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Majority rule outcomes and the structure of debate in one-issue-at-a-time decision-making*

SCOTT L. FELD
Department of Sociology, State University of New York at Stony Brook, NY 11790

BERNARD GROFMAN
School of Social Sciences, University of California, Irvine, CA 92717

Abstract. In many group decision-making situations, such as faculty hiring decisions, outcomes are often arrived at by deciding one issue at a time (e.g., first rank, then subfield). It is well known that procedures limiting votes to separate orthogonal dimensions always result in a unique outcome at the generalized median, the median of each separate issue dimension. Often, however, there is conflict within groups over what ought to be the relevant (orthogonal) dimensions within which choices will be made. We show that the way in which debate is structured (i.e., the way in which the dimensions of choice are specified) can have important consequences for what outcome gets chosen. However, we also show that the range of outcomes that could arise from alternative structurings of the decision process is bounded. These bounds are expressed relative to the yolk, a sphere located centrally in the Pareto set, whose existence was first noted by Tullock (1967: 262) and whose properties have recently been developed by McKelvey (1986) and Feld et al. (1987). We find that, in m orthogonal dimensions, the feasible outcomes must lie within $\sqrt{m}$ radii of the center of the yolk

1. Introduction

At least since Black and Newing (1951) it has been known that if we confine ourselves to a single issue dimension the outcome will be the median on that dimension; and that, for multidimensional issue spaces, if voters have convex indifference curves, if we proceed one orthogonal issue dimension at a time,

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the outcome will be at the generalized median (Kadane, 1972; Kramer, 1973), where the *generalized median* is simply the point consisting of the median voter projection on each of the separate issue dimensions. We go beyond earlier work by asking what happens when the choice of issue dimensions to structure the one-issue-at-a-time decision process is itself problematic, and we also briefly consider what happens if dimensions are not orthogonal.[1, 2]

The central question with which this paper deals is how easily manipulable are group decisions in which issues are considered one dimension at a time, but in which voter preferences over different issues need not be separable. To answer this important substantive question, we must first solve two technical problems which have not been previously addressed in the social choice literature (see note 1).

The first problem we need to solve is specifying the locus of the set of possible generalized medians, which we call the *Schattschneider set*.[3] The second problem we address is how the size of the Schattschneider set varies as a function of the number of voters and the number of issue dimensions.

Specifying the exact nature of the Schattschneider set is of considerable importance in understanding group decision making. First, many group decision processes (e.g., faculty hiring decisions) proceed one issue dimension at a time (e.g., decision as to level of appointment, then decision as to subfield; or conversely). Second, legislative decision making in the U.S. is commonly structured through committees with defined jurisdictions. When these committees make recommendations which can only be amended subject to a germaneness requirement, this is roughly equivalent to the legislature as a whole deciding one issue at a time (Krehbiel, 1984; see however, Sullivan, 1985).

One issue-at-a-time decision making is common, we believe, because it simplifies the collective process and ensures the selection of an apparently justifiable outcome – i.e., the selected alternative is majority preferred to all others on each salient dimension. When there is an alternative that is an overall majority winner, the procedure *always* finds it. When there is no such alternative, the procedure makes it likely that no one will notice that the selected alternative is majority inferior to some others that differ in several dimensions simultaneously.

1.1 Dimensionalization of the agenda: Germaneness

One form of constraints on agendas is that which is imposed on the nature of the choices that will be simultaneously compared. It seems to be common that alternatives are offered in a way that only requires votes on a single dimension at a time. Consider, for simplicity, a faculty hiring decision based on two dimensions of choice: Imagine that someone proposes that the group first decide on a collectively preferred level of seniority, and then make a choice among the candidates at the chosen seniority level. For purpose of illustration, once seniority had been decided, let us assume that faculty would pick among candidates at the chosen level of seniority on the basis of preferences for a theoretical vs. an empiricist orientation.

