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ON THE (SAMPLE) CONDORCET EFFICIENCY OF MAJORITY RULE: AN ALTERNATIVE VIEW OF MAJORITY CYCLES AND SOCIAL HOMOGENEITY

ABSTRACT. The Condorcet efficiency of a social choice procedure is usually defined as the probability that this procedure coincides with the majority winner (or majority ordering) in random samples, given a majority winner exists (or given the majority ordering is transitive). Consequently, it is in effect a conditional probability that two sample statistics coincide, given certain side conditions. We raise a different issue of Condorcet efficiencies: What is the probability that a social choice procedure applied to a *sample* matches with the majority preferences of the *population* from which the sample was drawn? We investigate the canonical case where the sample statistic is itself also majority rule and the samples are drawn from real world distributions gathered from national election surveys in Germany, France, and the United States. We relate the results to the existing literature on majority cycles and social homogeneity. We find that these samples rarely display majority cycles, whereas the probability that a sample misrepresents the majority preferences of the underlying population varies dramatically and always exceeds the probability that the sample displays cyclic majority preferences. Social homogeneity plays a fundamental role in the type of Condorcet efficiency investigated here.

KEY WORDS: Condorcet efficiency, majority cycles, representation, sampling, social homogeneity

1. INTRODUCTION

1.1. *Condorcet efficiency*

The Condorcet efficiency of social welfare functions remains a topic of great interest in the contemporary social choice literature (Gehrlein, 1999; Gehrlein and Lepelley, 1999, 2001; Gehrlein and Valognes, forthcoming; Lepelley et al., 2000). The basic idea behind this concept is to use majority rule as a benchmark to evaluate the outcomes of other social choice functions. For instance, the Condorcet efficiency of the Borda score is typically defined as the probability that the Borda winner coincides with the Condorcet (major-

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ity) winner, given that a unique Condorcet winner exists. Related concepts of Condorcet efficiency for the example of the Borda score are, among others: (i) the probability that the Borda loser is the Condorcet loser, given that a unique Condorcet loser exists; (ii) the probability that the ordering according to Borda scores is the same as the ordering by majority rule, given that this majority ordering is transitive.

Three important observations are in order: (1) The Condorcet efficiency is a conditional probability. (2) The Condorcet efficiency is usually derived from a hypothetical probability distribution over the actual preferences. This hypothetical distribution is often a uniform distribution over a (possibly constrained) set of preference relations. (3) The event in question, namely that the Borda outcome coincides with the Condorcet outcome, is the outcome of a random sampling process from the given hypothetical distribution.

More abstractly, the Condorcet efficiency of any social welfare function SWF is usually defined as the (conditional) probability that SWF and majority rule coincide in samples from some specified probability distribution over preference relations, given that the sample majority preference is not deteriorated (i.e., given that it is transitive or given that it has a unique top element). The prototypical case is the *conditional* probability that a social welfare function SWF (say, SWF is *plurality*, *Borda*, *approval voting*, the *single transferable vote*, or *anti-plurality*) elects the Condorcet winner, given that a unique Condorcet winner exists. To calculate such a (conditional) probability requires assuming a probability distribution that underlies the calculation. The most common underlying assumption is that preferences are distributed according to a uniform distribution over some set of preference relations (*impartial culture*). In statistical terms the standard definition of Condorcet efficiency is therefore the *conditional* probability that two sample statistics, namely the *sample SWF* outcome and the *sample majority outcome*, coincide, conditioned upon the event that the sample majority outcome satisfies certain conditions (unique Condorcet winner, or transitivity, etc.), in samples drawn from a specified distribution over preference relations.

This paper differs from previous work in two key respects: (1) We draw the reader's attention to a different type of Condorcet efficienc(ies)y, namely the probability that various sample statistics based on a social welfare function SWF coincide with the *population* majority preferences of the population that the sample of a given size was drawn from. (2) In line with Van Deemen (1999) we move from theoretical a priori distributions to realistic distributions of preferences, inspired by survey data.

We will start with the canonical case of the sample majority winner and the sample majority preference ordering being the two statistics under consideration. At first sight, it might appear that large samples would automatically have the same majority preferences as the distribution from which they are drawn. However, the standard literature on random samples from the impartial culture (on linear orders) – in which all linear orders are equally probable – proves the contrary. In the impartial culture, the majority preference relation is a complete tie, which even large samples of odd size can never recover. In other words, the most frequently studied case of odd size samples from the impartial culture is an odd example of a sampling process which entirely fails to be representative of the underlying population, even as sample size goes to infinity, because it rules out a majority tie in the sample. Thus, even in the limit, it is not entirely trivial how sampling affects the likelihood of accurate representation of majority preferences. For finite samples of the size as we typically encounter in reality, the effect of sampling is far from trivial. As we shall see, the accuracy of samples of different sizes is closely related to the size of the pairwise margins and, for any fixed sample size, varies dramatically across distributions.

To avoid ambiguities in terminology we do not use the expression *Condorcet efficiency* any further. Instead, we talk about *representation* and *misrepresentation*. We say that the social welfare function in the sample *represents* the population majority preferences when it coincides with the population majority preference and we say that it *misrepresents* the population majority preferences, when it differs from the population majority preference. As mentioned above, we focus here on the canonical case where the sample statistic itself is also majority rule. Following, e.g., Van Deemen's (1999) recommendations, we look at 'realistic' distributions of preferences inspired by empirical surveys instead of theoretical *a priori* distributions.

1.2. *Majority Cycles*

One of social choice theory's central classical preoccupations has been with majority rule *cycles* – in essence, the lack of transitive majority preferences, also called the *Condorcet paradox*. The simplest example of the paradox of cyclical majorities occurs with three voters choosing among three alternatives, here labeled *A, B,* and *C*. Let voter 1 have preference order *ABC*, let voter 2 have preference order *BCA* and let voter 3 have preference order *CAB*. Here a majority prefers *A* to *B* and *B* to *C,* which might lead us to expect that a majority prefers *A* to *C*. But this is not the case. Instead a majority prefers *C* to *A* and, therefore, these majority preferences form a cycle, i.e., they are not transitive. There is a considerable body of work estimating the probability of cycles. The theoretical literature suggests that the probability of cycles increases as we increase the number of alternatives, approaching a certainty of finding a cycle as the number of alternatives gets very large (Shepsle and Bonchek, 1997). In contrast, for only three alternatives, the probability of cycles has been estimated at around 8%. These estimates are derived from random samples drawn from the impartial culture, a uniform distribution over all linear orders (Black, 1958; Riker, 1982; Shepsle and Bonchek, 1997). For three alternatives drawn from a distribution in which all weak preference orders are equally probable the probability of a cycle has been estimated to be around 5% (e.g., Jones et al., 1995; Van Deemen, 1999). In contrast the empirical literature finds virtually no evidence of cycles among voter preferences over major candidates or parties, although there are only a few studies to draw on (see esp. Niemi, 1970; Dobra and Tullock, 1981; Dobra, 1983; Chamberlin et al., 1984; Niemi and Wright, 1987; Feld and Grofman, 1988, 1990, 1992; Radcliff, 1997; Regenwetter and Grofman, 1998a; Van Deemen and Vergunst, 1998; Kurrild-Klitgard, 2001; see also Brady, 1990). Furthermore, the few cycles that are observed tend to be among alternatives that are low-ranked and amongst which voters are not readily able to discriminate.

The failure to find almost any cycles in survey data (and equally few cycles in legislative roll call data, even when we include contrived cycles due to strategic voting of the sort described by Riker, 1958) led the economist Gordon Tullock (1981; see also Niemi, 1983; Grofman and Uhlaner, 1985) to ask the question: "Why so much stability?"

