RESEARCH NOTE

Jai-Alai Outcomes as a Function of Player Position and Player Skill Level

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Jai-Alai is one of the oldest known ball games, and is similar to racquetball or handball except that the ball must be caught in a long wicker basket (called a "cesta") attached to each player's right hand from which it is released in whiplike fashion, achieving speeds up to 125 miles per hour. As in tennis, play continues until the ball either is not caught or is not returned to a permissible area of the court. Matches may be played either between two players or as doubles competition. However, in the jai-alai sports arenas (called "frontons"—to be found in the U.S. in Las Vegas and Miami and some large metropolitan areas), the competition is not between two players (or two teams of players), but rather between eight players or doubles pairs. Players (or doubles teams) are randomly assigned a number from 1-8. The first match is between player/team 1 and player/team 2. The winner(s) of that match play(s) against the number 3 player/team; the loser(s) is/are moved to the bottom of the stack until his/their number comes up again, and so on. In the first part of the match, each time a

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player/team is victorious it scores one point; after seven rounds have been played, victories in the eighth and subsequent rounds are worth 2 points. The first player/team to reach 7 points wins the game. Second and third place in the game are determined by the points that have been scored before the first place victory occurs, unless there are ties—in which case runoffs continue among the tied contenders, with pairings occurring according to the order that the tied contender's numbers are currently in the stack.

In the jai-alai fronton, spectators may bet on the outcome of the games—e.g., buying win, place (2nd), or show (3rd place) tickets. Bettors have available to them information about the previous success records of the players and the (prerace) betting odds. Because, as we shall see, there are considerable advantages to players accorded to them because of their initial assignment in the stack, with low numbers being advantaged, the fronton also provides bettors with the past success records of positions 1-8.[1]

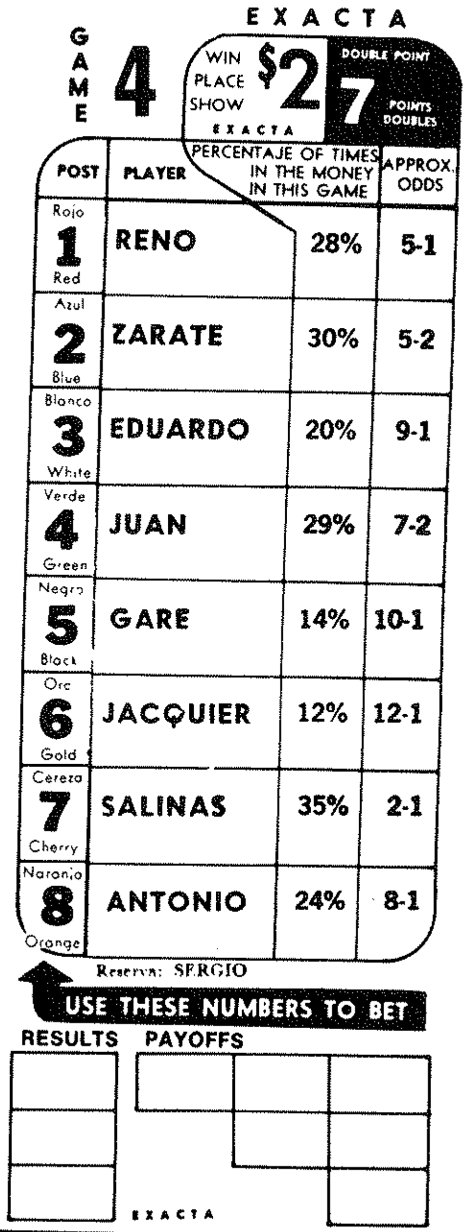
JAI-ALAI BETTING

If the *betting odds against* are j – i, which reflect i chances of winning, j of losing, then the *payoff probabilities* of bettors, which match up to those odds are

$$p(\text{winning}) = \frac{i}{i + j}$$

The return on investment on a winning bet = j/i; that is, for each i dollars bet you get j back. The *payoff* for x dollars invested in a *winning* bet = x(1 + j/i) = x(i + j/i); whereas the *net payoff* for x dollars invested in a *winning* bet is x(j/i).