The argument to be made for deciding one issue (seniority) first would be that this would avoid comparing apples and oranges (e.g., scholars at very different levels of seniority with very different approaches). In general, we would expect that many decision-making procedures in groups would involve one-issue-at-a-time decision making (cf. Farquarson, 1969).

If the group did vote one dimension at a time, the results would be determinate and located at the generalized median on the two dimensions, (x_m, y_m) as Black and Newing (1951), Kramer (1973), Shepsle (1979a) and others have pointed out. In other words, if we consider one dimension at a time, it can be shown that no matter which alternative we start from and no matter whether we first resolve the vertical differences or the horizontal differences, we always wind up with the point which is the median on each of the two dimensions as the collective choice.

Thus far, we have been assuming that the dimensions of choice are given, a common assumption in the social choice literature (Krehbiel, 1984). However, the dimensions themselves may be problematic. Research on preferences shows that often a small number of dimensions (usually one or two) accounts for almost all of the variance among individuals' preferences over alternatives. It is well known that if a given number of dimensions, e.g., two, account for the variance, then any rotation of those dimensions will similarly account for the variance. Thus, if the dimensions of methodological orientation (empiricist vs. theoretical) and rank (junior vs. senior) do account for most of the variation in preferences, then two other dimensions could do the same. Based upon the authors' experiences in political science and sociology, we suggest that two other pertinent dimensions might be scope (interest in microlevel research vs. interest in classic 'big' questions) and ideology (left vs. right).

For illustration purposes (and because it may not be too far from reality), assume that the ideology and scope dimensions are approximately orthogonal to one another and at an approximately 45 degree rotation from the original pair of rank and methodological orientation issue dimensions. See Figure 1.

Table 1 shows the hypothesized mapping between characteristics on the two different pairs of issue dimensions. Thus we are positing, for example, that left-oriented faculty who do applied research are more likely to be junior than senior; that senior faculty who do empirical work are more likely to be oriented toward the right than toward the left, etc.

Of course, this example is only intended to be illustrative. Other dimensionalizations can be imagined. For example, let the dimensions of preference be

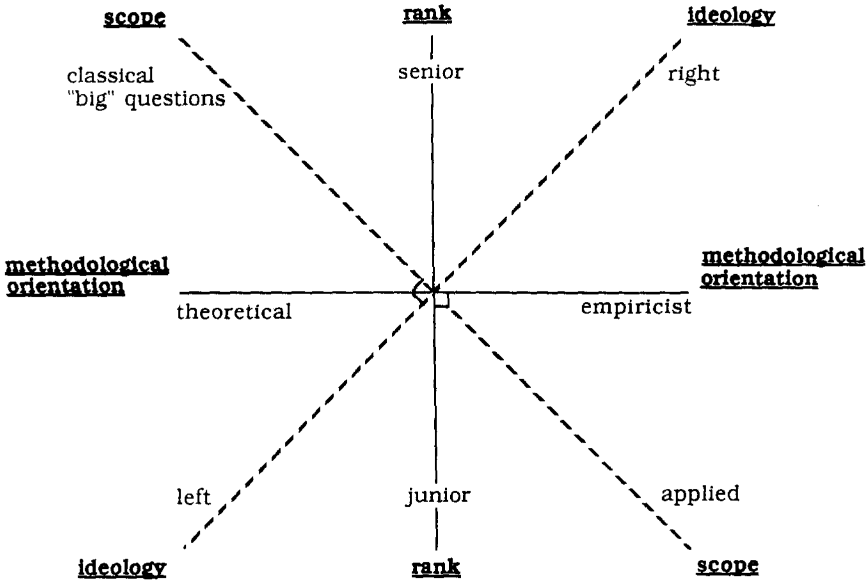

Figure 1. Two pair of orthogonal issue dimensions on which faculty can be classified (rank and methodological orientation; ideology and scope)

Table 1. Hypothetical relationships between two different dimensionalizations of faculty personnel choices

Methodology	Rank		Ideology	Scope
empiricist	senior	→	right	
empiricist	junior	→		applied
theoretical	senior	→		classic questions
theoretical	junior	→	left	

