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Modeling the Dropoff Between Minority Population Share and the Size of the Minority Electorate in Situations of Differential Voter Eligibility Across Groups*

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In any constituency partitioned into two discrete and mutually exhaustive groups, the difference between a group's share of the *eligible* electorate and its share of the *total* population will depend, on the one hand, on the differences in the age structure and eligibility rates of the two groups and, on the other hand, it will also depend upon the relative population sizes of the two groups. We model the exact nature of this relationship, providing formulae which allow us to ascertain the maximum difference between a group's share of total population and its share of the eligible electorate and to specify the percentage for 'effective voting equality', i.e. the population proportion in a constituency for a group whose members have lower rates of voter participation needed to equalize the sizes of each group's eligible electorate. We discuss the applicability of these results to remedial districting plans in black/Hispanic voting rights cases in the United States and their implications for the differential holding of identity papers among blacks and non-blacks in South Africa. Copyright © 1996 Elsevier Science Ltd.

In seeking to predict the likely political consequences of alternative districting schemes, one key factor is the estimated turnout of different sections of the potential electorate. In the United States (US), the difference between potential electorate and actual electorate is likely to be especially great in districts that have substantial minority (e.g. black or Hispanic) populations. Questions about differences

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between potential and probable minority electorates have been highly salient in the remedy phase of voting rights litigation brought under the Voting Rights Act of 1965 (as amended in 1982) because there is then a need to draw a remedial plan that ends minority vote dilution by creating constituencies in which it is realistically possible for the minority community to elect candidates of choice (see Grofman and Handley, 1992; Grofman *et al.*, 1992 for a review of the relevant case law and leading Supreme Court decisions). While these legal aspects of redistricting are peculiar to the US, estimating the probable consequences of redistricting plans in terms of the share of the potential electorate that is of a given race (or ethnicity) is an important issue in other countries where parties may be organized along racial or ethnic lines.¹

In any constituency, the difference between minority share of the eligible electorate and minority share of the population will depend, on the one hand, on the differences in the age and eligibility rates of minority and non-minority populations and, on the other hand, it will also depend upon the relative population sizes of the two groups. We build on earlier work (Grofman, 1982; Brace *et al.*, 1988) to model the exact nature of this relationship.² We show precisely what happens to the dropoff between minority share of the eligible electorate as we vary the relative eligibility to vote rates of minority and non-minority populations, on the one hand, and the minority percentage of the total population in a given constituency, on the other. We then illustrate our model with two examples, one involving California data on Hispanic eligibility, where there are differences in both age eligibility and citizenship status between Hispanics and non-Hispanics, and one involving data from South Africa, where we focus on differences (ca. 1993) in the proportions of age-eligible whites and blacks who had citizen identity papers that would permit them to vote in the April 1994 election.

Extreme Case Example

It is easiest to begin with an extreme case so as to provide an intuition about what drives the general results that we later give. To keep our examples simple, we confine ourselves to districts where there are only two groups.³ We will initially use black to refer to one group and white as the label for the other groups, and we assume that blacks have a lower rate of vote eligibility than do whites.

Imagine that 100 per cent of whites are eligible to vote but only 50 per cent of blacks are (e.g. imagine that all whites have identity papers but only 50 per cent of blacks do). If we start with a district with 100 blacks and 100 whites (i.e. one that is 50 per cent black and 50 per cent white), we will have 100 eligible whites but only 50 eligible blacks (100×0.5); hence the black share of the eligible electorate is only 33.3 per cent ($50/150$), compared to its 50 per cent share of the total population in the district. This is a dropoff of 16.7 percentage points ($50 - 33.3$).

In contrast, imagine that the district has 180 blacks and only 20 whites. We will have 90 eligible blacks and 20 eligible whites, and thus a black share of the eligible vote of 81.8 per cent ($90/100$), compared with a black population share of 90 per cent. Now we have a dropoff of only 8.2 percentage points. If whites rather than blacks make up 90 per cent of the population in this district, the dropoff caused by white-black eligibility differences will again be small. If we start with 180 whites and 20 blacks (a jurisdiction that is 10 per cent black), then the eligible population will be 180 whites and 10 blacks, i.e. 6.3 per cent black. This is a

dropoff of only 3.7 percentage points. (Note that the dropoff is not the same for a 90 per cent black district as for a 90 per cent white district.)⁴

The Basic Model

More generally, if k fraction of the whites are eligible to vote, but only a $(k - q)$ fraction of the blacks are eligible, then if B is the proportion black and W is the proportion white in the district (with $B = 1 - W$), when the dropoff (measured in percentage points) between black share of the population and black share of the eligible electorate will be given by $100f$, where:

$$f = B - \frac{(k - q)B}{(k - q)B + k(1 - B)} \quad (1)$$

After some algebra, we may rewrite equation (1) as:

$$f = \frac{q(1 - B)B}{k - qB} \quad (2)$$

It is easy to see from equation (2) that f will be zero if $B = 0$ or if $B = 1$. The function given in equation (2) reaches its maximum, i.e. the dropoff is greatest, when $f' = 0$.