1.3. *An alternative view of majority cycles and social homogeneity*

According to traditional arguments, as long as majority cycles cannot be ruled out, the concept of majority rule may be ill defined and, therefore, democratic decision making may be at risk. Although our work bears on the likelihood of cycles, results about cycle probabilities are not the most important part of our findings. (1) Our first central claim is that, for small numbers of alternatives and realistic distributions of preferences, the problem of *cycles* is much less important than the problem of *misrepresentation*, i.e., that the majority preferences in a sample diverge from those of the underlying population. Political representation is feasible, not when majority preferences in a sample are revealed to be transitive, but when the (transitive or intransitive) majority preferences so revealed are those of the underlying population. Our analyses reveal that, regardless of whether the probability of cycles is low or (relatively) high, the probability of a cycle is always lower than the probability of obtaining a transitive majority preference order that misrepresents the majority preferences of the society at large. (2) We also claim that the degree of homogeneity, i.e., the degree to which members of a population share common beliefs, knowledge, goals and preferences, is the key determinant for how representative the majority preferences in a random sample of given size will be of the majority preferences in the population of reference. While this statement may seem obvious, we draw the attention of the reader to the real and striking differences that have to be expected for different populations or different subgroups of a given population.

We focus on the collective preferences of national electorates – and, for illustrative purposes, on the preferences of six important subgroups within one of these electorates – over presidential candidates or political parties. The majority preferences of national electorates – as well as the preferences of important subgroups within these electorates – are important because, in order for group representatives to further their constituencies' interests, these representatives must be able to accurately gauge their constituents' majority preferences over the available alternatives. For instance, presidents must weigh the relative priorities their constituents assign to alternative policy goals such as fighting crime, maintaining a strong national defense, and clean air; party leaders in multiparty parliamentary democracies must be able to ascertain how their membership evaluates participation in alternative governing coalitions; union officials must be able to estimate how their union constituency evaluates the tradeoffs between maximizing wages, securing retirement benefits, and maximizing workers' vacation time, etc. In each of these cases, it is not enough for group representatives to infer that their constituencies have transitive majority preferences – they must also infer what the "correct" majority preference orders are.

We begin with voters' candidate/party preferences as reported in national election surveys in the United States (the 1992 and 1996 American National Election Studies), in Germany (the German National Election Studies of 1961, 1965, 1969, 1972 and 1976) and in France (the 1988 French Presidential Election Study). For each election we use respondents' thermometer ratings or rankings to infer their preference orders over three major parties or candidates. We report the distribution of voters' candidate/party rankings in the survey, and indicate what this distribution tells us about the electorate's collective majority preferences – i.e., whether or not there is a unique majority winner and a transitive majority preference order in the survey. For illustrative purposes, we also report similar information for six subgroups for the French survey. These are groups whose members either share common socio-demographic characteristics (here the working class and the middle class) and groups whose members share common political orientations (here leftist voters, right-wing voters, Communists, and Gaullists).

Ideally, representatives would determine constituency majority preferences by asking *every* member of their constituency. Although technological advances may eventually make this ideal feasible, for the foreseeable future representatives will be forced to employ more selective information gathering techniques, such as sample polls. These techniques all share the common feature that the representative infers his or her constituency's majority preferences from a sample of constituents' opinions. This raises the central question we pose in this paper: "How likely it is that the majority preference order of such a sample accurately reflects the preferences of the entire constituency from which the sample is drawn"? Similarly, how likely it is that such a sample contains a majority cycle? Given that polling is expensive, a closely related important issue is to determine a sufficient sample size for accurate representation of a population majority preference by a sample.

To tackle these questions, we take random samples (with replacement) from the surveys identified above. We repeat each sampling procedure 10 000 times and record the proportion of times a sample yields a majority cycle, the proportion of times a sample yields the correct majority winner, and the proportion of times a sample yields the correct majority preference order (where *correct* is being used to refer to the majority preferences in the full survey). To minimize confusion, since we are talking about sampling from data sets that are themselves random samples, we use the terms "survey" and "survey data" to refer to the relative frequency distribution in the national survey data.

To illustrate the overwhelmingly greater importance of representation compared to cycles in realistic distributions, and the role of social homogeneity, we now sketch some findings from three of our analyses. These examples together provide a snapshot of the tradeoff between social homogeneity and sample size in the quest for accurate representation. The first example illustrates a case where accurate representation is easy to obtain (i.e., requires moderate sample size). The second, less typical example, however shows that even large samples may not rule out the possibility of completely misrepresenting majority preference orders. It shows that for highly heterogeneous populations, correct representation is a real and challenging problem. The third example illustrates the other extreme case where accurate representation is amazingly good, even for a sample of only three voters, because virtually all members of that population think alike. All three examples also demonstrate that, contrary to the standard findings suggested by samples from the impartial culture, cycles are not a serious problem in random samples from these distributions. The issue is accurate representation.

1.4. *Three examples and their key properties*

(1) What happens when drawing small random samples of voters from the 1996 American National Election Study (ANES)? The

survey itself has a transitive majority preference relation, namely C beats D beats P (C: Clinton; D: Dole; P: Perot). If you draw a random sample of only 20 ANES respondents, then the probability of these 20 voters having cyclical majority preferences is less than 1% (i.e., majorities are transitive with 99% probability). However, the probability that they correctly rank order the candidates by majority rule is (only) 58% and the probability that they elect the correct majority winner is about 86%. Thus the likelihood that the 20-person sample yields the wrong majority first choice, 13% (i.e. 100%-86%-1%), is more than 13 times larger than the likelihood that the sample displays cyclical (intransitive) majority preferences (i.e. fails to find a majority first choice), which is, in turn, less than 1%.[1] When you randomly sample 101 ANES respondents, the probability that their majority preference order is CDP (i.e. matches the majority preference order in the original survey) is 92%. The probability that 101 randomly sampled respondents will have a cyclical majority is negligible, and the probability that they select the correct majority winner is over 99%.[2]

(2) What happens when drawing random samples from the 1976 German National Election Studies (GNES) survey? The survey itself has a transitive majority preference relation, namely S beats C beats F (S: SDP; C: CDU/CSU; F: FDP). If you draw a random sample of three voters from this distribution, the probability that these three voters end up with a majority cycle is 2% (i.e., majority preferences are transitive with 98% probability), whereas the probability of finding the correct majority preference order is only 11%, and the probability of generating the correct majority winner is only 46%. (In fact, there are three incorrect majority preference orders that are each at least twice as likely as the correct one!) In other words, there is again a tiny probability of a cyclical majority among the three voters, but now an enormous risk (100% – 2% – 46% = 52%) of selecting the wrong majority winner. For this survey, even a random sample of 2 000 voters has a probability of reaching the correct majority winner of only around 81%. On the other hand, for that sample size, the probability of a cycle is essentially zero.[3]

(3) What happens when drawing small random samples of communists from the 1988 French National Election Studies survey (FNES)? A majority (about 53%) of all survey respondents who

have identified themselves as communists state the preference ranking MBC (M: Mitterrand; B: Barre; C: Chirac). Thus, MBC is the majority preference order among communists in the survey. If you randomly sample three communists from the survey, the probability that their majority rule preference order is again MBC exceeds 76%, but the likelihood of a majority cycle is far below 1% (indeed we have not generated a single three-element sample with a cyclic majority, out of 10 000 samples). In addition, the probability exceeds 99% that the majority preference first choice is M for any three randomly sampled communists.[4]

For all eight election survey sets as well as all six analyses of the French data set at the level of sociodemographic/political subgroups we find that the threat of majority rule misrepresentation overwhelmingly outweighs the threat of majority cycles. The only time when the probability of a majority cycle gets even close to the probability of majority rule misrepresentation is when both probabilities are essentially zero, i.e., when correct majority representation is virtually guaranteed.[5]

In the remainder of the paper we present more detailed results about sampling majority preferences from various ‘realistic’ preference distributions. In particular, we record, for various sample sizes, whether or not a majority cycle occurs in a given sample and to what extend the majority preference order matches the majority preference order in the distribution that the sample was drawn from.