Table 1(a)

Ideology	Scope		Methodology	Rank
left	applied	→		junior
left	classic questions	→	theoretical	
right	applied	→	empiricist	
right	classic questions	→		senior

Table 1(b)

changed from the original two dimensions of methodological orientation and seniority to yet another new set of two dimensions, rigor (which tends to come only in those with low seniority and a strong focus on data) and philosophical maturity (which tends to come only with seniority and is associated with a theoretical rather than an empirical orientation). Clearly the old pair of issue

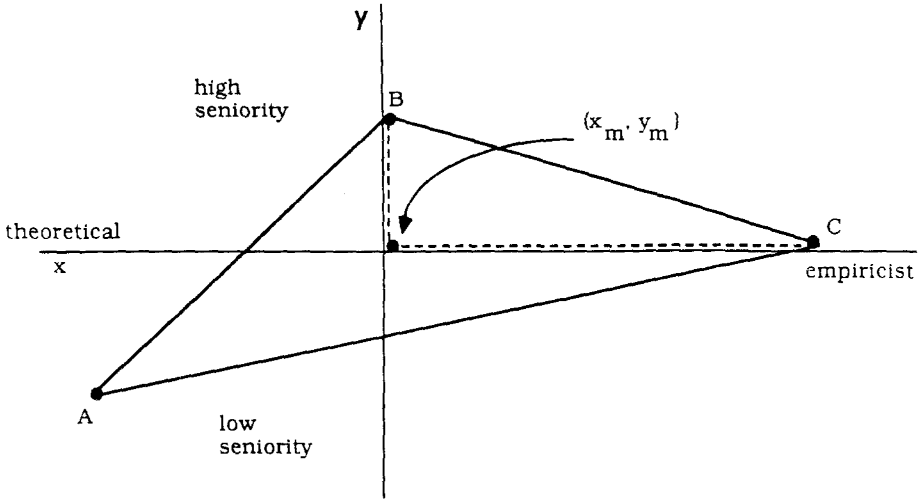

Figure 2(a)

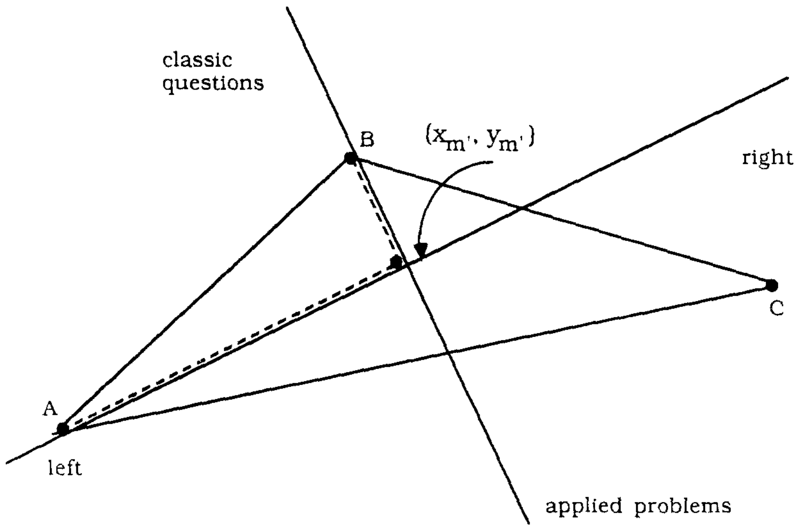

Figure 2(b)

Figure 2. Outcomes of two alternative dimensionalizations under one-issue-at-a-time decision making

dimensions and the new pair of issue dimensions are also interrelated. There are various other alternative ways of specifying dimensions of choice (e.g., subfield specialization would be an obvious candidate as one of the dimensions).

