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Research Note: The Accuracy of Group Majorities for Disjunctive and Conjunctive Decision Tasks

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For complex group decision tasks which can be described in terms of establishing the truth value of conjunctive (*and*) or disjunctive (*or*) composite propositions, the advantage for group judgmental accuracy of disaggregating the decision task into separate individual evaluations of the truth value of each component atomic proposition in the composite is investigated. For true propositions it is shown that disaggregation is preferable for conjunctive tasks, but not for disjunctive tasks; while the reverse is true for false propositions. © 1985 Academic Press, Inc.

In this paper we shall discuss how a group decision task can best be structured for different types of fact-finding situations. In particular we shall consider differences between the situation where an affirmative finding on *any one* of a number of different factors is *sufficient* to generate an overall affirmative judgment versus that where an affirmative finding on each of a number of different factors is *necessary* to generate an overall affirmative judgment. This question is directly relevant to group/jury decision making on complex factual issues. For example, in conspiracy trials there are usually several elements of proof required to establish the presence of a conspiracy, all of which must be shown. In other cases, e.g., civil rights trials brought under Section II of the Voting Rights Act of 1965 (as amended in 1982), either a proof of intent to dilute minority voting strength *or* a proof that an electoral plan has the effect of diluting minority voting strength is sufficient for a violation of the Act to be shown.

Consider a group (such as a jury) confronting a decision task which is to establish the truth value of a composite statement either at the conjunctive form "$x_1 \wedge x_2 \wedge \ldots \wedge x_n$," where $\wedge$ represents *and*, or of the

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disjunctive form "$x_1 \vee x_2 \ldots \vee x_n$" where $\vee$ represents *or*. (The components, x_i, of these statements may themselves be composites and may include such other logical symbols as negation, but we shall without great loss of generality disregard such complications in our exposition.) We may imagine two alternative procedures which the group (jury) could use. In the first procedure, each member of the group would be asked to assess the truth value of the composite expression (voting either T or F, i.e., yes, if the proposition is true, or no, if it is not), and these individual judgments would then be pooled in some fashion to form the group consensus as to the truth value of the composite expression (i.e., of the truth value of "$x_1, \vee x_2, \vee \ldots \vee x_n$" or "$x_1 \wedge x_2 \wedge \ldots x_a$," whichever might be under consideration). In the second procedure, each group member would be asked to evaluate the truth value of *each* of the atomic components $x_1, x_2, \ldots, x_n$, and the set of individual componential judgments would then be pooled in some fashion to form the group consensus as to the truth value of the composite expression. To make the calculations more readily tractable, we shall posit a particularly simple form of establishing group consensus, that of ordinary majority rule. While real-life juries often operate under supramajoritarian procedures (especially unanimity), the social psychological literature on jury decision making provides evidence that even under a de jure unanimity rule most (>90%) of all jury decision making can be predicted by considering only the predeliberation verdict preferences of the jury majority (Kalven & Zeisel, 1966; cf. Davis, 1973; Grofman, 1976, 1977; Lempert, 1975). Moreover, the majority rule case is of considerable interest in its own right as the most common decision mechanism for deliberate bodies in the United States and elsewhere.

Deriving our inspiration from the probabilistic ideas of Condorcet (1785; see Black, 1958; Grofman, 1975, 1979, 1981; Grofman and Feld, 1984; Grofman & Owen, 1985), we shall posit that the *i*th group member (juror) has a probability p_{ij}, $0 \leq p_{ij} \leq 1$, of correctly assessing the truth value of the proposition x_j and that juror competences across propositions are independent. Thus, the probability that the *i*th juror correctly assesses the truth value of conjunctive true proposition "$x_1 \wedge x_2$" is simply $p_{i1} \cdot p_{i2}$, and the probability the *i*th juror correctly assesses the truth value of the true disjunctive proposition "$x_1 \vee x_2$" is simply $1 - (1 - p_{i1})(1 - p_{i2})$. Without real loss of generality we can confine ourselves to a comparison of the "$x_1 \wedge x_2$" and "$x_1 \vee x_2$" cases. Also without great loss of generality we can initially consider a group of size $N = 3$. Let us simplify further by positing homogeneous abilities, i.e., $p_{ij} = p_{kj}$ for all i,k; and atomic propositions of identical difficulty levels, i.e., $p_{ij} = p_{il}$ for all j,l. Neither of these assumptions is really critical, but both together serve to dramatically simplify the formulas to be considered.

