call the *RM model with centered restraint*. It is defined by the utility function

$$U(\mathbf{V}, \mathbf{C}) = 2(1 - \beta)\mathbf{V} \bullet \mathbf{C} - \beta \mid \mathbf{C} \mid^2 \tag{6}$$

where β is estimated from the data. The second term imposes a simple reduction in utility as candidates recede from the neutral point, the effect of which is slight for small deviations but increasingly pronounced as the distance becomes larger.

It is simple to check that, for a fixed voter **V**, utility under this model is a quadratic function of **C** with maximum at $\mathbf{C} = \left[\frac{1-\beta}{\beta}\right] \mathbf{V}$, i.e., in the same direction as the voter. Maximum utility occurs a $\mathbf{C} = \mathbf{V}$ if $\beta = 1/2$ (as the proximity model would predict). It occurs at more extreme points as β decreases, tending to infinity as β approaches zero.

3. Comparison of voter choice under alternative models

3.1. *Indifference curves and "shadow" candidates*

The models we have considered give rise to quite different expectations as to the nature of voter utility functions. Nevertheless, the indifference curves for the Grofman discounted Downsian model (with status quo point at the neutral point and with the same discount factor, *d*, for each candidate) are identical to those for the RM model with proximity restraint if the mixing parameter, β, is equal to *d*, as can be seen geometrically in two dimensions (see Figure 1). With a suitable change of coordinates, we take the status quo point to be the origin, so that the discounted position of **A** is given by $\mathbf{A}_g = d\mathbf{A}$.[7] For two candidates, **A** and **B**, the indifference line (hyperplane in higher dimensions) in the undiscounted Downsian model is the perpendicular bisector of the segment, $\overline{\mathbf{AB}}$. In the discounted version, it is the perpendicular bisector of $\overline{\mathbf{A}_g\mathbf{B}_g}$ (see Figure 1). By elementary geometry, these indifference lines are parallel. As *d* moves from 0 to 1, the discounted indifference line moves proportionately from the origin to the perpendicular bisector of $\overline{\mathbf{AB}}$.

This indifference line is identical with that for the directional model with proximity restraint with mixing parameter $\beta = d$ since both are perpendicular

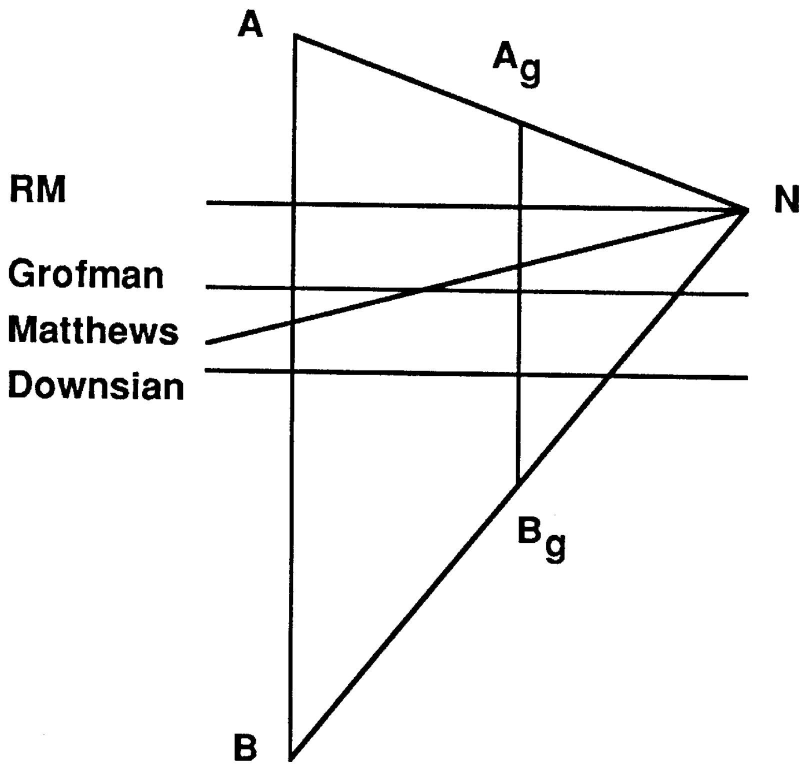

Figure 1. Indifference lines and shadow positions for four two-dimensional models, with neutral point, **N**, equal to the status quo point.