Once the dimensions of choice are determined, the collective outcome of one-at-a-time decision making is also determined, but choice of the dimensions has important consequences for the final decision reached. In particular,

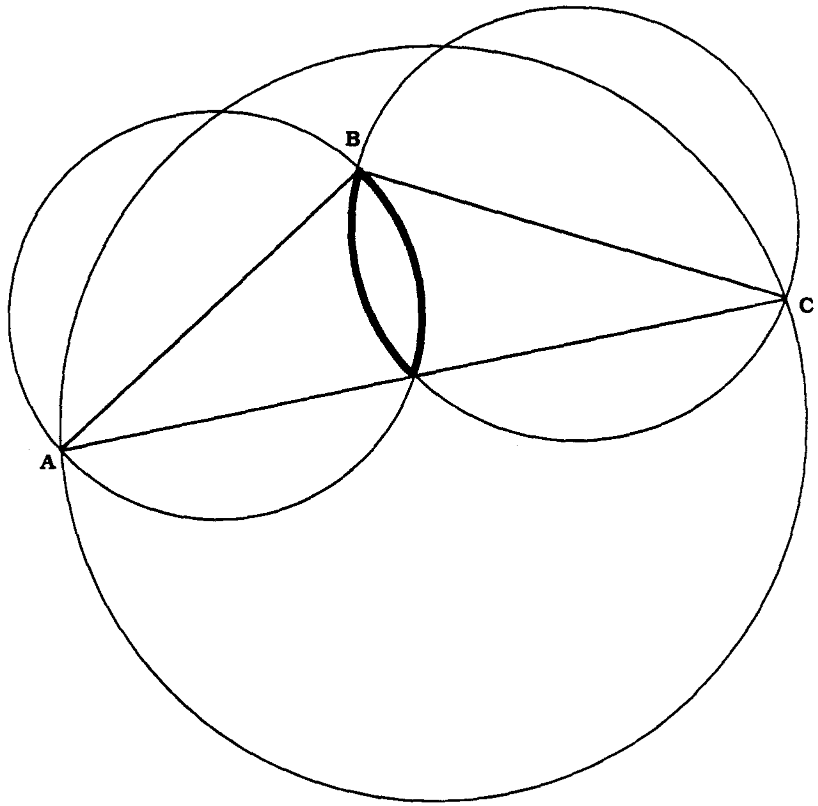

Figure 2(c). Schattschneider set for example of Figures 2(a) and 2(b)

for our two-dimensional example (x_m, y_m) in Figure 2(a) is not the same as (x_m', y_m') in Figure 2(b). We expect that an implicit understanding of this sort of dependence of outcome on the dimensions of choice can account for some of the seemingly irrational procedural haggling that often accompanies collective decision-making processes (at least among university faculty).

2. The geometry of the Schattschneider set

We consider spatial situations where each point in a space represents an alternative that might be chosen by the majority of a group. We illustrate our results for circular indifference curves where each individual in the group has an ideal point in the space and prefers alternatives that are closer to his or her ideal point to those which are further away. Such distance-based preference orderings are commonly assumed in modelling voting in the spatial context.

It can be shown that the set of alternatives that can arise under a germaneness rule of one-dimension-at-a-time decision making is much more limited than decisions with a completely free agenda. Figure 2(c) shows the Schattschneider set for every possible set of orthogonal dimensions for the three-voter two-dimensional example used in Figures 2(a) and 2(b).

Proposition 1: In two dimensions, the Schattschneider set (the set of possible generalized medians) is given by the arcs of circles which pass through each pair of voter ideal points.

Proof: In two dimensions, the Schattschneider set consists of points at the intersection of pairs of median lines. Since each median line passes through a voter ideal point, each Schattschneider point is at the perpendicular intersection of two median lines, each going through a voter ideal point. Thus, we wish to specify the locus of points which form the perpendicular intersection of median lines.