We show in Figure 1 the program card for the 4th game at the Tijuana Jai-Alai Games,[2] June 6, 1981. The odds shown in the last column are the approximate odds based on prerace betting. These odds are based on the relative amounts of money bet on each player (to come in first). Thus, if the odds (against) one player are j – i and the odds against another player are *l* – k, then the amounts bet on each are in the ratio [i/(i + j)]/[k/(k + *l*)]. Thus, the



EXACTA

GAME 4

WIN PLACE SHOW $2 EXACTA

DOUBLE POINT 7 POINTS DOUBLES

POST	PLAYER	PERCENTAJE OF TIMES IN THE MONEY IN THIS GAME	APPROX. ODDS
Rojo 1 Red	RENO	28%	5-1
Azul 2 Blue	ZARATE	30%	5-2
Blanco 3 White	EDUARDO	20%	9-1
Verde 4 Green	JUAN	29%	7-2
Negro 5 Black	GARE	14%	10-1
Oro 6 Gold	JACQUIER	12%	12-1
Cereza 7 Cherry	SALINAS	35%	2-1
Naranja 8 Orange	ANTONIO	24%	8-1

Reserva: SERGIO

USE THESE NUMBERS TO BET

RESULTS PAYOFFS

EXACTA

Figure 1: Game 4, Tijuana Jai-Alai Palace, June 6, 1981

odds reflect the perceptions of the bettors as to the relative probabilities of each player's winning. If there is no house cut, then odds of 5-1 shown for Player #1 (Reno) indicate a one chance in six that Player #1 will win the match, and five chances in six that Player #1 will not win.

By a *house cut,* we mean that a fixed proportion, $1 - c \,(0 \leq c \leq 1)$, of all money bet is siphoned off to the house, and thus the payoffs to individuals are correspondingly reduced—which is accomplished by reducing the odds (against) each player by the factor $1 - c$. We may calculate the house cut by summing up the payoff probabilities. These will sum to *more* than one, because the house will act as if each player's probability of victory was increased by the factor $1/(1 - c)$. In Game 4, the sum of the probabilities is 1.40; hence $1 - c = .714$, and the house cut is 28.6%. These odds are prerace odds and only approximate. The actual house cut was slightly less.

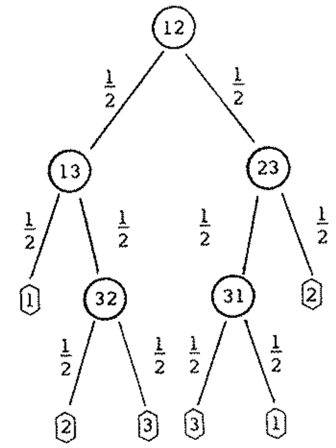
THE POSITIONAL ADVANTAGE IN JAI-ALAI

Even if all jai-alai players are equally skilled, the luck of the draw will affect the likelihood that each will be victorious, in that players early in the stack are more likely to win. We first show this with a simplified example, a jai-alai game including only three competitors, with one point achieved by a victory on rounds one or two, and two points earned by a victory on round three. The winner of the game is the first player to score two points.

Let p_{ij} denote the probability that player i will beat player j in any encounter. For simplicity assume the p_{ij} values remain constant throughout the game. Let us consider the case of three equally skilled players; that is, $p_{ij} = p_{ji} = 1/2$ for all i, j. We may represent the possible outcomes of this jai-alai game in the decision tree shown in Figure 2.

By multiplying through the probabilities of the branches in Figure 1 we see that

$$\text{probability of a win by player 1} = \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{3}{8}$$



*The ()'s represent wins terminal states of the decision tree.

Figure 2: Win Decision Tree for a Simplified Three-Player Jai-Alai Game

$$\text{probability of a win by player 2} = \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{3}{8}$$

$$\text{probability of a win by player 3} = \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{2}{8}$$

Of course, by symmetry, the win probabilities for players 1 and 2 must be identical. We see, however, that although all three players are equally skilled, players 1 and 2 are each 1 1/2 times more likely to win than is player 3.

