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Stability Induced by "No-Quibbling"

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Abstract.

We demonstrate the important consequence of one particular type of voter behavior: the "differentiation" ("no-quibbling") constraint that alternatives too similar to the alternative which they might replace will not be considered. We find that imposition of a sufficient differentiation norm leads to stable outcomes of decision making in a spatial context. We also briefly consider the potential effects of other possible constraints on feasible choices, especially as these might synergistically interact with choices based on "no-quibbling."

Key Words: social choice, equilibrium, social norm, spatial voting model, no-quibbling

1. Introduction

Recent theoretical work on majority-rule decision making has by and large replaced the early pessimistic views about the instability and potential manipulability of majority-rule processes (see especially McKelvey 1976, 1979; Schofield 1978; Riker 1982) with a more nuanced picture of the prospects for stability based on the "fine structure" of spatial social choice (McKelvey 1986; Feld et al. 1987; Feld, Grofman, and Miller 1988, 1989; Miller, Grofman and Feld 1989; Koehler 1990; Tovey 1992; Schofield and Tovey 1992); or stability induced by institutional arrangements (Shepsle 1979a, 1979b), by features of voter choice such as incumbency advantage generated by "benefit of the doubt" (Feld and Grofman 1991) by "finagling" on the part of politicians (Wuffle et al. 1989), by the costs of delaying agreement (Hoffman and Packel 1982), by the use of sophisticated voting strategies (Banks 1985; Krehbiel 1986), or by the imposition of supermajoritarian requirements (Schofield, Grofman, and Feld 1988). In this article we contribute to the literature on the stability of majority-rule processes by considering what happens when voters only support options that are "distinctive" from the pending alternative, a choice rule which we label "no-quibbling." We believe that many voters act in this way, thus not wasting time considering options that are only trivially different from one another.

The empirical prevalence of the "no-quibbling" rule among voters must be the topic of further research. Here we are concerned to demonstrate that "no-quibbling" behavior among voters has important consequences for the stability and centrality of majority-rule process. We show that, if there is a sufficiently large differentiation constraint, there will be an alternative or set of alternatives that cannot be defeated by another "feasible" alternative.

This set of alternatives falls within a delimited "absorbing zone" in the central portion of the space. Thus, the existence of "no-quibbling" behavior can lead to the development of a majoritarian consensus even in situations where there is no majority winner.

Like McKelvey (1986), we shall illustrate our definitions and results in two dimensions and for Euclidean preferences, i.e., alternatives can be characterized as points in some n-dimensional issue space, where voters have a "bliss point." Much group decision making can be analyzed under this rubric. Also, like McKelvey (1986) and many subsequent authors, we deal with sincere choices made under standard amendment procedure, the most common parliamentary procedure, in which each alternative is paired against the previous victor in a sequence of pairwise contests.[1] Thus, "quibbling" is defined with respect to the currently pending alternative.

2. Notation

$d(a_i, a_j)$ represents the (Euclidean) distance between a_i and a_j

$a_i\ P\ a_j$ means a_i is majority preferred to a_j.

3. Basic definitions

Definition 1: In the spatial context, a (majority rule) trajectory is said to be subject to a *differentiation* ("no-quibbling") constraint of $m_{\min}$ if $d(a_i, a_{i+1}) > m_{\min}$ for all i. In other words, a trajectory is differentiated if the only alternatives that voters see as eligible to replace the alternative in place are those which differ from it to a significant extent, i.e., are at least $m_{\min}$ units of distance away.

Definition 2: The *yolk* is the sphere of minimum radius which intersects all median hyperplanes (McKelvey 1986; Ferejohn, McKelvey, and Packel 1984; Miller, Grofman and Feld 1990; Feld and Grofman 1991).

In two dimensions, median "hyperplanes" are median lines, i.e., straight lines separating the space into two parts, each containing no more than half of the voter ideal points. (Note that each part can contain less than half of the ideal points, because some ideal points can be on the line itself.) In two dimensions, spheres are just circles. Thus, in two dimensions, the yolk is the circle of minimum radius that intersects all median lines. Figure 1 shows the yolk for five-voter ideal points. We shall work with this particular sample throughout the article. We shall always use r to refer to the radius of the yolk.