For each pair of voter ideal points I and J, for any line ℓ through I, there is only one line K which can be through J which is perpendicular to ℓ. The two lines ℓ and K will intersect at some point z. It is well known that the locus of points z is the circumference of a circle with diameter d(I, J). Let ℓ be a median line. Only one of the lines perpendicular to ℓ will also be a median line. The Schattschneider set will consist of a circle segment defined by the intersection of pairs of *median* lines. Because all median lines are lines through some voter ideal point, the median line perpendicular to any given median line will be a line through another voter ideal point. (Note that not all lines through a voter ideal point are median lines.) Thus, the Schattschneider set is given by the circle segments which pass through each pair of voter ideal points for the angles defined by median lines. Q.E.D.

In the case of three voters in two dimensions, as shown in Figure 2(c), all lines through voter ideal points that pass within the Pareto set (the triangle connecting the voter ideal points) are median lines; consequently, the portions of the three relevant circles that fall within the Pareto set define the Schattschneider set.[4]

Figure 3 shows three examples, a right triangle, an acute triangle, and an obtuse triangle. In each case, the circles through each pair of voter ideal points are shown, and the sections of them (in the Pareto set) which define the Schattschneider set, are darkened. In general the Schattschneider set consists of a connected set of arcs of circles, toward the center of the space defined by voter ideal points.

It is easy to show that the Schattschneider set is a singleton if and only if there is a core, i.e., an alternative capable of defeating all other alternatives in paired contest. It is of course, well known that in spatial voting games a core will not exist except under extremely restrictive symmetry conditions (Plott, 1967; McKelvey, 1979; Schofield, 1978). Thus, in general, the outcomes of majority rule spatial vote games are indeterminate until we specify an institu-

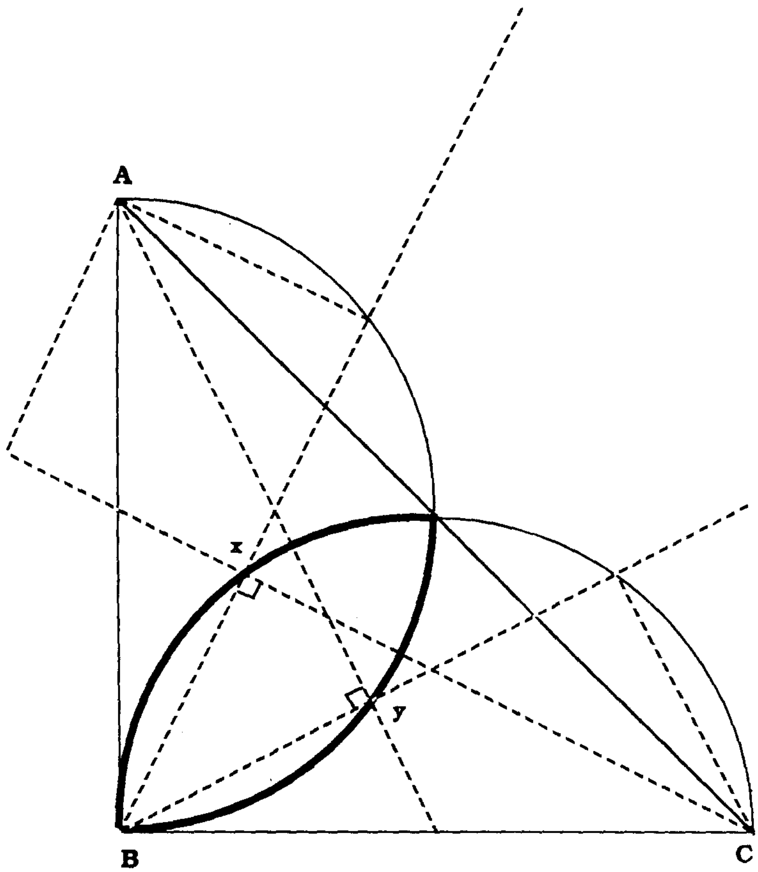

Figure 3(a). Surface of median points on median lines for a right triangle

tional structure. Nonetheless, in general the Schattschneider set will be relatively small.[5]

Figure 4 shows the Schattschneider set for a symmetric 5-voter example. Figure 4 readily generalizes for any symmetric polygons.