Let us first look at group judgments about "$x_1 \wedge x_2$." If the group considers the true composite "$x_1 \wedge x_2$," the probability that any individual juror correctly assesses the truth value of "$x_1 \wedge x_2$" is $p_{i1} \cdot p_{i2}$. Let us denote $p_{i1} \cdot p_{i2}$ as p. Hence, by our earlier simplifying assumptions, $p_{i1} = p_{j2} = p^{1/2}$. The probability that for a group of size $N = 3$, a group majority ($m = (N + 1)/2$) correctly evaluates the truth value of "$x_1 \wedge x_2$" can be shown to be given by

$$\sum_{h=2}^{3} \binom{3}{\mathrm{h}} p^h (1 - p)^{3-h} = 3p^2 - 2p^3. \tag{1}$$

On the other hand, if the decision task is disaggregated, the probability that an individual juror will correctly evaluate the truth value of any given component, x_i, is simply $p^{1/2}$, and the probability that the group majority will correctly assess that component is $3p - 2p^{3/2}$. (see Eq. (1)). For the conjunctive task, where components are independent, the probability that the group majority will correctly assess the truth value of "$x_1 \wedge x_2$" is the product of its judgmental accuracies on each component, and thus (under our simplifying assumptions) is given by

$$(3p - 2p^{3/2})^2. \tag{2}$$

We can show that, for all p ($0 < p < 1$),

$$(3p - 2p^{3/2})^2 > 3p^2 - 2p^3, \tag{3}$$

since, after some straightforward algebraic simplification (e.g., dividing both sides through by p^2), Eq. (3) reduces to

$$1 + p > 2\sqrt{p}, \tag{4}$$

which is always true for p: $0 < p < 1$. (The strict inequality is required because we have divided by p which, were it 0, would have been impermissible.) The implication of Eq. (4) is quite clear. If the conjunctive proposition is true, in dealing with a conjunctive decision task under majority rule procedures, it is better to disaggregate the components and require a judgment on the truth value of each component before assessing the truth of the composite conjunctive expression. Of course, the difference between the judgmental accuracy implied by Eq. (1) vs that of Eq. (3) will be relatively minimal for $p > .5$ especially as $p \to 1$ (see Eq. (4)).

Now let us look at the "$x_1 \vee x_2$" case. We shall repeat our earlier simplifying assumptions so as to have $p_{i1} = p_{j2} = p^{1/2}$ for all i,j and again look only at a group of size 3. The probability that any individual juror correctly assesses the truth value of the truth proposition "$x_1 \vee x_2$" is $1 - (1 - p^{1/2})^2$. For the composite assessment task, the probability that the group majority correctly evaluates the truth value of the true proposition "$x_1 \vee x_2$" is simply

$$3\left(1 - (1 - p^{1/2})^2\right)^2 - 2\left(1 - (1 - p^{1/2})^2\right)^3. \tag{5}$$

On the other hand, if we disaggregate, the probability that an individual juror will correctly evaluate any given component x_i is again simply $p^{1/2}$ and the probability that the group majority will correctly assess that component is $3p - 2p^{3/2}$; but for the disjunctive task, where components are independent, the probability that the group majority will correctly assess the truth value of "$x_1 \vee x_2$" will then be given by

$$1 - \left(1 - (3p - 2p^{3/2})\right)^2. \tag{6}$$

We may show that for $0 < p < 1$, the expression in Eq. (6) always exceeds that in Eq. (5). Hence, for majority rule decisions, accuracy on disjunctive tasks is opposite to that for conjunctive tasks—for true propositions, it is always preferable to ask jurors to evaluate the composite decision problem and then to pool the composite evaluations, rather than to disaggregate. Again, however, for $p > 1/2$, especially as $p \to 1$, the differences between Eqs. (5) and (6) are not that large. The policy implications of these findings exactly parallel those for the conjunctive case, only with the nature of the recommendations reversed.