2. DATA, METHODS AND FINDINGS

We study eight different data sets from three different countries. Each data set reports information on preferences or thermometer evaluations that we use to construct respondent rankings over the set of alternatives: either three parties or three presidential candidates. Our data pertain to presidential candidates in France in 1988 (Barre, Chirac, and Mitterrand) and in the U.S. in 1992 (Bush, Clinton and Perot) and 1996 (Dole, Clinton and Perot); as well as the three major parties (CDU/CSU, FDP, and SDP) in the German parliamentary elections of 1961, 1965, 1969, 1972 and 1976. The latter survey has also been studied by Norpoth (1979). From each data set we draw random samples of voter preference orders with replacement, with sample sizes (n) ranging from 2 to 10 000. To obtain our probability estimates, we conduct 10 000 draws for each sample size (and identify the proportion among 10 000 samples with the sampling probability). In order to construct (majority) preference orders from thermometer data we assume that party A is preferred to party B if and only if the respondent gives party A a higher thermometer rating than party B. We assume that a respondent gives identical thermometer ratings to two parties if and only if the respondent is indifferent between those two parties. For three elections (Germany 1969, 1972, and 1976) the rankings are taken directly from respondents' stated party preference rankings: in these three cases, the respondents were not permitted to express indifference. For these three elections the pool from which we are drawing samples is restricted to the six possible *linear orders* over the parties, i.e., rankings without ties. For the remaining elections, however, we draw from a set of *weak orders*, i.e., preference rankings in which ties are possible. When there are three choice alternatives, there are 13 possible weak orders (see, e.g., Jones et al., 1995, who study random samples drawn from the impartial culture on weak orders).

In addition to drawing samples from these elections at an aggregate level, i.e., from the entire survey, for illustrative purposes we also analyze samples at the 'subgroup' level for France in 1988. The French subgroups we analyze either shared sociodemographic characteristics (e.g., working class and the middle class) or common political orientations (e.g., leftist voters, right-wing voters, Communists, and Gaullists). We believe that looking at the subgroup level is politically important because representatives often are concerned about the political views of particular constituencies. Here, as for the electorate as a whole, representatives must often rely on information from samples. Also, we would anticipate that subgroups might differ significantly from the larger electorate sampled (and from each other) in terms of their preference homogeneity. Thus, if we wish to have a realistic view of how accurately information drawn from samples can be used to infer majority preferences and majority preference orders, we should look at samples drawn from specific subgroups and not just samples drawn from the electorate as a whole. One of the key features of these subgroups is that they are, by their very nature, more homogeneous than the population as a whole, in the sense that the members of the groups tend to share similar knowledge, concerns, convictions and opinions.

In our analysis, for each sample size and each of the elections/ subgroups studied, we report the likelihood of finding a majority rule cycle, the likelihood that the sample reflects the majority winner in the full survey, and the likelihood that the sample majority preference order matches the majority preference order in the full survey. As noted earlier, our central concern is not with cycles, but with correct representation, as demonstrated by an accurate reflection of the majority winner and majority preference order of the full survey data set. To illustrate our approach we will first focus on three of the elections: the 1996 U.S. presidential election, the 1976 German parliamentary election, and subgroup data from the French Presidential election of 1988 for those who identify themselves as Communists.

2.1. *The 1996 U.S. presidential election*

In the upper left panel of Figure 1 we give the relative frequencies of the thirteen possible preference rankings among Clinton (C), Dole (D) and Perot (P) in the 1996 ANES survey data set. To read the upper left panel in this figure, take for example, the entry in the center top, the rectangle showing D above C above P (referred to as DCP in the text). This represents the linear preference order < Dole preferred to Clinton preferred to Perot>. From the figure we can see that 9% of the respondents in the ANES survey expressed thermometer ratings that corresponded to this ranking. Note that the most common ranking was CDP, which occurred 26% of the time. The rankings enclosed in ovals are weak orders that involve ties; e.g., if we observe D and C on the same line above P enclosed in some oval, this indicates the preference statement <Dole and Clinton tied (i.e., indifference), but both preferred to Perot>. Twelve of the 13 possible linear and weak orders are shown around the outside edge of upper left panel in Figure 1; the last possible weak order is a diamond-shaped figure at the center, and reflects complete indifference among the three candidates. (For notational convenience the diamond-shaped figure representing complete indifference is simply left empty.) [6]

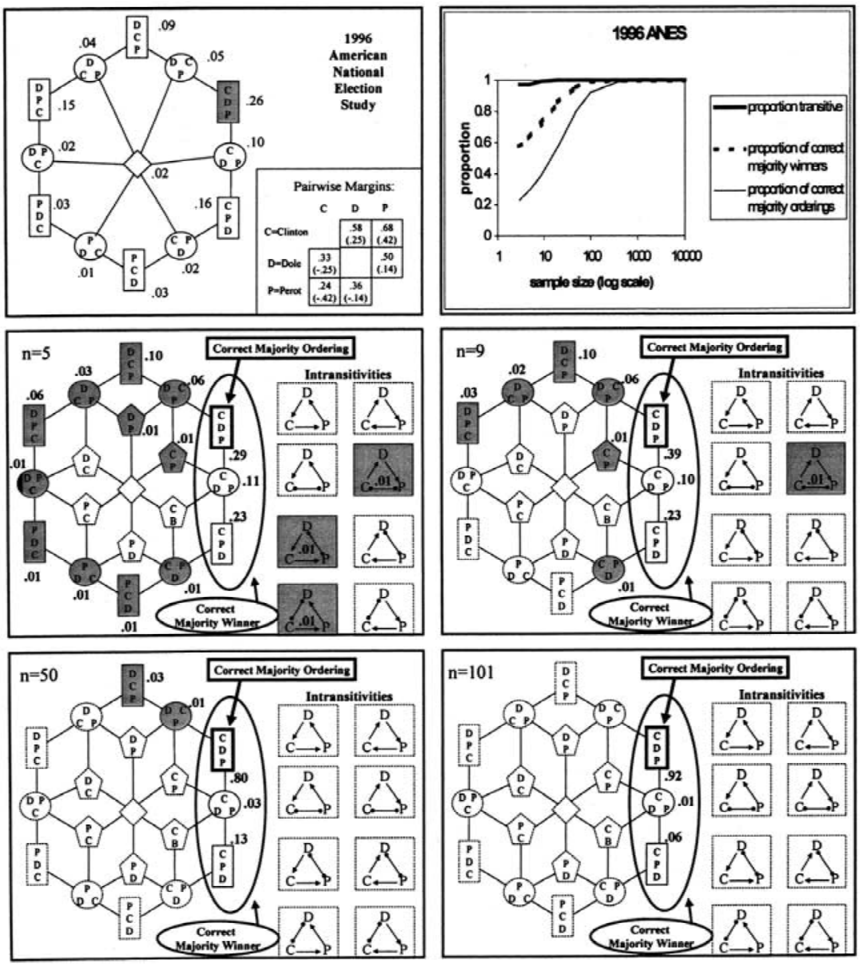

Figure 1. Overview of results for samples drawn from the 1996 American National Election Studies survey.