For more than two dimensions, Proposition 1 can be readily generalized.

Proposition 1' (Nicholas Miller, personal communication, June 1987). In w dimensions the Schattschneider set is given by the 'arcs' of hyperspheres passing through w-types of ideal points.

Definition: The *yolk* is the smallest sphere which intersects all median hyperplanes (McKelvey, 1986; Ferejohn, McKelvey and Packel, 1984). In two dimensions the yolk is the smallest circle which intersects all median lines.

The center of the yolk can be thought of as a 'central' point in the space. The idea of the yolk was first discussed (although not under that name) by Tullock (1967: 262–263).

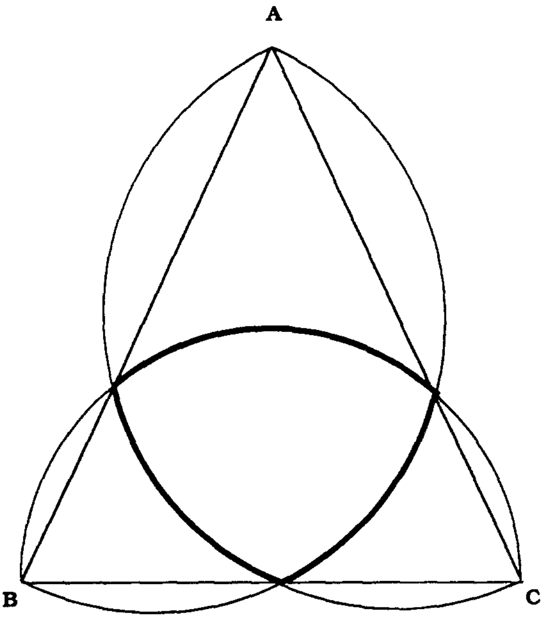

Figure 3(b). Surface of median points on median lines for an acute triangle

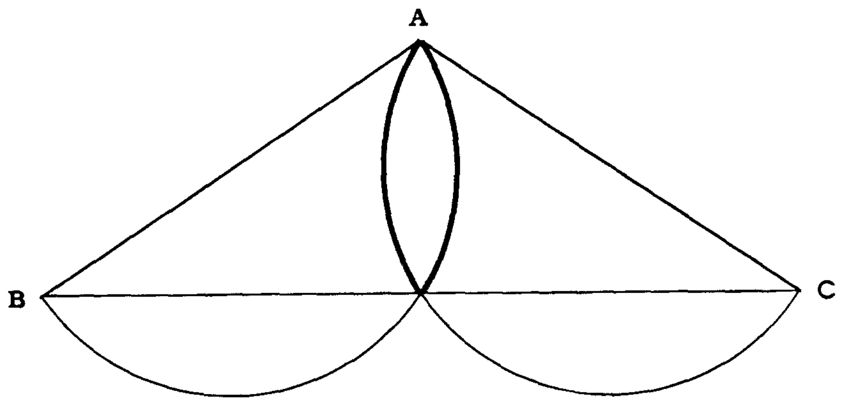

Figure 3(c). Surface of median points on median lines for an obtuse triangle

In each of our previous examples the Schattschneider set is small and located in a reasonably central part of the Pareto set. However these examples are special cases and confined to two dimensions. Proposition 2 states a general relationship between the Schattschneider set and the center of the yolk. It shows the Schattschneider set to be centrally located in the space whenever the number of dimensions is reasonably small.

