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Cycle avoiding trajectories, strategic agendas, and the duality of memory and foresight: An informal exposition*

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Abstract. This paper considers the notion of cycle avoiding trajectories in majority voting tournaments and shows that they underlie and guide several apparently disparate voting processes. The set of alternatives that are maximal with respect to such trajectories constitutes a new solution set of considerable significance. It may be dubbed the *Banks set*, in recognition of the important paper by Banks (1985) that first made use of this set. The purpose of this paper is to informally demonstrate that the Banks set is a solution set of broad relevance for understanding group decision making in both cooperative and non-cooperative settings and under both sincere and sophisticated voting. In addition, we show how sincere and sophisticated voting processes can be viewed as mirror images of one another – embodying respectively, "memory" and "foresight." We also show how to develop the idea of a "sophisticated agenda," one in which the choice of what alternatives to propose is itself a matter of strategic calculation.

1. Introduction

Recently there have been hints of a reversal of the deep pessimism, such as that expressed by Riker (1980, 1982), about the possibility of meaningful social choice. This pessimism was provoked by the generic instability and global

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cycling results of Plott (1967), McKelvey (1976, 1979), Schofield (1978), and others, which seemed to imply that "anything can happen" under majority rule and related collective choice processes. But, more recently, a number of voting theorists have argued that, even in the face of very large or all-encompassing top cycle sets, social choice based on majority rule may be quite well behaved. This reversal of pessimism is in turn based on two lines of theorizing that are distinct but by no means incompatible. The first line derives from the recognition that social choice is always embedded in some kind of institutional context, which can constrain choice processes so as to create equilibria that would not otherwise exist (see, in particular, Shepsle, 1979, and Shepsle and Weingast, 1982). The second line of theory focuses on the pure majority preference relation itself and attempts to find some deeper structure and coherence within the system of majority preference that may constrain or guide (perhaps in a probabilistic sense) voting processes, even in the face of pervasive cycles and more or less independently of particularly institutional arrangements. (See especially McKelvey and Ordeshook, 1976; Miller, 1980; Bordes, 1983; Shepsle and Weingast, 1984; Banks, 1985; McKelvey, 1986. Also see Kramer, 1977; Ferejohn, McKelvey, and Packel, 1984.)

This paper attempts to contribute to the second line of theorizing, by considering the notion of "cycle avoiding trajectories" in majority voting tournaments and by showing that they underlie and guide several apparently disparate voting processes.

2. Definitions

We work within the conventional setup for finite voting games established by Farquharson (1969) and followed by Miller (1977, 1980), McKelvey and Niemi (1978), Bjurulf and Niemi (1982), Banks (1985), and others. In particular, we assume that the set of alternatives available for choice, together with the majority preference relation P over the alternatives, can be represented by a tournament, i.e., a complete asymmetric digraph. This means that the set of alternatives – which, for convenience, we usually refer to as "points" – is finite, and that no ties exist in majority preference between distinct alternatives. (This would be the case, for example, if an odd number of voters all had strong perferences.) Otherwise no structure is assumed concerning voter preferences. If x P y, we say "x beats y." W(x) designates the "win set" of x, i.e., the set of points that beat x. X^* designates the "top cycle set" of X, i.e., the minimal subset of points such that every point in X^* beats every point not in X^*. In general we assume the top cycle set is not a singleton, i.e., that preferences are cyclical and there is no majority winner.

Now consider the following construction. We pick an arbitrary point x_1 in the tournament. We next pick a point x_2 that beats x_1. We then pick a third point x_3 that beats both x_2 and x_1, then a fourth point x_4 that beats x_3, x_2, and x_1, etc. Proceeding in this manner, we construct what may be called a *cycle avoiding trajectory*, i.e., a set of points among which there is no cycle. We continue to construct this cycle avoiding trajectory until we can proceed no further – that is, until the top element of the trajectory is x_k and there is no point in the tournament that beats x_k and all the points below x_k in the trajectory. We call a cycle avoiding trajectory that cannot be expanded upward a *Banks trajectory*, in recognition of the important article by Banks (1985) that first made use of such trajectories (in the context of sophisticated voting).