If we look at the proportions shown in Figure 1 (upper left), we can compute the majority preferences among the respondents in the ANES survey. In the paired comparison between Clinton and Dole, 58% of the respondents strictly preferred Clinton to Dole, 33% strictly preferred Dole to Clinton, and 9% were indifferent between the two. For example, to obtain the first of these numbers, the proportion who strictly prefer Clinton to Dole, we sum up the proportions associated with all orders that put Clinton strictly ahead of Dole (CDP, CPD, PCD, C(D=P),(C=P)D). In other words, taking into account rounding error, 0.58 = 0.26 + 0.16 + 0.03 + 0.10 + 0.02.

Since 58% is more than the proportion associated with the opposite strict preference, 33%, the survey data as a whole had a *majority preference* for Clinton over Dole.[7] The *pairwise preference margin* is 25% (i.e., 58%-33%) in favor of Clinton.

In the paired comparison between Clinton and Perot, 68% of the respondents preferred Clinton, 24% preferred Perot, and 8% were indifferent between the two. Hence, the survey data as a whole had a majority preference for Clinton over Perot. Because Clinton is preferred to both his opponents in pairwise contests, he is what is commonly called a *majority winner* (also known as the *Condorcet winner*, see, e.g., Black, 1958, and Saari, 1995) among the respondents surveyed. Turning now to the paired comparison between Dole and Perot we find that 50% of the respondents preferred Dole to Perot, 36% preferred Perot to Dole, and 14% were indifferent between the two. Hence, taking the information we now have, we determine that the majority preference order in the survey was CDP, i.e., Clinton majority preferred to Dole majority preferred to Perot. For the entire 1996 ANES survey, we include the pairwise preference probabilities and the pairwise margins (given in parentheses) in the upper left panel.

Now we turn from the survey as a whole to samples drawn (with replacement) from the survey. In the second and third row of Figure 1, we show some of the outcomes of our sampling process from the 1996 ANES survey to give the reader a feel for how the distribution of majority preference relations changes with the sample size. These figures show the probability distribution of each of the possible majority preference relations in the sample for sample sizes of 5, 9, 50, and 101, respectively. For individuals, only the 13 rankings identified in upper left panel are possible outcomes. However, once we consider (sample) majority preference relations, then 14 additional possibilities occur. These are the majority preference relations represented in the additional six pentagons and the eight intransitive majority preference relations shown in the boxes to the right (including the forward and backward cycles) in each panel of the second and third row of Figure 1. In the boxes, lines with arrows represent strict majority preferences; lines with dots at their ends indicate majority ties. In the pentagons, the notational convention is as follows: if, say, we find D above P, this translates as the preference ranking < Dole majority preferred to Perot, Dole and Clinton majority tied, Perot and Clinton majority tied>.[8] (The latter two aspects of the ranking are implicit: the indifference relationships are taken to apply between each of the indicated alternatives and the alternative that has been omitted.)

The shaded areas in the four lower panels are majority preference relations, with incorrect majority winners or intransitivities, found to have positive probabilities in the sampling process for samples of that size and rounded to the second significant digit. The observed proportions resulting from the sampling process are shown next to the relations. (Given that these proportions result from 10 000 repeated samples, we use them as proxies to the sampling probabilities and talk about them as probabilities.) In each panel, the rankings that are inside the large oval are those majority preference relations that yield the correct majority winner, i.e., for the 1996 ANES survey, all rankings in which Clinton is strictly in first place. The ranking which corresponds to the survey's majority preference ranking – here Clinton majority preferred to Dole majority preferred to Perot – is bordered in bold and identified with an arrow. Clearly, the shaded majority preference relations are "incorrect" majority preference relations (including the cycles as special cases). The lower four panels in Figure 1 allow us, in particular, to answer our three key questions: *what are the likelihoods that a sample drawn from the 1996 ANES survey data set of a given size (a) has transitive majority preferences, (b) has the same majority winner as the survey respondents as a whole, (c) has the same majority preference order as the survey respondents as a whole?*

Consider the middle left panel, which shows results for five-person samples drawn from the 1996 ANES survey (with replacement). We see that roughly 3% of the five-person samples exhibit intransitive majority preferences (see the boxes on the right-hand side of the panel), which implies that 97% of the majority orders are transitive. By summing the probabilities inside the oval (0.29 + 0.11 + 0.23), we see that approximately 63% of the five-person samples have the same (strict) majority first choice – namely Clinton – as the survey itself. This suggests that even a very small sample of respondents from the 1996 ANES survey would reproduce the full survey's majority winner more than half the time. However, we also see that only 29% of the five-person samples generate majority orders that match those of the full survey, i.e., CDP. It is apparent that the threat of nonrepresentativeness of the transitive sample outcomes (68%) dwarfs the threat of error due to cycles (3%).[9] Indeed, we are also far more likely to generate a five-person sample with a transitive majority preference order whose majority first choice fails to match that of the survey than we are to generate a sample with intransitive majority preferences (34 vs. 3%).[10]

Now let us compare this panel with the one underneath it, which has a sample size of 50. The first thing to note is that a larger sample size reduces variability in the majority preference distribution. Second, there are no longer any intransitivities, i.e., for samples from this survey, for even moderate sample sizes, the threat of a cycle is nonexistent. Third, the likelihood of obtaining the "correct" majority winner rises from 0.63 for $n = 5$ to 0.96 for $n = 50$, while the likelihood of recovering the complete "correct" majority preference order rises from 0.29 for $n = 5$ to 0.80 for $n = 50$. Thus, even for a sample size of 50, we still have a problem of misrepresentation, but we now have no problem of intransitivity. Finally, if we turn to a sample size of, say, 500, we again find no intransitivities. We also find that 500 person samples invariably hone in on the correct majority preference order, and thus, *a fortiori*, give us the correct majority winner, i.e., the probability of the sample majority preference order matching the survey majority preference order is 100%. (For that reason we have not included a panel for that sample size.) Completely contrary to expectations about sample size effects derived from the impartial culture (e.g., Shepsle and Bonchek, 1997), but in line with common sense, larger group (sample) sizes are better for majority rule decisions.

The upper right panel of Figure 1 summarizes the results from the lower four panels and similar panels that we did not bother to present. It shows how the observed probabilities of transitive majority preferences, correct majority winners, and correct majority preference order (where, *correct*, of course, is being used as synonymous with matching the pattern among the survey respondents as a whole) vary with the size of the sample. As we saw previously, the problems of intransitivity and of misrepresentation both diminish with sample size. Furthermore, for the full range of sample

