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ARROW'S IMPOSSIBILITY THEOREM

Kenneth Arrow's *Social Choice and Individual Values* (1951, 1963), one of the five "founding books" of the Public Choice movement, is a seminal work in social science. It reformulates the theory of social welfare in ordinal rather than cardinal terms; it demonstrates the power of an axiomatic approach to economic modeling; and it offers a new approach to traditional issues in democratic theory having to do with the nature of collective choice that has had enormous impact in political science, presaging later aspects of "economic imperialism" vis-à-vis the other social sciences. The key result in the book, Arrow's Impossibility Theorem, is arguably the best known purely mathematical result in the social sciences. Directly inspiring a huge literature, including numerous axiomatic formulations that were more in the nature of characterization or existence results than impossibility theorems, Arrow's work laid the reinvigorated foundations for the subfield of social choice and welfare that came to be exemplified in the journal of that same name. The impossibility theorem is also perhaps the most important of the many contributions which earned Arrow his Nobel Prize in Economics in 1972.

The Impossibility Theorem is deceptively simple. Arrow begins by insisting, seemingly incontrovertibly, that any social welfare ordering, R , be a (weakly) *transitive* collective preference ordering among the set of feasible alternatives; i.e., such that xRy and yRz together imply that xRz , thus requiring that the social welfare function be *acyclic*. Arrow then identifies a number of features of mechanisms for aggregating individual preference orderings into a collective preference that appear, on their face, noncontroversial and/or trivial. He then shows that no function that always generates a transitive social welfare ordering can simultaneously satisfy this set of seemingly reasonable and desirable features unless we are prepared to let a single individual, a dictator, make at least some of the choices for the society.

Arrow's original four conditions are: *unrestricted domain* (every possible combination of individual preference

orderings over the feasible set of alternatives is permitted); *weak positive responsiveness* (if individual preferences change so that some alternative is higher ranked than it used to be, then the collective ranking should place that alternative no lower than it used to be); *independence of irrelevant alternatives* (if some individual preferences change, but no individual changes the relative ordering of two alternatives, x and y , then the collective ordering of x and y also should not change); *citizen sovereignty* (choices over any pair, x and y , should not be externally imposed; i.e., there must be some sets of individual preference orderings for which x is collectively ranked ahead of y and some other sets of individual preference orderings for which y is collectively ranked ahead of x). The Impossibility Theorem states that these four seemingly minor conditions are not compatible with *nondictatorship* (an individual is said to be a *dictator* with respect to the relative ranking of x and y if his preferences govern that ranking regardless of the preferences of the rest of the collectivity; the nondictatorship condition requires that there be no pair of alternatives for which any individual, i , is a dictator). Thus, the horns of Arrow's dilemma: either give up on transitivity, or give up on "democratic" procedures for aggregating preferences and embrace dictatorship.

In the decades after Arrow's Theorem was announced, numerous variants of his theorem were propounded. For example, William Vickrey showed that we could replace weak positive responsiveness and citizen sovereignty with *unanimity* (if every individual has xRy , then so must the social welfare ordering), while still preserving the result that dictatorship was unavoidable. However, most of the energy of scholars has gone into attempts to escape the horns of the Arrowian dilemma.

There are five basic avenue of "escape" that can be pursued:

- (1) Seize the horns of the dilemma and either claim that transitivity is overrated, since collectivities are not individuals and thus need not be expected to generate a transitive social welfare ordering, or that dictatorship need not be so bad (e.g., by permitting dictators as long as they are *representative dictators*).
- (2) Seek to modify the fundamental framework in order to avoid an impossibility result, e.g., by weakening the requirement of transitivity (by replacing it with *quasi-transitivity* or with the requirement that the social welfare ordering only be a *semi-order*); or by requiring only that a unique best choice be selected but not that preferences be transitively ordered; or by shifting from an ordinal to a cardinal framework; or by introducing a lottery or other probabilistic elements.

- (3) Reject the implications of the result for the infeasibility of meaningful collective choice and for democratic theory, e.g., by emphasizing that Arrow's Theorem is about possibilities rather than probabilities and claiming that the theorem is generally irrelevant in that, in most *real-world* settings, there are a range of social welfare functions that can be expected to yield transitive orderings; or by claiming that, even though cycles occur, they will largely or entirely be among alternatives that are virtually indistinguishable from one another; or that the cycles that are present can be viewed as of little moment because collectivities do not even notice when they are there; or that we can cheerfully go on about our business even in the presence of cycles as long as outcomes are generated by rules that "legitimate" social choices.
- (4) Reject the appropriateness/desirability of one or more of the basic postulates of the Theorem other than domain restriction, and argue that once we remove the unsatisfactory postulates, aggregation methods exist that are quite satisfactory
- (5) Consider the possibility of domain restriction and search for plausible ways to limit feasible preference orderings in such a fashion that the impossibility result does not hold.

Trying to reject transitivity does not get us very far. The problem with this first line of attack is that the existence of cycling makes it hard to see what choices to make when the mechanism that is used to specify collective preferences does not yield a clear ordering among alternatives. Although we can simply choose a preferred alternative (perhaps at random) from among the elements in the *top cycle set* (each of whose elements is undominated by any element not in the set), often the top cycle set can be very large, perhaps even encompassing the entire set of feasible alternatives. Thus this line of approach will often not help us very much. Still, there has been considerable progress in understanding the structure of collective preference (especially for the case of majority rule) and in identifying the alternatives that lie in particular subsets of the top cycle set (such as the *uncovered set* or the *Banks set*) which have claims to be considered as "reasonable" choices.

