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A NOTE ON ABRAHAM LINCOLN IN PROBABILITYLAND

Abstract. Abraham Lincoln's dictum that "you may fool all of the people some of the time; and some of the people all of the time; but you can't fool all of the people all of the time", is interpreted in terms of a simple binomial model, and potential ambiguities in Lincoln's assertion are clarified.

> If you once forfeit the confidence of your fellow citizens, you can never regain their respect and esteem. It is true that you may fool all of the people some of the time; you can even fool some of the people all the time; but you can't fool all of the people all of the time.
>
> *Abraham Lincoln*

Let us consider Abraham Lincoln's famous dictum in the light of a simple combinatorial model. Consider a group of size N of mean competence p, where $(1-p)$ is taken to be the likelihood that an average individual will be fooled on any given occasion.[1] What's the probability that you can fool all of the people some of the time? Let us interpret some of the time the way logicians do, as meaning at least once. The probability that everyone is fooled on any given occasion is simply $(1-p)^N$. Even if p is very small, $(1-p)^N$ rapidly goes to zero as N gets larger. Thus, on any given occasion it is unlikely that everyone will be fooled. On the other hand, if we (simultaneously) confront the members of a group with a sufficiently large number of chances to be fooled (K times, say) then it's likely that on at least one of these occasions they will be fooled.

The probability that everyone in a group will be fooled at least once in K times is $1-(1-(1-p)^N)^K$. Regardless of p, for fixed N, this expression goes to one as $K \to \infty$; for fixed K, this expression goes to zero as $N \to \infty$. A natural question to ask is what happens when N and K simultaneously increase. Regardless of p, if $K = N$, the above expression goes to *zero* as N and K increase toward infinity. Even if there are considerably more

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opportunities to fool people than there are people to be fooled, as long as N rises proportional to K (i.e., $N = \alpha K$, $0 < \alpha \leqslant 1$), the probability of fooling all the people some of the time still goes to *zero* as $N \to \infty$, regardless of p. Thus it is *not* as easy to fool all the people at least some of the time as we (or Lincoln) might think.

Of course, if we interpret "fooling all of the people some of the time" as "fooling everybody at least once, but not necessarily at the same time"; then, under our assumptions, the probability of such an event may be expressed as $(1 - p^K)^N$. Thus

$$\lim_{\substack{N \to \infty \\ K \to \infty \\ K \propto N}} (1 - p^K)^N \to 1, \quad \text{for all } p.$$

Fooling everybody at least once, but not necessarily at the same time is considerably easier than fooling everybody at the same time at least once. Moreover, the former gets easier to do as N and K simultaneously increase; the latter gets increasingly more difficult!

Now, let us consider the case of fooling some of the people all of the time. The probability that any given member of the group will be fooled K times in a row is simply $(1 - p)^K$. As K gets larger, this probability rapidly goes to zero. Let us interpret "some people" as "at least one person". The probability that some set of one or more persons in a group of size N will be fooled K times in a row is simply

$$\text{(1)} \qquad \sum_{h=1}^{N} \binom{N}{h} ((1-p)^K)^h (1 - (1-p)^K)^{N-h}$$

This may be rewritten as

$$\text{(2)} \qquad 1 - \binom{N}{0} (1 - (1-p)^K)^N = 1 - (1 - (1-p)^K)^N.$$

This expression is equivalent to that obtained earlier for the probability of fooling all the people some of the time, except that K and N are interchanged. This interchange does not affect the behavior of the expression in the limiting case. Thus, similarly,

$$\lim_{\substack{N \to \infty \\ K \to \infty \\ K \propto N}} 1 - (1 - (1-p)^K)^N \to 0, \quad \text{for all } p.$$

If we do not require it to be the same person(s) being fooled each time, we may express the probability of "fooling some of the people all the time" as $(1 - p^N)^K$. In this case, we obtain quite opposite results, i.e.,

$$\text{(3)} \qquad \lim_{\substack{N \to \infty \\ K \to \infty \\ K \propto N}} (1 - p^N)^K \to 1, \quad \text{for all } p.$$

Abraham Lincoln was clearly quite right in asserting that one cannot expect to fool all of the people all of the time. Under our assumptions, the probability of fooling all of the people K times in a row is simply $(1 - p)^{NK}$. This expression goes *very* rapidly to zero as N or K increases.[2] However, our results suggest the need for caution in interpreting Abraham Lincoln's famous dictum. If we interpret "fooling all of the people some of the time" as "fooling each person at least once" and interpret "fooling some of the people all the time" as "fooling at least one person each time", the probabilities of each of these events is high, and the probability of each approaches certainty as N and K jointly approach infinity. On the other hand, if we interpret "fooling all the people some of the time" as "fooling everyone simultaneously at least once" and interpret "fooling some of the people all of the time" as "fooling some set of one or more persons each and every time", we obtain exactly opposite results!

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NOTES

[1] We assume that on each such occasion all persons in the group are given the opportunity to be fooled.

[2] The probability of fooling some of the people some of the time is simply $1 - (1 - (1 - p^N))^K = 1 - p^{NK}$. Of course, this probability goes very rapidly to 1 as N or K increases.

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