Using Bayes' Theorem, we can readily calculate the probabilities of a second-place finish for each player. The probability that player 1 will come in second is $3/4 \cdot 3/8 + 1/2 \cdot 2/8 = 13/32$. The probability that player 2 will come in second is, by symmetry, also 13/32. Hence, the probability that player 3 will come in second is only $6/32 = 3/16$. In similar fashion we can calculate the vector of third-place probabilities as (7/32, 7/32, 18/32). Note that the probability of player 1 coming in *either* first or second is 25/32; the same probability applies to player 2. The probability that player 3 will come in either first or second is only 14/32.[3]

We can calculate how much better player 3 must be than players 1 or 2 in order to have an *equal* chance of coming in first. Let $p_{13} = p_{23} = x$ and $p_{12} = 1/2$. Then

$$\text{probability of a win by player 1} = \frac{1}{2} \cdot x + \frac{1}{2}(1-x)x$$

$$\text{probability of a win by player 2} = \frac{1}{2}x + \frac{1}{2}(1-x)$$

$$\text{probability of a win by player 3} = \frac{1}{2}(1-x)^2 + \frac{1}{2}(1-x)^x$$

$$= (1-x)^2$$

We set

$$\frac{1}{2}x + \frac{1}{2}(1-x)x = (1-x)^2$$

that is,

$$x - \frac{1}{2}x^2 = 1 - 2x + x^2$$

$$0 = \frac{3}{2}x^2 - 3x + 1$$

$$0 = 3x^2 - 6x + 2$$

Using the quadratic formula we have

$$\frac{6 \pm \sqrt{36-24}}{6} = \begin{cases} 1 + \frac{3}{3} = 1.58 \\ 1 - \frac{3}{3} = .43 \end{cases}$$

TABLE 1

PROBABILITY OF FINISHING FIRST

		1	2	3	4	5	6	7	8
Simulation	(N = 40320)	.158	.157	.127	.119	.102	.111	.098	.127
1980-81 Actual	Exacta Races (N = 372)	.129	.169	.132	.116	.116	.121	.124	.094
1981 Actual	June (N = 99)	.182	.162	.091	.162	.121	.091	.091	.101

PROBABILITY OF FINISHING SECOND

		1	2	3	4	5	6	7	8
Simulation	(N = 40320)	.207	.203	.196	.118	.089	.057	.066	.064
1981 Actual	Exacta Races (N = 372)	.207	.167	.145	.159	.097	.102	.065	.046
1981 Actual	June (N = 99)	.232	.212	.131	.081	.131	.020	.101	.091

PROBABILITY OF FINISHING FIRST OR SECOND

		1	2	3	4	5	6	7	8
Simulation	(N = 40320)	.365	.360	.323	.237	.191	.168	.164	.191
1980 Actual	Quiniela Races (N = 1191)	.309	.340	.317	.249	.252	.169	.181	.181
1981 Actual	Quiniela Races (N = 1141)	.319	.350	.285	.279	.227	.191	.181	.167
June 1981 Actual	(N = 99)	.414	.374	.222	.242	.253	.111	.192	.192
1981 Actual	Exacta Races (N = 372)	.336	.336	.277	.275	.213	.223	.189	.140

PROBABILITY OF FINISHING FIRST OR SECOND OR THIRD

		1	2	3	4	5	6	7	8
Simulation	(N = 40320)	.533	.532	.492	.398	.305	.257	.226	.257
1981 June Actual	(N = 99)	.505	.525	.384	.404	.484	.192	.232	.273

The only answer that is acceptable is .43, because probabilities must be less than one. Hence, only if $p_{31} = p_{32} = .57$ will player 3 be as likely to win as players 1 or 2. If $p_{12} = p_{21} = 1/2$ and $p_{32} = p_{31} = 1 - x$, then the probability of a win for player 3 goes up more than linearly with $1 - x$; in fact it goes up as the square of $(1 - x)$. Nonetheless, relative to skill, player 3 is still disadvantaged. We

TABLE 2
Probability of Finishing in First Place by Weight and Position

WGT	POSITIONS 1	2	3	4	5	6	7	8	AVERAGE
.02	.001	.001	.002	.000	.000	.000	.000	.001	.001
.05	.021	.019	.014	.011	.009	.011	.009	.012	.013
.08	.058	.065	.043	.041	.034	.036	.029	.044	.044
.10	.093	.093	.071	.065	.052	.058	.054	.074	.070
.15	.196	.193	.151	.144	.124	.134	.112	.154	.151
.17	.235	.241	.191	.173	.146	.168	.154	.188	.187
.20	.299	.290	.250	.231	.195	.218	.194	.249	.241
.23	.359	.355	.291	.289	.259	.263	.235	.295	.293
.125	.158	.157	.127	.119	.102	.111	.098	.127	.125

NOTE: Simulation, N = 40, 320.

know that if $1 - x = .5$, then player 3's win probability = .25. If $1 - x = .6$, then player 3's win probability = .36. If $1 - x = .7$, then player 3's win probability = .49. If $1 - x = .8$, then player 3's win probability = .64, and so on.