Outcomes in the Schattschneider set must fall close to the center of the yolk. Specifically,

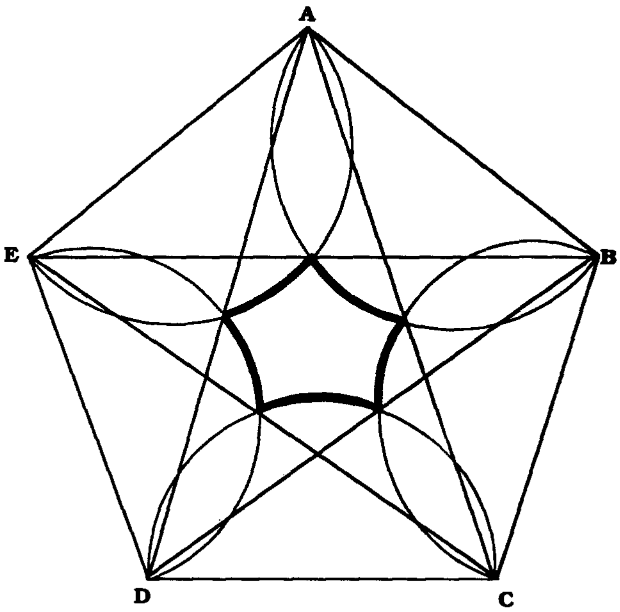

Figure 4. Surface of median points on median lines for a symmetric polygon

Proposition 2: All choices that could arise from one dimension-at-a-time decision making on some set of orthogonal dimensions fall within $\sqrt{m}$ radii of the center of the yolk, where m is the number of dimensions in the space.[6]

Proof: We shall first prove the result for two dimensions, and then show how it generalizes.

Since the yolk intersects all median lines the furthest ones must be tangent to the yolk. The furthest that a point of intersection of two orthogonal median lines can be from the yolk occurs when both median lines are tangent to the yolk. See Figure 5. Let r be the radius of the yolk. The diagonal of a square with side r is just $\sqrt{2}r$, which in this case is the distance from the center of the yolk to the intersection of the two orthogonal median lines.[7] Add a third dimension, and we want the hypotenuse of a right triangle with one side $\sqrt{2}r$ and the other side r; i.e., $\sqrt{3}r$, etc. Q.E.D.

If certain reasonable symmetry conditions are imposed, as the number of voters increases, the yolk will shrink (McKelvey, 1986; Feld, Grofman and Miller, forthcoming) and thus the area enclosed by the Schattschneider set relative to the Pareto set will shrink. As Tullock (1967) notes, for a large number of voters many of the difficulties with majority rule will be reduced in importance, because outcomes are likely to be reasonably well behaved even though not perfectly well behaved. Thus, *ceteris paribus*, it is when the number of voters is small that we could expect disagreement over the dimensionalization of issues to be greatest, since this is the situation in which the potential effect

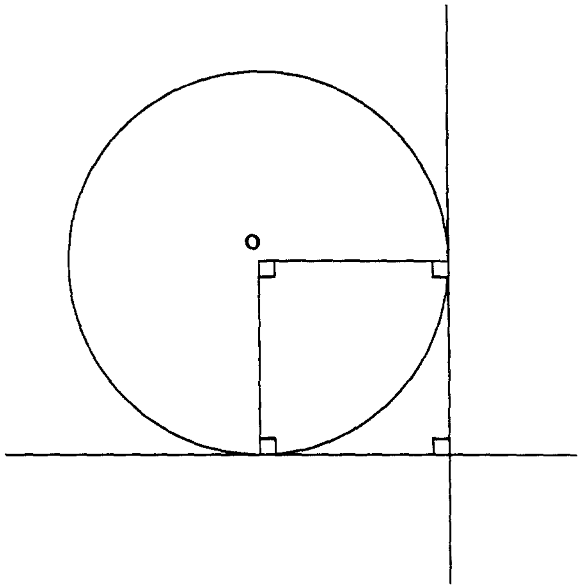

Figure 5. Construction used to prove Proposition 2 for the case of two dimensions

of choice of dimensions on outcomes is greatest. That is why we have concentrated on examples with a small number of voters.

3. Discussion

Our work can be considered a contribution to the literature on structure-induced equilibrium, but the results we discuss are intended to apply broadly to many types of group decision making, including legislatures or other formally structured institutional settings, as well as in more informal decision-making processes such as faculty meetings. Choice of dimensions under one-issue-at-a-time decision making is a special but important case of a structure-induced equilibrium.