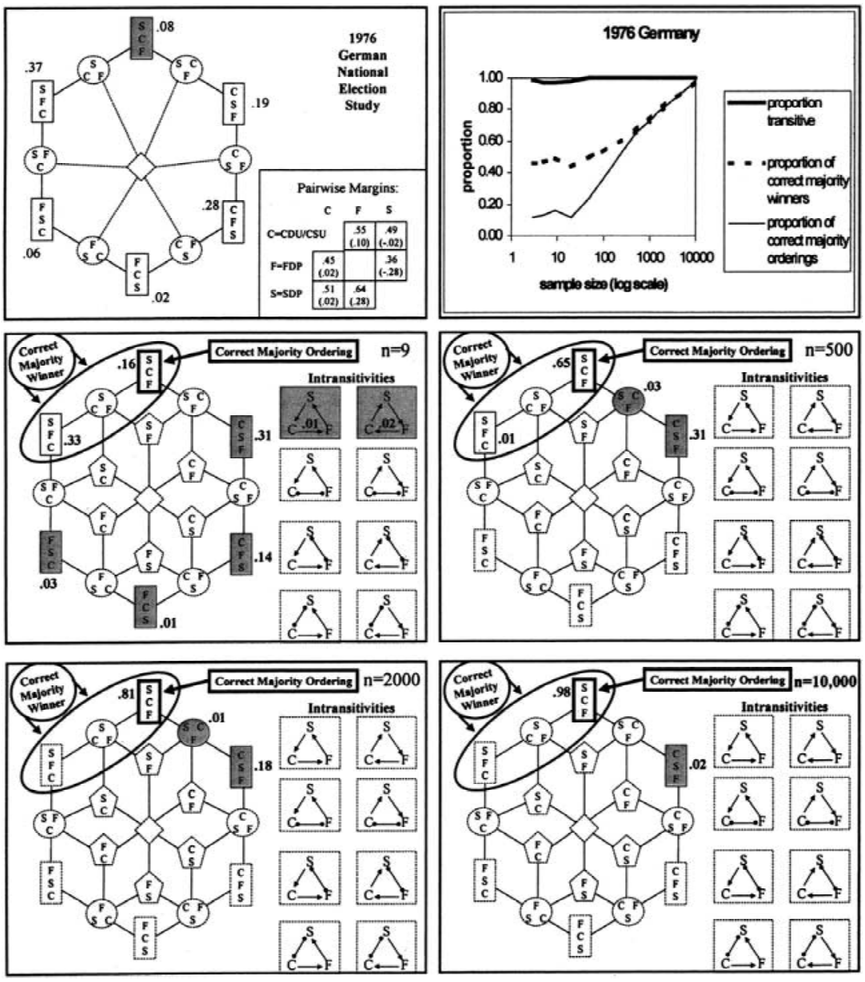

Figure 2. Overview of results for samples drawn from the 1976 German National Election Studies survey.

sizes, we are more likely to get a wrong majority winner than to get an intransitive majority preference. Of course, the probability of an incorrect majority preference order is always greater than the probability of an incorrect majority winner. Given the distribution of the 1996 ANES survey respondents' candidate preferences, we conclude that the possibility looms large that a sample does not represent the majority preferences in the survey, but the possibility that the sample displays cyclical majority preferences does not loom large at all.

2.2. *The 1976 German Parliamentary Election*

In the 1976 German National Election Study (GNES) respondents' rankings of the three major parties – the SDP (S), the CDU/CSU (C), and the FDP (F) – are taken directly from respondents' party preference rankings. Since the respondents were not permitted to express indifference, the distribution of the respondents' party rankings given in the upper left-hand panel of Figure 2 displays positive proportions for only the six strict linear party orders. It is interesting to notice that only 8% of the survey participants actually stated a preference ranking that coincided with the group majority preference order (SCF). The upper left panel of Figure 2 also collects the corresponding pairwise preference probabilities, plus the pairwise margins (in parentheses) and it displays the implied survey majority preference order SCF.

The lower four panels of Figure 2 show the probabilities of majority preference relations in samples of size 9, 500, 2000 and 10 000 drawn (with replacement) from the distribution given in the upper left panel of Figure 2. The panels reveal that while the likelihood of cyclical majority preferences in the samples is always small (no more than 3% for any given sample size), the likelihood that the sample's majority preference order fails to represent the underlying order in the survey is quite high, even for samples of several hundred voters. Thus, for a 500-voter sample (middle right in Figure 2), the probability of a cyclical majority is essentially zero, but the probability that the sample's majority preference order matches that of the survey is only 65%, and the probability that the sample preferences recover the correct majority winner in the survey is only 66%. For a 2000-voter sample (lower left in Figure 2), there is again approximately a 0% probability of a cyclical majority but a 19% probability that the sample fails to reflect the survey majority winner or majority order. The upper right panel of Figure 2 summarizes the key information for these and other sample sizes.

The conclusion we draw from our analysis of the 1976 GNES survey is consistent with our results for the 1996 U.S. presidential election: in both cases it is unrepresentative sample majority preferences, not cyclical sample majority preferences, that pose the greatest threat to political representation. Furthermore, for the 1976 German survey the problem of unrepresentative samples is severe even when the sample comprises thousands of voters. This conclusion goes hand in hand with the observation that the 1976 GNES is a highly heterogeneous distribution.

2.3. *The 1988 French Presidential Election: The preferences of Communist party identifiers*

To illustrate the issues involved in sampling from important subnational voting constituencies, we now move to the 1988 French National Election Study (FNES) survey data set, more specifically to the subgroup of respondents who identified themselves as Communists. The survey distribution in the upper left of Figure 3 shows that 53% of the French Communists had the preference order MBC (M: Mitterrand; B: Barre; C: Chirac), and that 92% strictly preferred Mitterrand to his two opponents. The upper left panel of Figure 3 also displays the pairwise preference margins and the implied survey majority preference order MBC. The large amount of agreement among Communists resonating from the upper left panel are clear indications that Communist respondents in the 1988 FNES can be regarded as highly homogeneous in their preference. The lower three panels show the resulting probabilities of the various majority preference relations for samples of size 3, 9, and 21, respectively. The upper right panel provides an overview for these and other sample sizes. In particular, we can see in that panel that any sample size virtually guarantees the correct majority winner and that even very moderate sample sizes virtually guarantee the full correct majority preference order. For instance, the middle right panel shows that samples of just nine voters will correctly recover the majority preference order among Communist survey respondents – MBC – with probability 0.95, and that these nine-voter samples recover the survey respondents' majority winner – Mitterrand – with a 100% probability. Even for three-voter samples (see middle left panel in Figure 3) the corresponding probabilities are 0.77 and 1.00. Meanwhile we note that for all sample sizes investigated the likelihood of a majority cycle was essentially zero.

Given the homogeneity of the Communist partisans' candidate rankings, the above results make perfect sense. Since the overwhelming majority of all Communist partisans strictly prefer Mitterrand, and an absolute majority display the candidate ranking MBC, even

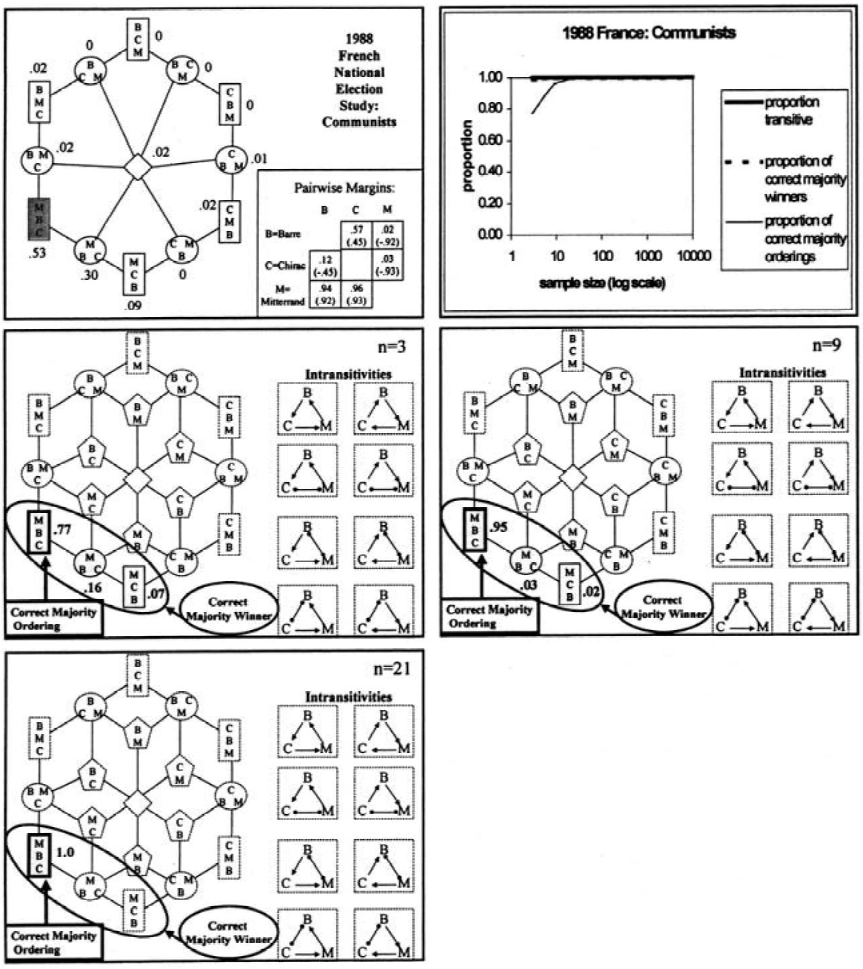

Figure 3. Overview of results for samples drawn from the Communist subpopulation in the 1988 French National Election Studies survey.