It might be thought that the striking discrepancies we observed in win, place, and show probabilities between positions at the top and at the bottom of the initial ordering for a three-player game of jai-alai would be reduced in magnitude for an eight-player game. This is not the case. We show in Table 1 the probabilities of finishing first, second, first or second, and in the money (first, second, or third) based on a simulation we did that randomly assigned each player a weight w_i (with $\Sigma\, w_i = 1$) such that the probability of player i defeating player j in a given pairing was taken to be $w_i / (w_i + w_j)$.[4] In this table we also show the observed probabilities in jai-alai games at the Tijuana fronton in 1980 and 1981. As can be seen from this table, player 1 (or 2) is about twice as likely to finish in the money as is player 8. The relative positional advantage of players 1 and 2 shows even more strongly in terms of likelihood of a second-place finish. It is also apparent that the assumption that player skills are randomly distributed

TABLE 3
Probability of Finishing in Second Place by Weight and Position

WGT	POSITIONS 1	2	3	4	5	6	7	8	AVERAGE
.02	.068	.062	.078	.014	.012	.004	.003	.004	.031
.05	.132	.137	.137	.054	.040	.026	.024	.036	.073
.08	.191	.182	.168	.096	.068	.046	.047	.056	.107
.10	.233	.231	.205	.118	.090	.059	.064	.072	.134
.15	.262	.259	.239	.157	.111	.079	.090	.084	.160
.17	.259	.249	.248	.157	.119	.076	.097	.085	.161
.20	.253	.261	.240	.166	.135	.078	.094	.086	.164
.23	.262	.244	.250	.177	.135	.090	.110	.091	.170
.125	.207	.203	.196	.118	.089	.057	.066	.064	.125

NOTE: Simulation, N = 40, 320.

TABLE 4
Probability of Finishing in Third Place by Weight and Position

WGT	POSITIONS 1	2	3	4	5	6	7	8	AVERAGE
.02	.138	.134	.137	.118	.033	.022	.013	.012	.076
.05	.179	.184	.179	.151	.076	.056	.035	.041	.112
.08	.192	.201	.185	.171	.113	.082	.051	.061	.132
.10	.190	.189	.187	.174	.112	.089	.069	.064	.134
.15	.182	.185	.173	.179	.140	.104	.078	.091	.141
.17	.170	.177	.181	.176	.155	.115	.081	.083	.142
.20	.151	.161	.165	.168	.136	.120	.083	.091	.134
.23	.137	.141	.153	.151	.141	.122	.086	.086	.127
.125	.167	.172	.170	.161	.113	.089	.062	.066	.125

TABLE 5
Probability of Finishing in the Money—First, Second, or Third by Weight and Position

WGT	1	2	3	4	5	6	7	8	AVERAGE
	POSITIONS								
.02	.207	.197	.217	.133	.045	.026	.016	.017	.107
.05	.331	.340	.330	.217	.125	.092	.067	.089	.199
.08	.441	.448	.396	.309	.215	.164	.127	.160	.283
.10	.515	.513	.462	.358	.254	.207	.187	.210	.338
.15	.641	.637	.563	.480	.375	.317	.280	.328	.453
.17	.664	.668	.620	.507	.420	.359	.332	.356	.491
.20	.702	.712	.655	.565	.466	.417	.370	.426	.539
.23	.758	.739	.694	.618	.536	.475	.431	.473	.590
.125	.533	.532	.492	.398	.305	.257	.226	.257	.375

NOTE: Simulation, N = 40, 320.

across positions is not perfectly met for actual games as the actual win, place, and show probabilities of players 1 and 2 are not in fact identical.

BETTING OUTCOMES IN JAI-ALAI

If you calculate the odds in your favor $[i'/(i'+j')]$ to be higher than the betting public does, that is, $i'/(i'+j') > i/(i+j)$ (or equivalently, if you calculate the odds against you to be lower than the betting public does, that is, $[i'/(i'+j) > j/(i+j)]$), then you ought to bet. If more than one bet satisfies this constraint, then you should pick the one such that $[i'/(i'+j')][(i+j)/i]$, is maximal. As noted above there is, ceteris paribus, a considerable advantage to being early in the jai-alai lineup. As can be seen from Tables 2-5, even when players in positions 6, 7, and 8 are considerably more skilled than their position 1 or 2 counterparts, their likelihood of finishing in the money may still be less. If bettors are sensitive to positional advantages, then their bets will reflect their assessments of both a player's innate capabilities and the advantages/disadvantages conveyed by his position in the lineup. If bettors are, in the aggregate, accurate in these assessments of skill/position, then the betting odds should be an accurate assessment of the end player's true probability of victory; that is, betting odds should be true odds.