Definition 3: The *win-set* of an alternative x (a point) is the set of alternatives, y, that are majority preferred to x, i.e., all y such that $y\ P\ x$.

The win-set of a particular point shown in Figure 1 is indicated in Figure 2 by the shaded "flower" patterns. Note that the win-set of a point need not extend in every direction from the point. In some directions, there may be no alternatives which beat it.

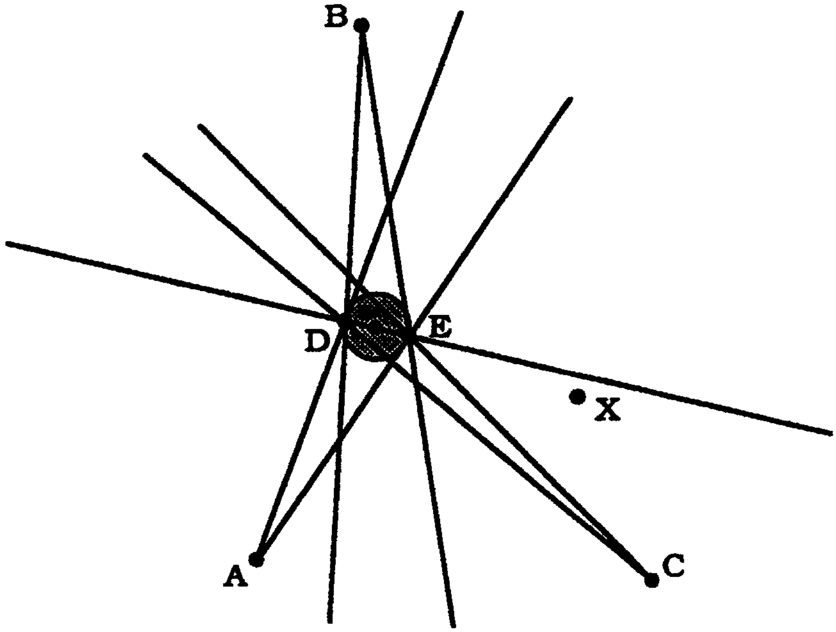

Figure 1. A five-voter example showing the yolk and alternative *x*.

Definition 4: When voters operate under a "no-quibbling" rule, we refer to the possible resultant agendas as *differentiated agendas*. In a differentiated agenda, voters require that each alternative differ from its predecessor by a distance of at least $m_{\min}$; i.e., voters don't bother about choices which are "trivially" different from one another.

Definition 5: By an *absorbing zone*, we mean an area of the space such that once we enter it no movement is possible that will take us out of this zone when we require that all trajectories satisfy some specified constraint (such as the differentiation constraint that the next element in the trajectory must be at least a distance $m_{\min}$ from the pending alternative).

If there is an absorbing zone, then it is the area within which outcomes can be expected to be found—although convergence may not be guaranteed.

4. Basic results

Theorem 1. If there is a differentiation constraint, $m_{\min}$, then, if $m_{\min} > 2r$, an absorbing zone exists and

(a) any alternative within the circle of radius d_1 around the center of the yolk with $d_1 = (m_{\min}/2) - r$, cannot be beaten by any alternative that can be reached by a permissible move;