A Banks trajectory has two important and distinct properties. The first property distinguishes a cycle avoiding trajectory from the broader concept of a *majority trajectory*, i.e., an ordered set of points $\{x_1, x_2, \ldots, x_k\}$ such that $x_h \text{ P } x_{h-1}$ for all $1 < h \leq k$. In a majority trajectory, each point beats the *immediately preceding* point, whereas in a cycle avoiding trajectory, each point beats *every* preceding point, i.e., $x_h \text{ P } x_g$ for all $1 \leq g < h \leq k$.

The second property distinguishes a Banks trajectory from the broader concept of a cycle avoiding trajectory. One way to express this property is the way Banks (1985) did and we did just above: a Banks trajectory has a maximal element – that is, the top point of the trajectory has the property that no other point can be placed on top of it to extend the trajectory upwards without violating acyclicity. This requirement is equivalent to saying that a Banks trajectory is an *externally stable set* – that is, every point outside the set is beaten by some point inside the set or, putting the matter the other way around, no point outside the set beats every point in the set (cf. Feld et al., 1987). The two properties of external stability and acyclicity set lower and upper bounds, respectively, on the length of Banks trajectories. For if we take some set of points that constitutes a Banks trajectory and begin to remove points, acyclicity necessarily is maintained but at some stage external stability is lost. Conversely, if we begin to add points, external stability necessarily is maintained but at some stage acyclicity is lost (assuming the full tournament is cyclic). Indeed, a set of points in a tournament is a Banks trajectory if and only if it is an acyclic externally stable set.

We call the maximal element of a Banks trajectory a *Banks point*. The *Banks set* consists of all Banks points. Banks (1985) demonstrated that the Banks set is a subset of the "uncovered set," defined in Miller (1980). The Banks set itself is externally stable but not acyclic – indeed, it contains a complete cycle (Miller, Grofman, and Feld, 1987).

Banks (1985) identified the set of points we have called the Banks set as the possible outcomes of a specific voting process, viz. sophisticated voting under standard amendment procedure. The purpose of the present paper is to demonstrate, in an informal fashion, that the Banks set is a solution set for majority voting tournaments of broader significance. We do this by examining four apparently disparate voting processes and showing that they all generate, either behaviorally or analytically, Banks trajectories and, accordingly, produce Banks points as their outcomes.

The first of these voting processes we dub the "cycle avoiding sincere process." It can be seen as a quasi-cooperative model of voting in a small committee. The second is sophisticated voting under standard amendment procedure, the process discussed by Banks (1985). We show that these two processes are mirror images, reflecting each other through the voting order and embodying, respectively, "memory" and "foresight" – a duality that has not, to our knowledge, been explicitly stated before.

The third process we examine involves agenda building where the agenda process is both "open" – that is, any voter may place any alternative on the agenda at any time – and "backwards built" (cf. Shepsle and Weingast, 1984) – that is, alternatives are subsequently voted on in the reverse of the order in which they are proposed. For such a process, we look at the problem of creating a "sophisticated agenda," i.e., an agenda in which the decision as to which alternatives to propose is itself a matter of strategic calculation by the voters. We show that the agenda building and voting stages of this process are mirror images and that, under reasonable assumptions, the agenda that is constructed will constitute a Banks trajectory in the whole tournament.

Finally, we reexamine the cooperative voting model presented in Miller (1980) and show that, analytically, it too is based on Banks trajectories and that, behaviorally, its outcomes are confined to the Banks set (and not just to the uncovered set, as previously established by Miller).

3. The cycle avoiding sincere process

A voting body of a particular type might proceed in this fashion. There is an existing status quo point, and the voting body considers various alternatives to the status quo at a time. When the voting body finds some alternative that it prefers, on the basis of majority preference, to the status quo, that alternative is adopted and it becomes the new status quo, or "provisional decision." Then, if other alternatives remain to be considered, the search for better alternatives continues. But, at each stage, in searching for better alternatives, the voting body compares new proposals, not only against the current provisional decision, but also against each prior provisional decision, so that the new alternative is accepted if and only if it represents a clear improvement over all points the body has already accepted, i.e., if and only if the new alternative is majority preferred to the current and every prior provisional decision. The voting body continues in this manner until every alternative has been considered.