to $\overline{\mathbf{AB}}$ and located at a distance proportional to β (= *d*) from **N** to the bisector of $\overline{\mathbf{AB}}$. This follows (Merrill, 1993) because – under the mixed model – a voter **V** is indifferent between candidates **A** and **B** if and only if

$$\left[\mathbf{V} - \beta\frac{(\mathbf{A}+\mathbf{B})}{2}\right] \bullet [\mathbf{A} - \mathbf{B}] = 0, \tag{7}$$

i.e., if **V** lies on the hyperplane perpendicular to **A**–**B** passing through $\beta\,\frac{(\mathbf{A}+\mathbf{B})}{2}$, or equivalently, on the perpendicular bisector of the segment connecting $\mathbf{A}_g$ and $\mathbf{B}_g$, where $\mathbf{A}_g = \beta\mathbf{A}$ and $\mathbf{B}_g = \beta\mathbf{B}$ (see Figure 1). In other words, voter choice is as if the candidate positions were shrunken (or dilated) to $\mathbf{A}_g$ and $\mathbf{B}_g$ and voters behaved as under a proximity model, selecting not the nearer candidate but the candidate whose "shadow," $\mathbf{A}_g$ of $\mathbf{B}_g$, is nearer.

A similar calculation for the RM model with centered restraint shows that (7) defines indifference if β is replaced by $\frac{\beta}{1-\beta}$, i.e., if **V** lies on the per-

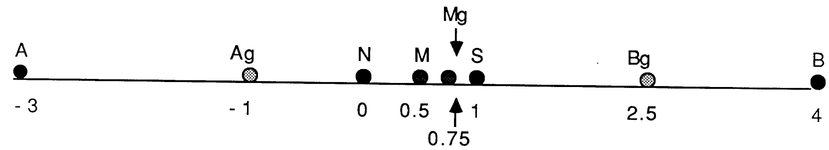

Figure 2. Indifference points for four one-dimensional models.

pendicular bisector of the segment connecting $\mathbf{A}'$ and $\mathbf{B}'$, where $\mathbf{A}' = \frac{\beta}{1-\beta}\mathbf{A}$ and $\mathbf{B}' = \frac{\beta}{1-\beta}$ $\mathbf{B}$. Thus, for $\beta < 1/2$, voters may behave as if the candidates are more clustered about the origin and are hence more likely to support relatively extreme candidates than under a pure proximity model, where $\beta = 1/2$.

Thus, if there is a single discount factor, voter choice for the Grofman variant of the proximity model is the same as for a mix of the Downsian proximity model and the RM model.[8] We summarize in the following proposition.

Proposition 1: The indifference curves are identical for the Grofman discounting model (with common discounting factor, *d*) and for the directional model with proximity restraint (with mixing parameter, β) provided that $\beta = d$ and the status quo is located at the neutral point.

Thus, if the status quo is identical with the neutral point and the discounting and mixing parameters agree, the Grofman discounting model and the directional model with proximity restraint are indistinguishable on the basis of indifference curves and hence on voter choice, although their utility curves are quite disparate.[9]

3.2. *Geometry in one dimension*

In one dimension, to compare the two directional models (Matthews and RM) with the standard Downsian proximity model and with the Grofman discounted proximity model is straightforward. In Figure 2 we show an example where the predictions of the four models differ. We let the Grofman proximity discount factor be .5 applied to both candidates; we let the Grofman status quo point be +1, the neutral point be 0, and the two candidates, **A** and **B**, be located at –3 and +4, respectively.

The Downsian proximity model has all voters to the left of the midpoint of $\overline{\mathbf{AB}}$, 0.5, voting for **A** and all to the right of 0.5 voting for **B**. The Grofman discounted proximity model with a discount factor of .5 applied to the locations of both **A** and **B** relative to the status quo point, 1, moves **A** from –3 to –1 (i.e., .5(–3) + .5(1)), which we denote $\mathbf{A}_g$; while it moves **B** from 4 to

2.5 (i.e., .5(4) + .5(1)), which we denote $\mathbf{B}_g$.[10] Thus, the midpoint of $\overline{\mathbf{A_g B_g}}$ is 0.75, which we denote $\mathbf{M}_g$. Hence, in the Grofman discounted proximity model, all voters to the left of 0.75 should vote for **A** and all to the right of 0.75 should vote for **B**.