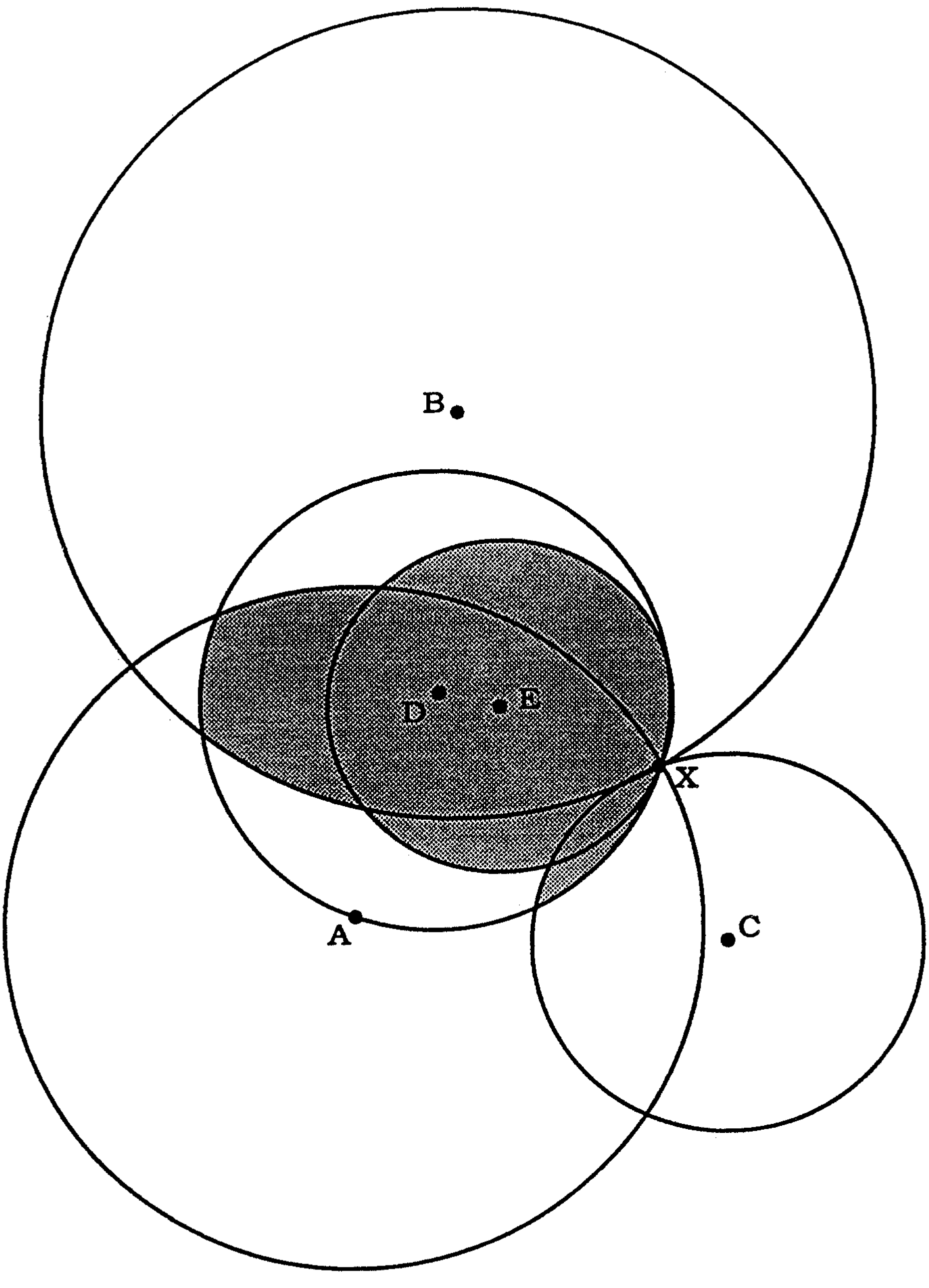

Figure 2. The unrestricted win-set for alternative x.

(b) any alternative outside the circle of radius d_2 around the center of the yolk, with $d_2 = (m_{min}/2) + r$, must be beaten by some alternative that can be reached by a permissible move.

In other words, if $m_{min} > 2r$, there exists an absorbing zone in the space which includes the circle with radius $d_1 = (m_{min}/2) - r$ around the center of the yolk, and this zone must be contained within the concentric circle of radius $d_2 = (m_{min}/2) + r$.

For proof, see Appendix.

Corollary 1 to Theorem 1. If, for some differentiation constraint, m_{min}, there is an absorbing zone, this zone always includes the center of the yolk.

Consider the case of a differentiation constraint where $m_{min} = 2r$. The absorbing zone includes the center of the yolk and falls within a circle of radius $2r$ around the center of the yolk. If m_{min} is larger than $2r$, the absorbing zone expands (i.e., every element that is absorbing when m is small remains absorbing for larger values of m). If $m_{min} = 4r$, then the absorbing zone includes the entire yolk and must be included within a circle of radius $3r$ around the center of the yolk. If r is small (as we would normally expect it to be, especially if the set of voters is large), then even small values of m_{min} can create an absorbing zone. Of course, the smaller the m_{min}, the smaller the zone.

