Notes 1

A COMMENT ON "SINGLE-PEAKEDNESS AND GUTTMAN SCALES: CONCEPT AND MEASUREMENT"

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In a recent article in this journal, Niemi and Weisberg (1974) have compellingly shown the falsity of the plausible assumption (made by Grofman, 1968 and others: see citations, Niemi and Weisberg, 1974) that Guttman scalability (see Stouffer, et al., 1950 MacRae, 1970) is equivalent to single peakedness (see Black, 1958 Arrow, 1962, and Coombs, 1964). They have further demonstrated that, in general, information about individual votes on a series of items found to be Guttman scalable does not yield the locations of those individuals' ideal points in some space, unidimensional or otherwise.

Niemi and Weisberg (1974) focus on the question of whether Guttman scalability implies single-peaked preferences. If this implication would be demonstrated then various theorems on single-peakedness and equilibria in voting outcomes (e.g., Black, 1958 Arrow, 1962) would have been shown to be applicable to the legislative or judicial context where sizeable subsets of Guttman scalable issues have been found (MacRae, 1970; Rhode, 1972). Furthermore, spatial models of voter choice which postulate single-peaked preferences (e.g., Downs, 1957) could be applied to data on attitude dimensions shown to be Guttman scalable (e.g.,

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data on attitudes to the Vietnam War, such as that discussed in Page and Brody (1972)).

Niemi and Weisberg (1974) show that Guttman scalability does not imply single-peaked preference either for single alternative response tasks, where an individual is asked to respond favorably or unfavorably to each of a series of alternatives; or for paired alternative response tasks, where the individual responds to a pair of alternatives by indicating which he prefers more. Elsewhere, (Weisberg and Niemi, 1971, Theorem 3, p. 20, Theorem 4, p. 24A) they have shown that Guttman scalability is equivalent to single-peaked preferences only for the highly restrictive case where (a) all possible paired comparisons are made, (b) all preferences are both single-peaked and symmetric and strictly monotonic.[1]

Clearly we almost never have available to us data on all possible paired comparisons. A natural question to ask is "Are there particular subsets of paired comparison data of the sort likely to be provided us by various procedures now in use (e.g., legislative procedures for handling amendments, or scaling techniques for eliciting attitudinal data) such that, for those subsets, single-peaked preferences would imply Guttman scalability?" In other words, while the inference from Guttman scalability to single-peakedness is a hazardous one that can be made with certainty only under very special circumstances, is there a way to *rule out* single-peakedness on the basis of the absence of Guttman scalability? This is clearly a more modest ambition. Even if we found domain restriction such that for those comparisons single-peakedness required Guttman scalability, an empirical finding of the absence of Guttman scalability in cases satisfying these domain assumptions would tell us merely that theorems about single-peakedness and equilibria conditions could *not* be applied to these cases.

Weisberg and Niemi (1971) provide us with one useful suggestion along this direction. They point out (1971, p. 27) that "the minimum number of paired comparisons which would be necessary to detect deviations from single-peakedness among n alternatives is achieved when the n-1 pairs of adjacent alternatives are employed." However, they further point out that, alas, the utility of this result is limited since "for a researcher to employ only the n-1 pairs of adjacent alternatives would require that he know the order of the alternatives on the dimension, *whereas that information is generally sought from the analysis.*" (Weisberg and Niemi, 1974, p. 27, emphasis ours)

One possibility that suggests itself is to look at standard amendment procedure (SAP) in a legislature (see Grofman, 1969) and assume that voters vote sincerely. Assume preferences are single-peaked preferences along the continuum *ABCDE*. If alternatives are proposed in order ADBEC (with C taken to be the status quo) then, under SAP they will be noted on in pairwise fashion A vs. D, the winner of that contest (D) vs. B, the winner of thst contest (D) vs. E, and finally, the

[1]Implicit also in the proof of these theorems is the assumption that the distribution of voters' ideal points is such as to generate all possible scale positions consistent with Guttman scalability.

winner of the last contest (D) versus the status quo of C. Consider five individuals with single-peaked preferences ABCDE, BCADE, CDBAE, DCBAE, and EDCBA respectively. These will give rise to the pattern given in Table 1 which can with permutation of columns 3 and 4 (an admissible procedure), be converted to perfect Guttman scalability. We are not always so lucky. Consider preferences identical to the above, except that member two has single-peaked preference ordering BCDEA.