This story should be taken primarily as heuristic. While the aggregate process has been fully specified, we do not here try to fill in details with respect to individual behavior.[1] Our motivating idea is that a voting body may seek what we might call majority consensus by generating a sequence of decisions that moves from some starting point in the direction of continued and unambiguous (i.e., consistent or transitive) improvement with respect to majority preference and continues to do so as long as such improvement is possible. Thus, we assume that in some fashion, the voting process has "memory" built into it, in that current proposals are compared, not only with the current provisional decision, but also with whatever the body has previously accepted. We think that this may be a plausible description of how certain voting bodies, especially smaller committees with relatively informal internal processes, may actually operate.

One can imagine, for example, behavioral mechanisms that could support such a process, in terms of what Riker (1983) has dubbed "heresthetics." Suppose a new alternative is being considered which in fact a majority of voters prefer to the current provisional decision, though a minority have the opposite preference. If it were further true that the new alternative was inferior, with respect to majority preference, to some prior provisional decision, its present opponents would have reason to invoke the heresthetic argument that "we should not adopt a proposal x that, we agree, is worse than some prior status quo z, when we have already found some other alternative y (i.e., the current provisional decision) that, we agree, is better than z."[2]

We may call the voting process described above the *cycle avoiding sincere process*. It is a sincere process in that it is driven by the "honest" majority preference relation applied sequentially from the first to final vote – and not by the kind of backwards induction that characterizes a sophisticated voting process (such as that described in the next section). That the sequence of adopted alternatives (or provisional decisions) is a cycle avoiding trajectory is immediate.[3] It should also be apparent that the Banks set is precisely the set of all possible voting outcomes under the cycle avoiding sincere process. In contrast, sincere voting under standard amendment procedure (Black, 1948, 1958) has no such guaranteed acyclicity.

4. A sophisticated process

Again a voting body is to select one alternative from the set X as its decision, and it does so under standard amendment procedure. But now we assume the voting order is publicly announced in advance and voters know each others' preferences or, at least, the majority preference graph. Thus voters are able to identify and use sophisticated voting strategies, in the sense of Farquharson

(1969) or, more particularly, in the sense of McKelvey and Niemi (1978). That is, they can anticipate the outcome of every possible final vote and can thereby identify the effective choices (or "sophisticated equivalents"), and thus their own best choices, at each next-to-last vote, and so forth.

In Miller (1977), increasingly stringent necessary conditions for an alternative to be the sophisticated voting decision under amendment procedure were identified. These conditions were used in Miller (1980) to demonstrate that the sophisticated decision always belongs to the "uncovered set." It can be checked that application of these conditions generates a cycle avoiding trajectory with a maximal element corresponding to the sophisticated decision.[4]

More recently, Shepsle and Weingast (1984) have devised the elegant "sophisticated voting algorithm." Given a set of alternatives $\{x_1, x_2, \ldots, x_m\}$ whose subscripts indicate the voting order, the sophisticated agenda $Z = (z_1, z_2, \ldots, z_m)$ is defined as follows:

$$z_i = x_i \text{ iff } x_k \in \bigcap_{j=i+1}^{m} w(z_j), \text{ and } z_i = z_{i+1} \text{ otherwise.} \tag{1}$$

Trivially, $z_m = x_m$; then the remaining elements of the sophisticated agenda may readily be computed by progressing backwards from z_m to z_1. Alternatives that do not appear in the sophisticated agenda Shepsle and Weingast dub *innocuous*; it follows that an alternative is innocuous if and only if it fails to beat every noninnocuous alternative following it in the voting order.

Proposition 1: Let $(d_1, d_2, \ldots, d_m)$ be a sequence of provisional decisions under the cycle avoiding sincere process. The sophisticated agenda algorithm (1) is identical to the following algorithm which generates outcomes of the cycle avoiding sincere process:

$$d_1 = x_1 \text{ iff } x_i \in \bigcap_{j=1}^{i-1} w\,(d_j), \text{ and } d_i = d_{i-1} \text{ otherwise.} \tag{2}$$

Proof: To see the equivalence between (2) and (1) we need merely replace the symbols d_k with z_k and reverse the order of voting.[5] Q.E.D.

Analytically, therefore, the cycle avoiding sincere process and sophisticated voting under amendment procedure are mirror images of one another – what is true for one is true of the other when voting order is reversed.