Corollary 2 to Theorem 1. If $r = 0$, i.e., if there is a Condorcet (majority) winner, then, for any m_{min}, the minimal absorbing zone equals the maximal absorbing zone and is the circle at a radius $m_{min}/2$ around the center of the yolk.

Thus, if there is a majority winner and agendas are subject to a differentiation constraint, we may miss choosing the majority winner, because the absorbing zone created by the differentation constraint will generally include not only the majority winner but many other alternatives around it as well. However, the absorbing zone must be within $m_{min}/2$ of the majority winner and all trajectories are acyclic and must take us toward the absorbing zone.

Theorem 1 is, in our view, an important result. In the common case where there is no Condorcet (majority) winner, no-quibbling behavior can create the possibility of stability where otherwise there would be none.[2]

If there is an absorbing zone and power over agenda setting is shared, then whenever the collective choice deviates more than $2r$ beyond the absorbing zone, anyone who can offer an agenda item can move the collective choice to the absorbing zone in one step. Since any collective choice within the absorbing zone is the final choice of the group, individuals have incentive to offer alternatives in the absorbing zone that are preferred by them at their earliest opportunity. Thus, even if some powerful agenda setters are clever enough to manipulate agendas, any sharing of agenda-setting power makes it unlikely that they would be able to move the collective choice more than $2r$ beyond the absorbing zone. Even if there is a monopolistic agenda setter who can completely control the consideration of alternatives, the manipulability of outcomes is subject to the limitations discussed in the previous section; each move away from the yolk can go only a short distance, at most $2r$, and a majority of others can usually abort the proceedings if they perceive the trajectory as moving in an undesirable direction for them.