In the RM directional model, all voters to the left of the neutral point 0 should vote for **A** and all to the right of 0 should vote for **B**. In the Matthews directional model, all voters to the left of the status quo point, 1, should vote for **A** and all to the right of 1 should vote for **B**.

Thus, in one dimension, for this example, the location of the indifference line in the four models ranges from a location at 0 (the RM directional model) to a location at 1 (the Matthews directional model), with the other two models intermediate (0.5 for the Downsian proximity model and 0.75 for the Grofman discounted proximity model). The extreme locations of the indifference lines for the two directional models is, however, coincidental. For appropriate choices of *N*, *S* and the Grofman discounting factor, any relative ordering of indifference line locations among the four models is possible.

3.3. *Geometry in two dimensions*

If we let the neutral point, **N**, of the RM model coincide with the status quo point, **S**, in the Grofman discounted proximity model and the Matthews directional model, for two dimensions we may embed these three models along with the standard Downsian proximity model within a common framework. In Figure 1, we show the indifference lines defined by the standard Downsian proximity model (the perpendicular bisector of the line $\overline{\mathbf{AB}}$); the Matthews directional model (the bisector of the angle, **ANB**); the RM directional model (the perpendicular to $\overline{\mathbf{AB}}$ through **N**); and the Grofman dicounting model with $d = .5$ (the line halfway between the lines for the Downsian and RM models and parallel to each).

As is apparent from inspection of Figure 1, voters with ideal points that lie in the cone defined by the RM and Matthews indifference lines are predicted to vote differently by the two models. Voters between the parallel indifference lines for the RM, Grofman discounting, and Downsian models vote differently between one or more pairs of these models, etc.

4. Nash equilibria

We now extend some standard results about candidate equilibria in a Downsian setting to the models developed in the previous section.

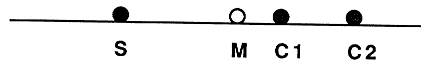

Figure 3. Nash equilibrium under the Grofman discounted model.

4.1. *Nash equilibria under the Grofman discounting model*

4.1.1. *One dimension*

Under the Grofman discounting model for one dimension, a Nash equilibrium is obtained if the candidates locate at points C_i,i = 1,2, such that

$$M = d_i C_i + (1 - d_i) S$$

where M is the median voter, d_i is the discounting factor for candidate C_i, i = 1,2, and S is the status quo (see Figure 3). In other words the candidates locate at points such that the electorate expects them, after discounting, to implement policy at the position of the median voter, because in turn the voters will behave as if both candidates are located at M. If S is taken as the origin, then $M = d_i C_i$. M may be interpreted as the (common) shadow of both C_i,i = 1,2.

Accordingly, the Nash equilibrium strategies are given by

$$C_i = S + (M - S)/d_i$$

and thus differ by the quantity $(M - S)(\frac{1}{d_1} - \frac{1}{d_2})$. Hence they may be divergent, but typically by only a modest amount if discounting is not excessive (see Figure 3 where d_1 = 0.75 and d_2 = 0.5). If the discounting factor is the same for both candidates, then the equilibrium strategies are convergent. Thus, the optimal strategies for the Grofman discounted model appear more like that of the proximity model than the extreme predictions of the RM theory.[11] We summarize in the following proposition.

Proposition 2: Under the Grofman discounted model in one dimension, a Nash equilibrium exists and is given by $C_i = S + (M–S)/d_i$. If the discounting factors are between 0 and 1, the equilibrium strategies cannot diverge by more than the quantity $\frac{|M-S|}{d_1 d_2}$.

4.1.2. *Two dimensions*

For the proximity model, in two or more dimensions, there is in general no Nash equilibrium; all medians need not intersect at a single point. The construction known as the yolk (defined by McKelvey, 1986, as the smallest disc which intersects all medians), however, retains some vestigial properties of the core. For example, any point outside the yolk can be beaten (or tied) by some candidate in the yolk. A similar construction can be made for the Grofman discounting model.