As noted before, the cycle avoiding sincere process has "memory" built into it in some unspecified way. Sophisticated voting is based on a model of individual behavior (noncooperative rationality) which explicitly incorporates "memory" in reverse – that is, "foresight"; i.e., voters look to the endpoints of the voting tree, anticipate the outcome associated with each node, and on this basis deduce the appropriate choice at each vote.

As Banks (1985) observed, Shepsle and Weingast's sophisticated agenda, or its truncated version (see note 5), is a cycle avoiding trajectory. Further, every possible voting order will, through the corresponding sophisticated agenda, generate every possible cycle avoiding trajectory. From these facts, Banks derived the insight that a necessary and sufficient condition for an alternative x to be the sophisticated voting decision under some voting order is that x be the maximal element of some cycle avoiding trajectory in the majority preference tournament (and not, as Miller, 1980, conjectured, merely that x be uncovered). That is, considering all possible voting orders, the set of sophisticated voting decisions under standard amendment procedure is precisely the Banks set of alternatives. By virtue of the duality result asserted in Proposition 1, the possible outcomes of the cycle avoiding sincere process must also be precisely the Banks set.

5. An agenda building process

We now consider sophisticated voting under standard amendment procedure in which the choice of which items to place on the agenda is also a matter for strategic calculation by the voters.

Let the set X be the set of feasible alternatives that might enter the voting, but now let us posit that not all alternatives need actually be voted on. However, some point x_1, perhaps representing the status quo, is automatically on the agenda. Members of the voting body can propose other alternatives – that is, select points out of the feasible set X and place them on the agenda for consideration under standard amendment procedure. As a result, a voting agenda $A \subset X$ is built up. We assume, to use the language of Shepsle and Weingast (1984), that the agenda is "built backwards" – that is, alternatives enter the voting order in the reverse of the order in which they are proposed; this implies in particular that the status quo x_1 enters the voting last. (In ordinary parliamentary procedure, agendas are built more or less backwards in this sense.) We assume that the agenda is fixed before any voting takes place.

If all voters have complete information concerning the structure of the majority preference tournament, then they can use this information at both the agenda building stage and at the voting stage of the process. However the agenda is built, if voting is sophisticated, by Banks's (1985) result any voting outcome will be a Banks point in the subtournament $A \subset X$. But we can show a new and stronger result.

Proposition 2. The agenda building process described above – though open and in some measure indeterminate – can be expected to work in such a way that the backwards built agenda will be a Banks trajectory, and thus the sophisticated outcome of the agenda building process will be a Banks point in the full tournament X (and not just in the subtournament $A \subset X$).

Proof: If no voter has an incentive to expand the existing agenda by proposing additional alternatives, we shall say that we have an *equilibrium agenda*. (This usage is similar, though not identical, to the same term as used by Ferejohn, Fiorina, and McKelvey, 1987.) Given an existing agenda, some alternative as yet not on the agenda is either noninnocuous, i.e., it can beat every noninnocuous alternative already proposed (and that will follow it in the voting order given by the backwards built agenda), or it is innocuous, i.e., it fails to beat every noninnocuous alternative already proposed.

No voter can have a strategic incentive to propose an alternative that is, given the existing agenda, innocuous, since adding an innocuous alternative to the agenda can have no effect on the sophisticated voting outcome, regardless of how many and which alternatives may subsequently be added to the agenda. In particular, adding an innocuous alternative to the agenda can have no effect on the sophisticated voting outcome of the agenda so expanded, and it can have no effect on which other alternatives not yet on the agenda are innocuous or noninnocuous given the agenda so expanded. Thus, rational agenda builders propose only noninnocuous alternatives. This implies that an open backwards built agenda will correspond to a cycle avoiding trajectory.[6]

On the other hand, if some alternative x yet to be proposed is noninnocuous, given the existing agenda, x beats every noninnocuous alternative already on the agenda, including the alternative that would be the sophisticated voting decision in the event the agenda were expanded no further. Thus some voters – indeed a majority of voters – have an incentive to expand the agenda by proposing x.