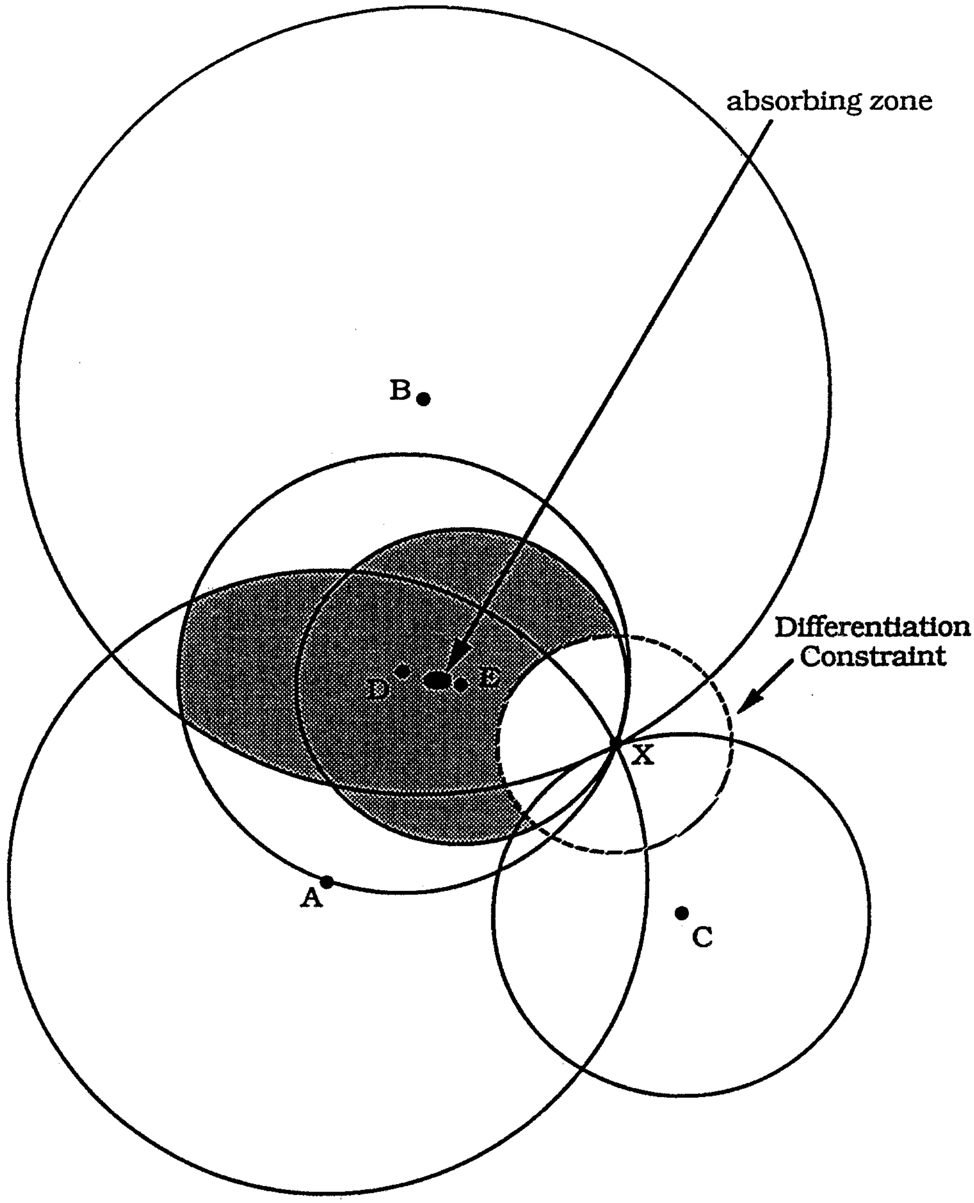

Figure 3. The permissible alternatives subject to a differentiation constraint with an absorbing zone (shape and location of absorbing zone only approximate).

We show in Figure 3 the approximate location of the absorbing zone if there were a differentiation constraint of the given magnitude shown for the five-voter example which we presented in Figures 1 and 2.

5. Other voter-choice rules and their interaction with no-quibbling

A differentiation constraint, requiring that new alternatives be sufficiently distinctive from previous alternatives, does not generally make it more difficult to move away from the yolk, but it does create an absorbing zone around the center of the yolk, such that once one reaches an alternative in that zone there are no permissible alternatives that can beat it. We can consider the consequences for stability and centrality of social choice of two other possible voter choice rules: the "incrementalist" rule that alternatives too distant from those which they might replace will not be considered, and the "no-skipping" rule that requires that desirable intermediate alternatives not be passed over.

More formally:

Definition 6. In the spatial context, a (majority-rule) trajectory (sequence of agenda items) is said to be subject to an *incremental change constraint* if $d(a_1, a_{a+1}) \geq m_{max}$ for all i. In other words, a trajectory is incremental in nature if the movement of any step from the alternative in place to any new alternative that might replace (defeat) it is at most an incremental shift of m_{max} units of distance, because no voter will vote for any greater change.[3]

Definition 7: A *direct trajectory* exists between any pair of alternatives a_i and a_k if and only if a_k P a_i and there is no alternative a_j on the line between a_i and a_k such that both a_j P a_i and a_j P a_k. In other words, a trajectory between two alternatives is direct if there are no intermediate alternatives that have been skipped over which are majority preferred to both the original alternative and the new alternative.

If groups are unwilling to consider alternatives too distinct from the prevailing consensus or pending status quo, then an incrementalism norm can be desirable. The "no-skipping" requirement, the least intuitive of our potential norms of voter choice, implements the commonsense notion that desirable options should not be passed by in favor of inferior ones.

An incremental constraint, requiring small steps, makes it especially difficult to move from an alternative closer to the yolk to one further away. Requiring trajectories that do not pass over intermediate preferred points has a similar type of consequence. Together these two constraints make it especially difficult to go from alternatives close to the yolk to those further away. Incrementalism and the no-skipping rule each tend to create strong centripetal pressures of a sort whose existence was first conjectured by Tullock (1967).

Combining a sufficiently strong differentiation constraint with one or both of the other two constraints can, by limiting the set of alternatives that can be considered at each stage of the process, make it especially likely that outcomes will be located within the absorbing zone, centrally located in the space. The requirement of direct trajectories in the context of a differentiation constraint makes it more difficult to move away from the yolk, and increases the size of the absorbing zone. Consequently, it increases the likelihood that alternatives in the absorbing zone will be reached. When the differentiation constraint is strict enough (i.e., requiring long moves) and therefore the absorbing zone is large enough, then all direct trajectories must lead to the absorbing zone. When both the incrementalist constraint and the differentiation constraint are imposed, the constraints do not interact with one another, but simply act independently.