Given a status quo point, **N** which we take to be the origin, a common discounting factor, *d*, and the yolk, **Y**, define the *pseudo-yolk*, $\mathbf{Y}'$, as the set of points **A** such that *d***A** lies in the yolk. Hence an alternative lies in the pseudo-yolk if and only if voters believe s(he) will implement policy in the yolk. In other words the yolk plays the role of shadow of the pseudo-yolk. The following proposition is then immediate.

Proposition 3: Under the Grofman discounting model, any candidate not in the pseudo yolk can be beaten (or tied) by some candidate in the pseudo-yolk.

4.2. *Nash equilibria under restrained directional models*

We have seen that voter choice for the Grofman discounting model with common discount factor, *d*, and **S** = **N** is the same as that for the RM model with proximity restraint, provided $\beta = d$, and the same as that for the directional model with centered restraint if $\frac{\beta}{1-\beta} = d$. It follows that the equilibrium analysis in Section 4.1 for the Grofman discounting model applies equally to these restrained directional models. Necessary and sufficient conditions for equilibrium for the pure directional models are provided in a separate paper (Merrill, Grofman, and Feld, 1996).

5. Discussion

We have seen that voter choice under a number of apparently disparate spatial models can be viewed as selection by proximity to “shadow” candidates, where shadow positions are obtained by a simple transformation of the actual positions. Models for which the shadow positions are obtained by simple multiplication by a constant (pure contraction or dilation) include proximity itself, the RM model with proximity restraint, RM with centered restraint, and the Grofman discounting model with common discounting factor and status quo at the origin.

Writing this transformation as $\mathbf{C_i'} = k\mathbf{C_i}, i = 1, 2$ for some constant, *k*, we note that the indifference hyperplane is the perpendicular bisector of the segment from $\mathbf{C}'_1$ and $\mathbf{C}'_2$. Thus it passes through $k\left[\frac{\mathbf{C_1}+\mathbf{C_2}}{\mathbf{2}}\right]$, i.e., lies in a position between that for a pure proximity and pure RM model, in proportion to *k*. For the pure proximity model, *k* = 1; for the RM model with proximity restraint, *k* is just the mixing parameter; for the Grofman discounting model,

k is the discounting factor; and for the RM model with centered restraint, $k = \frac{\beta}{1-\beta}$.

Under this class of models, for one dimension, a Nash equilibrium is obtained if the candidates locate at points $C_i, i = 1,2$, such that $M = kC_i$, where M is the core voter and k defines the shadow transformation. In other words the candidates locate at points such that their shadow positions fall on the median voter, because the voters will behave as if both candidates are located at M. Accordingly, the Nash equilibrium strategies are given by $C_i = M/k, i = 1, 2$.

We have shown that, in one dimension, the Grofman discounted model has the general implication of moderate divergence in party positions, compared to complete convergence or extreme divergence for the pure proximity and the pure RM model, repectively. In higher dimensions, insofar as competition under proximity tends to be constrained to the yolk (McKelvey, 1986), competition in the directional and discounting models studied here will tend to occur in the pseudo-yolk.

Notes

1. See also Cohen and Matthews (1980) for the Matthews model. The RM and related models have been further developed in Listhaug, Macdonald and Rabinowitz, l991; Macdonald, Listhaug and Rabinowitz, l991; Rabinowitz, Macdonald and Listhaug, l991; Rabinowitz, Macdonald and Listhaug, l993; and by others (Iversen, 1994; Merrill, 1993, 1994, 1995). A utility function, identical to that of the RM model, was used by Carroll (1972) and DeSoete and Carroll (1983), who refer to it as the “wandering vector” model.
2. This feature of the model is not made explicit in the model’s original presentation.
3. In the Matthews (1979) formulation of the pure directional model, all voters and candidates are restricted to the unit sphere (with the exception of totally indifferent voters at the origin). Our initial definition permits voters and candidates to assume any position in space but define utility in terms of the normalized positions, $\mathbf{V}/|\mathbf{V}|$ and $\mathbf{C}/|\mathbf{C}|$, so that political actors are assumed to behave *as if* they lay on the unit sphere. We will return to the Matthews formulation when convenient.
4. The RM utility is the product of the Matthews directional utility and an intensity factor, $|\mathbf{V}||\mathbf{C}|$, that is $U(\mathbf{V},\mathbf{C}) = \mathbf{V} \bullet \mathbf{C} = \left(\frac{\mathbf{V} \bullet \mathbf{C}}{|\mathbf{V}||\mathbf{C}|}\right)(|\mathbf{V}||\mathbf{C}|)$ and hence defines a mixed model, of which the Matthews directional function represents the pure directional component.
5. For example, Damgaard (1969) notes that in multiparty systems it has often been the case that communist parties or right wing monarchical parties are considered *uncoalitionsfahig*, i.e., so extreme as to be unacceptable as possible coalition partners for the other parties.
6. Both the mixed model of Rabinowitz and Macdonald,