Thus rational agenda builders continue to expand the agenda so long as noninnocuous alternatives remain to be proposed. An equilibrium agenda is created when and only when no noninnocuous alternatives remain to be proposed. Put otherwise, an equilibrium agenda is an externally stable set vis-à-vis the entire set X of alternatives. By the previous conclusion, any existing agenda, whether or not it has expanded to equilibrium size, is a cycle avoiding trajectory. Thus an equilibrium agenda, constructed in an open backwards built process by rational agenda builders, is a cycle avoiding trajectory with some maximal element x, i.e., a Banks trajectory. Moreover x, a Banks point in the full tournament X is the outcome of the sophisticated agenda building (and sophisticated voting) process. Q.E.D.

We may note two additional points. First, define C(z) as the set of alternatives in X that are maximal in cycle avoiding trajectories all of which have z as their minimal (or lowest ranked) element. Clearly, for all z in X, C(z) is a subset of the Banks set. Then the set of possible outcomes of the sophisticated agenda building plus voting process considered above is $C(x_1)$. $C(x_1)$ may be a one-element set, in which case the process is fully determinate; but it may also be a multi-element set, in which case the (agenda building) process is to that extent indeterminate – that is, the outcome depends on who happens to propose what noninnocuous alternative when.[7] But given the assumptions made, $C(x_1)$ establishes bounds on the range of indeterminacy.

The second point is to note that two rationality assumptions were made concerning the agenda building process. The first was that only noninnocuous alternatives would be proposed. The second was that, so long as noninnocuous alternatives remain to be proposed, they will be added to the agenda. Both assumptions are necessary to conclude that the agenda will form a Banks trajectory. But only the second – and probably weaker – assumption is necessary for the more important conclusion that the outcome of the overall agenda building plus voting process will belong to the Banks set. (That is, even if voters "irrationally" add innocuous alternatives to the agenda – perhaps for reasons of "position taking" in the sense of Mayhew (1974) – the voting outcome is unchanged.)[8]

6. A cooperative process

We now assume that voters can cooperate fully – that is, they can communicate and, before any actual voting takes place, make binding agreements or contracts implemented through coalitions. The cooperative voting decision thus depends on the outcome of "pre-play" negotiations, which results in members of some decisive (majority) coalition agreeing to make some alternative the voting decision; once such an agreement has been struck, voting itself is a mere formality to be played out (e.g., by members of the majority coalition voting as a bloc at each vote). But of course the structure of the majority preference tournament over alternatives influences the course of these preplay negotiations.

In general, following Miller (1980), we may suppose that any tentative agreement or contract among members of one majority coalition to make some alternative z the decision may be upset by another tentative contract among members of another (necessarily overlapping) majority coalition to make some alternative in W(z) the decision. This process of "recontracting" continues until an agreement is reached that cannot be upset, i.e., an agreement on x such that $W(x) = \emptyset$ (i.e., the Condorcet winner, if any), or, in the absence of such an alternative, until the process is broken off essentially arbitrarily (e.g., under the constraint of time). Thus the cooperative decision depends on the majority preference tournament and the fact that the procedure makes a coalition decisive if and only if it is a majority coalition. The exact nature of the voting procedure, and the order in which alternatives are voted on, are irrelevant under fully cooperative conditions.

Thus we may speak of a *recontracting trajectory* as a sequence of alternatives with some arbitrary starting point and then following some majority trajectory. The question is: where may such a cooperatively determined trajectory end up? In other words, what alternatives may be the voting decision under this cooperative process? Miller (1977) conjectured that the answer to this question was the top cycle set. Miller (1980) sharpened this by demonstrating that the end points of recontracting trajectories must always lie in the uncovered set (a subset of the top cycle set).

But the upshot of the argument was that the next alternative after z in the recontracting trajectory would be some alternative in C(z) where y is in C(z) if and only if there is a sequence of alternatives $(x_1, x_2, \ldots, x_k)$ where $x_1 = z$, x_h e $\underset{g=1}{\overset{h-1}{\epsilon}} W(x_g)$ for all such that $2 \leq h \leq k$, $x = y$, and $\bigcap_{g-1}^{k} w(x_g) = \emptyset$, that is – though this characterization was not used – if and only if there is a Banks trajectory with z as its minimal (lowest ranked) element and y as its maximal element. It was then shown that, for all z in X, C(z) is a subset of the uncovered set.