Our results indicate that while any particular dimensionalization of a decision results in a determinate collective choice, how decisions are dimensionalized can change the collective choice. Consequently, the choice of issue dimensions should be done cautiously; and participants should be aware that the decision may have more than symbolic implications. (Cf. Plott and Levine, 1978.) In actual decision-making groups, the often-found quibbling over the structure of decision making may partially result from some participants' tacit recognition of the implications of particular dimensionalizations.

However, our results also show that the amount of variation among collective choices resulting from different dimensionalizations of the same situation is usually not great. If the 'yolk' of the voter ideal points is small (as it usually is), then the maximal distance between Schattschneider points is limited. Thus, while participants might recognize that dimensionalization has effects, they will often find that the effects are small and might be better tolerated than fought. Our work reinforces a conclusion reached by Tullock (1967: 270) many years ago. 'The majority voting process normally leads to a determinate outcome' and 'this outcome is apt to be reasonably satisfactory.' Tullock (1967: 270) goes on to note that 'this will surprise no practical man.' We agree.

Finally, the facts that one-issue-at-a-time decision making (1) results in alternatives which are close to the center of the yolk, (2) finds a majority winning alternative whenever there is one, and (3) results in an apparently justifiable alternative otherwise, provide strong arguments for using such a method.

Notes

1. Subsequent to completing this paper we have learned that results similar to those we present in the first part of this paper are discussed in upublished work by Brian Humes. Our work is also related to Riker's notion of heresthetics (Riker 1982, 1984) and Shepsle's work on structure induced equilibria (Shepsle, 1979a; see especially Shepsle, 1986: 156).
2. Consider the task of partitioning an n-dimensional space into n orthogonal axes. If there is no majority winner, then by rotating the basis vectors for the space we can change the location of the generalized median.
3. We have labeled this set of generalized medians the Schattschneider set because Schattschneider was one of the first political scientists to emphasize the importance of the choice of dimensions over which political conflicts are to be carried out, and because the idea of this set came to the second-named author while rereading Schattschneider (1968). Schattschneider (1968: Ch. 4) argues that the nature of the axes of cleavage determines in large part which winning coalition will form and thus what outcomes can be expected to result.
4. Even in two dimensions, for more than 3 voters, not all arcs through pairs of voter ideal points which pass through the Pareto set will lie in the Schattschneider set.
5. For three voters we know the Schattschneider set to be cyclic, and we conjecture that this is always true for any number of voters and dimensions. If the Schattschneider set is cyclic, then it is impossible to avoid the problem of the absence of a core by switching the problem to one of the group choosing from the set of possible generalized medians. While manipulation of the set of committee jurisdictions can be used (together with a germaneness requirement) to impose stability, if the problem of choice of jurisdictional arrangements is made endogenous rather than treated exogenously as it normally is (see e.g., Shepsle's 1979a, discussion of structure-induced equilibria), then the problem of indeterminacy of group decision making in spatial majority rule games reemerges.
6. McKelvey (1986) has shown that the uncovered set (Miller, 1980) can be located within 4 radii of the yolk. (See also Feld, et al., forthcoming.) For two dimensions, the Schattschneider set is a subset of the uncovered set (Feld, et al., 1987). The result we give is a tighter bound than the 4 radii result if $m < 16$.
7. Even when dimensions are not perfectly orthogonal, bounds on the Schattschneider set can be found. In particular, if two dimensions are at an angle θ ($0 \leq \theta \leq 90°$), the points in the

Schattschneider set can be no further than r / sin(θ / 2) from the center of the yolk, an expression which is minimum when sin θ = 90°. Thus, as long as θ is near 90°, the Schattschneider set is relatively central. For example, for θ = 60°, the Schattschneider set can be no further than 2r from the center of the yolk. (Of course, for θ = 0, the above bound offers no useful constraint.) Also, in more than two dimensions, the bounds on the Schattschneider set can extend outside the Pareto set. (See Proposition 2.) However, other bounds on the Schattschneider are possible. (See note 6.)

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