modest-sized samples of Communist partisans are likely to reflect the majority preferences of the Communist survey data as a whole.

Hence, for the French Communists we conclude that neither majority cycles in voter samples, nor unrepresentative majority preferences among samples, are a serious problem for political representation, provided the sample size reaches double digits. And for extremely small samples (e.g., $n = 3$) the problem is again unrepresentative sample majority preference orders, not cyclical majority preferences.

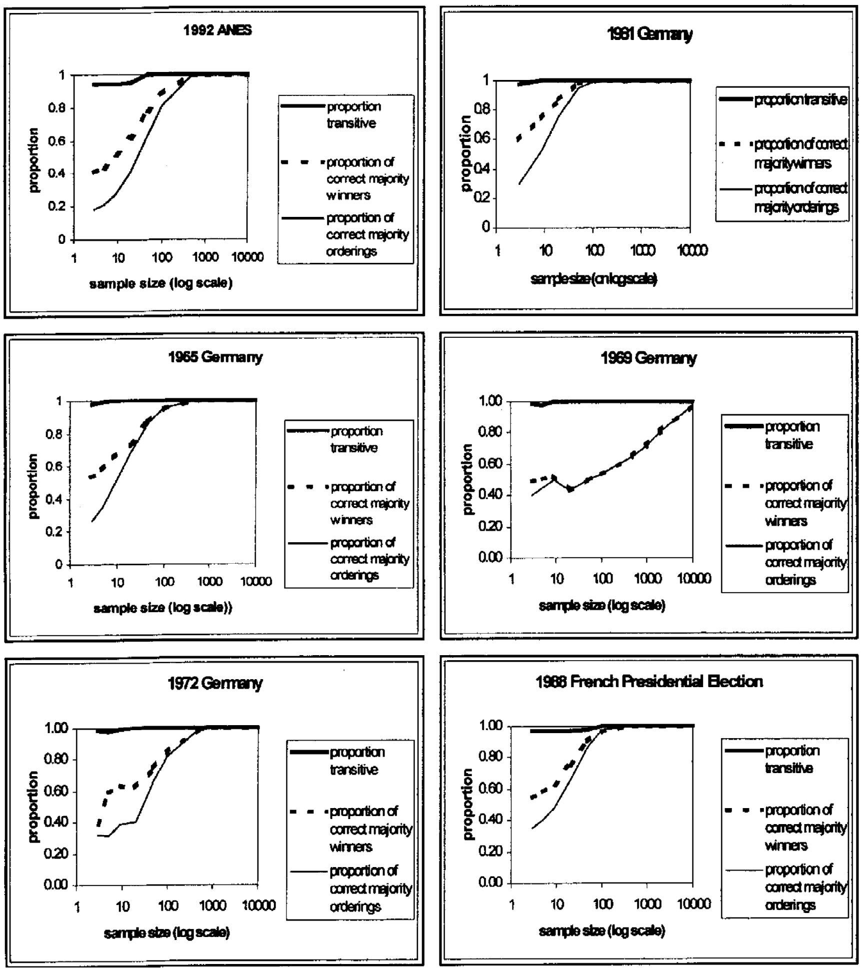

Figure 4. Summaries of the proportion transitive, proportion correct majority winners and proportion correct majority orderings plotted against log sample size for 1992 ANES, 1991 GNES, 1965 GNES, 1969 GNES, 1972 GNES and 1988 FNES.

2.4. *Results for the remaining American, French, and German National Election Studies*

We have carried out similar analyses as the ones above for the 1992 ANES, 1961 GNES, 1965 GNES, 1969 GNES, 1972 GNES and 1988 FNES surveys. For the sake of brevity we omit the large number of figures that it would require in order to display the survey probabilities and the sample majority preference probabilities for various sample sizes. Instead, the six panels of Figure 4 summarize the convergence properties we find as we vary the sample sizes. We only mention here that the dispersion of majority preference orders displayed in Figure 1 for the 1996 ANES, in Figure 2 for the 1976 GNES and in Figure 3 for the 1988 FNES Communists give a representative idea of the diversity in probability distributions over sample majority preference orders that we find overall. Figure 4 provides the key information, namely that in all cases, cycles play a rather minor or even negligible role. This figure also indicates that the threat of misrepresentation is substantial for small sample sizes, and that in some cases (such as the 1969 GNES and to lesser extents the 1972 GNES and the 1992 ANES) misrepresentation remains a severe problem even for larger samples of several hundreds, and even thousands, of respondents.

2.5. *Summary of key findings*

We reach two important conclusions about sampling voter preferences from these election surveys – and, predict by extension, that the same conclusions extend to the outcomes of random sampling from other real-world distributions:

(1) *Misrepresentation of the majority winner and of the majority preference order occurs frequently in small samples. By contrast, majority cycles are extremely rare for all sample sizes.* This strongly suggests that misrepresentation, not cycles, poses the most serious threat to political representation when we are seeking to judge the preferences of an electorate from incomplete or noisy data.

(2) *Increasing the sample size generally increases the probabilities of finding both the correct majority winner and the correct majority preference order.* With respect to national electorates and as shown below (especially) with respect to important subnational voting constituencies, we find that the problem of misrepresentation becomes markedly less severe as the sample size increases. *As the sample size gets very large we are virtually guaranteed to find the correct majority preference order in the sample.* However, “very large” may mean tens of thousands of voters.

Note that the latter finding is diametrically opposite the conclusions that are routinely reached by researchers who are sampling from the impartial culture. There, for n odd, increasing sample size *increases* the likelihood of a cycle. Furthermore, for odd sample size and linear order preferences, correct representation is impossible when drawing from the impartial culture, even as sample size goes to infinity.

Similarly, we note that each survey diverges widely from the impartial culture. This observation supports the common-sense intuition that in actual elections, political representation is best studied by exploring the preference structure of the electorate under review, not by extrapolating from the impartial culture or other *a priori* distributions (Van Deemen, 1999).[11]

Figure 5 also displays the sample convergence information for various subgroups of the 1988 FNES, namely the middle class, the working class, the left, the right, and the UDF, respectively. This figure shows that points 1 and 2 above extend from national electorates to subnational groups. The figure also reveals interesting patterns, which suggest that the sampling process is most likely to recover the correct majority order when the subgroup members display similar candidate preferences. For instance, the left, the working class, the UDF and the Communists display highly homogeneous preferences, in the sense that group members have ranking probabilities that are highly concentrated in one small area of the weak order graph. Consequently these groups show amazingly fast convergence to the correct majority preference relation as the sample size increases even slightly over a range of very small values. By contrast, more heterogeneous groups, such as the middle class or the right have weak order probabilities that are very 'spread out' throughout the weak order graph. Accordingly, they display convergence patterns more similar to the entire FNES survey.