$$U(\mathbf{V},\mathbf{C}) = 2\mathbf{V} \bullet \mathbf{C} - \beta[|\ \mathbf{V}\ |^2 + |\ \mathbf{C}\ |^2],$$

and the Iversen (1994) formulation,

$$U(\mathbf{V},\mathbf{C}) = s\mathbf{V} \bullet \mathbf{C} - (1-s)\ |\ \mathbf{V} - \mathbf{C}\ |^2,$$

are equivalent to (4). To see that (4) is equivalent to the Rabinowitz-Macdonald form, note that (4) may be expanded as

$$2\mathbf{V} \bullet \mathbf{C} - 2\beta\mathbf{V} \bullet \mathbf{C} - \beta[| \mathbf{V} |^2 - 2\mathbf{V} \bullet \mathbf{C} + | \mathbf{C} |^2]$$
$$= 2\mathbf{V} \bullet \mathbf{C} - 2\beta\mathbf{V} \bullet \mathbf{C} - \beta[| \mathbf{V} |^2 + | \mathbf{C} |^2] + 2\beta\mathbf{V} \bullet \mathbf{C},$$

which simplifies to the Rabinowitz-Macdonald form. The same β appears in both forms. To see that (4) is equivalent to the Iversen form, set $\beta = 2(1–s)/(2–s)$ and note that this makes $1–\beta = s/(2–s)$. Thus, (4) becomes

$$\frac{2s}{2-s}\mathbf{V} \bullet \mathbf{C} - \frac{2(1-s)}{2-s} | \mathbf{V} - \mathbf{C} |^2 = \frac{2}{2-s}[s\mathbf{V} \bullet \mathbf{C} - (1-s) | \mathbf{V} - \mathbf{C} |^2],$$

which is equivalent to the Iversen expression.

7. Note that the RM neutral point no longer need be the origin.
8. This general point is noted in passing in Rabinowitz, Macdonald, and Listhaug (1993) at footnote 3, but without any formal mathematical development of the exact link, such as is provided below in the next section.
9. Denote by U_{DP} utility under the mixed directional and proximity model. Utility under the Grofman discounting model is

$$\begin{aligned} U_G &= - | \mathbf{V} - \beta\mathbf{C} |^2 = -(\mathbf{V} - \beta\mathbf{C}) \bullet (\mathbf{V} - \beta\mathbf{C}) = -\mathbf{V} \bullet \mathbf{V} + 2\beta\mathbf{V} \bullet \mathbf{C} - \beta^2\mathbf{C} \bullet \mathbf{C} \\ &= - | \mathbf{V} |^2 + \beta^2 | \mathbf{V} |^2 + \beta[-\beta | \mathbf{V} |^2 + 2\mathbf{V} \bullet \mathbf{C} - \beta | \mathbf{C} |^2] \\ &= \beta U_{DP} - (1 - \beta^2) | \mathbf{V} |^2 . \end{aligned} \tag{8}$$

(To see that the expression in brackets is equal to U_{DP}, expand the scalar products in equation 4.) Thus, for a fixed voter, each utility is a linear function of the other. It follows that any voter indifferent between two candidates under one model will likewise be indifferent under the other.

10. We apply a uniform discounting factor to both **A** and **B** for illustrative convenience only. Grofman (l985) suggests that greater discounting might occur to the positions of some candidates than of others, e.g., some candidates might be more credible in their promises. In their study of U.S. presidential elections, Enelow, Endersby and Munger (l993) find evidence for discounting of challenger positions but not of the positions attributed to incumbents.
11. Grofman (1985) does not discuss this feature of his model.

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