If voting is sincere under SAP and the alternatives are posed in the order AEBDC, then A beats E, B beats A, D beats B, and C beats D. This leads to the scale pattern given in Table 2. This pattern cannot be rearranged so as to be Guttman scalable. Thus, we are compelled to reject the conjecture that sincere voting under SAP when preferences are single-peaked will necessarily give rise to a Guttman scalable pattern. Moreover, we can construct nonsingle peaked preference ordering which will, under these assumptions (for particular ordering of alternatives) give rise to Guttman scalable patterns. This line of attack appears then to be a dead end. Another tack is to fix one element and to look at all pairwise choices versus that element. For example, for the above preferences, we may look at the results of A vs. B, C vs. B, D vs. B, and E vs. B on the assumption of sincere voting to obtain the scale pattern shown in Table 3.

By polarizing column 1 (i.e., converting pluses to minuses and vice versa – an admissible operation), we may convert this pattern into one of perfect Guttman scalability. Similarly, if we look at all pairwise comparisons involving C, we obtain the scale pattern shown in Table 4. If we polarize columns 1 and 2 (i.e., polarize all columns which contain an element to the left of C on the continuum *ABCDE*) we again obtain a pattern of perfect Guttman scalability. This leads us to the following proposition.

If a set of preferences is single-peaked along some unidimensional continuum, then the scale patterns generated by all paired comparisons from alternatives along that continuum vs. some one fixed alternative from the continuum will be Guttman scalable when the pairwise choices are left-right ordered according to each alternative's position along the continuum, and the polarity of all columns involving choices located the left of the one fixed alternative have their polarity reversed.

This result follows from the fact that the choices in each pairwise comparison will be such that if a voter with ideal point to the left (right) of the fixed alternative prefers that point to a point located to the right (left) of the alternative, he must prefer it to all points further right (left). Similarly, a voter with ideal point to the left (right) of the fixed alternative who prefers it to some point located to the left (right) of the alternative, must prefer it to all points further left (right).

This result suggests the following possibility: isolate the set of pairs which have some particular element common to them all. If decisions over this set are not Guttman scalable, then (assuming sincere voting) the underlying perferences for alternatives in the set cannot be single-peaked.[2]

[2]Since in sincere majority pairwise voting, a Condorcet choice if one exists will (by definition) defeat all other alternatives, and single-peakedness guarantees the existence of a

However, it is easy to show that even in this special case, Guttman scalable responses can be generated by preferences which are not single-peaked. Consider the case of a series of items proposing changes in, say, social welfare policy. As Weisberg and Niemi point out (1971, p. 31, with a change in sentence order):

> "Even when asking a person of whether he approves of some proposed policy, he is likely to respond in terms of his choice between that proposal and the status quo. 'Would you support minimum guaranteed family income of $2500 for a family of four?' is likely to receive a very different response from many people when the present minimum guaranteed level is $3000 than when it is $1600, even when individual ideal points are the same in the two instances. Legislative votes are a prime instance. The legislator must decide whether he prefers the situation resulting from the passage or the defeat of the motion on which he is to vote."

It is possible to interpret some roll-call data and also some public opinion poll data as representing a series of choices in which various alternatives are compared to the status quo.[3] If the underlying preferences for these alternatives (including the status quo) are single-peaked, then such choices will generate Guttman scalable responses when the appropriate reordering and polarizations are performed.[4] Moreover, if preferences are in fact single-peaked, in determining the point to the left of which columns must be polarized to generate a Guttman scalable pattern, we are actually determining the perceived location of the status quo point.

Conclusion

As Weisberg and Niemi (1971) are careful to point out, the relationship between single peakedness and Guttman scalability depends upon whether the response task is a single stimulus situation or a paired comparison. Even in the context of the latter, in general, except under very restrictive conditions, Guttman scalability does not imply single-peakedness, nor conversely. If, however, we confine ourselves to the special (but empirically relevant) case in which responses may be viewed as being generated by an explicit or implicit series of paired comparisons with some fixed alternative (such as a fixed status quo point), then we have shown that when voters' preferences for all alternatives (including the status quo) are single-peaked, the response patterns can be adjusted so as to form a perfect Guttman scale pattern in such a way as to reveal the location of the (implicit) status

Condorcet winner (Black, 1958), such a winner whenever introduced into the balloting under SAP will remain an alternative in each succeeding pair.

[3]Another possibility would be a proximity scale. See Weisberg and Niemi (1971), pp. 33-34.

[4]Note: For preference to be single-peaked for all alternatives including the status quo requires that all individuals agree on the location of the status quo point with respect to the underlying one dimensional continuum. This may be a strong requirement. E.G., liberals and conservatives may differ as to how liberal (conservative) the status quo really is.

quo point on the underlying continuum.[5]

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[5] An anonymous referee has suggested that these results may be generalized to the multidimensional case. See Carroll (1972).