For the five-voter example in Figure 1, whose yolk we showed in Figure 2, and the approximate location of whose absorbing zone, given a differentiation constraint greater than $2r$, we showed in Figure 3, we provide in Figures 4 and 5 the consequences of imposing an incrementalism constraint and a no-skipping constraint, respectively, in addition to the differentiation constraint identified in Figure 3.

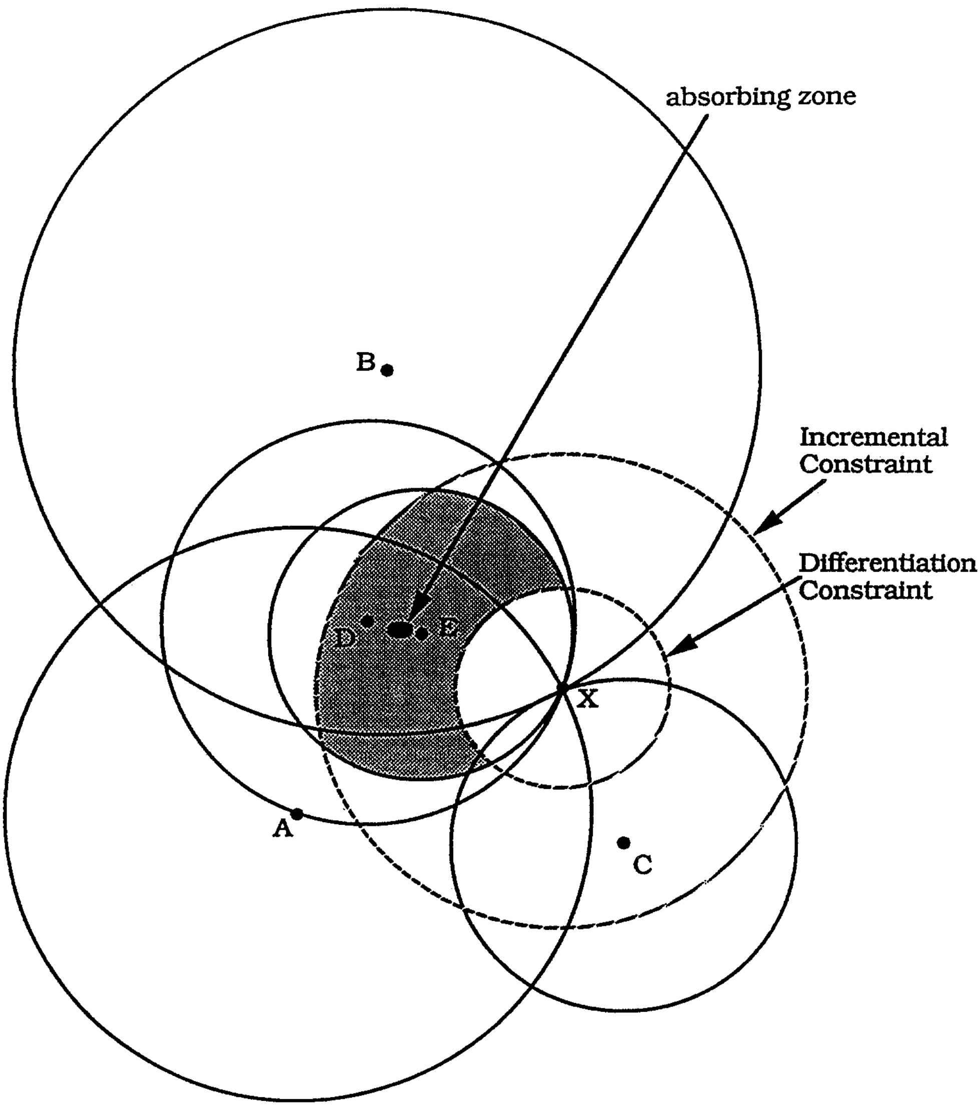

Figure 4. The permissible alternatives subject to incremental and differentiation constraints with an absorbing zone (shape and location of absorbing zone only approximate).