To find f' we use the chain rule and, after some algebra, when we set f' equal to zero we find the value of B that maximizes f , we obtain a quadratic function: $qB^2 - 2kB + k$.

By using the familiar solution to a quadratic equation, we may solve to obtain:

$$B = \frac{2k (+ \text{ or } -)\sqrt{4k^2 - 4qk}}{2q} \quad (3)$$

We may simplify this function as:

$$B = \frac{k (+ \text{ or } -)\sqrt{(k^2 - qk)}}{q} \quad (4)$$

Because k and q are positive, and because $k \geq q$, the only relevant root is the negative one, i.e. the desired optimum is given by:

$$B = \frac{k - \sqrt{(k^2 - qk)}}{q} \quad (5)$$

If $k = 1$ and $q = 0.5$, as in our initial extreme case example, then the maximum dropoff is roughly 17.2 percentage points and occurs when black population is equal to $2 - \sqrt{2}$, which is roughly 58.6 per cent, i.e. with a black population of 58.6 per cent the black share of eligibles will be only 41.4 per cent. Note that the dropoff at 58.6 per cent black population, at 17.2 percentage points, is only slightly greater than the dropoff at 50 per cent black population, which we earlier calculated to be 16.7 percentage points.

One other formula that is useful to give is what Grofman (1982)⁵ refers to as the "effective voting equality" proportion, i.e. the proportion of the population that needs to be black in order for the black share of the eligibles to be equal to that of the white voters (50 per cent). To find the proportion needed for effective voting equality we solve the equation below:

$$k(1 - B) = (k - q)B \quad (6)$$

to obtain

$$B = k/(2k - q).$$

If again $k = 1$ and $q = 0.5$, as in our initial extreme case example, then the necessary value of B is 0.667. In other words, under the above assumptions, only if the black population share is two-thirds will the number of blacks eligible to vote be equal to the number of whites eligible to vote.

Los Angeles County Example

A challenge to the districting lines of the five member Los Angeles County Board of Supervisors was brought in 1989 and decided in 1990 in favor of the Hispanic plaintiffs who had alleged intentional vote dilution in the way that 1981 district lines had been drawn to split the (East Los Angeles/San Gabriel Valley) Hispanic population concentration among three different districts. As part of the testimony in that case data was presented on the ethnic demography of the County.⁶ That data showed that the County population was 27.6 per cent Hispanic in origin, but only 23.3 per cent Hispanic in terms of voting age population and only 14.6 per cent Hispanic in terms of citizen voting age population.⁷ Thus, there was a dramatic difference between the Hispanic proportion in the County and the proportion of the eligible electorate that was Hispanic. From the data given in the opinion, we can calculate that 77.2 per cent of the non-Hispanic population in Los Angeles County was of voting age but 61.4 per cent of the Hispanic population was of voting age. Also, 92.3 per cent of the non-Hispanic voting age population were citizens, but only 52.0 per cent of the Hispanic voting age population were citizens.

Let us first look at the differences between voting age shares of the Hispanic and non-Hispanic populations. From the data above, we have values of $k = 0.77$ and $q = 0.16$ for the model in the section immediately above. Thus, the dropoff between Hispanic population share and Hispanic share of age-eligible voters in any given constituency with Hispanic proportion H will be given by:

$$f = \frac{0.16(1 - H)H}{0.77 - 0.16H}. \quad (7)$$

From our earlier results, we can find that the maximum value of this function occurs at $H = 0.563$. For this value of H , the dropoff, as given by equation (7), is 5.8 percentage points. In other words, in a Los Angeles district where the Hispanic population is 56.3 per cent, under the above assumptions, the Hispanic share of the age-eligible electorate will be 50.5 per cent.⁸ In this example the Hispanic percentage needed for effective voting equality is given by equation (6) as only 55.8 per cent if voting age were the only issue. Of course, only citizens are eligible to vote.

When we look at citizenship among the voting age population, then $k = 0.92$ and $q = 0.40$. Hence, using calculations like those above, the maximum dropoff between Hispanic share of voting age population and Hispanic share of citizen voting age population occurs when H is approximately 0.50. This maximum dropoff is roughly 14 percentage points, i.e. in a constituency where Hispanics and non-Hispanics are in roughly identical proportions in terms of age eligibles, then Hispanics will make up only 36 per cent of the citizen voting age population. We can also calculate the proportion of Hispanic voting age population sufficient to give rise to equal

numbers of Hispanic and non-Hispanic citizens in a constituency as 63.9 per cent.

We may combine these two calculations by looking at Hispanic and non-Hispanic rates of citizenship among the *total* population. Because we have previously reported the ratio of Hispanic voting age to total Hispanics and the ratio of Hispanic citizen voting age to Hispanic voting age (and the corresponding ratios for non-Hispanics), to obtain the desired ratios all we need do is multiply each pair of our two previous ratios, since the intermediate factor (that is, voting age) will cancel out. Hence, we find that only 31.9 per cent (61.4×52.0) of the Hispanic population is both a citizen and a voting age eligible, while, in contrast, 71.3 per cent of the non-Hispanic population in Los Angeles County is both a citizen and of voting age. For this combined calculation, $k = 0.71$ and $q = 0.39$. The maximum dropoff between population and citizen voting age population is 19.7 percentage points and occurs when Hispanic population is 59.8. We would require a Hispanic population of 68.9 per cent to equalize Hispanic and non-Hispanic citizen voting age populations.