Because of these findings we add one more obvious but important conclusion to the previous list.

(3) *Even for small or moderate sample sizes, representation will be accurate when there is a high degree of social homogeneity among members of the population from which we sample.*

Thus the problem of misrepresentation is most acute when the underlying population is heterogeneous, in the sense that voters dis-

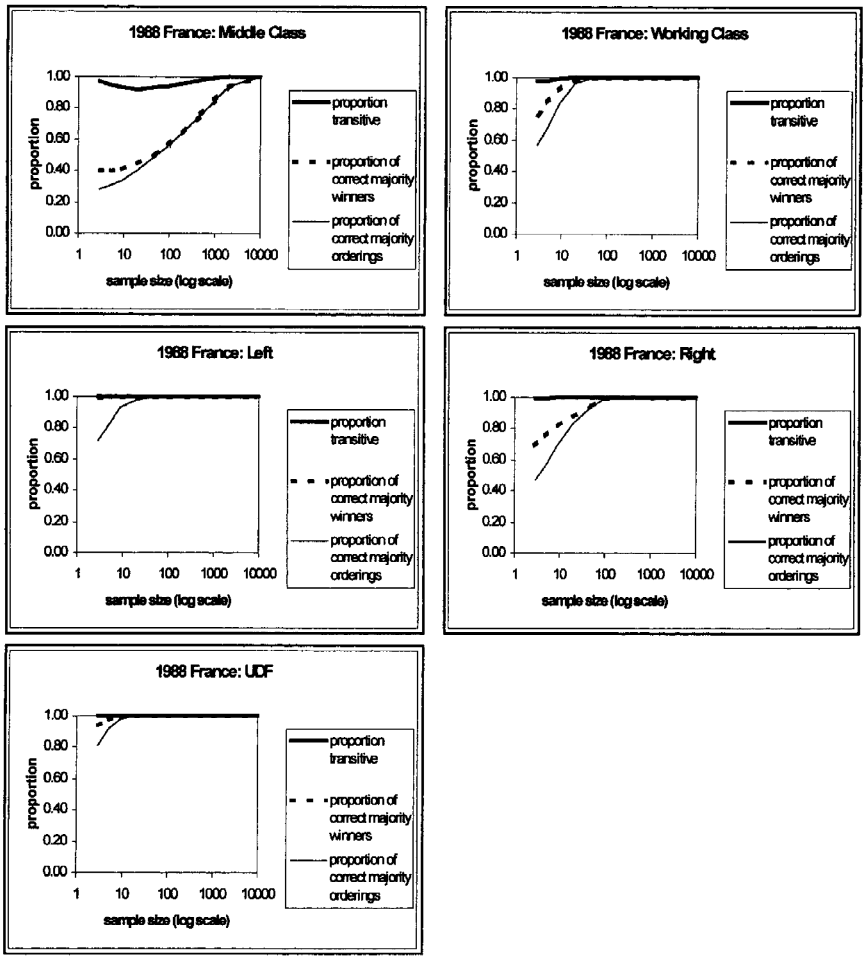

Figure 5. Summaries of the proportion transitive, proportion correct majority winners and proportion correct majority orderings plotted against log sample size for various subgroups of the 1988 FNES.

play high levels of disagreement about the merits of the competing alternatives.

2.6. *Questions raised by the findings*

Why do the majority preferences of relatively modest sample sizes of 100–500 respondents so consistently reflect the majority preferences of our American, French and German survey respondents (with the important exceptions noted above)? Why, as we increase the sample size, are the samples in some cases more quickly representative of the survey majority preference orders than in others? How 'concentrated' do the weak order probabilities need to be to guarantee a fast convergence?

We believe the answer to the first question revolves around the large pairwise preference margins observed in some of the survey data, i.e., the fact that the respondents' majority preferences in pairwise choices over the candidates/parties diverged significantly from 50%. Given these large pairwise margins, it is extremely likely that the collective preferences of any moderately large voter sample will reflect the majority direction (if not the majority margin) of the full survey; and it is the direction of majority preferences that determines the group's majority preference order. This observation cuts both ways, i.e., it also accounts for our finding that even moderately large random samples may not be representative of the German survey data in 1969 or 1976. In these two surveys, survey respondents were almost evenly divided in their preferences between two of the parties – the SDP and the CDU/CSU – and in such cases even slight sampling error may reverse the apparent direction of the group's majority preferences.

The above argument also helps answer the second and the third question. For our subgroup survey data, even extremely small voter samples typically reflect the majority preferences of the subgroups of French survey respondents from which they are drawn. We can account for this finding by observing that these subgroups tend to exhibit larger pairwise margins than did the survey as a whole (see Table I). These subgroups were selected on the basis of common sociodemographic characteristics or common partisan affiliations and, therefore, it is not surprising that they display high levels of interpersonal agreement, as manifested in larger pairwise margins. This is consistent with the well-known fact (see Berelson et al., 1954) that voters with common sociodemographic characteristics tend to display similar voting patterns. Similarly, any grouping of voters according to their ideological predilections is likely to impose a considerable degree of homogeneity on the group's partisan/candidate preference order. In many cases, at least for subgroups identified vis-a-vis their political affiliations, the subgroup pairwise margins in our 1988 French survey are *so* large that the direction of the subgroup's

TABLE I

Minimum and maximum pairwise margin for each survey

	Minimum pairwise margin	Maximum pairwise margin
1992 USA	0.14	0.27
1996 USA	0.14	0.44
1961 Germany	0.26	0.61
1965 Germany	0.16	0.62
1969 Germany	0.02	0.58
1972 Germany	0.11	0.59
1976 Germany	0.02	0.28
1988 France	0.2	0.22
1988 France-Middle Class	0.03	0.2
1988 France-Working Class	0.32	0.51
1988 France-Left	0.4	0.93
1988 France-Right	0.03	0.7
1988 France-UDF	0.55	0.83
1988 France-Communists	0.35	0.93

majority preference is virtually certain to be reproduced in samples of only a few dozen voters or even less.

We can make this observation about the link between the pairwise margins and the representativeness of a given sample size more precise. Table I shows the minimum and maximum pairwise majority margin in each of our fourteen survey data sets (eight national surveys plus six subgroups of the 1988 French data). For samples of various sizes ($n = 5, 20, 101$), Figure 6 plots the relationship between the minimum pairwise margin in the various surveys (or survey subgroups) and the proportion of samples that correctly recover the survey majority order.

It is obvious from Figure 6 that, for each sample size, as the *minimum* pairwise margin in the survey data increases, in general,

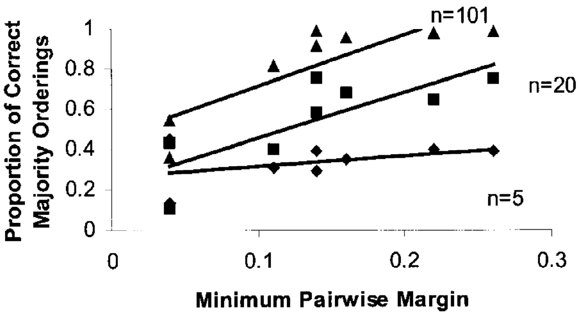

Figure 6. Proportion of correct majority orders versus minimum pairwise margin for various sample sizes

so does the probability that a sample from that data set correctly recovers the majority preference order of the full survey.[12]

3. DISCUSSION

We have looked at the following criteria of representativity in a majority rule framework:

(1) *How often does the majority rule preference order in a random sample drawn from a given distribution coincide with the majority rule preference order of the distribution that the sample was drawn from?*

(2) *How often does the majority winner in a random sample drawn from a given distribution coincide with the majority winner in the distribution that the sample was drawn from?*

(3) *How often does a random sample from a given distribution display a nontransitive majority preference relation?*

Our approach is similar to a few previous articles (e.g., Feld and Grofman, 1986a, 1988, 1990; Tanguiane, 1991: esp. Chap. 9; Adams and Adams, 2000) that emphasized the potential for majority rule representativeness, and it resonates as well with traditional political science concerns for substantive representation (e.g., Weissberg, 1978 and numerous other authors). Our results reinforce with new and extensive analyses the findings of earlier research that cycles are simply not as much of a problem in the real world as they might appear to be from the theoretical social choice literature.