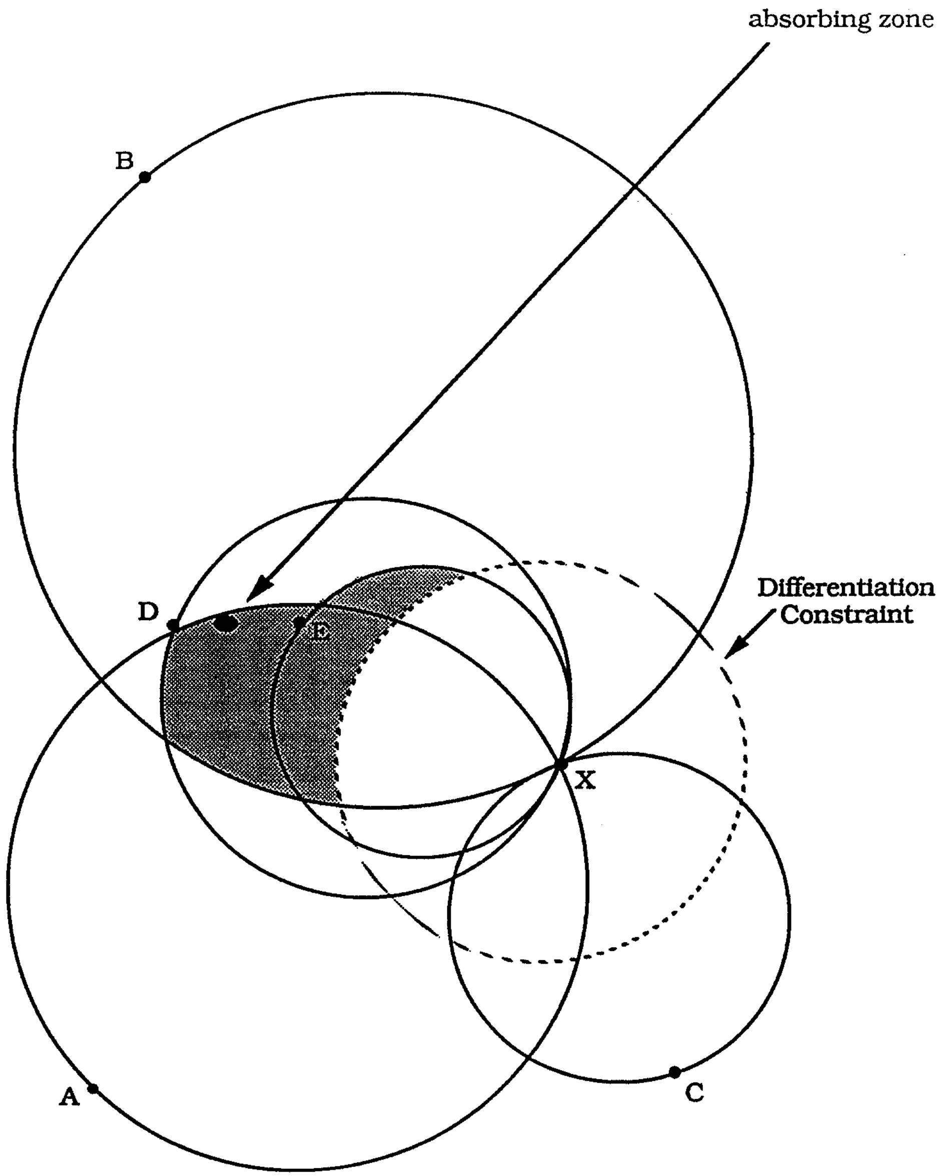

Figure 5. The permissible alternatives subject to "no-skipping" and differentiation constraints with an absorbing zone (shape and location of absorbing zone only approximate).

The key effects of the three different types of constraints on the minimum length of trajectories leading away from the yolk are briefly summarized in Table 1.[4] Results on the size of the absorbing zone under various types of constraints are given in Table 2.[5]

Table 1. Effects of incrementalist, differentiation, and no-skipping constraints on minimum possible agenda lengths from a point x at a distance d_1 from the center of the yolk to a point y at a distance d_2 from the center of the yolk.