South African Example

Reynolds (1993) presents data that suggests that (as of May 1993) only roughly 70 per cent of the blacks living in South Africa who would be eligible to vote on the basis of their age hold valid identity papers, while virtually all whites and colored hold citizen identity papers. This example gives us values of $k = 1$ and $q = 0.3$ for the model above. In this case, if we look at differences in identity papers among the age-eligible voters, and assume that those without such papers will be unable to vote, then the dropoff between black voting-age population share and black share of eligible voters will be given by:

$$f = \frac{0.3(1 - B)B}{1 - 0.3B} \quad (8)$$

From our earlier results, we can find that the maximum value of this function occurs at:

$$B = \frac{1 - \sqrt{(1 - 0.3)}}{0.3} = 0.544. \quad (9)$$

For this value of B , the dropoff, as given by equation (8), is 8.9 percentage points. In other words, in a South African district where the black population is 54.4 per cent, under the above assumptions, the black share of the eligible electorate will be only 45.3 per cent. In this example the black percentage needed for effective voting equality is given by equation (6) as 58.8 per cent.

Is the Maximum Dropoff Always Near 50 Percent?

For the three cases we have calculated above we observe that the maximum dropoff between eligible electorate and population share occurs when the group with lower eligibility is near 50 per cent of the population (58.6 per cent for the extreme case example, 54.4 per cent for the South African example, and 59.8 for the Los Angeles example).

Is this an accident of the particular values we have chosen for k and q , or is it a more general feature of the function f ?

The value of B for which f' is zero (and thus dropoff maximized) is given by:

$$B = \frac{k - \sqrt{(k^2 - qk)}}{q}. \quad (10)$$

But we can show that equation 10 is close to 0.50.

It is easy to see that:

$$-\sqrt{(k^2 - qk)} \approx q/2 - k, \quad (11)$$

because:

$$k^2 - qk \approx (q/2 - k)^2 = k^2 - qk - q^2/4.$$

But then, the desired result follows immediately upon substitution into equation 10. Equation (11), provides a good approximation if q is small.

Discussion

Our basic result is that, regardless of the level of discrepancy in eligibility rates between two population groupings, the dropoff between eligible electorate and total population will be greatest in constituencies where the two groups are found in *roughly* equal numbers. However, only under very special circumstances will this dropoff be greatest in districts where the groupings are *exactly* equal in population. Moreover, the dropoff will usually be relatively insensitive to population share in the immediate range around 50 per cent.

We have shown that, for actual data from the US and South Africa, maximum dropoffs can vary considerably, from a little over five percentage points to as large as nearly 20 percentage points. But we have also shown that dropoffs can be expected to be trivial when one or the other group makes up the bulk of the population. The formulae developed above should prove useful in any situation in which we are trying to anticipate potential political consequences of alternative districting plans where we are dealing with groups who differ in their eligibility to vote or in their expected turnout rates. This includes a wide range of situations, as illustrated by our examples of passbook identity papers among black and white South African and citizenship among Hispanics and non-Hispanics in the US.

Notes

1. For a discussion of this issue for South Africa, see Reynolds (1993).
2. We confine ourselves to differences in eligibility to vote. The same ideas can be extended to deal with differential turnout levels of two (or more) groups. Grofman and his colleagues model turnout in terms of a four step process. First the dropoff between total population and voting age population, then the dropoff between voting age population and citizen voting age population, then the dropoff between citizen voting age population and registration, and then the dropoff between registration and actual turnout. The work by Grofman and his colleague formalizes ideas advanced by a sociologist, James Loewen, testifying in a 1966 voting rights case in Hinds County, Mississippi.
3. Extending the analysis to more than two groups is straightforward but tedious and we will omit these essentially technical complications.
4. Alternatively, we might think about dropoff relative to initial minority population. In the first example we started with, a 50 per cent minority population, then the 16.7 percent-

age point dropoff we calculated above becomes a 33.34 per cent dropoff relative to the initial minority population. Here we will again see dropoff greatest when the two groups are near equal in size.

5. See also Brace *et al.* (1988).
6. *Garza v. Board of Supervisors of Los Angeles County*, No. CV 88-5143 KN (U.S. District Court, California, 4 June, 1990). This case was joined by the United States Department of Justice as a plaintiff-intervenor.
7. *Garza*, Op. cit., p. 16. The term Hispanic is used to refer to those who self-identify themselves as of Spanish origin on the US census.
8. We have provided calculations using countywide figures for illustrative purposes. We could substitute constituency-specific calculations as needed to the extent that the data are available for smaller levels of census aggregation.

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