But now we can go further and say that C(z) is a subset of the Banks set, for, by definition, the Banks set is C(z), where the union has taken over all z in X. Thus, the second element of any recontracting trajectory belongs to the Banks set, all recontracting trajectories cycle entirely within the Banks set, and – whenever this process is broken off – the cooperative decision belongs to the Banks set.

It should be noted that a recontracting trajectory is not itself cycle avoiding; indeed it is necessarily cyclical. Rather adjacent elements z and y in a recontracting trajectory are linked by a cycle avoiding trajectory (reflecting a bargaining process) with minimal element z and maximal element y.

7. Discussion

We have looked at four different processes for group decision making: (1) a sincere cycle avoiding process which implicitly assumed an informal norm of majoritarian consensus building; (2) a noncooperative voting process positing individual sophisticated voting; (3) a two-stage process which involved sophisticated considerations by voters both of what was to be on the agenda and of how to vote on agenda items once the agenda was set; and (4) a fully cooperative voting game in which competing (overlapping) coalitions bargained over alternatives in a sequential fashion. We have demonstrated that all four of these processes generate Banks trajectories and lead to outcomes in the Banks set.

We believe the Banks set to be an extremely important solution concept which is relevant to both cooperative and noncooperative settings and to both sincere and sophisticated voting. The Banks set in turn contains various subsets such as Schwartz's (1986) "tournament equilibrium set," Grofman and Feld's (1986) "Schattschneider set" (see Feld and Grofman, 1988), and the "bargaining equilibrium set" (Miller, Grofman, and Feld, 1987), which are important in their own right. The identification of the Banks set and of its properties as a solution concept goes a long way toward demonstrating that there is an "internal structure" to majority rule top cycles which makes outcomes of majority rule processes considerably more predictable than has often previously been supposed.

Notes

1. In particular, it will not do just to assume that an *individual voter* i votes for a new proposal x if and only if i prefers x to the current and every prior provisional decision. If preferences are diverse, such behavior could result in x being rejected even if x is majority preferred to the current and every prior provisional decision.
2. We are indebted to Thomas Schwartz for calling our attention to this heresthetical argument. It is only fair to report, however, that Schwartz would argue further that a new proposal should, under this argument, be compared, not only with previously adopted alternatives, but with all alternatives previously considered, even – or especially – those that were immediately rejected (and so never became a provisional decision). (A voting process that worked in this way would lead to alternatives in the top cycle set, not the Banks set.) In some circumstances, this might well be reasonable. But in other circumstances, where each provisional decision receives some kind of special recognition – possibly in the form of temporary implementation – the sequence of provisional decisions would be salient points of comparison, whereas alternatives that were immediately rejected would not be.
3. Indeed the story told just above provided the original motivation for the cycle avoiding trajectory construction presented in the introduction.
4. Reid (1988a, 1988b) turns these conditions into a formal algorithm and demonstrates that this algorithm identifies the same sophisticated decision as the Shepsle-Weingast "sophisticated voting algorithm" discussed just below.
5. Shepsle and Weingast work primarily with what they call the *trucated sophisticated agenda*, in which repeated entries are eliminated; it is straightforward to create an analogous "truncated" cycle avoiding sincere agenda.
6. This consideration is implied by the early discussion in Tullock (1967: 44–45) and is explicit in the recent Banks and Gasmi (1987) three-voter spatial model of agenda formation. If standard amendment is backward built in this fashion (so that the set of alternatives $\{x_1, x_2, \ldots, x_k\}$ is a cycle avoiding trajectory and the first vote is between x_k and x_{k-1}, the next is between the

winner of the first vote and x_{k-2}, and so forth; put otherwise, so that no alternative is innocuous), it may readily be checked that the "sophisticated equivalents" at the two nodes in the voting tree immediately following a given decision node are just the alternatives that are overtly paired at that vote (cf. McKelvey and Niemi, 1978). This in turn implies that, though we assume sophisticated behavior at the voting stage, such sophisticated voting on a strategic backwards-built agenda will be observationally equivalent to sincere voting (cf. Austen-Smith, 1987).

7. In this event, interesting strategic complexities can arise, but these are beyond the scope of the present paper.
8. Obviously, this is not true if *voting* behavior, as opposed to *agenda setting* behavior, is influenced by position taking or similar considerations; cf. Denzau, Riker, and Shepsle (1985).

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