Type of Constraint	Minimum Possible Agenda Lengths for Agendas Away from the Yolk
1. No agenda constraints	$\frac{d_2 - d_1}{2r}$
2. Incrementalist constraint m_{max}	$\frac{d_2^2 - d_1^2}{2m_{max}}$
3. Direct trajectory ("no-skipping") constraint	$\frac{d_2^2 - d_1^2}{r^2}$
4. Differentiation constraint	No lower limit beyond that shown in (1) above (but creation of an absorbing zone)
5. Incrementalism plus direct trajectories constraints	$\frac{d_2^2 - d_1^2}{2rm_{max} - m_{max}^2}$
6. Differentiation plus direct trajectories ("no-skipping") constraints	$\frac{d_2^2 - d_1^{2}*}{2rm_{min} - m_{min}^2}$ (and creation of an absorbing zone)
7. Incrementalism plus differentiation constraint	The effects are independent.

*Note: If $m_{min} \geq 2r$, then agenda cannot move away from the yolk.

Table 2. Radius bounds of an absorbing zone arising from a differentiation constraint of m_{min}, with and without a no-skipping constraint.

	Inner bound	Outer bound
Differentiation constraint only	$(m_{min}/2) - r$	$(m_{min}/2) + r$
Differentiation constraint plus No-Skipping (direct trajectory) constraint	$m_{min} - r$	$m_{min} + r$

Appendix A

Proof of Theorem 1. (a) By known results on the maximal bound on the win-set of a point, p (see Lemma 3), we know that the furthest point from p that might beat p is the one directly opposite p on the opposite side of the yolk and $2d_1 + 2r$ away from p, where d_1 is the distance of p from o, the center of the yolk. If $m_{min} > 2d_1 + 2r$, no alternative which can be reached by a permissible move can defeat p; but, by straightforward algebra, this is equivalent to the condition that $d_1 < (m_{min}/2) - r$.

(b) By known results on the minimal bound on the win-set of a point (see Lemma 3), we know that there must be a point at least $2d_2 - 2r$ away from p that beats p. If $m_{min} < 2d_2 - 2r$, then there must be some alternative which can be reached by a permissible move than can defeat p. This condition may be reexpressed as $d_2 > (m_{min}/2) + r$. ■

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Notes

1. Unlike some public choice theorists, we believe that sincere voting is characteristic of most groups, especially informally structured ones. For most groups, sophisticated voting in the sense of Farquharson (1970) is simply impossible, since sophisticted voting requires fixed and known agendas, and fixed and known preferences. Much group decision making occurs in the context of loosely structured group discussion where votes are not even formally taken. Moreover, even for a fixed agenda, the sorts of calculations required to vote sophisticatedly in a legislative setting are complex ones which few will be capable of without "training," and those calculations require majority preferences to be known in advance. Voters usually lack detailed knowledge about each other's preferences. Finally, votes are public statements. Individuals may lose credibility with others if they vote in contradictory ways. This encourages sincere voting even where strategy is possible. There are a few cases in formally structured groups such as legislatures where blocs voting sophisticatedly influence the fate of major pieces of legislation; and often legislative committees may behave in a sophisticated manner vis-à-vis the floor (Krehbiel 1986). However, even in legislatures, representatives may be constrained to vote sincerely because constituents see only the votes and not the strategic intent that lies behind them (Denzau, Riker, and Shepsle 1985).
2. However, if there is a differentiation constraint, but with $m_{\min} < 2r$, it is easy to show that no absorbing zone exists.
3. As far as we are aware, in the formal literature on spatial models, an incrementalism norm has previously been considered only in the limiting case of local cycles (Schofield 1978). These constraints may be thought of as sequential search norms (see Plott 1967).
4. It is interesting to note the close resemblance between the results for incremental and differentiated trajectories. The limitations are identical for incremental and differentiated trajectories if and only if

$$(m_{\min} - r) = (r - m_{\max}).$$

Thus, direct trajectories can be similarly limited by requiring each step to be either a long direct trajectory or a short direct trajectory.
5. The proofs of other results shown in Tables 1 and 2 are similar in form to those for the case of the no-quibbling norm, given in Appendix A and available from the authors upon request.

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