The second of these approaches, the attempt to reformulate either the transitivity condition or dictatorship in a way that will make the impossibility result go away, has led to numerous mathematically sophisticated approaches that have greatly enhanced our understanding of the nature of preference aggregation mechanisms but that, in my view, have all turned out to be dead ends vis-à-vis being ways to

plausibly avoid the horns of the Arrowian dilemma (see the excellent review in Plott, 1976).

The third approach, arguing for the practical irrelevance of Arrow's Theorem, is found in many of my own relatively recent works in social choice theory [see e.g., Uhlaner and Grofman, 1985; Feld and Grofman, 1992; Regenwetter et al. (forthcoming)]. Suffice it to say that I regard this work as an important strand of the debate over the meaning/importance of Arrow's Theorem.

The fourth approach, rejecting one of the axioms used to establish the impossibility result other than domain restriction, often chooses the criterion of independence of Irrelevant Alternatives (CIIA) as the axiom to reject. For example, Donald Saari (1994) argues that CIIA is an undesirable property for social welfare functions/voting rules to have in that it forces the decision process to throw away information. Furthermore, Saari has emphasized that, if we limit the social welfare function to only using information on pairwise choices, it will not distinguish between transitive and intransitive individual preference inputs.

In the context of majority rule processes, the criterion of independence of irrelevant alternatives, when combined with the other elements of Arrow's Theorem, implies that the only feasible decision rules are those that can be thought of as *Condorcet extension methods*, i.e., rules that will always pick the *Condorcet winner* (a.k.a. the *majority winner*), that alternative, if any, that is undefeated against each and every other alternative in paired contest. Saari's views about CIIA and Arrow's Theorem can in many ways be thought of as resuscitating the debate between the two eighteenth century figures who are key progenitors of modern social choice theory, the Marquis Nicolas Caritat de Condorcet and John Charles Borda — with Arrow in the role of Condorcet, emphasizing how things can go wrong, and Saari as the defender of the Borda method.

The *Borda method* is a *scoring rule* assigning points to alternatives based on how far up in each individual ranking they are found, as a reasonable way to generate social welfare orderings/social choices. We can use each alternative's Borda scores to rank order the alternatives in a transitive fashion, and the Borda method can also be given a variety of plausible axiomatic underpinnings. However, the Borda rule is not a Condorcet extension method. In the context of democratic processes, the logic of choosing a majority winner is quite compelling if we are fortunate enough to have a Condorcet winner that could be chosen. Indeed, arguably, this is what majority rule is all about. By dropping independence of irrelevant alternatives we are giving up on rules that guarantee that we will pick majority winners when such exist. (For the history of the Condorcet-Borda debate see Black, 1958.)

The fifth approach seeks to avoid the impossibility result by imposing constraints on admissible preferences. One way to do this is to imagine that individual preferences are based on some common, but imperfectly understood, evaluative criterion, such as a notion of the public good (Grofman and Feld, 1988). Another way is to limit the number of alternatives. If there are only two alternatives then Arrowian impossibility results do not apply. Similarly, if voters are limited to classifying alternatives into two categories (say, satisfactory and unsatisfactory), then we can also avoid impossibility results. However, the most common way in which constraints on preferences are imposed is by limiting preferences over each triple of alternatives in some way, such as by the Black *single-peakedness condition* (referred to by Amartya Sen as *the NW condition*). Arrow (1963) states a possibility theorem for single-peaked preferences, showing that we can create a transitive social welfare function while avoiding dictatorship and satisfying positive responsiveness, CIIA, and citizen sovereignty, if we restrict ourselves to single-peaked preferences. In particular, the single-peakedness restriction guarantees that majority rule will be transitive. (We will discuss domain restriction conditions in more detail in the essay in this volume on Black's single-peakedness condition.)

We would offer one final observation on Arrow's Theorem. Arrow does not explicitly make use of a game-theoretic framework, nor connect his work to considerations of strategic misrepresentation of preferences (see, however, pp. 20–21). Nonetheless, as Craven (1992) observes, there is a deep mathematical connection between the impossibility result and later results on manipulability of social choice processes, such as the Gibberd-Satterthwaite Theorem.

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AN 'AUSTRIAN' PERSPECTIVE ON PUBLIC CHOICE

[F]rom the economic viewpoint, such 'goods' as family, church, love and the like are merely linguistic devices for a totality of concretely useful renditions of services. (Bohm-Bawerk, 1881, 133)

It is impossible to draw a clear-cut boundary around the sphere or domain of human action to be included in economic science. (Knight, 1934, 110)

The Austrian school of economics and the Virginia school of political economy have a long history of interdependence — methodological, analytically, and ideologically. While Austrians typically focus on an analysis of market processes, Virginia Political economists analyze the dynamics of the political realm. Despite their seemingly disparate fields of interest both Austrians and members of the Virginia school share a commitment to methodological individualism and the development of a rational choice paradigm in the human sciences; a commitment to a catalytic or exchange based model of social phenomena that emphasizes the dynamic processes of adjustment by individuals within the process and not exclusively the static efficiency of equilibrium states; and a deep appreciation of the classical liberal tradition and its teachings about the necessary institutional pre-requisites for a society of free and responsible individuals.

1. The Domain of Economic Explanation

Both the Austrian and Virginia schools consider the discipline of political economy a subset of the larger, more general science of human action called *praxeology* (see Buchanan and Tullock, 1962, 16–30). By employing the praxeological approach, Austrians and public choice economists are able to apply the "economic way of thinking"

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