If, however, the betting odds are the *true* odds and there is no house cut, then the *expected net payoff* for x dollars invested in *any* given bet equals

$$\left(x\,\frac{i}{i+j}\right)\left(\frac{j}{i}\right)-\left(\frac{j}{i+j}\right)x=0$$

that is, no matter what player/team you bet on, if the betting odds reflect the true probability of winning then your expected net gain is zero because your expected payoff will be exactly equal to the amount you bet. If, on the other hand, there is a house cut, and betting oddings are true odds, then the expected payoff for each dollar invested is only (1 − c) dollars and it would never make sense to bet. Because of the house cut, even if betting odds are not "true" odds you must still be considerably more accurate in assessing relative odds than the betting public if you hope to come out ahead, because the house will automatically levy an off-the-top tax of c dollars for each dollar you would have won in the absence of a house cut.

If betting odds are true odds, then, as we have seen, it will not matter what bet you place because the expected return will be at most zero (if there is no house cut) and will be negative if there is a house cut. If betting odds are true odds, then all bets should have an identical return.[5] We show in Table 6 the actual payoffs made for bets at the Jai-Alai Palace, June 9-20, 1980. The payoffs shown are returns for each $198.00 bet (99 x $2.00 bet).

As can be seen from Table 6, only the bets on either player 1 or player 2 to place would have earned more than the $198.00 that was bet. These positive returns are anomalies, occurring because of the fact that players 1 and 2 came in first or second more often during the June 9-20, 1981 period than was true for the season as a

TABLE 6
Total Payoffs of Bets made in the Tijuana Jai-Alai Fronton

ON PLAYER	TO WIN	TO PLACE	TO SHOW
1	$175.40 (18)	$242.00 (23)	$167.40 (9)
2	179.40 (16)	201.40 (21)	191.80 (15)
3	147.20 (9)	144.60 (13)	109.40 (16)
4	182.40 (16)	159.80 (8)	152.60 (16)
5	141.80 (12)	113.80 (13)	185.40 (23)
6	152.00 (9)	75.00 (2)	81.00 (8)
7	164.00 (9)	146.40 (10)	101.40 (4)
8	133.20 (10)	127.00 (9)	104.00 (8)
Average Return/dollar	$0.81	$0.76	$0.69
House cut	19%	24%	31%

NOTE: N = 99, June 9-20, 1981.

whole, or than would be expected on the basis of our simulation (see Table 1). It does appear, however, that the higher numbered positions were underbet somewhat relative to the lower numbered positions, because the average return for bets on players 1 and 2 is higher than the average return for bets on players 6, 7, and 8.[6] Because the higher numbered positions are favored, underbetting these positions relative to their payoffs is analogous to underbetting favorites—that is, assessing the probabilities of victory of alternatives (e.g., horses/players) that have a high probability of victory as being lower than they in fact are—a phenomenon that has been found to be true for race track bettors. (See Griffith, 1949; Ali, 1977; Synder, 1978; and Harville, 1973.) Of course, because of the house cut, bettors as a whole must lose money, and this will be true for each and every category of bet.

NOTES

1. There are also special bets that require the bettor to correctly pick both the winner(s) *and* the second place player/team in a given game in the exact order of finish (the

exacta) or regardless of order of finish (the quiniela). The number of times each possible winning order has occurred during the past season is information available to the bettor. Exacta and quiniela bets are not available in all matches, however.

2. The Tijuana Jai-Alai fronton is known as the Jai-Alai Palace. It is right next to one of Tijuana's better known restaurants and discos, Tijuana Tillie's, on the main shopping street, Ave. Revolucion.

3. The sum of these probabilities is, of course, 2.

4. This win probability is consistent with Luce's Choice Axiom. See Luce, 1959.

5. See Eisenberg and Gale, 1959. This claim is analogous to those made in the theory of efficient markets in economics. See, for example, Fama, 1970; Synder, 1978.

6. The situation for bets on player 3 is also perhaps anomalous because, as can be seen from Table 1, players in that position came in the money less often during June 9-20, 1981, than during the rest of the season, and less often than would be expected from our simulation results.

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