The distributions that we have drawn our samples from are realistic distributions in the sense that they reflect the distribution of preferences in large real world surveys. As noted previously, these real-world distributions are also quite unlike symmetric distributions with pairwise majority ties such as the impartial culture. They are also different from the preference distributions considered by Lepelley et al., (2000) in the present journal.

One referee pointed out about the impartial culture (IC) that it has been "well known since the work of Williamson and Sargent" (1967) "that the expectation of ties rapidly breaks down for large samples as soon as even a slight perturbation from IC takes place" and that nobody has ever written that "IC represents reality". While we fully agree with these points, we argue that, nonetheless many authors continue to cite studies based on the impartial culture as if they generated predictions that are relevant to the real world. The deviations between the conclusions suggested by the impartial culture and those suggested by our realistic distributions are dramatic.

In practice, many real-world choices are from menus of choices that have been drastically limited via institutional or cultural mechanisms. The work we present here strongly reinforces the view that in such realistic distributions, at least for a small number of feasible choice alternatives, cycles must be rare or nonexistent. But that is only an incidental point in this paper.

Our key finding is that, for a small number of alternatives, while we need not worry much about majority cycles when dealing with realistic distributions, especially if our sample is very large; we do, however, need to worry about misrepresentation, especially when dealing with heterogeneous groups and small sample sizes. As mentioned in passing earlier, this observation also held up for the case of five candidates that we looked at.

We believe that we will find the right answers to the puzzles surrounding democratic decision making only if we ask the right questions. We have emphasized an explicit sampling framework (based on preference distributions inspired by real survey data) in which we have looked for the degree of representativeness and not just the prevalence of cycles. We believe that the key question should be: *When and why are random samples from realistic distributions representative of the majority preference orders that the samples are drawn from?* This calls for political, sociological and historical analyses in order to account for the underlying patterns of preference homogeneity (especially among subgroups) that we observe in election survey data such as the ones from Germany, France, and the United States studied here. It also calls for the study of a statistical type of Condorcet efficiency: What is the probability that a given *sample* statistic (Condorcet, Borda, or other) correctly represents the *population* majority (Borda or other) preferences of a population of reference? While the present paper lays the conceptual foundations and points out some striking examples of accurate or inaccurate representation, we leave it for subsequent work to develop an analytical sampling and inference framework for social choice functions (see, e.g., Tsetlin and Regenwetter, in press, for some analytical results). We emphasize, however, that it is crucial also for subsequent work to broaden the analysis away from highly artificial theoretical distributions to realistic distributions like the ones we expect to find in the real world.

We conclude with an analogy between our results and the 2000 U.S. presidential election. Clearly, nobody was ever concerned about the possibility of a majority cycle among Bush, Gore and Nader. On the other hand, the question whether or not the outcome of the election might 'accidentally' reverse the top two choices has for some time dominated the national and international headlines. Clearly, one can think of the ballot casting and counting processes as noisy processes, which correctly record a given voter's preference with probability less than one. Clearly we can think of voter turnout as a probabilistic process. Clearly, the certified ballot counts themselves are thus the outcome of a noisy process. Clearly, the narrow margins in the ballot counts in various states – particularly Florida – raise questions about which candidate is the 'correct' or the 'incorrect' winner in these states, and thus in the electoral college. (Indeed this is the justification which underlies the statutes mandating election recounts in various American states whenever the winning candidate's margin of victory falls below some specified threshold.) Hence, the central questions in the 2000 presidential elections concern *representation*, not *cycles*. Correct and incorrect representation in general, due to sampling and other probabilistic components, is therefore not just of theoretical interest, but of acute practical importance as well.

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NOTES

1. To keep things simple, we use the terms *cyclical* and *intransitive* interchangeably, even though, strictly speaking, they are not the same. The same holds for *cycle* and *intransitivity*.
2. The reader who wishes to skip ahead to the presentation of these data should consult Figure 1. The notation used in the figure is explained in the text.
3. The reader who wishes to skip ahead to the presentation of these data should consult Figure 2.
4. The reader who wishes to skip ahead to the presentation of these data should consult Figure 3.
5. The reader may also be assured that we have not just selected data sets that are consistent with our point and omitted others. While we have looked at some additional data sets (e.g., an additional six subgroups in the 1988 French data set), every real-world survey data set we have examined has shown that, for three alternatives, the likelihood of cycles tends to vanish as we increase sample size (contrary to samples drawn from the famous *impartial culture*). Moreover, we always find that the issue of misrepresentation of majority preference orders is by far a more serious one than the problem of cycles. While we study only the case of three choice alternatives here, we conjecture that

our points will only become more dramatic when future work investigates multiple candidate cases. Clearly, with more candidates, the opportunities for misrepresentation grow much faster than the opportunities for cycles. We have looked at the 1988 FNES also by including four or five candidates instead of the three discussed here, and find no cycles at that level either.

6. The 13 weak order rankings shown in the upper left panel of Figure 1 are possible both as individual rankings and as the rankings achieved by an aggregation of preferences according to majority rule decision-making. However, once we move to the level of collective (majority rule) choices, fourteen additional majority preference relations become possible. All 27 possible majority preference relations are presented in the lower four panels of Figure 1 and subsequent figures. See the discussion in the text following Figure 1.
7. Note that even if the proportion preferring Clinton to Dole had not been above 50%, so long as the proportion preferring Clinton to Dole exceeds the proportion preferring Dole to Clinton, Clinton is the majority winner among the two.
8. The rankings shown in the pentagons are known as *semiorders* (Luce, 1956; Brady and Ansolabehere, 1989).
9. We obtain the figure of 68% by observing that this is the percentage of outcomes that were transitive but did not match the survey's transitive order (68% = 100% – 29% – 3%), where the 3% is the percentage of intransitive (cyclic) majority preference orders.
10. We obtained 34% by subtracting both the proportion of cycles (3%) and the proportion of correct majority orders (63%) from 100% to determine the residual category of incorrect transitive orders.
11. In response to a referee comment, we point out that we do not mean to suggest that voters should cast their vote first and that the tally procedure should be determined after the fact, based on the collected votes. Instead, we argue that the descriptive analysis of political representation ought to incorporate analyses of the performance of actual tally procedures on actual ballots. Similarly, the choice of future ballot formats and future tally procedures should take into account the actual performance of competing methods in real world settings in past elections, rather than be driven by theoretical assumptions that have never been tested (or have not held up) against real world data.
12. For contests involving only two candidates the accuracy of a sample of a given size in reproducing the majority preference in some larger data set is a monotonic function of the absolute difference in support for the two candidates observed in the full data set. The monotonic function in question is a simple binomial function that can be approximated as the area under a normal curve (cf. Grofman, 1975). However, when we have more than two candidates the statistical inference issues become more complex. These issues are considered in more detail in Tsetlin and Regenwetter (in press).

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