If instead of looking at true propositions we look at false propositions, we obtain by symmetry (since the formulas are otherwise identical) the opposite result: that under majority decision making for a disjunctive task, it is better to disaggregate the components and require a judgment on each component before judging the whole; while for conjunctive tasks judgment of the composite is to be preferred. Thus, if either our a priori probability is greater than one half that a proposition is true, or we are more concerned with falsely rejecting a true proposition than with wrongly accepting a false one, then we should act as if the proposition were true and disaggregate decision making for conjunctive tasks and deal with evaluations of the composite issue for disjunctive tasks. On the other hand, if our a priori probability is greater than one half that a proposition is false, or we are more concerned with wrongly accepting a false proposition than with wrongly rejecting a true one, then we should act as if the proposition were true and disaggregate decision making for disjunctive but not conjunctive tasks.

Although we have formally proved the result only for the two-component case, extension to the n-component case is straightforward. We can show the result for two of the n components, and then proceed by adding one new component each time. Some preliminary exploration suggests that for the majority rule case, for false (true) propositions superiority in accuracy of the aggregated (disaggregated) over the disaggregated (aggregated) decision making increases as we increase the number of component factors which must be found present for an affirmative judgment to be reached. Extension of the results to the case where group members/jurors are not identical in their competence can be done by taking advantage of the result (Grofman, Owen, & Feld, 1983) that, where the p_{ij} values are symmetrically distributed or where N is large, as long as all $p_{ij} \geqslant 1/2$, using $\bar{p}_{ij}$, the group's *mean* competence, to calculate the probability that a group majority will obtain the correct answer provides a *very* good approximation to the true group majority competence, i.e., we may treat the mean competence value as if it were the competence of *each* member of the group.

REFERENCES

Black, D. (1958). *The theory of committees and elections.* London: Cambridge Univ. Press.

Condorcet, N. C. de (1785). *Essai sur l'application de l'analyse a la probabilite des decisions rendues a la pluralite des voix.* Paris.

Davis, J. H. (1973). Group decision and social interaction: A theory of social decision schemes. *Psychological Review,* **80**(2), 97–125.

Grofman, B. N. (1975, Spring). A comment on "Democratic theory: A preliminary mathematical model." *Public Choice,* **21**, 100–103.

Grofman, B. N. (1976). Not necessarily twelve and not necessarily unanimous: Evaluating the impact of *Williams v. Florida* and *Johnson v. Louisiana.* In G. Bermant, C. Nemeth, & N. Vidmar (Eds.), *Psychology and the law: Research frontiers* (pp. 149–168). Heath.

Grofman, B. N. (1977). Jury decision-making models. In S. Nagel (Ed.), *Modeling the criminal justice system, Sage criminal justice system annuals* (Vol. 7, pp. 191–203). Beverly Hills, CA: Sage.

Grofman, B. N. (1979). Preliminary models of jury decision-making. In G. Tullock (Ed.), *Frontiers of economics* (Vol. 3, pp. 98–110).

Grofman, B. N. (1981). Mathematical models of juror and jury decision-making. In B. D. Sales (Ed.), *The trial process* (pp. 305–352). New York: Plenum.

Grofman, B. N. & Feld, S. L. (1984). Group size and the performance of a composite group in multi-item decision tasks. *Organizational Behavior and Human Performance,* **33**, 350–359.

Grofman, B. N., & Owen, G. (1984). *Information pooling and group decision-making.* Westport, CT: JAI.

Grofman, B. N., Owen, G., & Feld, S. L. (1983). Thirteen theorems in search of the truth. *Theory and decision,* **15**, 261–278.

Kalven, H. Jr., & Zeisel, H. (1966). *The American jury.* Boston: Little, Brown.

Lempert, R. O. (1975, March). Uncovering nondiscernible differences: Empirical research and the jury-size cases. *Michigan Law Review,* **73**, 644–707.

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